Low-Complexity Fractional Turbo Receiver for Space-Time BICM over Frequency-Selective MIMO Fading Channels

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Abstract—This paper is devoted to wireless communications over frequency-selective and fading multiple-input multiple-output (MIMO) channels. Space-time bit-interleaved coded modulation (STBICM) is considered. The receiver is of the turbo type. A soft-input soft-output (SISO) space-time equalizer is designed for the minimum mean squared error criterion (MMSE). The main features of this equalizer is that it is filter based, of low complexity and of the fractionally-spaced type. This equalizer followed by a soft demapper, exchanges information with the SISO decoder. Simulation results are reported, which show the effectiveness of the turbo receiver, its ability to account for inter-symbol interference, co-antenna interference and its capability to exploit transmit, multipath and receive diversity.

I. INTRODUCTION

MIMO communications systems are currently receiving a lot of attention. It has been shown that MIMO channels offer increased outage capacity and are useful to combat fading and to achieve high bit rate wireless transmission. Most of the work reported in the literature has been devoted to flat fading MIMO systems. However, when high bit rate is targeted, the MIMO channel may become frequency selective. Such channels will be considered in the present contribution.

When using a frequency-selective MIMO communications system, a number of issues have to be addressed. First of all, to benefit from transmit diversity, appropriate space-time (ST) coding needs to be used. Second, a receiver structure has to be proposed which is able to handle the inter-symbol interference (ISI), the co-antenna interference (CAI) and the receive-diversity capability. Regarding transmit diversity, we consider an approach based on bit interleaving proposed by [1] and named space-time bit-interleaved coded modulation (STBICM) in [2]. The information bits are encoded by means of a convolutional coder. The coded bits are interleaved and distributed to the different antennas where they are mapped on complex symbols and transmitted. The receiver is turbo based. It jointly performs turbo space-time equalization to counteract ISI and CAI, and turbo demodulation.

Most of the turbo detectors are based on optimal a posteriori probability evaluation and are thus trellis based. This leads to excellent error rate performance but the detector complexity is huge and basically prohibitive if multiple antennas and/or multilevel/phase modulations are used. Less complex solutions have been proposed, like the max-log-MAP detector ([2]) or reduced-state approaches. Further complexity reduction is possible using filter-based (FB) solutions. Ariyavisitakul has used a low-complexity FB detector in [1] but this detector does not fully exploit the available a priori information. In the CDMA context, Wang and Poor have proposed in [3] a multiuser FB detector that can be shown to be the exact MMSE solution. In the current paper, a low-complexity FB solution will be considered. It is based on the approached MMSE solution proposed by Tüchler et al. in [4] for single-input single-output channels. A second feature of the FB solution proposed here is that it is of the fractionally-spaced (FS) type. Usually, turbo receivers are applied after matched filtering and noise whitening. Instead, the proposed turbo receiver can be applied on the received signal sampled at a higher rate after appropriate bandlimiting filtering. In short, the solution put forward in the current paper can be viewed as an extension to FS equalization and to multilevel MIMO transmission over frequency-selective channels, of the low-complexity FB scheme proposed in [1]. It can also be seen as a simplified version of the FB MMSE solution proposed by Lu and Wang in [5] for ST block codes (STBC) and ST trellis codes (STTC). The effectiveness of the solution will be demonstrated thanks to simulations performed for multipath MIMO channels affected by Rayleigh slow fading.

II. TRANSMITTER MODEL

We focus in this paper on a MIMO wireless communications system with \(n_T\) antennas at the transmitter and \(n_R\) antennas at the receiver. Although only one user is considered, the proposed scheme can easily be extended to a multiuser scenario if space division multiple access (SDMA) is used. The transmitter structure considered in this paper has been called STBICM in [2]. To our knowledge, this scheme was first proposed in [1].

The considered transmitter is depicted in Fig.1. The information bits are organized in frames. A frame of information bits \(u_k\) is first encoded by a rate-\(r\) convolutional encoder. The coded bits are then interleaved by a random permutation. The frame of interleaved coded bits is then split into \(n_T\) sub-blocks (corresponding to the \(n_T\) transmit antennas). Within each of these sub-blocks, the bits are grouped and mapped to one of the \(M\) possible complex symbols in the considered multilevel/phase constellation \(S\) (e.g. M-PSK, M-QAM,...). On each transmit antenna \(i\) \((i = 1, \ldots, n_T)\), the resulting length-\(L_s\) frame of complex symbols \(s_{k}^{(i)} \in S (k = 0, \ldots, L_s - 1)\)

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is transmitted over the wireless channel. The symbols are zero mean and have variance $\sigma_i^2$. After pulse shaping with a square-root Nyquist filter $g(t)$, the symbols $s_k^{(1)}, \ldots, s_k^{(nr)}$ are sent at time $kt$, where $T$ is the symbol period.

