A conceptual and methodological framework for psychometric isomorphism: Validation of multilevel construct measures

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Abstract

The conceptual and methodological framework for measurement equivalence procedures has been well established and widely used. Although multilevel theories and methods have been widely used in organizational research, there is no comparable framework for measurement equivalence of multilevel constructs, or psychometric isomorphism. In this paper, we present a conceptual and methodological framework for understanding and testing various forms of isomorphism. Within this framework, we explicate (a) the different types of psychometric isomorphism, (b) the conditions where psychometric isomorphism is appropriate and necessary, (c) how psychometric isomorphism corresponds with different composition models and estimation methods, and (d) the analytic procedures that can be used. Using simulated data, we also illustrate how the proposed procedures may be applied via two analytic methods – item response theory and factor analysis. We conclude with a discussion of theoretical and methodological implications provided by the proposed framework of psychometric isomorphism.
A conceptual and methodological framework for psychometric isomorphism: Validation of multilevel construct measures

Over the last decade, journals such as Journal of Applied Psychology, Academy of Management Journal, and Organizational Research Methods have published over 135 articles that address issues related to collective phenomena with a multilevel or cross-level approach (e.g., Bliese, Chan, & Ployhart, 2007; Felps et al., 2009; Gelfand, Nishii, & Raver, 2006; Henderson, Wayne, Shore, Bommer, & Tetrick, 2008; Morrison, Wheeler-Smith, & Kamdar, 2011; Ostroff, Kinicki, & Clark, 2002; Sacco & Schmitt, 2005). With the increasing use of multilevel theory and methods in the organizational research (Aguines, Pierce, Bosco, & Muslin, 2009), researchers have been seeking to bring clarity to the structural and functional nature of multilevel constructs (Chen, Mathieu, & Bliese, 2004; Morgeson & Hofmann, 1999). Significant advances in both theories and methods of multilevel constructs have been made, but a number of issues are yet to be resolved (Bliese et al., 2007). In particular, the field is still in need of a better conceptual and methodological understanding of cross-level isomorphism.

In the current article, we propose that for multilevel constructs that are assumed to be conceptually similar across levels, cross-level isomorphism can be psychometrically tested. With this perspective, the overarching goal of this paper is to provide conceptual and methodological clarification of psychometric isomorphism. The current literature calls for a careful integration of different views of isomorphism that have been advanced in the past several years. Some researchers have emphasized the importance of understanding and testing isomorphism (e.g., Zyphur, Kaplan, & Christian, 2008), whereas others have noted that it need not be assumed nor tested (e.g., Bliese et al., 2007; Chen, Bliese, & Mathieu, 2005). Therefore, in the first part of the paper we address the following conceptual questions: “What is isomorphism?” and “Why and
when is it important to test for isomorphism psychometrically?”. In the second part, we present methodological considerations and propose analytic procedures for testing psychometric isomorphism. More specifically, we 1) define and explain different types of psychometric isomorphism, 2) describe certain conditions when a psychometric test of isomorphism is appropriate and necessary, 3) prescribe different estimation methods to be used for specific conditions, and 4) explicate the analytic procedures that can be used for testing psychometric isomorphism in those conditions. In the third part, we illustrate one of the proposed procedures on simulated data using two analytic methods – item response theory (IRT) and factor analysis (FA). In the last part, we conclude with a discussion of theoretical and methodological implications of the proposed framework of psychometric isomorphism and emphasize the need for more research in this area.

**CLARIFYING THE CONCEPT OF ISOMORPHISM**

**What is Isomorphism?**

Generally defined as the “similarity or one-to-one correspondence between two or more elements” (Bliese et al., 2007, p. 553), the concept of cross-level isomorphism implies that higher-level constructs have similar meanings and properties as their lower-level counterparts. Assuming isomorphism is useful for multilevel theory development. When psychological constructs are discussed as group characteristics (e.g., team efficacy), these higher-level constructs are thought to be embodied by individuals within the collective units. Positing a higher-level construct without bestowing an anthropomorphic nature to the collective entity triggers the question of whether collective units should be compared on dimensions that are fundamentally psychological and realistic only at the individual level. For such cases, the isomorphism assumption offers a theoretical resolution by positing that the characteristics of a
higher-level entity are conceptually similar to those of the lower-level counterpart, and that this conceptual similarity allows us to quantify higher-level units in the same manner as individuals using a shift in the levels-of-analysis. Further, the notion of cross-level similarity also allows us to infer external relations at the collective level using individual-level theories.

