Benefits of a combined micro-macro approach for managing rail systems in case of disruptions

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Abstract

Optimisation and simulation tools are vital for planning and managing rail systems, providing performance analysis and evaluation of interactions without any kind of disturbance to the service. We may distinguish between combinatorial optimisation and simulation models which can be also classified into macroscopic and microscopic models. The former models describe the network and the timetable in a simple way by means of a simplified graph. The latter models consist of the specification of all technical characteristics related to infrastructure, rolling stock and signalling system as well as timetable data. Macroscopic models are useful during the planning process when the design of service frequencies and capacity to satisfy demand are carried out. The major benefit of this approach is the possibility to consider jointly several features of the rail system obtaining reliable results. By contrast, microscopic models reproduce the network as closely as possible to the ‘real world’; they allow evaluating the interactions among trains and the performance of the network precisely. The aim of this paper is to propose a new approach for planning and managing the rail system combining both approaches macroscopic and microscopic. In particular, an optimisation model, based on a macroscopic approach, represents the kernel of the procedure and it is used as a first step to study any kind of scenario. The microscopic simulation model, by contrast, generates detailed (and precise) data, such as headways or running times, to overcome the approximations of the macroscopic model. Above all, in case of disruptions, the combination of the two models provides reliable results taking advantage of the benefits of the two approaches. Numerical applications have been applied in a realistic case taken from a real metro network located in the south of Italy; the preliminary results show the effectiveness of the proposed approach.

Keywords: Rail; macro-optimization approach; micro-simulation approach; disruption; public transport management.

1. Introduction

The management of the rail service in the case of disruptions is an important process for any Train Operating Company (TOC). Especially in the case of dense rail networks characterised by high frequencies and short headways, recovering promptly from any kind of disturbances can prevent the propagation of knock-on delays to

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other trains (Corman and D’Ariano, 2012). Solving these problems in metro rail contexts is even more complicated since the configuration of the line and the stations does not give so many alternatives to re-establish the ordinary conditions.

For this reason tools are necessary to provide performance analysis and evaluations of interactions in order to define plans and strategies to be adopted in real time. In the literature, both macroscopic and microscopic models have been adopted to solve the so-called rescheduling problem (see for instance Corman et al., 2012b; Cadarso et al., 2013; Quaglietta et al., 2013). However, the macroscopic approach, which provides outputs in short computational times and allows the implementation of optimisation models, is not able to evaluate precisely the interactions between trains. The microscopic approach, by contrast, produces accurate results but needs a great number of input data and requires long computational time. For this reason our proposal is to combine the macroscopic and the microscopic approach to provide reliable dispatching solutions taking advantage of the benefits of the two methods (Eickmann et al., 2003). Moreover, our methodology analyses the effect of these rescheduling strategies on passenger demand: TOCs are interested in increasing service quality for improving the attractiveness of their services. In fact, recovery strategies focus on just operational aspects (i.e., reliability of the service) considering erroneously that it is always the best solution to achieve (Quaglietta et al., 2011). Indeed, since neglecting user satisfaction during the rescheduling process could provide strategies which increase the disutility perceived by customers (D’Acierno et al., 2012), the aim of the paper is to present an innovative procedure for managing rail and metro networks giving more importance to passenger needs.

The rest of the paper is organised as follows. In section 2 the state of the art regarding rail system management is provided. Section 3 describes the problem in detail. In section 4 the proposed methodology is analysed providing the analytical formulation. Section 5 presents the preliminary application on a real network and finally, in section 6, conclusions and research prospects are summarised.

2. State of the art

Planning and management are key steps for any TOC so as to keep high service quality standards. Many authors have indeed focused on these topics proposing different kinds of methods which are based either on optimisation or rail simulation approaches.

Caprara et al. (2010) for instance, proposed a robust routing model to assign paths to trains through a station by means of an optimisation procedure. A macroscopic simulation framework was also presented to validate the outputs of the proposed approach.