The key role of the coder and the interleaver has to be emphasized. First, the interleaver greatly reduces the correlation between successive coded bits. Together with the coder, it enables the use of the turbo principle at the receiver. Second, the system benefits from transmit diversity because the coding is made before the frame is split between the antennas. So there is no need to use STBC or STTC to take advantage of transmit diversity.

Since we assume that the transmitter does not have any knowledge of the channel impulse responses, the antennas send signals with identical powers. They also use the same shaping filter and the same modulation and mapping rule.

### III. CHANNEL MODEL

We consider frequency-selective MIMO fading channels modeled as tapped-delay lines. The different paths are characterized by gains (independent zero-mean complex gaussian random variables with tap-specific variances) and delays. We assume quasi-static Rayleigh fading, which means that the channel taps remain constant over a frame of transmitted symbols (which is a reasonable assumption for relatively small frames). From one frame to another, the channel taps are assumed to vary independently.

The lowpass equivalent channel impulse response (including the shaping filter $g(t)$ and the physical multipath channel) between transmit antenna $i$ ($i = 1, \ldots, n_T$) and receive antenna $j$ ($j = 1, \ldots, n_R$) is denoted by $h_{i,j}(t)$. We assume $h_{i,j}(t)$ can be truncated without loss of accuracy. We only keep its values $\forall i, \forall j$ from $-L_1T$ to $L_2T$. At receive antenna $j$, $r_{i,j}(t)$ denotes the received signal and $n_{i,j}(t)$ is the complex envelope of an additive white gaussian noise with two-sided power spectral density (psd) $N_0/2$.

We use fractional sampling at the receiver. This allows us to avoid the expensive implementation of the matched filter and of the noise-whitening filter required in the symbol-spaced receiver. Samples are taken at rate $M_s/T$ after low-pass filtering. For $l = -\infty, \ldots, +\infty$ and $m = 0, \ldots, M_s - 1$, the samples $r_{i,m}^{(j)} = r_{i,j}(IT + mT/M_s)$ are sufficient statistics. We also define $h_{k,m}^{(i,j)} = h_{i,j}(kT + mT/M_s)$ for $k = -L_1, \ldots, L_2$ and $n_{i,m}^{(j)} = n_{i,j}(IT + mT/M_s)$.

A compact representation may be obtained if we denote by $\mathbf{s}_k$ (for $k = 0, \ldots, L_R - 1$), the vector of transmitted symbols and by $\mathbf{r}_l$ (resp. $\mathbf{n}_l$) (for $l = -L_1, \ldots, L_2 + L_R - 1$) the vectors of received samples (resp. noise samples):

$$\mathbf{s}_k \triangleq [s_k^{(1)}, \ldots, s_k^{(nr)}]^T (n_R \times 1),$$

$$\mathbf{r}_l \triangleq [r_l^{(1)}, \ldots, r_l^{(nR)}, r_l^{(nR+1)}, \ldots, r_l^{(nR+nL-1)}]^T (n_R M_s \times 1),$$

$$\mathbf{n}_l \triangleq [n_l^{(1)}, \ldots, n_l^{(nR)}, n_l^{(nR+1)}, \ldots, n_l^{(nR+nL-1)}]^T (n_R M_s \times 1).$$

The observation model then becomes:

$$\mathbf{r}_l = \sum_{p=-L_1}^{L_2} H_{p,l} \mathbf{z}_{l-p} + \mathbf{n}_l,$$

where

$$H_{p,l} = \begin{bmatrix}
  h_{1,0}^{(1,1)} & \cdots & h_{1,0}^{(nR,1)} \\
  \vdots & \ddots & \vdots \\
  h_{n_R,0}^{(1,1)} & \cdots & h_{n_R,0}^{(nR,1)} \\
  \vdots & \ddots & \vdots \\
  h_{1,M_s-1}^{(1,n_R)} & \cdots & h_{1,M_s-1}^{(nR,n_R)} \\
  \vdots & \ddots & \vdots \\
  h_{n_R,M_s-1}^{(1,n_R)} & \cdots & h_{n_R,M_s-1}^{(nR,n_R)}
\end{bmatrix} (n_R M_s \times n_R).$$

### IV. ITERATIVE RECEIVER

The receiver is iterative and makes use of the turbo principle. For the sake of simplicity, we assume perfect channel knowledge and perfect synchronization. As represented in Fig. 2, the receiver is made of the association of two SISO stages, separated by bit (de)interleavers, exchanging extrinsic information under the form of bit log-likelihood ratio’s (LLR’s).

