Mobile robot localization in an unknown environment using sonar sensors and an incidence angle based sensors switching policy - Experimental results

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Abstract—In this work the mobile robot localization problem in an unknown environment is faced. To solve this problem, an Extended Kalman Filter, based on measurements taken from ultrasonic sensors and only on local data, with no assumption on robot’s working environment, is proposed. A new model for the ultrasonic sensors is proposed and validated through experimental tests. A switching sensors activation policy, based on the physical characteristics of ultrasonic sensors, is devised. Such policy allows power saving still achieving good estimation performance. Experimental tests, using the robot Khepera III, show the effectiveness of the proposed sensors switching policy in solving the mobile robot localization problem.

I. INTRODUCTION

In recent years, due to the increasing use of mobile robots in various applications, the robot localization problem has been faced in different ways. The aim is to localize the robot using all the available information by sensors and by a-priori knowledge on the environment where the robot moves. In [10] the localization problem has been solved using a sonar rangefinder to determine the two dimensional position and orientation of a mobile robot inside a room. The algorithm works by correlating straight segments in the range data against the room model and then eliminating implausible configurations using a defined sonar barrier test based on physical constraints on sonar data. In [12] single sonar rays are used as the primary inputs to the localization algorithm. A well known technique used for robot localization is the Extended Kalman Filter which helps to estimate robot (position and orientation) state given a model for robot evolution and sensors measurements. In [1] the authors face the problem of fusing information provided by a set of ultrasonic sensors placed in the environment. In [14] the localization problem has been solved in a real perfectly known environment using a Khepera III robot [13] and an Extended Kalman Filter based on the measurements provided by the onboard ultrasonic sensors.

In contrast of [14], in this work no assumption is done about the environment where the robot moves and only sensors’ measurements are used to obtain the robot pose. The basic idea behind the proposed solution is that robot state can be estimated using only information about the environment portions that interact with robots sensors. The above concept is the basis of the implemented Neighbours Based Algorithm and will be illustrated in Section IV.

In many works the mobile robot localization problem has been solved using a set of ultrasonic sensors. Such sensors use sound waves to detect a target and they measure (within tolerance) the distance to the surface intercepted by their beam. As exploited in [10] and [11] one of the disadvantages of the ultrasonic sensors is the influence, on the provided measurements, of the incidence angle between the measured target and the sonar beam.

In order to localize the robot, it seems intuitive that the more sensors you use, the better estimate you get. However, especially when ultrasonic sensors are used, there are reasons that suggest not too exceed with their number or use. The most important is that sensors employ robot’s batteries, and an intensive use reduces the robot autonomy. Moreover there are also situations where multiple sonar sensors cannot operate simultaneously, for example when they use the same frequency band [2]. When the filter is not run onboard the robot, which is the most frequent case, one more issue is bandwidth consumption and possible collisions when transferring measurements from the sensors to the filter. Thus, using an appropriate sensors switching policy, only a part of all available sensors will be used, and the energy consumption will be reduced. However, this may result in a lower accuracy of the robot state estimation.

Due to the above considerations the problem is related to finding the “best” sequence of activation of a small (fixed) part of the available sensors to obtain the “best” (in some statistical sense) robot pose estimation.

Within this framework, in this work, assuming to use ultrasonic sensors, a new sensors switching policy is presented. Such policy is based on the sonar sensor physical characteristics and, on the contrary of the switching rule presented in [14], it has a very low computational cost.

The paper is organized as follows: in Section II the model of the mobile robot is presented; in Section III the new sonar sensor model is presented; in Section IV the Neighbours Based Extended Kalman Filter is proposed; in Section V a sensors switching technique to improve the state estimation accuracy is presented. Finally in Section VI and VII experimental results are presented and discussed and some possible extensions for future investigations are outlined.