Cadarso and Marín (2010, 2011 and 2014) presented sequential and integrated optimisation models to solve the routing and rolling stock circulation problems in rapid transit networks. The authors gave importance to minimise possible delays during the service which could propagate throughout the network in short time. In fact, due to the constant increase in capacity consumption, it is likely that conflicts between trains arise after single deviations from the planned timetable (Goverde 2010), and therefore robustness is an important factor to assure during the timetable planning process. To reach this aim, Goverde (2007) adopted a timetable linear description by means of max-plus algebra theory and time event graph, describing a real time procedure to provide sensitivity analysis to delays.

Schlechte et al. (2011) dealt with the timetable optimisation problem through the combination of macroscopic and microscopic approaches. In particular, the authors presented an algorithm to transform a microscopic network into a macroscopic representation so as to optimise a given timetable by means of an integer programming model. Then, a comeback to the microscopic level assures that a conflict free timetable is obtained.

However, unforeseen events may cause perturbations; therefore, rescheduling actions must be adopted to restore schedule feasibility. D’Ariano (2008) proposed a real-time procedure based on microscopic models to minimise delays, once some deviations from the original timetable occur. The same method was then extended to coordinate two dispatching areas (Corman et al., 2010) or multiple areas (Corman et al., 2012a) so as to reschedule jointly a wider traffic zone.

Kecman et al. (2013) presented four macroscopic models to deal with rescheduling actions in the case of wide networks such as, national rail networks. In particular, the authors focused on computational time in order to produce dispatching actions very quickly considering the whole rail traffic flow and not just a small part of it.
However, the majority of the presented works concerns operational aspects of the rail service during disruptions and do not take into account the effects of proposed strategies on passenger demand. Indeed, since customers are considerably influenced by the consequences of designed strategies (Gallo et al., 2011), it is always worth considering user satisfaction.

In this context, Schöbel (2007) used an integer programming approach to solve the delay management problem minimizing the delay perceived by passengers. Schachtebeck and Schöbel (2010) further improved the previous model by including headway and capacity constraints.

Cadarso et al. (2013) described a user oriented optimisation model to recover promptly from disruptions in rapid transit networks. The authors analysed passenger reaction to service interruptions taking into account the cost they perceived as a results of possible rerouting strategies or increase in waiting times.

Kanai et al. (2011) developed a new procedure to deal with the delay management problem considering user’s viewpoint. In particular, through the combination of simulation and optimisation modules, both train traffic and passenger flow are simulated and even the influence of crowding on dwell times at station is computed.

D’Acierno et al. (2012) presented an off-line Decision Support System (DSS) to manage rail and metro networks in the case of failure scenario. The authors adopted a micro-simulation approach and introduced also capacity constraints to rolling stock so as to provide more realistic outputs. Further applications of this DSS demonstrated the dependence of intervention strategies on breakdown severities (D’Acierno et al., 2013a) and on travel demand levels (D’Acierno et al., 2013b). Recently, Quaglietta et al. (2013) analysed the stability of rail dispatching solutions combining a tool for rescheduling plans (D’Ariano, 2009; Corman et al., 2012b) with a stochastic microscopic simulation software (Quaglietta, 2013). The model evaluates the robustness of the intervention strategies providing more stable plans.

3. Problem description

In this section, the recoverability problem in metro networks is described in detail. First, a summary of possible disruptions is presented. Then, the metro infrastructure is introduced. Next, we describe train services. Finally, we explain how we treat the passenger demand for disturbed scenarios.

3.1. Disruptions

During the daily operations of a metro network, incidents may cause the traffic to deviate from the planned operations. These incidents may make it impossible to operate the schedule as it was originally planned. The first task after noticing an incident is to determine whether or not it requires a substantial active intervention. If it does, the operations are said to be disrupted, and plans must be designed in order to recover from the disrupted situation. In such a situation the operator needs to adjust the timetable and the rolling stock assignment for the time interval of the incident, and to carry out further recovery steps in order to get back to the original schedules.