The SISO binary decoder is the classical outer SISO stage found in turbo receivers built in analogy with the decoding of a serial concatenated turbo code. It is classically implemented using an a posteriori probability (APP) algorithm in the logarithmic domain, based on the BCJR algorithm [6].

The inner SISO stage has to mitigate intersymbol interference (ISI), to cancel co-antenna interference (CAI) and to properly demodulate the symbols. These first two tasks are achieved by the space-time equalizer. Assuming independence of the interleaved and demultiplexed coded bits at the transmitter, space-time equalization and demodulation may be split without
the definition of the following length-
will derive in the sequel a low-complexity solution based on
ation, and of the number of transmit and receive antennas. We
viously, this becomes rapidly intractable with the increase of
stion, the a priori information

The symbol a priori probabilities can be computed from the available a priori bit LLR’s. On the basis of the symbol extrinsic
sic probabilities obtained at the output of the SISO space-time
equalizer, the demapper (demodulator) can optimally compute
the extrinsic LLR’s of the coded bits. The details of these op-
loss of optimality, as represented in Fig.2. On the basis of the received samples defined in section III and of the symbols a pri-
ori probabilities \( P_a(s_k^{(i)}) \) (\( i = 1, \ldots, n_T; k = 0, \ldots, L_s - 1 \)), a SISO ST equalizer outputs the symbol extrinsic probabilities
\( P_e(s_k^{(i)}) \triangleq \kappa_s P_p(s_k^{(i)}) / P_a(s_k^{(i)}) \), where \( P_p(s_k^{(i)}) \) are the APP of symbol \( s_k^{(i)} \) and \( \kappa_s \) is a normalization constant.

A BCJR-based implementation of the space-time equalizer optimally required would lead to a very high complexity. Ob-
viously, this becomes rapidly intractable with the increase of the channel length, of the number of symbols in the constellation, and of the number of transmit and receive antennas. We will derive in the sequel a low-complexity solution based on
filters rather than on a trellis.

Introducing the parameter \( N \triangleq N_1 + N_2 + 1 \), we begin by the definition of the following length-\( N \) sliding-window model of the received signal (4):

\[
x_k = H s_k + n_k,
\]

with:

\[
s_k \triangleq [s_k^{T} L_2 - N_1 \cdots s_k^{T} L_N - N_2]^{T} (n_T(N+L-1)\times 1),
\]

\[
R_k \triangleq [L_1 - N_1 \cdots L_1 - N_2]^{T} (n_T n_R M_s \times 1),
\]

\[
n_k \triangleq [n_k^{T} L_2 - N_1 \cdots n_k^{T} L_N - N_2]^{T} (n_T n_R M_s \times 1),
\]

and the channel matrix \( H \) (of size \( n_R M_s \times n_T(N + L - 1) \)) defined as:

\[
H \triangleq \begin{bmatrix}
H_{L_2} & \cdots & H_{L_N - L_1} & 0 & \cdots & 0 \\
0 & H_{L_2} & \cdots & H_{L_1} & \cdots & \ddots \\
0 & \cdots & 0 & H_{L_2} & \cdots & H_{L_N - L_1}
\end{bmatrix}.
\]

In this context, a linear filter is used in order to produce an es-
imitted symbol \( s_k^{(i)} \) cannot be used in order to compute its es-
timate \( \hat{s}_k^{(i)} \), so that the extrinsic probabilities \( P_e(s_k^{(i)}) \) approx-
imated on the basis of \( \hat{s}_k^{(i)} \) will not depend on \( P_a(s_k^{(i)}) \).

Respecting this constraint, we search the optimal solution accord-
ning to the MMSE criterion (i.e. minimizing the mean squared
error \( E[(\hat{s}_k^{(i)} - s_k^{(i)})^2] \)). For the sake of conciseness, we re-
fer the reader to [7] for a complete derivation of what follows.

