ADDRESSING THE UNCERTAINTY ASSESSMENT FOR REAL-TIME STAGE FORECASTING
Barbetta S.¹, Brocca L.¹, Melone F.¹, Moramarco T.¹, Singh V.P.²

¹Research Institute for Geo-Hydrological Protection, National Research Council, ²Department of Biological & Agricultural Engineering, and Department of Civil & Environmental Engineering, Texas A & M University

ABSTRACT
A Flood Monitoring and Warning System (FMWS) operating in real time is the main non-structural measure for reducing risk in flood-prone areas. During the last years significant efforts were addressed to improve the reliability of the forecast quantities mainly by assessing the forecast uncertainty in order to avoid the “illusion of certainty” for decision-makers. In this context, the purpose of this study is to assess the forecast uncertainty for a simple stage forecasting model, called STAFOM-RCM, currently operative within the FMWS of the Upper-Middle Tiber River basin (central Italy). Firstly, the model reliability is tested for three river reaches considering several floods and assuming as performance evaluation measures the Nash–Sutcliffe coefficient, the error on peak stage and time to peak and the coefficient of persistence. Then, the estimate of model uncertainty is addressed by determining the 95% confidence band (CB) on the basis of the errors between observed and forecast stages after their Box-Cox transformation. The results of this preliminary analysis show that the method furnishes the 95% CBs including, on the average, 94.6% and 93.8% of observed stages with the Box-Cox transformation factor calibrated through all and half of the selected floods, respectively.

1. INTRODUCTION
Many communities owe much of their prosperity to advantages offered by adjacent streams. Adverse effects, however, occur when significant flood events hit the flooding-prone areas causing loss of lives, destruction of property, loss of agricultural production and disruption of transport and services. In order to reduce these effects, structural measures, such as construction of river banks, flood walls, dams and flood storage areas, can be used. However, these types of measures cannot eliminate the hydraulic risk and, hence, non-structural measures, typically consisting in Flood Monitoring and Warning Systems (FMWSs) to be actuated in real time for flood damage reduction, have also to be implemented. A FMWS requires reliable flood forecasts with a sufficient forecasting horizon and its effect depends, obviously, on the accuracy of the forecast estimate, but also on turning the forecast into a warning which, moreover, has to be properly interpreted. Therefore, one of the fundamental system components is a forecast model able to forecast flows and stages using as input data measurements recorded hours early by the hydro-meteorological network operating on-line. Forecast models aim at reducing the uncertainty on the evolution of future events thus allowing decision-makers to take the most effective decisions under uncertainty (Todini, 2004). This means that, to be properly
integrated within an operational flood forecasting methodology, models not only require to be timely, sufficiently accurate within pre-determined time horizons but also to provide an usable quantification of the forecasting uncertainty (Todini, 2009). This last can be considered as an asset of the forecast (Montanari, 2007) which enables the authorities to set risk-based criteria for flood warning, furnishes information for making rational decisions and offers potential for additional economic benefits of forecasts to decision-makers (Krzysztofowicz, 2001).

Mathematical models applied for forecast purpose generally belong to either conceptual or black-box approaches. For medium-sized catchments, conceptual rainfall-runoff models of semi-distributed type seem to be most reliable for operational activities provided that an updating procedure and rainfall forecasts are adopted. To reduce the complexity of the forecasting system, simplified models based only on flood routing can be used for flood-prone sites located downstream of gauged river sections. In this context, Franchini and Lamberti (1994) proposed a simple stage forecasting model, of Muskingum type, which has been enhanced by Moramarco et al. (2006) through a procedure for adapting in real time the parameter linked to lateral inflows. This last model, called STAFOM (STAge FOrcasting Model), provided accurate results for most of the analysed flood events observed in different river reaches of the Upper-Middle Tiber River basin, in central Italy. However, a thorough analysis of the results showed that the forecast stage should be enhanced when sudden modifications affect the upstream and downstream hydrographs recorded in real-time. Therefore, the original model was improved by coupling it with a procedure based on the Rating Curve Model (RCM) able to relate local stage and remote discharge also with significant lateral inflows (Moramarco et al., 2005). The modified model (STAFOM-RCM) was tested for the reaches of the Tiber River (Barbetta et al., 2010). On average, a significant improvement in the forecast reliability was observed, particularly the peak prediction and mainly for long river channels. However, the approximation with which the flood event is described along with the parameters value and the input data introduce a degree of uncertainty that has to be quantitatively evaluated to support decision-makers.