Regardless of the cause of a disruption, it has an impact on the system. The impact is generally in the form of a change in the system settings, a change in resource availability, or both. In this paper, we study a disruption that involves a change in resource availability. Therefore, we need to re-plan the current operations to apply only the available resources which may include giving up some of the planned services.

3.2. Metro infrastructure

In the macro-optimisation approach, the metro network is studied as a graph composed of nodes and directed arcs linking different stations. It consists of tracks and two types of stations, namely passenger stations and depot stations. The first type is characterized by train services that only attend to passenger demand. In depot stations shunting operations can also be performed. Between two stations, two different arcs exist, one for each direction of movement. Therefore, every arc is defined by its departure and arrival station and by its length.

In the microscopic simulation model, the arcs contain the highest level of information such as slopes, curves as well as the Automatic Train Protection (ATP) signalling system. The stations are represented with both transit and recovery tracks, while a timetable is defined to provide the rail service during the whole day.
Our case study in Section 5 is drawn from “Line 1” of Naples metro system. The line, operated by MetroNapoli, has recently been updated with a new service from Piscinola station to Garibaldi station which consists of 17 stations. However, since survey data of the new service are not available yet, we focus on the previous service, namely the track sections between Piscinola and Dante stations (Figure 1).

3.3. Train services and rolling stock

There are two types of train services ($\ell \in L$): the planned train services ($\ell \in L^p \subset L$) and the emergency services ($\ell \in L^e \subset L$). The former are the trains scheduled for a regular situation and the latter are the trains inserted to the schedule during the disruption in order to alleviate its negative effects in passengers.

A planned train service is a passenger train travelling from a depot station to another depot station stopping at a number of intermediate stations. They are characterized by their departure depot station; their arrival depot station; every arc they travel on and their departure time. Planned services may be cancelled due to some disruptions. For emergency services, the model will decide whether they are used or not. An emergency service represents a feasible movement between depot stations, and it is defined by a departure station, an arrival station, every intermediate arc and the departure time. We define a feasible movement as a physical movement in the network once the disruption has started. For planned and emergency train services the headway must be maintained in every infrastructure they come through.

We also account for empty services; these are defined by an origin, a destination and a departure time. Empty services can help satisfy both capacity and rolling stock material availability in depot stations.

Every service (i.e., train and empty services) will be assigned at most one composition. A composition is a sequence of train units of the same type, which are self-propelled with a driver seat at both ends.

3.4. Passenger demand

The passenger demand is estimated by a common three phases procedure (see Cascetta, 2009). In the first step, Origin and Destination of each trip (O-D Matrix) is computed by adopting four sub-models: an emission model (which evaluates the number of passengers starting their trip during a time period), a distribution model (which computes the number of passengers going to a generic place during a time period), a mode choice model (which simulates user choices about the mean of transport) and a path choice model (which, in the case of public transport, provides line choices and the number of boarding and alighting passengers at stops or stations). The second step consists in surveying passenger flows at each station. Finally, a travel demand correction procedure (Marzano et al., 2008) provides the OD Matrix which produces user flows on the platform closest to surveyed data. The adopted assumption on users’ arrival on the platform is based on a Poisson process with a constant arrival rate.

4. Modelling approach

In this section, our modelling approach is described in detail. First, the macro-optimisation approach is presented. Then, the micro-simulation approach is introduced. Finally, we explain how we interact both approaches.