The equalizer structure derived here is an extension to a MIMO context of the results reported in [3], [4] and [7].

The solution makes use of the symbol mean \( S_k^{(i)} \) and variance \( V_k^{(i)} \), which can be estimated on the basis of the available a priori
information (see [7]). We define the length-\( n_T(N + L - 1) \) vector \( \mathbf{S}_k^{(i)} \) and the square matrix \( \mathbf{R}_{ss,k}^{(i)} \) of size \( n_T(N + L - 1) \):

\[
\mathbf{S}_k^{(i)} \triangleq [s_k^{(i)} L_2 - N_1 \cdots s_k^{(i)} L_N - N_2]^{T},
\]

\[
\mathbf{R}_{ss,k}^{(i)} \triangleq \text{diag}[V_k^{(i)} L_2 - N_1 \cdots V_k^{(i)} L_N - N_2].
\]

After some calculation, the solution may be expressed as follows:

\[
\hat{s}_k^{(i)} = H_k^{(i)H} [\mathbf{R}_k - H_k^{(i)H} \mathbf{S}_k^{(i)}].
\]

The time-varying complex filter \( w_k^{(i)}(1) \) has the following expression:

\[
w_k^{(i)} \triangleq \sigma_n^2 (\mathbf{H}_k^{(i)} \mathbf{R}_{ss,k}^{(i)} \mathbf{H}_k^{(i)H} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{H}_k^{(i)} \mathbf{e},
\]

where \( \mathbf{e} \) denotes a length-\( n_T(N + L - 1) \) vector of all zeros except for the \( (n_T(N + L_2 + i)) \)th element which is 1, and where \( \mathbf{I} \) is the identity matrix of size \( n_R M_s \times n_R M_s \).

For each transmitted symbol, a matrix inversion is required in order to compute its estimate as in (11) (i.e. \( n_T L_s \) matrix inversions per iteration). A careful implementation based on the recursive procedure proposed in [4] leads to a complexity evolving as \( O(n_T M_s N (n_T M_s + n_T L_s + n_T L_T)) \). An ef-
cient approximation [4], leading to a further complexity re-
duction with weak performance degradation, may be obtained
by computing the mean a priori variance of all the transmitted
symbols in the frame:

\[
\psi \triangleq \frac{1}{n_T L_s} \sum_{i=1}^{n_T} L_s - 1 \sum_{k=0}^{\psi_k^{(i)}} \psi_k^{(i)},
\]

which allows to define a constant covariance matrix \( \mathbf{R}_{ss,k}^{(i)} \) and to compute the estimates with only one matrix inversion per
and\(−2\)

interleaver is chosen randomly and varies from frame to frame

PSK modulation with Gray mapping. The pulse shaping filter

This leads to a complexity per symbol (not accounting for the

implementation. As a matter of fact, the number of states in the

rates (BER) or the frame error rate (FER). The results are re-

As in [3], we make the assumption the estimate \(s_k^{(i)}\) is the

The parameters \(\mu_k^{(i)}\) and \(\nu_k^{(i)2}\) can be calculated for each trans-

The demapper can finally compute the bit extrinsic LLR’s

V. SIMULATION RESULTS

This section is devoted to the presentation of performance re-

Table I

TABLE I

GSM TYPICAL URBAN (TU) CHANNEL MODEL

<table>
<thead>
<tr>
<th>Path Delay (μs)</th>
<th>0.0</th>
<th>0.2</th>
<th>0.5</th>
<th>1.6</th>
<th>2.3</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Power (dB)</td>
<td>−3.0</td>
<td>0.0</td>
<td>−2.0</td>
<td>−6.0</td>
<td>−8.0</td>
<td>−10.0</td>
</tr>
</tbody>
</table>

Fig. 3. FER with four transmit and four receive antennas. Frequency-selective Rayleigh fading channel (GSM TU profile). Coder [155₈, 117₈, 123₈]. 8-PSK.

for the average bit energy to noise power spectral density ratio

In the sequel, several simulations are presented for a

Table I shows the path power and delay profiles of this channel. Each one of the \(n_RM_T\) physical multipath impulse responses has taps selected according to these profiles. Each tap is modelled as an independent (from all others) zero-mean complex gaussian random variable. The space-time equalizer parameters \(N_1\) and \(N_2\) are chosen to be \(N_1 = L_1 = 4\) and \(N_2 = L_2 = 5\).