II. ROBOT MODEL

We consider a battery-powered mobile robot with two independent driving wheels and a castor wheel: a concrete example is the Khepera III robot [13], which has been used
in the experiments reported in Section VI. For such robot, an approximated, discrete-time model, which neglects motors and frictions dynamic, is [3]:

\[
\begin{align*}
x^1_{k+1} &= x^1_k + V_k T \cos(\theta_{k+1}) + w^x_k \\
x^2_{k+1} &= x^2_k + V_k T \sin(\theta_{k+1}) + w^y_k \\
\theta_{k+1} &= \theta_k + \Delta_k + w^\theta_k,
\end{align*}
\]

where:
- \((x^1_k, x^2_k)\) is the robot position at time \(t_k\);
- \(\theta_k\) is the angle between the robot axle and the \(x\)-axis;
- \(V_k = r (\omega^r_k + \omega^l_k)/2\) is the robot linear velocity;
- \(\omega^r_k\) and \(\omega^l_k\) are the wheels angular velocities;
- \(w^x_k, w^y_k, w^\theta_k\) are zero-mean uncorrelated Gaussian noises;
- \(r\) is the wheels radius;
- \(d\) is the distance between the active two wheels;
- \(\Delta_k = r (\omega^r_k - \omega^l_k) T/d\) is the rotation within \([t_{k-1}, t_k]\);
- \(T = t_k - t_{k-1}\) is the sampling period.

The input variables of the model are the wheels angular velocities \(\omega^r_k\) and \(\omega^l_k\), and they have been precomputed so that the robot follows the desired trajectories. The model’s state variables are \(x_k = [x^1_k \ x^2_k \ \theta_k]^T\). The Gaussian disturbances take into account unmodeled dynamics, friction, wheels slipping and also, if the case, external disturbances such as wind.

III. SONAR SENSOR MODEL

Ultrasonic sensors are widely used in many applications thanks to their simplicity, availability, and low cost. Such sensors measure (within tolerance) the distance to the surface intercepted by their beam. Ultrasonic sensors generate high frequency sound waves and evaluate the received back echo. Sensors calculate the time interval between sending the signal and receiving the echo to determine the distance to an object.

As remarked in [10] and [11], the measurements provided by these sensors are very influenced by the angle of incidence between the “sensor beam”, whose typical shape is shown in Figure 1, and the surface intercepted. When the sensor is perpendicular to a flat surface, the measurement provided is the true range, within tolerance, to the surface. However, the measurement error can be much larger when the beam strikes a surface at a different incidence angle, \(\gamma\). In this work the main purpose is to reduce such disadvantage choosing the sensors to use on the basis of their related incidence angles.

A. Incidence Angle

Consider an ultrasonic sensor, \(S\), which provides the distance, \(y_i\), from a surface \(U\), as depicted in Figure 1.

Let \(l(x^1, x^2)\) be the tangent line to the boundary of the surface in the intersection point, \(A\), between the incidence surface and the sensor axis. Defining \(\vec{r}\) as the unit vector of the sensor axis and \(\vec{b}\) as the unit vector of \(l(x^1, x^2)\), it is possible to define the incidence angle \(\gamma\) as

\[
\gamma = \arccos(\vec{r} \cdot \vec{b})
\]

The more the incidence angle is near to \(\frac{\pi}{2}\) rad, the better the measurement provided by the ultrasonic sensor is. Using (2) the following incidence parameter can be defined:

\[
p = \cos \gamma = \vec{r} \cdot \vec{b}
\]

since the incidence angle is \(\gamma \in (0, \frac{\pi}{2})\) rad, the incidence parameter \(p \in [0, 1]\). For \(p \rightarrow 0\), the sensor axis become orthogonal to the incidence surface; for \(p \rightarrow 1\) the sensor axis become parallel to the incidence surface.

B. Measurement Model

To take in consideration the influence of the incidence angle into the measurement model related to the ultrasonic sensor, we model the measurement provided by such sensor as \(y_M = y_R + \xi(p)\)

where \(y_R\) is the real distance between the sensor and the incidence surface, \(y_M\) is the measurement provided by the ultrasonic sensor and \(\xi(p)\) is a Gaussian noise of zero mean and covariance matrix \(V(p)\) related to the parameter \(p\). Such noise takes into account the incidence angle influence on the measurement \(y_M\) along with the measurement noise.