Researchers have had several different views on isomorphism. One perspective equates isomorphism to the similarity of relations with other constructs: Constructs correspond across levels because they are embedded in conceptually equivalent nomological nets. Representing this view, Morgeson and Hofmann (1999) endorsed a pragmatic perspective to understanding hypothetical entities such that constructs are known by their similarity of inputs and outputs. Rousseau (1985) also pointed out that “nomological networks give meaning to abstractions and place them in context” (p. 8). To the extent that these nomological linkages are similar, we are able to establish a greater degree of isomorphism. Tests for this form of isomorphism (also termed ‘homology’) have been established by Chen, Bliese, and Mathieu (2005).

Other scholars emphasize that isomorphism is founded on the similarity of individual scores within collective units. This is commonly known as interrater agreement and reliability of scores (LeBreton & Senter, 2008), which we refer to as (within-group) score similarity in this article. Kozlowski and Klein (2000) succinctly summarized this view in their statement, “isomorphism means that the amount of elemental content is essentially the same for all individuals in the collective” (p. 62). The homogeneity of lower-level elements such as personality, mental models, or performance, result in collective phenomena that represent the same characteristics as lower-level units. A corollary of perfect convergence is that individual-level scores are interchangeable with collective unit scores. If all individuals have the same rating on job performance, the collective unit score (via aggregation) will be equivalent to any
single chosen member. Conversely, large score divergence would demonstrate a lack of isomorphism because collective constructs are not representative of individual-level elements. Various indices for interrater agreement and interrater reliability have been formulated for investigating this form of isomorphism (Bliese, 2000; LeBreton & Senter, 2008).

Yet another definition of isomorphism underscores the measurement equivalence of constructs between levels. Some scholars have proposed that isomorphism is operationally defined by factorial equivalence across levels (Chen et al., 2005). Other scholars such as Dyer, Hanges, and Hall (2005) propose that isomorphism further entails the similarity in the magnitude of measurement relations (e.g., loadings) between indicators and constructs (across levels). This perspective is in part driven by current measurement equivalence standards (Vandenberg & Lance, 2000). However, the equivalence of measurement relations across levels has not been uniformly endorsed by researchers (e.g., Chen et al., 2004) because of a few conceptual concerns that are yet to be resolved (we elaborate on these concerns in the next section). Consequently, no framework has been established for testing measurement equivalence across levels.

In sum, while isomorphism is commonly recognized as a term that describes the similarity of construct properties across levels, at least three distinct concepts of isomorphism have been advanced. We propose that isomorphism in multilevel constructs, in its ideal form, involves all three aspects: (1) nomological similarity across levels; (2) score similarity within collective units; and (3) measurement equivalence across levels. Although the first two forms of isomorphism have been well emphasized in the existing literature, the importance of the third form, cross-level measurement equivalence, has not been fully explicated. In the following section, we propose that psychometric isomorphism, which entails measurement equivalence, is vitally important and essentially undergirds the first two forms.
Psychometric Isomorphism – Why and When is it Important?

Some view that there is no theoretical grounding for measurement equivalence isomorphism (or psychometric isomorphism) because the emergent processes underlying the constructs are distinct (e.g., Chen et al., 2005). However, it is important to note that measurement equivalence across levels primarily assesses the similarity of internal structures of multilevel constructs, rather than of their emergent processes. As Morgeson and Hofmann (1999) noted, constructs are hypothetical concepts or heuristic devices. Measurement procedures such as factor analysis posit that latent constructs summarize observed correlations among indicators and are invoked on the basis of parsimony. The goal is not to infer whether there is correspondence in how constructs come to be. From an analytic perspective, latent variable methods employ covariance matrices of indicators; covariability at a slice in time offers little insight on when and how constructs emerge. As such, our construct labels and definitions do not typically entail an elaboration of underlying processes.

Our position is that methodology surrounding construct equivalence (measurement equivalence or homology) does not speak directly to ontology, that is, the nature of constructs and how they come to be. The implicit desire to match emergent processes is best resolved in the realm of philosophy and theory. For example, if we assume a constructivist approach to constructs, one may argue that the emergence of group and psychological constructs occur in the same manner, perhaps through process of collective agreement fundamentally rooted in the minds of individuals. We do not seek to address this theoretical issue which lies beyond the scope of the current paper. In the foregoing presentation, we approach isomorphism from a pragmatic perspective (see Morgeson & Hofmann, 1999). In other words, given known practices
in construct validation techniques, how would we determine multilevel isomorphism? From this perspective, we propose that multilevel isomorphism requires psychometric isomorphism.