Fig. 3 reports simulation results (FER) for a \((n_1, n_R) = (4, 4)\) setup and the GSM TU channel. We use a rate-1/3 convolutional coder with constraint length 7 and octal generator polynomials \([155₈, 117₈, 123₈]\). If the excess bandwidth of the pulse-shaping filter is neglected (what will be done in the sequel), the spectral efficiency is 4 bps/Hz. The curves represent the results obtained after 0 to 5 iterations. We can see the gain achieved by the iterative processing and notice that it classically increases with the \(E_b/N_0\) ratio. With the same simulation parameters and for a similar receiver complexity, our scheme outperforms the scheme proposed in [1] by nearly 7 dB at the fifth iteration.

Fig. 4 shows the performance improvement when the number

of transmit antennas \(n_T\) increases in a frequency selective fading channel. There are four receive antennas \((n_R = 4)\) but \(n_T\) takes the values 1, 2 and 4. We use a rate-1/2 convolutional coder with constraint length 5 and generator polynomials \([2₃₈, 3₅₈]\); the corresponding spectral efficiencies are thus
Our scheme is thus able to benefit from transmit and receive diversity over a frequency-selective fading channel.

The coder has generator polynomials \( [23_8, 35_8] \). 8-PSK.

Fig. 4. BER with four receive antennas. Frequency-selective Rayleigh-fading channel (GSM TU profile). Coder \([23_8, 35_8]\). 8-PSK.

1.5, 3 and 6 bps/Hz. Therefore, for a fixed \( E_b/N_0 \) ratio, the total transmitted power is proportional to \( n_T \). Adding one more transmit antenna enables to increase the spectral efficiency with a linear growth of the total transmitted power instead of the exponential power growth that is usually needed. Moreover, this enables to increase transmit diversity. The drawback is the introduction of CAI. If the receiver cannot manage to suppress CAI, the performance can be seriously degraded. Fig. 4 shows that without iteration our receiver cannot suppress CAI: the more transmit antennas, the poorer the performance. However, when increasing the iteration number, a large part of the CAI can be canceled. We see that with four transmit antennas, both the spectral efficiency and the BER are better than with a single one. Our scheme is thus able to benefit from transmit diversity even over a frequency-selective fading channel.

Fig. 5 shows the BER curves when \( n_R \) and \( n_T \) are identical and increase over the GSM TU channel. The antenna number is 1, 2 and 4. The coder has generator polynomials \([23_8, 35_8]\). The spectral efficiencies are respectively 1.5, 3 and 6 bps/Hz again.

For the sake of comparison, we have also plotted the BER curve with \( n_T = n_R = 4 \) over the flat Rayleigh-fading channel. With one antenna, the performance is very poor but it dramatically improves as the antenna number increases. Three reasons can be identified. The first one is the array gain: each time \( n_R \) is doubled, the SNR increase is 3 dB. The second improvement is due to the receive diversity: all the receive antennas are not likely to fade simultaneously. The third reason is the transmit diversity that has been previously explained. At a 3 \( 10^{-5} \) BER and after the fifth iteration, we can see a 17 dB gap between the 1- and the 4-antenna curve. As there is a 6 dB array gain, the 11 dB additional gain is due to the transmit and receive diversities. This shows that our scheme is able to benefit from transmit and receive diversity over a frequency-selective fading channel.

Over fading channels, multipath propagation is a potential source of degradation but it can lead to improved performance if multipath diversity can be exploited. In a few articles, the receiver structures is not adapted: ISI dominates and multipath propagation degrades the performance. In [1] too, no gain is perceptible with the GSM TU channel. On the contrary, our iterative receiver manages to suppress a large part of the ISI and takes advantage of the multipath diversity to lower the BER. As shown in Fig. 5, at iteration 5 and at a \( 2 \times 10^{-5} \) BER, the GSM TU curve is indeed 3.5 dB better than the flat fading one.

VI. CONCLUSION

We have proposed an FS MMSE-based turbo receiver suited to MIMO transmission. It leads to drastic complexity reduction w.r.t. the optimal implementation, but it still offers excellent performance. The reported simulation results have shown its effectiveness, its ability to account for inter-symbol interference, co-antenna interference and its capability to exploit transmit and receive diversity. Moreover, it is able to benefit from multipath diversity, so that the BER is even lower over frequency-selective channels.

REFERENCES