This study mainly aims to address the issue of forecast stage uncertainty by defining confidence bands (CBs) on the basis of the errors between observed and forecast variables after their transformation (Box and Cox, 1964), as proposed by Misirli et al. (2003).

2. THE STAFOM-RCM MODEL

For the sake of completeness, the main theoretical background of the stage forecasting model is briefly described in what follows.

The STAFOM model formulation for stage forecasting is provided by:

\[ h_d(t_f + \Delta t^*) = \left( \frac{t_f}{\xi} \right) \left[ C_i (Q_u(t_f) + q_i(t_f) L + \gamma L) + C_2^* \xi h_d^\delta(t_f) \right]^{1/\delta} \]  

(1)

where \( h_d \) is the downstream stage; \( C_i^* \) and \( C_2^* = 1 - C_i^* \) are the Muskingum parameters depending on the Muskingum coefficients \( K \) and \( \theta \) and to the forecasting lead-time \( \Delta t^* = 2K\theta \); \( t_f \) is the time of forecast; \( \xi \) and \( \delta \) are the parameters of the downstream rating curve; \( q_i \) is lateral contribution per unit channel length and \( L \) is the total reach length.
length (Moramarco et al., 2006); γ is an additive error for q which is determined through the continuity equation in the characteristic form (Moramarco et al., 2005):

\[(A_d(t) - A_u(t - T_L)) / T_L = q_l\]  

(2)

with \(T_L\) wave travel time (equal to \(\Delta t^*\)); \(A_u\) and \(A_d\) upstream and downstream flow area, respectively. Eq. (1) requires the assessment of all the involved parameters. γ is updated through an automatic procedure based on the last stages acquired in real time. \(\xi\) and \(\delta\) are inferred through the downstream rating curve. \(K\) is evaluated as the mean wave travel time of the branch estimated through different flood events (\(K = T_L\)) and is assumed as the forecasting lead-time, \(\Delta t^*\), which implies a value of 0.5 for \(\theta\). The STAFOM model is coupled with the Rating Curve Model (RCM) able to reconstruct the discharge hydrograph at a river site where only the stage is monitored whereas the discharge is known at another section (Moramarco et al., 2005):

\[Q_d(t) = \alpha [A_d(t) / A_u(t - T_L)]Q_u(t - T_L) + \beta = \alpha X + \beta\]  

(3)

\(\alpha\) and \(\beta\) are the model parameters which were found almost constant from one flood event to another, as well as \(T_L\). Therefore, these quantities are assumed as characteristics of the investigated river reach. Eq. (3) provides a very strong information, that is each flood along the branch lies on the identified linear relationship.

The STAFOM-RCM model consists of the following procedure (Barbetta et al., 2010): i) at each time of forecast, \(t_f\), STAFOM provides the future estimate of the downstream stage, \(h_d\), at time \((t_f + \Delta t^*)\) through the application of Eq. (1); ii) the downstream flow area corresponding to \(h_d(t_f + \Delta t^*)\), \(A_d(t_f + \Delta t^*)\), is computed using the stage-area relationship derived from the cross-section geometry; iii) the term \(X = [A_d(t_f + \Delta t^*) / A_u(t_f)]Q_u(t_f)\) is easily computed depending on the observed quantities, \(A_u(t_f)\) and \(Q_u(t_f)\), and on \(A_d(t_f + \Delta t^*)\) estimated in step ii); iv) with \(\alpha\) and \(\beta\) known, Eq. (3) provides the corrected forecast discharge; the corrected forecast stage is then derived through the downstream rating curve.

