Electron dynamics in superlattices subject to crossed magnetic and electric fields

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Abstract

The motion of charge particles in a superlattice structure subject to the crossed magnetic and electric fields, both applied in the in-plane direction, is discussed with respect to the novel type of terahertz oscillations suggested recently, [M. Orlita, R. Grill, L. Smrčka, M. Zva´ra, Phys. Rev. B 74 (2006) 125312]. The effects of the finite size of the superlattice is considered in comparison with the standard Bloch oscillations.

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The external electric field induced motion of charged particles in periodic systems is of an intensive interest many years. A lot of attention has been especially paid to the experimental verification of Bloch oscillations, the phenomenon predicted by Zener in 1930s \cite{1}, which was first found in superlattices (SLs) suggested by Esaki and Tsu \cite{2}. Nowadays, the investigations are aimed to the practical utilization of these oscillations, i.e. to the preparation of efficient tunable emitters of the THz radiation \cite{3,4}.

Recently, a novel type of terahertz oscillations \cite{5} has been suggested for SLs subject to the crossed electric \( F \) and magnetic fields \( B \), both applied in the in-plane direction. The frequency of these oscillations \( \omega_{B_x} = 2\pi v_d / \lambda \), where \( v_d = F / B \) represents the drift velocity, is tunable by both electric and magnetic fields.

To sketch the model suggested in \cite{5}, let us first briefly discuss the energy spectrum of SL with the growth axis oriented along the \( z \)-direction, subject to the in-plane magnetic field \( B = (0, B_z, 0) \) described by the gauge \( A = (B_z z, 0, 0) \). Having the periodic SL potential \( V(z + \lambda) \), where \( \lambda \) stands for the SL period, we can simply write the one-dimensional Hamiltonian describing the motion of an electron in this SL \cite{5}:

\begin{equation}
H = \frac{\hbar^2}{2m} \left( k_x + \frac{|e|B_z}{\hbar} \right)^2 + \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 d^2}{2m \, dz^2} + V(z),
\end{equation}

where parameters \( k_x \) and \( k_y \) denote the components of the electron in-plane momentum. This Hamiltonian is invariant under the following transformation:

\begin{equation}
H(z, k_x) \rightarrow H(z + \lambda, k_x - K_0),
\end{equation}

where \( K_0 = |e|B_1 \lambda / \hbar \). Taking account of this symmetry, we get the \( K_0 \)-periodic dispersion \( E_n(k_x) = E_n(k_x + K_0) \), where the index \( n \) enumerates the Landau subbands, \( n = 1, 2, 3, \ldots \). In the following, we will consider the lowest lying subband \( E_1(k_x) \equiv E(k_x) \) only. Note that we use the term subband instead of a common level to express the fact that degeneracy of the Landau level in \( k_x \) is lifted due to the SL periodic potential.

In this paper, we quasi-classically analyze the motion of electrons in this periodic subband structure, induced by the electric field applied along the \( x \)-direction, taking account of the finite size of a real SL. We do not consider the problem of the Zener-like tunneling between Landau
subbands, as this topic is addressed elsewhere [6]. Instead, we limit ourselves to the statement that the Zener-like tunneling should disappear in the suggested system, as the energy levels in the corresponding ladder represent true eigenstates and not the resonances from which the standard Wannier–Stark ladder consists.

The energy spectrum of an infinite SL subject to no external field is given by the well-known dispersion $E(k_z) = E(k_z + 2\pi / \Lambda)$, where momentum $k_z$ is a continuous parameter taken within the chosen Brillouin zone. Taking into account the finite size of the sample and having thus a limited number of quantum wells (QWs), the Born–von Karman boundary conditions are usually employed. The momentum $k_z$ is then no longer a continuous quantum number, instead its discretization appears. Going further to a few-QWs SL, the energy spectrum looses its quasi-continuous character and a spectrum consisting of isolated energy levels comes up. Experimentally, the number of QWs in SL mostly ranges from 30 to 100 [7,8] to keep the quasi-continuous energy spectrum of the SL.

Apparently different situation is obtained for SL in a finite $B_\parallel$. The periodic dispersion in a real SL remains quasi-continuous even in few-QWs SL, as it is formed due to the anti-crossing behavior of dispersion parabolas, which correspond to different QWs and which are at finite $B_\parallel$ mutually shifted in the reciprocal space. Hence, the dispersion $E(k_z)$ remains in a real SL partly periodic, each of its minimum corresponds to one considered QW. The shape of $E(k_z)$ changes into a parabolic one when the first or the last (edge) QWs are considered.

This situation is depicted in the lower part of Fig. 1. The curves in this figure were calculated numerically within the tight-binding approximation employed already in [5].

The parameters of this model $\Lambda$, $m$ and the tunneling coefficient $t$ are given directly in the figure. The upper part of the figure shows the particle velocity $v_x$ in the $x$-direction. The asymmetric profile of the electron velocity is the result of the non-cosine shape of $E(k_z)$. The inset in Fig. 1 shows the quasi-classical trajectory of an electron undergoing motion in the part of $E(k_z)$ which is marked by a gray-filled area. The corresponding $x$-component of the velocity is selected in the upper part of Fig. 1 by the gray color as well.

The electron, having at given time the momentum $k_x$, starts to oscillate with the frequency $\omega_B$ in the $x$-direction and to move with the drift velocity $v_d$ along the $z$-axis. After two periods of the oscillations, the electron achieves the last QW and its motion changes into an in-plane one. In other words, the electron steady drift along the sample growth direction, which accompanies the oscillations, changes into the motion along $x$-axis. Being pushed by the Lorentzian force into the outer barrier, the electron remains in the last QW and is removed into the electrical contact.

A note should be addressed to the motion of an optically created electron–hole pair, as time-resolved optical experiments appeared to be decisive in the evidence of standard Bloch oscillations [7,8]. Both electric and magnetic parts of the Lorentzian force are sensitive to the charge sign; the holes drift in the same direction as electrons both with the same drift velocity $v_d$. Only the oscillations are of different amplitudes and are shifted in the phase by amount of $\pi$.

To conclude, we addressed in detail the quasi-classical motion of electron in SLs subject to crossed electric and magnetic fields in order to bring the theoretical considerations [5] closer to their experimental realization. Especially, the influence of the finite size of a real SL structure has been addressed, showing the possibility to observe suggested oscillations even in few-QWs SLs.

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References