In order to satisfy the described characteristics about the incidence angle and its influence on the sensor measurement, the incidence function \(\xi(p)\) has to be chosen such that:

1) its standard deviation is monotonic increasing in \(p\);
2) \(V(0)\) has to be equal to the nominal covariance of the used sensor that is the covariance of the measurement noise when the incidence angle is \(\gamma = \frac{\pi}{2}\) rad.

C. Observation structure

The robot is supposed to be equipped with five ultrasonic sensors (this is the case of the Khepera III), located as shown in Figure 2 and denoted by \(S_i, i = 1, \ldots, 5\).

In contrast of [14], the robot is assumed to be placed in an unknown environment. Since there is no a-priori knowledge on the environment, to yield the output equation related to the onboard sensors, a model for the boundary of the environment is needed. The environment will be modeled as a set of segments such that each of them intersects at least at one point of the boundary of the environment (Figure 2).

Each sensor \(S_i\) provides the distance between the center of gravity of the robot, denoted by \(P = (x^1_i, x^2_i)\), and one point on the environment, denoted by \(P_i = (\tilde{x}^1_i, \tilde{x}^2_i)\).

Considering the model used for the environment, the measurement provided by the \(i\)-th sensor, \(S_i\), is approximated,
Fig. 2. Observation model.

as shown in Figure 2 for the case $S_i = S_3$, by the distance between $P$ and the intersection, denoted by $P_i = (\pi_1, \pi_2)$, between the axis of the sensor $S_i$ and one of the representative segments of the world.

Marking the axis of the sensor $S_i$ as $x^2 = a_i x^1 + q_i$ and denoting the axis of the representative segment as $x^2 = c_i x^1 + s_i$, the intersection $P_i$ is given by

$$
\pi_1 = \frac{s_i - q_i}{a_i - c_i}, \quad \pi_2 = \frac{a_i s_i - c_i q_i}{a_i - c_i},
$$

therefore the measurement, $y_i$, provided by sensor $S_i$, is modeled as

$$
y_i = \left((x_k^1 - \bar{x}^1)^2 + (x_k^2 - \bar{x}^2)^2\right)^\frac{1}{2},
$$

and approximated by

$$
y_i \approx \left((x_k^1 - \bar{x}^1)^2 + (x_k^2 - \bar{x}^2)^2\right)^\frac{1}{2}.\tag{6}
$$

Since the robot structure is given, the orientation, $\alpha_i$, of each sensor $S_i$, with respect to robot axis (placed on third sensor axis, orthogonal to the wheel axes), is known. Thus the axis of the sensor $S_i$ is given by:

$$
a_i = \tan(\theta_k + \alpha_i), \quad q_i = x_k^2 - a_i x_k^1. \tag{7}
$$

By replacing (6) with (7), an observation function, $y_i$, depending only on robot state and segment $(s_i, c_i)$, can be obtained as

$$
y_i = h((x_k^1, x_k^2, \theta_k), (s_i, c_i)), \quad i = 1, \ldots, 5
$$

These relationships define the output equation of the dynamical model of the robot and will be written in the more compact form

$$
y_k = h(x_k, (\bar{s}_k, \bar{c}_k)) + \xi(\bar{p}_k), \tag{8}
$$

where the dimension of vector $y_k$ ranges from one to five, depending on how many sensors are used, the vector $\bar{p}_k$ collects the incidence parameters for each used sensor and the function $\xi(\cdot)$ is the incidence function previously defined. $\xi(\cdot)$ is a Gaussian noise and it is assumed uncorrelated with $w_k = [w_k^x w_k^y w_k^\phi]^T$. The vectors $\bar{s}_k$ and $\bar{c}_k$ contain one segment, $(s_i, c_i)$, for each used sensor at time $k$.