3. FORECAST UNCERTAINTY ESTIMATE

The issue of the uncertainty estimate for the forecast stage is here addressed by defining the CB on the basis of the errors between the observed stage, \(h_{\text{obs}}(t)\) at the time \(t\), and the corresponding stage, \(h_{\text{for}}(t)\), forecast \(\Delta t^*\) hours in advance. In particular, the approach proposed by Misirli et al. (2003) is adopted which involves the following steps: i) for each instant in time, the transformed values of the observed and forecast stages \((z_{\text{obs}}(t)\) and \(z_{\text{for}}(t)\), respectively) are calculated through the Box-Cox transformation (Box and Cox; 1964):

\[z_{\text{obs}}(t) = \left[\log(h_{\text{obs}}(t) + 1)\right]^\lambda - 1 / \lambda; \quad z_{\text{for}}(t) = \left[\log(h_{\text{for}}(t) + 1)\right]^\lambda - 1 / \lambda \quad \lambda \neq 0\]  

\[z_{\text{obs}}(t) = \log(h_{\text{obs}}(t) + 1); \quad z_{\text{for}}(t) = \log(h_{\text{for}}(t) + 1) \quad \lambda = 0\]  

(4)

ii) the error \(\varepsilon(t)\) between the observed and forecast values on the transformed plane is calculated:
\( \varepsilon(t) = z_{\text{obs}}(t) - z_{\text{for}}(t) \) \hspace{1cm} (5)

(iii) through the errors calculated for all the events of the calibration set the standard deviation, \( \sigma_{\varepsilon} \), is computed;

(iv) for the assigned confidence level \( \alpha \) of the transformed variable \( z \), the end points of the CB are estimated as:

\( z_{\text{for}}^+(t) = z_{\text{for}}(t) + u_{\alpha/2} \sigma_{\varepsilon} \) and \( z_{\text{for}}^-(t) = z_{\text{for}}(t) - u_{\alpha/2} \sigma_{\varepsilon} \)

where \( u_{\alpha/2} \) is the standard normal variable associated with a probability of exceedance equal to \( \alpha/2 \);

(v) the end points of the CB on the natural plane are finally derived:

\[
\begin{align*}
    h_{\text{for}}^+(t) &= \left[ (z_{\text{for}}^+(t) \cdot \lambda + 1)^\frac{1}{\lambda} - 1 \right], \\
    h_{\text{for}}^-(t) &= \left[ (z_{\text{for}}^-(t) \cdot \lambda + 1)^\frac{1}{\lambda} - 1 \right]
\end{align*}
\] \hspace{1cm} (6)

The steps i)-v) are repeated for different values of \( \lambda \) and that minimizing the efficiency criterion Forecast Range Error Estimate (FREE) proposed by Misirli et al. (2003) is selected.

### 3.1 Forecast Range Error Estimate (FREE) criterion

The efficiency criterion Forecast Range Error Estimate (FREE) (Misirli et al.; 2003) summarizes the model performance in terms of both the inclusion of the observed data (desirable as large as possible) and the width of the CB (to be as small as possible while maximizing inclusion). FREE is the sum of the absolute values of two distances: FREE_POS and FREE_NEG:

\[
\begin{align*}
    \text{FREE}_{\text{NEG}} &= \left| \text{sum of negative dist} \right| / \text{number of negative dist} \hspace{1cm} (7) \\
    \text{FREE}_{\text{POS}} &= \text{sum of positive dist} / \text{number of positive dist} \hspace{1cm} (8) \\
    \text{FREE} &= \text{FREE}_{\text{POS}} + \text{FREE}_{\text{NEG}} \hspace{1cm} (9)
\end{align*}
\]

\[
\text{dist}^{(t)}(j) = \begin{cases} 
    h_{\text{for}}^+(t) - h_{\text{obs}}^+(t), & (h_{\text{obs}}^+(t) - h_{\text{for}}^+(t)) \geq 0 \\
    h_{\text{obs}}^+(t) - h_{\text{for}}^+(t), & (h_{\text{obs}}^+(t) - h_{\text{for}}^+(t)) < 0 \\
    h_{\text{obs}}^-(t) - h_{\text{for}}^-(t), & (h_{\text{obs}}^-(t) - h_{\text{for}}^-(t)) \geq 0 \\
    h_{\text{for}}^-(t) - h_{\text{obs}}^-(t), & (h_{\text{obs}}^-(t) - h_{\text{for}}^-(t)) < 0 
\end{cases}
\] \hspace{1cm} (10)

where \( n_j \) = total number of forecast times of the event \( j \); \( M \) = number of events of the calibration set.