4.1. Macro-optimisation approach

The INtegrated TiMeetable and ROlling Stock Rescheduling Model (INTIROSRM) aims at computing the timetable and the rolling stock schedule for a disrupted metro network accounting for passengers flows. The INTIROSRM is based on the model presented in Cadarso et al. (2013).
The most central decision variables are $x_{c,\ell} \in \{0,1\}$, defined for $\ell \in L, c \in C$. They take value 1 if composition $c \in C$ is scheduled for service $\ell \in L$; 0, otherwise. $dp_{a,\ell} \in \mathbb{R}^+$ are defined for $a \in A, \ell \in L_a$ ($L_a$ are the train services attending arc $a$), to denote the number of denied passengers in arc $a$ and train service $\ell$. $y_{\ell} \in \{0,1\}$ are defined for $\ell \in L$, to indicate whether service $\ell \in L$ is cancelled; $y_{s,t}^c \in \mathbb{Z}^+$ are defined for $s \in SC, t \in T, c \in C$, to denote the number of compositions $c$ in station $s$ at $t$ period (SC is the set of depot stations); $em_{s',t,s}^c \in \{0,1\}$ are defined for $s, s' \in SC, t \in T, c \in C$, to indicate whether an empty service is performed at $t$ period from station $s$ to $s'$ with composition $c$.

4.1.1 Objective function

The multiobjective function (1) minimizes the following costs: the incurred system costs (operating and cancellation costs) and the passenger inconvenience (denied passengers).

$$\min z = \sum_{c \in C} \sum_{\ell \in L} c_{c,\ell} x_{c,\ell} + \sum_{s,t \in T} \sum_{c \in C} c_{s,t}^c y_{s,t}^c + \sum_{c \in C} \sum_{a \in A} dpc_{a,\ell} dp_{a,\ell} + \sum_{c \in C} \sum_{\ell \in L} canc_{c,\ell} y_{\ell}$$

(1)

The objective terms, in the given order, penalize the following quantities:

- operating costs of planned and emergency services: $c_{c,\ell}^c$ is the operating cost of service $\ell$ with composition $c$;
- operating costs of empty movements: $c_{s,t}^c$ is the operating cost from $s$ to $s'$ with composition $c_{s,t}^c$;
- cancelation of services; here $canc_{c,\ell}$ is the cancelling cost for service $\ell$;
- and denied passengers; here $dpc_{a,\ell}$ is the cost per denied passenger in each arc $a$ and service $\ell$.

4.1.2 Passengers constraints

$$\sum_{c \in C} \sum_{\ell \in L} cap_{c} x_{c,\ell} \geq pf_{a,\ell} - dp_{a,\ell} \quad \forall a \in A, \ell \in L$$

(2)

Constraints (2) ensure that there will be enough capacity for passengers in each arc $a$ belonging to each service $\ell$. $cap_{c}$ is the passenger capacity in composition $c$.

4.1.3 Timetabling Constraints

The following set of constraints enforces the headway requirements. They say that any arc during any interval of length the headway time ($\phi$) can accommodate at most one service. $LCS_{s,t}$ denotes the time period during which service $\ell$ comes through station $s$.

$$\sum_{\ell \in L} \sum_{s \in S} \sum_{t \in T} x_{c,\ell} \leq 1 \quad \forall s \in S, t \in T$$

(3)

4.1.4 Rolling Stock Constraints

$$\sum_{c \in C} x_{c,\ell} + y_{\ell} = 1 \quad \forall \ell \in L$$

(4)

$$\sum_{c \in C} x_{c,\ell} \leq 1 \quad \forall \ell \in L$$

(5)

Constraints (4) state that each planned service $\ell \in L^p$ is either cancelled or it gets exactly one composition. Constraints (5) express that emergency services $\ell \in L^e$ get at most one composition.
Composition conservation constraints (6) ensure the train units’ flow balance. The schedule is given by $\alpha_{\ell,s,t}$, which takes the value 1(-1)((0)), if train service $\ell$ arrives (leaves)((stays)) in station $s$ at period $t$. $\ell_{s,s'}$ is the travel time between stations $s$ and $s'$.