One reason why psychometric isomorphism is important is because it is a logical prerequisite for homology. From a construct validation standpoint, it is well-known that measurement equivalence needs to be established before predictive equivalence (Vandenberg & Lance, 2000). This is because we need to determine whether a measurement equivalent construct is being assessed between groups before we can make any inferences on predictive equivalence (Drasgow, 1984). The same logic applies in a multilevel context. It has been emphasized that the same number of factors in our multilevel constructs needs to be borne out at the individual and group level for a valid inference on homology (Chen et al., 2004). Because factor structure is a function of loadings (e.g., pattern of loadings fixed vs. not fixed at zero in CFA), the equivalence in the magnitude of loadings across levels necessarily reflects a stronger case of isomorphism.

Two additional points regarding the relationship between homology and psychometric isomorphism are noteworthy. First, there are degrees of psychometric isomorphism, just as there are degrees of homology. For example, it has been proposed that there is configural, scalar, and metric similarity in homology (Chen et al., 2005). Gradation in measurement equivalence isomorphism is also consistent with prior theoretical work that proposes isomorphism as an ideal type with many different multilevel constructs falling along the isomorphic-nonisomorphic continuum, or composition-compile continuum (Kozlowski & Klein, 2000). Second, measurement equivalence isomorphism identifies whether variations among entities at individual and group levels are dimensionalized using a similar combination of linear indicator weights. Quantitatively comparing groups based on dimensions that are originally used to describe individuals is valid only if these dimensions are present at the group level. For instance, when
groups are conceptualized to differ on personality dimensions such as the Big Five, the dimensional structure of personality at the collective level is assumed to be similar to that at the individual level (Hofmann & Jones, 2005; McCrae & Terracciano, 2008; Steel & Ones, 2002). With greater similarity across levels, we can be more certain that the differentiation of units occur along a parallel spectrum. By extension, comparing strength of relations across levels will be more valid when there is greater psychometric isomorphism. This is important when contextual analysis is conducted and coefficients across levels are compared (Enders & Tofighi, 2007; Zyphur, Kaplan, & Christian, 2008; Hofmann & Gavin, 1998).

Establishing psychometric isomorphism is also important for establishing score similarity within collective units. As mentioned earlier, measurement equivalence isomorphism serves to elucidate whether variability between levels are dimensionalized in a similar manner. Techniques that apply comparisons of within- and between-group variability (Bliese, 2000) for establishing score reliability at the collective level (e.g., intraclass correlation coefficient; ICC) inherently assume complete measurement equivalence isomorphism. Specifically, between-group variability is dimensionalized in the same manner as within-group variability. To the extent that constructs do not have measurement equivalence, inferring score similarity from these statistics would be less valid.

Therefore, psychometric isomorphism is a fundamental form of isomorphism and needs to be tested when one seeks to determine if constructs are isomorphic – whether in the form of nomological or score similarity. Additionally, tests for cross-level measurement equivalence need to be conducted on constructs that are proposed to be theoretically similar across levels. Typically, these higher-level constructs follow a composition or fuzzy composition process (see Kozlowski & Klein, 2000) and the resultant multilevel construct is defined by the shared
content of constructs across levels of analysis while recognizing differences in their emergence. Specifically, shared content refers to the same concept with differences in its referent (i.e., individual versus group). When similar conceptual labels differentiated by their referents – such as individual and collective efficacy – are applied to lower- and higher-level constructs, conceptual isomorphism is often invoked or assumed (e.g., Ostroff et al., 2002; Park & DeShon, 2010). For instance, individual efficacy is derived from a personal belief that one can achieve an outcome, whereas a sense of collective efficacy may stem primarily from multiple individuals being able to work together to achieve an outcome. Both individual and collective efficacy share the same content – confidence in achieving an outcome – while involving antecedent processes that are unique to each level. Therefore, when we posit constructs that are theoretically similar, we are invoking isomorphism. Establishing measurement equivalence isomorphism would demonstrate that the features of the multilevel construct as indexed by indicators are indeed similar across levels-of-analysis. We note that at a minimum, multilevel constructs should have similar dimensions across levels. More restrictive forms of loading equivalences would represent greater degrees of isomorphism.