A positive deviation, FREE_POS, refers to the distance to the CB if the observed value is within the confidence band and a negative deviation, FREE_NEG, refers to the same distance if the point is outside the band. Smaller FREE_NEG means that more points are within the CB; the FREE-POS is desired to be smaller too meaning band not too wide while containing data.

### 4. RESULTS AND DISCUSSION

Three river reaches were selected along the Tiber River, in central Italy, bounded by Pierantonio, P.te Felcino and P.te Nuovo gauged sections at upstream end and M.te Molino equipped station at downstream end. The main properties of the investigated river reaches are summarized in Table 1 from which it can be inferred that significant intermediate drainage areas are present. The reliability of the CBs for the STAFOM-RCM model was tested through 16 floods whose details are summarized in Table 2. The variation range and the mean of the percentage lateral inflow contribution on the downstream flood formation are shown in Table 3 along with the optimal forecasting.
model parameters. As can be see, the observed wave travel times of the selected floods allowed to assume a forecasting lead-time equal to 9, 7 and 4 hours for the three river reaches, respectively.

### Table 1. Main properties of the selected river reaches: area subtended by the upstream, $DA_u$, and downstream section, $DA_d$; intermediate drainage area, $DA_{int}$; channel length, $L$.

<table>
<thead>
<tr>
<th>River reach</th>
<th>$DA_u$ (km²)</th>
<th>$DA_d$ (km²)</th>
<th>$DA_{int}$ (km²)</th>
<th>$L$ (km)</th>
<th>$S_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pierantonio-M.te Molino</td>
<td>1805</td>
<td>5279</td>
<td>3474</td>
<td>71.06</td>
<td>0.0011</td>
</tr>
<tr>
<td>P.te Felcino- M.te Molino</td>
<td>2035</td>
<td>5279</td>
<td>3244</td>
<td>56.22</td>
<td>0.001</td>
</tr>
<tr>
<td>P.te Nuovo- M.te Molino</td>
<td>4145</td>
<td>5279</td>
<td>1134</td>
<td>30.83</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