IV. NEIGHBOURS BASED EXTENDED KALMAN FILTER (NEKF)

Nonlinear filtering is the problem of estimating the state of a nonlinear stochastic system from noisy measurements. For discrete-time systems such framework is represented by the equations

$$
x_{k+1} = f(x_k, u_k) + w_k
$$

$$
y_k = h(x_k) + v_k,
$$

where $x$ is the state to be estimated, $y$ is the measured output, $w$ and $v$ are the system and measurements noises.

Ignoring $(\bar{s}_k, \bar{c}_k)$, model (1), (8) falls within this framework on defining

$$
x_k = [x_k^1 x_k^2 \theta_k]^T, \quad u_k = [\omega_k^x \omega_k^y]^T
$$

$$
w_k = [w_k^x w_k^y w_k^\phi]^T, \quad v_k = \xi(\bar{p}_k).
$$

The matrices $W$ and $V(\bar{p}_k)$ will denote the covariance matrices of the noises $w_k$ and $\xi(\bar{p}_k)$, respectively. These matrices will be assumed known. Moreover we suppose we are given an initial estimate of the state $\hat{x}_{0|0}$ and the estimation error covariance matrix $P_{0|0}$, related to the “reliability” of $\hat{x}_{0|0}$.

The observation structure (8) is well-posed only if $(\bar{s}_k, \bar{c}_k)$ are known. The Neighbours Based Algorithm (NBA) has been devised to approximate them.

A. Neighbours Based Algorithm

The core idea behind the NBA is proximity among acquired measurements; two points $P_1, P_2$ are defined neighbours iff $||P_1 - P_2|| < R$, where $R$ is a given algorithm parameter. Moreover, the following set-valued closeness function $N$ has been defined:

Given a set of points $A$ and a point $P$,

$$
B = N(P, A) = \{P_i \in A : ||P_i - P|| < R\}
$$

that is the subset of points of $A$ which are neighbours, in a radius $R$, of $P$.

Given a new measurement $y_i$ acquired by sensor $S_i$, and given the actual estimate of the robot state, an approximation of the environment point $\hat{P}_i$, hit by the sensor beam, can be computed. This approximation, denoted by $P_i^*$, differs from the actual point because of both the estimation and the measurement errors.

The NBA then computes the closeness function of $P_i^*$ on the set of the previously identified boundary points, denoted by $M$. At this point the $LMS(\cdot)$ function has been defined as follows:

Given a set of points $A$

$$
LMS(A) = (q, m)
$$

returns the Least Mean Squared approximation, $(q, m)$, of the straight line, $x^2 = m x^1 + q$, through points in $A$.

Using the defined functions, the parameters $(s_i, c_i)$ of the Least Mean Square line which approximates the points that are neighbours of $P_i^*$ are computed as $(s_i, c_i) = LMS(N(P_i^*, M))$. Finally, the point $\hat{P}_i$ is computed as the intersection between the Least Mean Square line and the sensor beam.
The NBA can be summarized as follows:

### Neighbours Based Algorithm

At each step, given \( \hat{x}_i^{k}, \hat{x}_i^{k+1}, \theta_i, M_k, I_k, J_k, \{(\hat{s}_i, \hat{c}_i), i \in I_k\} \) do:

1. for \( i \in I_k \)
   - acquire measure \( y_{i,k} \) from sensor \( S_i \)
   - \( \left[ \begin{array}{c} P_{i,k}^{a} \\ P_{i,k}^{a+1} \end{array} \right] = \hat{x}_i^{k} + y_{i,k} \cos(\theta_i + \alpha_i) \)
   - \( \left[ \begin{array}{c} P_{i,k}^{a+1} \\ P_{i,k}^{a+2} \end{array} \right] = \hat{x}_i^{k} + y_{i,k} \sin(\theta_i + \alpha_i) \)
   - \( P_i^k = (P_{i,k}^{a+1}, P_{i,k}^{a+2}) \)

2. \( M_{k+1} = M_k \cup \{P_i^k, i \in I_k\} \)

3. for \( i \in I_k \)
   - \( (\hat{s}_i, \hat{c}_i) \in \text{LMS}(N(P_i^k, M_{k+1})) \)

end

4. return \( (\hat{s}_i, \hat{c}_i), i \in J_k \)

where:

- \( I_k \) is the set of indexes related to the sensors used at step \( k \);
- \( J_k \) is the set of sensor indexes whose intercepted segments are needed at step \( k \);
- \( x_i^k, \hat{x}_i^k, \theta_i \) is the robot state estimation at step \( k \);
- \( \alpha_i \) is the orientation of each sensor \( S_i \) with respect to the robot axis;
- \( M_k \) is the set of previously acquired environment points;
- \( (\hat{s}_i, \hat{c}_i) \) are the approximations, at step \( k \), of the parameters of the segment intercepted by sensor \( S_i \) axis.

From now on we will use the NBA as a function: \((\hat{s}, \hat{c}) = NBA(I_k, J_k, k)\) where \((\hat{s}, \hat{c})\) represent the output of NBA at time \( k \) and for each sensor \( S_i, i \in J_k \).

### Neighbours based Extended Kalman Filter

The Extended Kalman Filter (EKF) is based on the linearization of the nonlinear maps \((f, h)\) of (9) around the estimated trajectory, and on the assumption that the initial state and measurement noises are Gaussian and uncorrelated each other.

From the computational point of view the EKF is simply a time-varying Kalman filter where the dynamic and output matrices are given by:

\[
A_k = \frac{\partial f(x, u_k)}{\partial x} \bigg|_{x = \hat{x}_k} , \quad C_k = \frac{\partial h(x)}{\partial x} \bigg|_{x = \hat{x}_k[k-1]}, \quad (10)
\]

and its output is a sequence of state estimates \( \hat{x}_k \) and matrices \( P_k[k] \); starting from given \((\hat{x}_{0[k]}, P_{0[k]}\)) and using the observation structure (8), the following new version of the EKF can be defined:

### Neighbours based Extended Kalman Filter

\[
\begin{align*}
\hat{x}_{k+1[k]} &= f(\hat{x}_{k[k]}, u_k) \\
P_{k+1[k]} &= A_k P_{k[k]} A_k^t + W \\
(\hat{s}_k, \hat{c}_k) &= NBA(I_k, I_k, k) \\
K_{k+1} &= P_{k+1[k]} (C_{k+1} + P_{k+1[k]} C_{k+1}^t + V(P_{k+1}))^{-1} \\
\hat{x}_{k+1[k+1]} &= \hat{x}_{k+1[k]} + K_{k+1}(y_{k+1} - h(\hat{x}_{k+1[k]}, (\hat{s}_k, \hat{c}_k))) \\
P_{k+1[k+1]} &= P_{k+1[k]} - K_{k+1} C_{k+1} P_{k+1[k]} \\
\end{align*}
\]

where \( \hat{x}_{k+1[k]} \) represents the estimate of \( x_{k+1} \) before getting the observation \( y_{k+1} \), \( \hat{x}_{k+1[k+1]} \) represents the estimate after getting that observation. Moreover the matrix \( V(\overline{p}_{k}) \) is the time varying covariance matrix related to the Gaussian noise \( \xi(\overline{p}_{k}) \) and the set \( J_k \) has been taken equal to the set \( I_k \).

It is well known that the EKF is prone to diverge, mainly for bad initial estimates and high noises [5], but no testable convergence conditions are known to our knowledge.

### V. SENSORS SWITCHING POLICY

Assuming to have \( j \) sensors available, choosing at any time instant to activate \( q \) out of them \((q \ll j)\) is an old problem. The interest about such problem was motivated, more than two decades ago, by the limited capacity of the transmission medium and the low computational power of the processors. The above limitations imposed to select at any instant only a few sensors.

Although nowadays such difficulties are vanished, due to the huge developments in the transmission and computational devices, a new and important problem, especially in mobile robot applications, is related to save the sensors’ batteries lifetime. To this end it could be very meaningful to devise a policy that uses a small part of the available sensors to limit power consumption. Obviously such policy has an impact on the quality of the estimate since there is a trade off between the number of sensors used and the performance of the resulting estimation algorithm. Thus the best policy is the one that, for a number \( q \ll j \) of sensors, gives the best estimate, in some statistical sense.

