Irreversible Capital Accumulation and Nonlinear Tax Policy: A Note

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We analyze the influence of tax progression on optimal investment policy and its value. We show that three possible optimal regimes arise, depending on the nature of the tax policy. If the exogenously given progression threshold lies between the optimal capital stocks in the case of higher and lower marginal profit taxes, then the optimal investment policy is independent of profit tax rates. But outside this corner solution, the optimal investment policy is conventional.

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1. Introduction

It is reasonable to argue that most major investments are at least partially irreversible because firms cannot disinvest without costs after having carried out their investment decision. Physical capital is of course industry-specific, but also often firm-specific. In the recent literature using the framework of irreversible investment under uncertainty, various justifications for a neutral tax system have been analyzed by using real option theory. In most studies corporate taxation has been assumed to be proportional, meaning that the marginal and the average tax rate are constant and therefore equal. But if there is tax exemption, then even if the marginal tax rate is constant, the average tax rate will increase with the tax base, so that in this case taxation is linearly progressive. Corporate taxes are progressive in many countries, and therefore it is important to study their influence on capital accumulation policy and value formation (see, e.g., OECD Tax Database 2000 and Bordignon...
et al., 2001). Alvarez and Koskela (2005) have studied the impact of tax progression – defined as an average tax rate that is increasing in the tax base – on irreversible investment under uncertainty. They have shown that if the tax exemption is smaller than the sunk cost of investment, a higher tax rate will decelerate optimal investment by raising the optimal investment threshold, while if the tax exemption is greater than the sunk cost of investment, then – interestingly – three different regimes with respect to the optimal investment threshold arise. First, for a set of sufficiently low volatilities of underlying value dynamics, higher volatility decreases the optimal investment threshold, but the threshold is independent of the tax rate. Second, for intermediate volatilities the optimal investment threshold does not depend either on the tax rate or on the volatility. Finally, when the volatility is high enough, the optimal investment threshold will depend positively on volatility but negatively on tax rate, so that in this case we have a tax paradox.

In the case of proportional taxation under certainty, Samuelson (1964) showed that under a uniform marginal tax rate the present value of the return on an investment project is not affected by the tax rate, so that investment neutrality prevails if tax depreciation is equal to economic depreciation and debt interest is deductible. Since there might be difficulties in implementing this because the true economic depreciation may not be observable, a cash-flow tax emerges as an interesting alternative neutral tax. The idea is to write off new investment expenditures completely in the year of acquisition, but provide no later deduction for interest rate or depreciation (see Smith, 1963). Sandmo (1979) has shown that this is valid only when the tax rate is constant over time (see also Sinn, 1987, section 5). The allowance for corporate equity (ACE) corporate tax system is an alternative system with well-known neutrality properties (cf. Bordignon et al., 2001, and Devereux and Freeman, 1991).

The purpose of our note is to abstract from uncertainty and study the influence of taxation on irreversible investment and capital accumulation in the situation where taxation is not proportional, but progressive. We assume that the marginal profit tax rate increases at an exogenously given progression threshold. We characterize the optimal investment policy and its value in the presence of progressive taxation and show that three possible optimal regimes arise, depending on the nature of the tax policy. Our new result is the following: If the progression threshold lies between the optimal capital stocks in the case of higher and lower marginal profit tax rates, then the optimal investment policy is independent of the tax rates. But if the progression threshold is outside this corner solution, the optimal capital accumulation policy is the conventional one, which has been analyzed in the literature.

We proceed as follows. In section 2 we incorporate a progressive tax system, where the marginal profit tax rate increases after a progression thresh-
old, and analyze its impact on optimal capital accumulation. In section 3 we illustrate explicitly our general findings concerning the effects of profit taxation both on the optimal capital accumulation policy and the value of the firm. Finally, there is a brief concluding section.