### Table 2. Main characteristics of the flood events: peak stage, $h_p$; peak discharge, $Q_p$.

<table>
<thead>
<tr>
<th>Event</th>
<th>Pierantonio</th>
<th>P.te Felcino</th>
<th>P.te Nuovo</th>
<th>M.te Molino</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 1996</td>
<td>4.34 380.5</td>
<td>4.22 364.6</td>
<td>4.63 728.4</td>
<td>6.22 703.6</td>
</tr>
<tr>
<td>Apr 20, 1997</td>
<td>4.67 429.4</td>
<td>4.57 445.6</td>
<td>5.16 502.1</td>
<td>5.19 525.1</td>
</tr>
<tr>
<td>Jan 19, 1998</td>
<td>3.14 223.1</td>
<td>3.23 225.3</td>
<td>3.83 307.6</td>
<td>3.62 299.0</td>
</tr>
<tr>
<td>Feb 9, 1999</td>
<td>4.66 427.9</td>
<td>4.52 436.6</td>
<td>6.61 763.8</td>
<td>6.39 735.3</td>
</tr>
<tr>
<td>Dec 25, 2000</td>
<td>5.52 565.9</td>
<td>5.25 365.8</td>
<td>7.17 879.1</td>
<td>6.80 808.3</td>
</tr>
<tr>
<td>Apr 7, 2001</td>
<td>3.28 239.7</td>
<td>3.23 225.3</td>
<td>3.78 301.1</td>
<td>3.50 284.1</td>
</tr>
<tr>
<td>Nov 25, 2005</td>
<td>6.70 779.0</td>
<td>6.92 958.0</td>
<td>8.28 1131.4</td>
<td>9.00 1192.6</td>
</tr>
<tr>
<td>Dec 30, 2005</td>
<td>4.59 417.4</td>
<td>4.34 405.1</td>
<td>6.10 665.7</td>
<td>6.37 731.6</td>
</tr>
<tr>
<td>Mar 5, 2006</td>
<td>2.89 194.5</td>
<td>3.00 193.9</td>
<td>3.63 278.0</td>
<td>4.31 391.5</td>
</tr>
<tr>
<td>March 13, 2007</td>
<td>1.84 92.8</td>
<td>2.13 96.7</td>
<td>2.88 173.1</td>
<td>2.62 180.7</td>
</tr>
<tr>
<td>Mar 21, 2008</td>
<td>1.62 77.6</td>
<td>1.94 78.5</td>
<td>3.10 202.9</td>
<td>3.28 258.5</td>
</tr>
<tr>
<td>Dec 5, 2008</td>
<td>5.13 501.4</td>
<td>3.60 310.9</td>
<td>5.28 521.7</td>
<td>5.28 539.7</td>
</tr>
<tr>
<td>Dec 11, 2008</td>
<td>6.51 742.9</td>
<td>4.98 542.5</td>
<td>7.29 904.9</td>
<td>8.20 1052.7</td>
</tr>
<tr>
<td>Feb 6, 2009</td>
<td>4.26 369.0</td>
<td>3.19 246.8</td>
<td>3.91 319.4</td>
<td>4.00 348.6</td>
</tr>
<tr>
<td>Nov 21, 2010</td>
<td>6.30 703.8</td>
<td>4.84 518.7</td>
<td>5.48 555.2</td>
<td>5.97 658.1</td>
</tr>
<tr>
<td>Nov 28, 2010</td>
<td>5.08 493.4</td>
<td>4.75 503.4</td>
<td>6.71 783.8</td>
<td>7.87 995.1</td>
</tr>
</tbody>
</table>

### Table 3. Forecasting model parameters ($\xi$ and $\delta$=downstream rating curve parameters; $K$ and $\theta$=Muskingum parameters; $\Delta t^*$=forecasting lead time; $\alpha$ and $\beta$= RCM model parameters).

<table>
<thead>
<tr>
<th>River reach</th>
<th>$\xi$</th>
<th>$\delta$</th>
<th>$K$ (h)</th>
<th>$\theta$</th>
<th>$\Delta t^*$ (h)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$T_L$ (h)</th>
<th>Lateral Inflow (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pierantonio-M.te Molino</td>
<td>36.04</td>
<td>1.63</td>
<td>9.0</td>
<td>5.0</td>
<td>1.003</td>
<td>25</td>
<td>2.0±11.5</td>
<td>(m=8.4)</td>
<td>25.3±76 (m=51)</td>
</tr>
<tr>
<td>P.te Felcino- M.te Molino</td>
<td>7.0</td>
<td>0.92</td>
<td>18</td>
<td>9.0</td>
<td>1.082</td>
<td>-27</td>
<td>0.5±5.0</td>
<td>(m=3.7)</td>
<td>1.4±30.7 (m=13)</td>
</tr>
<tr>
<td>P.te Nuovo- M.te Molino</td>
<td>4.0</td>
<td>9.0</td>
<td>-27</td>
<td>4.0</td>
<td>6.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The accuracy of the STAFOM-RCM model can be deduced by the results summarized in Table 4, showing the variation range and the mean absolute value of the error on peak stage, $e_{hp}$, error on time to peak, $e_{tp}$, coefficient of persistence for the rising limb, $PC_r$, ($Kitanidis and Bras, 1980$) and Nash-Sutcliffe coefficient, NS ($Nash and Sutcliffe, 1970$). In particular, for Pierantonio-M.te Molino reach the model provided satisfactory forecasts with $e_{hp}$ nearly always lower than 10% (mean absolute value equal to 0.38 m) and a mean absolute value of $e_{tp}$ equal to 2.3 hours. Moreover, the $PC_r$ and NS coefficients, with a mean value equal to 82.2 and 86.2,
respectively, suggest the usefulness of the forecasts provided 9 hours in advance. Referring to P.te Felcino- M.te Molino reach, the model provided good results with the error on peak stage nearly always lower than 10% (mean absolute value equal to 0.25 m) and the mean absolute $e_{hp}$ equal to 2.2 hours. Also in this case, $PC_r$ and NS coefficients, with a mean equal to 80.8 and 88.4, respectively, show a satisfactory accuracy of the model with forecast stages provided 7 hours in advance. Finally, the model was found more reliable for P.te Nuovo- M.te Molino reach. Specifically, $e_{hp}$ was always lower than 8% (mean absolute equal to 0.19 m), $e_{tp}$ was, on the average, equal to 1.8 hours and the mean value of $PC_r$ and NS coefficients was equal to 79.2 and 94.8, respectively.