To formalize the problem, suppose that at each instant only \( q \) out of \( j \) sensors are activated. A \( q \)-valued output equation \( h(\cdot) \) and a \( q \times n \) output matrix \( C_k \) related to such sensors can be evaluated by Eq. (8) and (10). Changes in the activation sequence clearly return different estimates of \( x_k \) for the EKF.

Our aim is to obtain the best estimate, in some statistical sense, using only \( q \ll j \) sensors.

The proposed switching policy is based on the sonar sensor model presented in Section III. The main idea is:

Among the \( j \) sensors, choose the subset of \( q \) ones related to the best incidence angles.

Given the set of available sensors \( \{S_i, i = 1, \ldots, j\} \), each of them is related to an incidence angle \( \gamma_i \) and then to an incidence parameter \( p_i \). The \( q \) sensors related to the lower values of \( p_i \) are chosen.

If no assumption on the environment is made and the environment is totally unknown, the incidence angles are not known \( a-priori \). To use the proposed sensors switching rule an estimate of the incidence angles, \( \{\hat{\gamma}_i, i = 1, \ldots, j\} \), or which is completely equivalent, of the associated incidence parameters \( \{\hat{p}_i, i = 1, \ldots, j\} \), has to be found.

To obtain such estimate the information provided by the NBA can be used.

Marking the axis of the sensor \( S_i \) as \( x^2 = \alpha_i x^1 + q_i \) and ignoring the translation \( q_i \), a vector on such line can be parametrized as \( r_i(g) = [g, \alpha_i g]^t, g \in \mathbb{R} \), thus, the unit vector, \( \hat{r}_i \), related to the sensor’s axis is \( \hat{r}_i = r_i(g)/||r_i(g)|| \).

Using the current estimate \( \hat{x}_{k[k]} \), an estimation of \( \alpha_i \) can be found as \( \hat{\alpha}_i = \tan(\theta_{k[k]} + \alpha_i) \). Thus an estimation of \( r_i(g) \) is \( \hat{r}_i(g) = [g, \hat{\alpha}_i g]^t \) and finally the estimated unit vector is

\[
\hat{r}_i = \hat{r}_i(g)/||\hat{r}_i(g)|| \quad (11)
\]
The Neighbours Based Algorithm provides a set of lines, \( \{(s_i, c_i)\} \), related to an approximation of the parameters of the segments intercepted by the used sensors’ axes. The line \( (s_i, c_i) \) is related to the sensor \( S_i \) and can be assumed as an estimation of the tangent line, \( l_i(x^1, x^2) \), to the boundary of the portion of the environment intercepted by such sensor’s axis. Therefore, for each used sensor \( S_i \), an approximation of the unit vector \( \hat{b}_i \) of the line \( l_i(x^1, x^2) \) can be found as

\[
\hat{b}_i = \frac{\hat{b}_i(g)}{|\hat{b}_i(g)|} 
\]

(12)

where \( \hat{b}_i(g) \) is the approximation of the parametrized vector on the line \( l_i(x^1, x^2) \), that is \( \hat{b}_i(g) = [g \cdot \hat{c}_i g]^T \). At this point, using the equations (11) and (12) along with the equation (3), it is possible to obtain an approximation of the incidence parameter related to each sensor \( S_i \) as

\[
\hat{p}_i = \hat{r}_i \cdot \hat{b}_i 
\]

(13)

Thanks to the above equation, at each time step \( k \) the following steps can be performed to use the presented switching logic:

**Sensors Switching Algorithm**

Given \( I_k \) and \( (\tilde{s}_k, \tilde{c}_k) = NBA(I_k, \{i = 1, \ldots, j\}, k) \), do