$$
\begin{align*}
\sum_{(s',s) \in SC} x_{s,s'} + \sum_{s \in SC} \sum_{t \in T} \sum_{c \in C} em_{s,s',t,c} &= \sum_{(s',s) \in SC} x_{s',s} + \sum_{s \in SC} \sum_{t \in T} \sum_{c \in C} em_{s,s',t,c} \\
\forall s \in SC, t \in T, c \in C
\end{align*}
$$

(6)

Fleet size constraints (7) ensure that the number of train units used is limited by the size of the fleet $\chi_m \cdot C_m$ is the set of compositions belonging to material type $m$. Depot capacity constraints (8) ensure that the total capacity is not overpassed. $tu_c$ is the number of train units in composition $c$. Each train service time duration is given by $\beta_{\ell,t}$, which takes value 1, if train service is rolling at period $t$; 0, otherwise. Similarly, $\xi_{s,s',t,c}$ gives information about performance time of an empty train service, which departed from $s$ during $t$ and is going to $s'$.

$$
\begin{align*}
\sum_{s \in SC} \sum_{c \in C_m} tu_c \cdot y_{s,t,c} &+ \sum_{(\ell,s) \in SC} \sum_{c \in C_m} tu_{\ell} x_{\ell,s} + \sum_{s \in SC} \sum_{c \in C_m} \sum_{t \in T} \sum_{c \in C_m} tu_c \xi_{s,s',t,c} \cdot em_{s,s',t,c} \leq \chi_m \\
\forall m \in M, t \in T
\end{align*}
$$

(7)

$$
\sum_{c \in C} tu_c \cdot y_{s,t,c} \leq \text{cap}_{s,t} \\
\forall s \in SC, t \in T
$$

(8)

4.2. Micro-simulation approach

The Micro-simulation approach is based on the Decision Support System (DSS) presented by D’Acierno et al. (2013a; 2013b); we introduce some changes due to the necessity to work in combination with the macro-optimisation method. In particular, just the Service Simulation Model (SSM) and the On-Platform Model (OPM) are considered so as to assign travel demand to the rail network.

4.2.1 Service simulation model

The SSM consists of a system of differential equations whose numerical solution provides service performance (i.e. running times, headways) depending on rail infrastructure, rolling stock, signalling system and timetable. The system of differential equations may be represented by the following analytical formulation:

$$rnp = SSM \left( in, rs, ss, pt, unf \right)$$

(9)

where $rnp$ is the vector of rail network performance, $in$ is the vector of the infrastructure characteristics, $rs$ identifies the rolling stock, $ss$ describes the signalling system, $pt$ is the planned timetable and $unf$ is the vector of user network flows, that is the number of boarding passengers for each train and for each station.

The resolution of this model cannot be provided by an analytical formulation, but it requires the adoption of numerical procedures carried out by commercial software. In our case, we adopt OPENTRACK® software (Nash & Huerlimann, 2004) which combines continuous events (i.e., train motion) with discrete events (i.e., modifications to network conditions due to the signalling system or to delays) in order to simulate rail service with high details.

4.2.2 On-platform model

The OPM analyses passenger behaviour once arrived at the station. In particular, the model evaluates the number of users who manage to board the train taking into account the maximum capacity of each train service. Indeed, the model checks whether the number of boarding passengers at station $s$ exceeds the residual capacity of the train coming from the previous station. In analytical terms, the residual capacity can be evaluated as follows:

$$rc_s = mc - obp_{(s-1,s)} + ap_s,$$

(10)
where \(rc_s\) is the residual capacity of the train approaching station \(s\), \(mc\) is the maximum capacity of the train, \(obp_{s-1,s}\) is the number of on-board passengers between station \(s-1\) and \(s\) and \(ap_s\) is the number of alighting passengers at station \(s\).

If the residual capacity is not sufficient, just a portion of travel demand can board the train while the residual part has to wait for the following train services on the platform. In this case, a FIFO (First In – First Out) rule is adopted which means that these passengers have priority to board the following train services.