Table 4. Variation range and mean absolute value, $m$, of the forecasting model results (error on peak stage, $e_{hp}$; error on time to peak, $e_{tp}$; coefficient of persistence for rising limb, $PC_r$; Nash-Sutcliffe coefficient, NS).

<table>
<thead>
<tr>
<th>River reach</th>
<th>$T_1$ (h)</th>
<th>$e_{hp}$ (%)</th>
<th>$e_{tp}$ (h)</th>
<th>$PC_r$ (%)</th>
<th>NS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pierantonio- M.te Molino</td>
<td>2.0±11.5 (m=8.4)</td>
<td>-16.0±16.0 (m=6.6)</td>
<td>-8.0±5.5 (m=2.3)</td>
<td>22.6±97.9 (m=82.2)</td>
<td>43.2±96.9 (m=86.3)</td>
</tr>
<tr>
<td>P.te Felcino-M.te Molino (Δt=7 h)</td>
<td>3.0±9.0 (m=6.7)</td>
<td>-12.6±11.1 (m=4.5)</td>
<td>-7.0±2.5 (m=2.2)</td>
<td>43.8±97.2 (m=80.8)</td>
<td>72.9±97.4 (m=88.4)</td>
</tr>
<tr>
<td>P.te Nuovo-M.te Molino (Δt=4 h)</td>
<td>0.5±5.0 (m=3.7)</td>
<td>-8.0±7.3 (m=3.2)</td>
<td>-6.0±1.0 (m=1.8)</td>
<td>51.2±96.8 (m=79.2)</td>
<td>89.7±98.7 (m=94.8)</td>
</tr>
</tbody>
</table>

4.1 Confidence band (CB)

As concerns the confidence band, the Misirli et al. (2003) approach was applied for 95% CB estimate. The transformation factor, $\lambda$, was calibrated by considering two different events sets: the first one refers to all the 16 selected floods and the second one to the first 8 events (see Table 2). In the latter case, the remaining 8 events were used for CB validation. The variation of FREE and its positive and negative components with $\lambda$ is shown in Figure 1 for P.te Felcino-M.te Molino reach. According to Misirli et al. (2003), the obtained results suggested to assess the transformation factor by minimizing FREE and FREE_POS.

Table 5. Range of optimal $\lambda$ factors and corresponding standard deviation, $\sigma_\lambda$, assessed with the FREE criterion.

<table>
<thead>
<tr>
<th>River reach</th>
<th>$\lambda$</th>
<th>$\sigma_\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pierantonio-M.te Molino</td>
<td>0.4±0.6</td>
<td>0.4±0.6</td>
</tr>
<tr>
<td>P.te Felcino-M.te Molino</td>
<td>0.5</td>
<td>0.1±0.4</td>
</tr>
<tr>
<td>P.te Nuovo-M.te Molino</td>
<td>0.2±0.4</td>
<td>0.1±0.3</td>
</tr>
</tbody>
</table>