2. Progressive Taxation and Optimal Investment

We now investigate the optimal investment problem (cf. Arrow, 1968)

\[
V(k) = \sup_{I \in \Lambda} \int_0^\infty e^{-rs} \left[ \pi(r_1, r_2, kI_s) - qdI_s \right] ds \quad \text{(1)}
\]

subject to the standard capital accumulation rule

\[
dk_I^t = dI_t - \delta k_I^t dt, \quad k_0 = k . \quad \text{(2)}
\]

In the objective function (1) \( r > 0 \) denotes the discount rate, \( q > 0 \) denotes the unit cost of investment, and \( \pi(r_1, r_2, k) \) denotes the after-tax revenue flow (cf. Kari, 1999; Alvarez and Koskela, 2005), which is defined as

\[
\pi(r_1, r_2, k) = (1 - r_1) \hat{\pi}(k) - (r + \delta)qk - (r_2 - r_1)(\hat{\pi}(k) - \hat{\pi}(k^*)) , \quad \text{(3)}
\]

where \( \hat{\pi}(k) \) is the before-tax revenue flow and \( k^* \) is an exogenously determined constant progression threshold at which the profit tax rate increases from \( r_1 \) to \( r_2 \) (for a definition of tax progressivity, see the seminal paper by Musgrave and Thin, 1948). As usual in the neoclassical literature, \( \delta > 0 \) denotes the depreciation rate of the capital stock. For the sake of generality, we assume that the class \( \Lambda \) of admissible accumulation policies is constituted by nonnegative and nondecreasing functions depending on the stock \( k \). Thus, we do not initially exclude potentially discontinuous accumulation rules from the class of admissible investment policies (even though the resulting optimal accumulation policy turns out to be continuous except for a possible initial jump). In accordance with standard neoclassical studies of the firm, we assume that the short-run profit flow \( \hat{\pi}(k) \) is continuously differentiable, increasing, and strictly concave and that it satisfies the standard Inada conditions. Given these assumptions, the instantaneous yield \( \theta(k) \) reads as

\[
\theta(k) = \begin{cases} 
\theta_2(k), & k > k^*, \\
\theta_1(k), & k \leq k^*, 
\end{cases}
\]

where

\[
\theta_1(k) = (1 - r_1)\hat{\pi}(k) - (r + \delta)qk
\]

and

\[
\theta_2(k) = (1 - r_2)\hat{\pi}(k) - (r + \delta)qk + (r_2 - r_1)\hat{\pi}(k^*).
\]
Denote now by $k_i$ the capital stock at which the yield $\theta_i(k)$ is maximized for $i = 1, 2$. Our assumptions imply that

$$\theta_i'(k_i) = (1 - \tau_i)\pi_i'(k_i) - (\delta + r)q = 0$$

for $i = 1, 2$. In particular, since $\tau_2 > \tau_1$, we find that

$$\pi'(k_2) = \frac{(r + \delta)q}{1 - \tau_2} > \frac{(r + \delta)q}{1 - \tau_1} = \pi'(k_1),$$

implying, by the strict concavity of $\pi(k)$, that $k_2 < k_1$. Thus, we find that in the presence of the considered progressive tax system three possible cases may arise, depending on the level of the progression threshold $k^*$. Our main results on the nature of the optimal investment policy and the value of the firm are now summarized in the following.

**Theorem 1** The value of the rationally managed firm reads as

$$V(k) = \begin{cases} qk + (R_0\theta)(k) + \frac{1}{2}(R_0\theta)(\Bar{k})k^{\Bar{k}}, & k > \Bar{k}, \\ qk + \frac{\alpha k\varphi}{\delta}, & k \leq \Bar{k}, \end{cases}$$

where

$$\quad R_0\theta(k) = \frac{k^{r+\delta}}{\delta} \int_0^k y^{r+\delta-1}\theta(y) \, dy$$

denotes the cumulative present value of the flow $\theta(k)$, and the optimal investment threshold reads

$$\Bar{k} = \text{argmax}(\theta(k)) = \begin{cases} k_2 & \text{if } k^* < k_2 < k_1, \\ k^* & \text{if } k_2 < k^* < k_1, \\ k_1 & \text{if } k_2 < k_1 < k^*. \end{cases}$$