1) evaluate \( \hat{r}_i, i = 1, \ldots, j \), using (11)
2) evaluate \( \hat{b}_i, i = 1, \ldots, j \), using (12)
3) evaluate \( \hat{p}_i = \hat{r}_i \cdot \hat{b}_i, i = 1, \ldots, j \), using (13)
4) \( I_{k+1} = MIZ(\{\hat{p}_i, i = 1, \ldots, j\}, q) \)

where, \( I_k \) is the set of indexes used at previous step by the switching rule and the function \( MIZ \) is defined as:

\[
\text{Given a set of variables } \{z_i\}, i = 1, \ldots, m, \text{ and } z \in \mathbb{N} \text{ such that } z \leq m,
MIZ(\{z_i, i = 1, \ldots, m\}, z)
\]

returns the indexes related to the \( z \) variables with lower value than the others.

Other possible sensors switching logics can be found in [7] for the state estimation of linear systems and in [14] for the state estimation of nonlinear systems using the Extended Kalman Filter. Such switching logics are based on choosing the subset of sensors whose measurements have maximum effect on the current estimation. These policies have a combinatorial computational cost on the number, \( j \), of available sensors.

On the contrary, the proposed policy has a polynomial computational cost on the number, \( j \), of sensors. This cost is mainly related to the use of the NBA in the switching algorithm. Moreover, using the proposed policy along with the NEKF algorithm, the computational cost of the filter is not increased since the parameters \( (\tilde{s}_k, \tilde{c}_k) \) obtained by the filtering algorithm can be used also by the switching policy. In such situation, the additional cost due to the switching policy, in the worst case, is only related to sorting the elements \( \hat{p}_i, i = 1, \ldots, j \), and it is \( O(j \log j) \). Clearly such cost is less than the NEKF cost.

In principle \( q \) could depend on \( k \), but here we will assume it constant for simplicity. Since there is no guarantee on the optimality of the proposed choice to find the optimal switching sequence (finding it would be a problem of combinatorial complexity), we look at this criterion as a heuristic method to improve the battery life without affecting the obtained estimation.

**VI. EXPERIMENTAL RESULTS**

To evaluate the performance of the proposed algorithm for the mobile robot localization problem, some experimental tests have been made.

First of all, to estimate the noise covariance function, a series of experiments has been performed, each one related to a different value of the incidence angle, from \( 90^\circ \) to \( 60^\circ \). For each angle value, a set of 1000 measurements has been taken, and the corresponding standard deviation has been estimated. The obtained results are shown in Figure 3.

![Fig. 3. standard deviation function estimation](image)

Using the obtained data, the standard deviation of the incidence function \( \xi(\cdot) \) can be estimated through an LMS approximation of the values by a polynomial curve. As it can be seen, such standard deviation function can be well approximated by a second order polynomial function. Such function is monotonic increasing in \( p \) and thus the experimental data confirm the correctness of the model used for the ultrasonic sensors. The obtained estimate of the standard deviation function of \( \xi(\cdot) \) can be used to evaluate the covariance matrix \( V(\hat{p}_k) \) used in the NEKF algorithm.

At this point, to evaluate the performance of the proposed sensors switching rule two series of experiments have been performed, each one refereeing to a different trajectory followed by the robot Khepera III. Each trajectory is a circle of radius \( 20m \) centered in \( (0,0,0) \pm 30 \) m and continues in \( 0 \leq m \leq 50 \). The trajectory starts from \((0,0,0)\) and continues in the counterclockwise. The second one is a square of 0.3m with \((0,0,0)\) as starting point. For each of them, 20 experiments have been performed. A sample time \( T = 1 \) second has been taken. Both trajectories are completed in \( k_f = 100 \) seconds. For each trajectory a trapezoidal profile has been imposed to the wheels angular velocities.