Obviously, the performance of the system strongly influences passenger flows at stations. In fact, the more is the delay of a train and therefore the headway between two train services, the more the platform will be crowded according to the above mentioned assumptions on travel demand.

The OPM function can be expressed as follows:

\[
unf = OPM (upf, rs, rnp),
\]

where \(unf\) is the vector of user network flows, \(upf\) is the vector of users on the platforms, \(rs\) is the rolling stock of the line and \(rnp\) is the vector of rail network performance.

4.3. Iterative approach

We combine the macro-optimisation and the micro-simulation approaches by means of an iterative procedure. In particular, according to the estimated travel demand, the macro-optimisation model provides the schedule and the rolling stock composition considering both operating cost and passengers’ needs.

This strategy may be adopted for re-establishing undisrupted plans as soon as possible and reducing the disutility perceived by the users (once the cause of the disruption has disappeared).

The macro-optimisation model treats the demand heuristically. The model is unable to trace individual passengers; instead, it considers demand on the arcs (i.e., between successive stations) and it is not linked on successive arcs. Therefore a denied passenger still shows up in the demand of later arcs. However, Cadarso et al. (2013) demonstrate that the denied demand is very well approached whenever the passenger costs are part of the objective.

In order to evaluate the macro-optimisation model’s strategy we adopt the micro-simulation approach: it computes the feasibility of the proposed solution either in terms of operating service (i.e., adoption of recovery tracks, precise time requirements for shunting operations) or in terms of user inconvenience (i.e., number of denied passengers). The micro-simulation model provides precise results which guarantee that the strategy can be implemented to the rail service. These precise results provide feedback to the macro-optimisation model in terms of penalties: these penalties penalise either the schedule (if the strategy turns out to be infeasible as evaluated by the micro-simulation approach) and/or the passenger inconvenience (in order to minimise the number of denied passengers). Therefore, we enter an iterative process. At each iteration, we obtain a strategy which is evaluated. The iterative process stops when the convergence criterion is reached. We adopt the criterion of measuring the Mean Absolute Percentage Error (MAPE) in the number of denied passengers between two iterations:

\[
MAPE_{\text{pax}} = \frac{\sum_{a,c} \sum_{e} |dp_{a,c}^{n} - dp_{a,c}^{n-1}|}{\sum_{a,c} \sum_{e} dp_{a,c}^{n-1}},
\]

where \(n\) is the iteration number. Figure 2 describes the iterative process.

5. Computational results

The proposed methodology has been applied to Naples metro system. We study the following disrupted scenario in Line 1 (Figure 1): the rolling stock material performing run 801 (i.e., the run which starts from Dante station at 7:16 a.m.) breaks down at Rione Alto station. Hence, all the passengers are forced to alight the train and the faulty train is driven to the closest recovery track, which is located in Colli Aminei station (so as not to disrupt the rest of the train services). We assume that the time needed to fix the faulty train is known: 1 hour. Obviously, this train
cannot longer follow the planned timetable. Therefore, our approach aims at determining which actions should the operator take in order to reschedule the system as soon as possible and considering both, operator and passenger costs.

Table 1 shows the solutions provided by the macro-optimisation and microscopic approach. The first column shows the iteration number. The rest of the columns show different characteristics of the given solution. Column TU gives the number of train units used by the solution. Columns TSOC and EMOC give the total operational costs for passenger train services and empty movements, respectively. Column #DP gives the number of denied passengers as estimated by the macro-optimisation model (note that the macro-optimisation model presented here is arc based and hence overestimates the number of denied passengers). Column ST gives the solution time in seconds. The last column shows the number of waiting passengers (#WP) as calculated by the microscopic approach.