**Proof.** See the appendix.

Theorem 1 characterizes the optimal investment policy and its value in the presence of progressive taxation. According to Theorem 1 the optimal accumulation policy is constituted by a potentially significant initial lump-sum investment taking the capital stock instantaneously to a desired level, after which the optimal policy is to maintain the stock at this optimal level by setting the gross investment equal to the depreciated stock. Moreover, we find that in the presence of progressive taxation three possible regimes arise, depending on the magnitude of the progression threshold. More precisely, in the case where $k^* \in (k_2, k_1)$ the optimal investment policy is independent of the tax rates, while outside this region the optimal capital accumulation policy is conventional.

**3. Illustration**

In order to illustrate explicitly our general findings concerning the effect of progressive profit taxation on the optimal capital accumulation policy and
its value, we assume that the revenue flow $\hat{\pi}(k)$ now reads as $\hat{\pi}(k) = ak^b$, where $a > 0$ and $b \in (0, 1)$ are exogenously given constants. It is now a simple exercise in nonlinear programming to establish that if $\tau_1 < \tau_2$ then in the present example

$$k_1 = \left( \frac{(1-\tau_1)ab}{(r+\delta)q} \right)^{1/(1-b)} > \left( \frac{(1-\tau_2)ab}{(r+\delta)q} \right)^{1/(1-b)} = k_2,$$

(7)

implying for $i = 1, 2$ the familiar comparative statics

$$\frac{\partial k_i}{\partial \tau_i} = -\frac{k_i}{(1-b)(1-\tau_i)} < 0.$$

(8)

This states that higher profit taxation should decelerate rational investment demand by decreasing the optimal capital stock threshold at which reinvestment becomes optimal. In particular, Theorem 1 now implies that there are three different optimal regimes, depending on the precise magnitude of the exogenously determined threshold $k^*$. Moreover, the value of the optimal policy now reads as in (4), where in our explicit case the cumulative present value of the flow $\theta(k)$ can be expressed as

$$(R, \theta)(k) =
\begin{cases}
(1-\tau_2) \frac{ak^b}{r+\delta} - (\tau_1 - \tau_2) \frac{ak^b}{r+\delta} \left(1 - \frac{\delta}{r}(\frac{k}{k^*})^r\right) - qk, & k \geq k^*, \\
(1-\tau_1) \frac{ak^b}{r+\delta} - qk, & k < k^*,
\end{cases}
$$

(9)

so that

$$(R, \theta)'(k) =
\begin{cases}
(1-\tau_2) \frac{ak^{b-1}}{r+\delta} - (\tau_1 - \tau_2) \frac{ak^{b-1}}{r+\delta} \left(\frac{k}{k^*}\right)^{r/b-1} - q, & k \geq k^*, \\
(1-\tau_1) \frac{ak^{b-1}}{r+\delta} - q, & k < k^*.
\end{cases}
$$

(10)

The growth rate of the excess returns accrued from following an optimal irreversible capital accumulation policy and the optimal investment thresholds are illustrated in figure 1 (under the assumption that $a = 1, b = 0.75, \tau_1 = 0.15, \tau_2 = 0.3, q = 0.45$).

As our numerical illustration indicates, the optimal investment threshold now reads as

$$\tilde{k} =
\begin{cases}
1.85 & \text{if } k^* < 1.85, \\
k^* & \text{if } 1.85 \leq k^* \leq 4.03, \\
4.03 & \text{if } k^* > 4.03.
\end{cases}
$$

Thus, the optimal capital accumulation policy is independent of the profit tax rate as long as the progression threshold satisfies the inequality $1.85 \leq k^* \leq 4.03$. Outside this corner solution the optimal capital accumulation policy is a conventional one.
We have analyzed the influence of profit tax progression – defined as an increase of the marginal profit tax rate at an exogenously given progression threshold – on the optimal investment policy and its value. We showed that three possible optimal regimes arise, depending on the progression threshold. More precisely, our new results establish that if the progression threshold lies between the optimal capital stocks in the cases of higher and lower marginal profit taxes, then the optimal investment policy is independent of the profit tax rate. Outside this corner solution, the optimal capital accumulation policy is conventional.