Table 5 summarizes the calibrated transformation factors. As can be see, for the first case study the range of optimal $\lambda$ assessed by considering 16 and 8 events are the same, with lower standard deviation for 8 events analysis depending on the higher accuracy of the forecast obtained by STAFOM-RCM for these events. For P.te Felcino-M.te Molino reach, the factor calibrated by considering 16 events is 0.5, whereas the analysis with 8 events identified a range between 0.1 and 0.4. Finally, for the third reach the variability range of $\lambda$ referring to 16 and 8 events is quite similar. The analysis showed that the percentage of observed stages within the
CB is, on the average, near 95% for \( \lambda \) and the corresponding \( \sigma \) assessed with both 16 and 8 events (see Table 6). As concerns the validation events, the mean percentage of observed points within the confidence bounds was found lower 95%, but nearly always greater than 90%. As the width of CBs increases by decreasing \( \lambda \), all the optimal transformation factors were analyzed in order to select the minimum width of the CB while maximizing the inclusion of observed data.

Figure 1. P.te Felcino-M.te Molino reach: FREE measure and its components for different values of \( \lambda \) considering: a) 16 calibration events and b) 8 calibration events.

Figure 2. Pierantonio-M.te Molino reach: frequency of events with the percentage of the observed stages within the 95% CB determined through 16 and 8 calibration events for a) \( \lambda=0.4 \); b) \( \lambda=0.6 \).

Table 6. Percentage of observed stages within the estimated 95% CB.

<table>
<thead>
<tr>
<th>( \lambda ) (16)</th>
<th>( \lambda ) (8)</th>
<th>( \lambda ) (16)</th>
<th>( \lambda ) (8)</th>
<th>( \lambda ) (16)</th>
<th>( \lambda ) (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

A=mean for calibration events; B=mean for validation events

For each river reach and each optimal \( \lambda \), the 95% CBs were then verified by analyzing the number of observed stages within the band itself. For a fixed \( \lambda \) factor the CBs assessed by considering all the 16 floods were found more reliable than the ones assessed with only 8 calibration events. Obviously, this can be expected because standard deviation generally increases by increasing the number of
calibration events, as can be seen in Figure 2 showing the results for Pierantonio-M.te Molino reach. However, this produces wider CBs as can be seen in Figure 4 where the comparison between the observed and forecast stages along with the estimated CBs is shown for four events.

![Figure 3](image1)

**Figure 3.** As Figure 2, but for a) P.te Felcino-M.te Molino reach and b) P.te Nuovo-M.te Molino reach.

![Figure 4](image2)

**Figure 4.** Pierantonio-M.te Molino reach: comparison between observed and forecast stages for a) February 9, 99; b) November 25, 05; c) December 5, 08 and d) December 11, 08. The 95% CBs computed with λ=0.6 and the input hydrograph are also shown.

Figure 3 shows the frequency of events with the percentage of the observed stages within the 95% CB for P.te Felcino-M.te Molino and P.te Nuovo-M.te Molino reaches. It has to be underlying that selecting the λ value corresponding to the more
narrow CBs identified with 16 and 8 events, the CBs themselves were found very similar (Figures 5 and 6).

**Figure 5.** As Figure 4, but for P.te Felcino-M.te Molino reach and for a) December 30, 05; b) November 21, 10. (λ=0.5 and 0.4 for 16 and 8 events analysis, respectively).

**Figure 6.** As Figure 4, but for P.te Nuovo-M.te Molino reach and for a) April 7, 01 and b) December 5, 08 (λ=0.4 and 0.3 for 16 and 8 events analysis, respectively).

### 5. CONCLUSIONS

The uncertainty of the forecast stages provided by a simple model, involving a correction procedure based on a methodology able to relate local stage and remote discharge along natural channels, was addressed in this study. The model accuracy was verified for three river reaches along the Tiber River, in central Italy, considering several flood events. In particular, the issue of the forecasting uncertainty assessment was addressed by defining the 95% CB through the approach proposed by Misirli et al. (2003) based on the errors between observed and forecast variables. The analysis was carried out by estimating the Box-Cox transformation factor through both all the 16 selected events and only 8 calibration floods. The results show that the identified CBs contain, on the average, a percentage of observed stages near to 95%, even for the validation events. The results provided by this preliminary analysis suggest that the method used for the CB assessment could be reliable for real-time uncertainty estimate although the width of the CB is quite large.
The application and the comparison of different methodologies proposed in the
literature it will be carrying forward.

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