The macro-optimisation approach (accounting for the feedback provided by the micro-simulation approach at each iteration) provides a recovery strategy which reduces the number of denied passengers. Obviously, the operator needs to schedule greater capacities so as to serve more passengers and thus its operational costs increase (TSOC and EMOC). Note that the iterative approach stalls: it behaves in a cyclic way from the third iteration onwards.

The microscopic simulation model receives as input data the timetable and the rolling stock composition from the macro-optimisation approach. This provides a great advantage in terms of computational time because, previously, the micro-simulation model was based on a “what-if” approach which required the analysis of all the possible strategies to be implemented. Our new proposal largely reduces the number of microscopic simulations and it converges to a solution in a few steps. The microscopic simulation provides feedback on the feasibility of the solution together with the load diagrams of each train and the actual number of denied passengers on the platforms. The effectiveness of the solution obtained is confirmed by the micro-simulation model, since the number of passengers who are forced to wait on the platform (passengers who cannot travel at their desired departure time) during the whole day service drastically decreases from 21919.37 to 5399.76 across iterations (best solution in terms of waiting passengers, indicated as #WP in Table 1). Note that the macro-optimisation model cannot estimate waiting passengers because it is arc based; instead, it assumes that a passenger waiting more than a given time, i.e. 10 minutes, is denied. It takes around 400 seconds to compute the micro-simulation model solution at each iteration.
Table 1: Solutions of the macro-optimisation and the microscopic approaches

<table>
<thead>
<tr>
<th>Iteration</th>
<th>TU</th>
<th>TSOC</th>
<th>EMOC</th>
<th>#DP</th>
<th>ST</th>
<th>#WP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>35</td>
<td>54945.28</td>
<td>1079.20</td>
<td>3594.62</td>
<td>3.31</td>
<td>21919.37</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>61813.44</td>
<td>1884.80</td>
<td>1980.42</td>
<td>2.38</td>
<td>6586.95</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>61813.44</td>
<td>2067.20</td>
<td>1705.58</td>
<td>2.80</td>
<td>5479.84</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>60756.80</td>
<td>1915.20</td>
<td>1792.95</td>
<td>2.03</td>
<td>5399.76</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>59964.32</td>
<td>1854.40</td>
<td>1758.72</td>
<td>2.06</td>
<td>5857.25</td>
</tr>
</tbody>
</table>

Figure 3 shows the variation of (12) across the iterative procedure. Note that its value drops rapidly to less than 5% within the first three iterations indicating that our solving approach yields reasonably accurate results within a handful of iterations. After this threshold, it continues decreasing but at a slower rate. Figure 3 also shows the mean absolute percentage error in train services compositions (13) and the relative error in the objective function values (14). All of them show similar behaviour.

$$MAPE_{services_c} = \frac{\sum_{t \in T} \sum_{c \in C} |x_{t,c}^v - x_{t,c}^{v-1}|}{\sum_{t \in T} \sum_{c \in C} x_{t,c}^{v-1}}$$  \hspace{1cm} (13)

$$RE_{z_v} = \frac{|z^v - z^{v-1}|}{z^{v-1}}$$  \hspace{1cm} (14)

![Figure 3: Trend in solutions across iterations.](image)

6. Conclusions

In this paper we study the recovery problem of metro networks. When dealing with a disruption, the operator wants to offer a good service quality while the system is being recovered to the original planning. The main contribution with respect the literature is that we combine a macro-optimisation approach – which decides on the timetable and on the rolling stock schedule using an integrated optimisation model accounting for the passenger demand behaviour – with a micro-simulation approach – which evaluates the solution given by the macro-optimisation approach and provides feedback to it in order to improve the solution. We embed our two-step approach in an iterative framework. The preliminary computational tests conducted on realistic instances of Naples metro show that our method is able to find solutions with a very good balance between the managerial goals: the number of denied passengers are drastically reduced while operating costs do not increase too much. As future
research prospects, this method could be applied in the case of more complex metro-rail networks considering a wider set of train breakdowns.

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