An interesting and natural extension of our approach would be to analyze the influence of progressive taxation on the optimal investment policy under tax-policy and price uncertainty. Such an extension would provide valuable information on the relative strength of the effects of taxation and market uncertainty on optimal investment policies. Such an analysis is out of the scope of the present study and left for future research.

5. Appendix

Proof of Theorem 1. Consider first the cumulative present value of the flow $\theta(k)$. It is now clear that
\[(R, \theta)(k) = \int_0^{\infty} e^{-rs} \theta(ke^{-\delta s}) \, ds.\]

Making the change of variable \(y = ke^{-\delta s}\) implies that \(e^{-rs} = \frac{y}{k}\) and therefore that the cumulative present value \((R, \theta)(k)\) can be expressed as in (5). Given this observation, consider the functional
\[J(k) = \frac{\delta}{r} k^{\pi/b+1}(R, \theta)(k) = \frac{1}{r} k^{\pi/b} \theta(k) - \frac{1}{\delta} \int_0^k y^{\pi/b-1} \theta(y) \, dy.\]

If \(0 < k < z < \tilde{k}\), then
\[J(z) - J(k) = \frac{1}{r} \left[ z^{\pi/b} \theta(z) - k^{\pi/b} \theta(k) \right] - \frac{1}{\delta} \int_k^z y^{\pi/b-1} \theta(y) \, dy \geq \frac{1}{r} k^{\pi/b} [\theta(z) - \theta(k)] \geq 0.\]

Similarly, if \(\tilde{k} < k < z < \infty\), then
\[J(z) - J(k) = \frac{1}{r} \left[ z^{\pi/b} \theta(z) - k^{\pi/b} \theta(k) \right] - \frac{1}{\delta} \int_k^z y^{\pi/b-1} \theta(y) \, dy \leq \frac{1}{r} k^{\pi/b} [\theta(z) - \theta(k)] \leq 0.\]

Hence, the mapping \(J(k)\) is increasing on \((0, \tilde{k})\), is decreasing on \((\tilde{k}, \infty)\), and attains a unique global maximum at \(\tilde{k}\).

Denote now the proposed value function as \(V_\rho(k)\). Since \(V_\rho(k)\) is attained by applying the admissible singular investment strategy defined as \(I_0 = (\tilde{k} - k)^+\) and
\[I_t = \delta \tilde{k}, \quad t \geq \inf \{t \geq 0 : k^t = \tilde{k}\},\]
we observe that \(V(k) \geq V_\rho(k)\). In order to prove the opposite inequality we first observe that the proposed value function is nonnegative and continuously differentiable. Since \(-\delta k V_\rho'(k) - r V_\rho(k) + \pi(k) = 0\) on \((\tilde{k}, \infty)\) and
\[\frac{\delta}{r} k^{\pi/b-1} [J(k) - J(\tilde{k})] \leq 0,\]
by the assumed monotonicity of \(\theta(k)\) we find that \(-\delta k V_\rho'(k) - r V_\rho(k) + \pi(k) \leq 0\) for all \(k \in \mathbb{R}_+\). Moreover, standard differentiation implies that for all \(k \in (\tilde{k}, \infty)\) it holds that
\[V_\rho'(k) = q + \frac{r}{\delta} k^{\pi/b-1} [J(k) - J(\tilde{k})] \leq q,\]
since \(\tilde{k} = \arg\max J(k)\). Consequently, \(V_\rho'(k) \leq q\) for all \(k \in \mathbb{R}_+\). Hence, the proposed value function satisfies the sufficient variational inequalities, and therefore \(V_\rho(k) \geq V(k)\).
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