In each experiment, three different kinds of filter have been tested. The first one uses two sensors \( s_i, s_j \), keeping them fixed along the path; this test has been repeated for all the \( \binom{5}{2} = 10 \) possible sensor configurations. The second one uses two sensors \( q = 2 \) out of the five available, switching between sensors using the above discussed switching policy \( (I_k) \) is chosen at each step using the proposed sensors
switching algorithm in order to include the measurements with the best incidence angles). The third one uses all five sensors together ($I_k = \{1, \ldots, 5\}$, $k \in \{0, \ldots, k_f\}$), and it is used as a reference. The following values for the covariance parameters and the initial conditions were common to all experiments:

- $W = \text{diag}\{0.00029, 0.00019, 0.09274\}$, which corresponds to a standard deviation of 0.017 m on $x^1$, 0.014 m on $x^2$ and of 17° on $\theta$.
- $V(0) = \text{diag}\{0.081^2, 0.123^2, 0.061^2, 0.096^2, 0.050^2\}$.
- $\hat{x}_{0|0} = [0.59, 0.31, 0]^T$; the true initial state of the robot is $\tilde{x}_0 = [0.6, 0.3, 0]^T$ for each trajectory.
- $P_{0|0} = \text{diag}\{0.01, 0.01, 0.003\}$, which corresponds to a standard deviation of 0.1 m on the robot position and 1° on the orientation.

The NBA requires as a parameter the radius $R$, illustrated in Section IV, and $R = 0.2m$ has been adopted in all experiments done. With regard to the initialization of the set $M$, of acquired environment points, and of the parameters $\{(\hat{s}_i, \hat{c}_i), i = 1, \ldots, 5\}$, we start our filters 15 steps after the beginning of each experiment. During these initial steps measurements are acquired to form the initial condition for $M$ and $\{(\hat{s}_i, \hat{c}_i), i = 1, \ldots, 5\}$. To evaluate the performance of the filters the following index has been introduced

$$
\varepsilon[\%] = \frac{1}{k_f + 1} \sum_{k=0}^{k_f} \frac{||x_k - \hat{x}_{k|k}^*||}{||x_k||} \times 100
$$

where $\hat{x}_{k|k}^*$ is the estimated state with one of the tested filters and $x_k$ is the real value of the robot pose. Such index gives information about the state estimation error. In Tables I the obtained results are reported; the values are averaged over the 20 experiments. In this table, "Switching" refers to the proposed switching policy with $q = 2$; "All" refers to the case where all sensors are used together; "$\tilde{z}_i$" indicates a generic pair of sensors among the $\binom{5}{2}$ available.

As it can be seen by the obtained results, the proposed switching rule provides very good estimation results for both the trajectories yielding to a very low value of the estimation error $\varepsilon$. Please note that due to the high measurement noise, using all the sensor is not the best possible choice. Obviously if the used sensors are not too noisy, the minimum value of the estimation error would be obtained using all the sensors simultaneously.

To further test the proposed switching rule, a set of 1000 numerical simulations, based on the same standard deviation function of $\xi(\cdot)$ and parameters of the experimental setting, has been performed. Simulation tests show the effectiveness of the proposed switching policy. In the performed numerical tests, the filter based on the switching policy and $q = 2$ sensors performs better than each other possible fixed pair of sensors.

### VII. CONCLUSIONS AND FUTURE WORKS

In this work the mobile robot localization problem in an unknown environment has been faced. To solve this problem, an Extended Kalman Filter, based on measurements taken from ultrasonic sensors, has been used. The algorithm is based only on local data and does not require any assumption on robot’s working environment. Moreover a new sensor model has been proposed and validated by experimental data. A switching sensors activation policy, based on the physical characteristics of ultrasonic sensors, has been devised. Real experiments using the robot Khepera III have been performed and the obtained results have shown that the proposed switching rule yields to very high estimation performance. Please note that using the proposed sensors switching rule, the filter performs well whatever is the trajectory performed by the robot, as shown by the experimental results.

Actually a larger number of experiments are in progress to look for better estimation results and to further validate the ultrasonic sensor model used.

As future research directions loss of information on the data transmission channel will be considered and the proposed switching rule will be adapted to other extensions of the Kalman filter, such as the Unscented Kalman Filter. Moreover studies on an incidence angle based switching rule related to a time varying number, $q = q(k)$, of used sensors, are in progress.

### REFERENCES