In view of this, organizational research would greatly benefit from a framework delineating the forms of psychometric isomorphism and prescribing procedures to test for it. Organizational phenomena are fundamentally multilevel in nature (Kozlowski & Klein, 2000) and this issue of isomorphism is relevant in many content areas such as leadership (e.g., Henderson et al., 2008), work teams (e.g., Park & DeShon, 2010), diversity (e.g., Sacco & Schmitt, 2005), group or organizational personality (e.g., Hofmann & Jones, 2005), climate (e.g., Morrison et al., 2011), and culture (Taras, Kirkman, & Steel, 2010). Yet, very limited attention has been given directly to the topic of isomorphism in organizational research because it was not
regarded as an essential form of isomorphism. By verbalizing a clear impetus for measurement equivalence we seek to move our field toward a rigorous examination and factorial testing of multilevel constructs. We also envision that akin to individual-level measurement equivalence research (Vandenberg & Lance, 2000) this will sow the seeds for future methodological research.

Interestingly, the issue of psychometric isomorphism has gained considerable interest in other research areas such as culture and values (e.g., Cheung, Leung, & Au, 2006; Fontaine & Fischer, 2011; van de Vijver & Leung, 2001; van de Vijver & Watkins, 2006), national personality (e.g., McCrae & Terracciano, 2008; Steel & Ones, 2002), and subjective well-being (e.g., Tay & Kuykendall, 2013). One common theme across these areas is whether individual-level dimensions are isomorphic with country-level dimensions. Researchers have recognized the need to establish the generalizability and meaningfulness of individual-level constructs to the levels of country and/or culture in order to make commensurate comparisons. This is because dimensions used to describe individuals are used to compare higher-level units such as countries and cultures. However, the shift in the level-of-analysis may lead to a change in the meaning structure, evidenced from differences in the number of dimensions between levels (Fontaine & Fischer, 2011). For example, Schwartz’s (1992, 2006) values research has demonstrated 10 value types at the individual level, and 7 value types at the cultural level. Therefore, it is misleading to construe that cultures differ on generalized dimensions from individual-level value types.

Despite the awareness that psychometric isomorphism is important in other research areas, there has not been substantial advance on the nature of psychometric isomorphism and an analytic framework for testing and evaluating isomorphism using factor analytic models.

In the existing measurement literature, the application of cross-level measurement equivalence analysis has been applied to contextual analysis where individual-level indices are
aggregated to approximate higher-level (or contextual) constructs (e.g., Ludtke et al., 2008; Ludtke, Marsh, Robitzsch, & Trautwein, 2011). The main purpose of contextual analysis is to model individual ratings of higher-level phenomena (e.g., Dedrick & Greenbaum, 2011), and to correct for the unreliability of higher-level constructs due to sampling and measurement unreliability (e.g., Ludtke et al., 2011). In this framework, demonstrating measurement equivalence across levels is used to show that the single higher-level factor can be partitioned into individual-level and higher-level variation (Skrondal & Rabe-Hesketh, 2004). For example, using individual ratings of team-efficacy, one can examine the extent to which team efficacy scores vary across teams and within teams. As such, cross-level measurement equivalence in the extant measurement literature is primarily concerned with the measurement of higher-level constructs rather than the isomorphic correspondence of constructs between levels. Therefore, cross-level measurement equivalence analysis as it currently stands does not directly address the issue of psychometric isomorphism as we conceptualize and describe in the present article.

Lastly, we note that the systematic advancement of psychometric isomorphism in multilevel research has been severely hampered due to the lack of an overarching framework that prescribes all the necessary conceptual and methodological considerations. At present, the application of psychometric isomorphism is rare and has been limited to certain types of composition models. For example, little is known about how to examine psychometric isomorphism when different sets of indicators are used (e.g., self-efficacy and team-efficacy may be assessed using two different sets of indicators – the former referenced to self and the latter to team). Similarly, a number of macro-level constructs such as innovation, social capital, diversity, and mobility may be only accessed through indicators other than individually-referenced ratings provided by members themselves. The extant literature does not offer a clear guidance on how to
evaluate psychometric isomorphism in such cases. In order to address these concerns, the second part of this article presents specific ways of assessing psychometric isomorphism – as part of multilevel construct validation – after delineating multiple forms of measurement equivalence and composition models.

**Psychometric Isomorphism and Multilevel Construct Validation**

In the previous section, we propose that the purpose of psychometric isomorphism is not to establish the similarity in underlying processes. Take the example of between-group measurement equivalence analysis, the utility of equivalence is that construct scores of groups can be meaningfully compared and used for scientific inferences; using equivalence to show uniformity in cognitive processes is limited because it requires multiple inferential leaps from between-individual scores to within-individual processes (Borsboom, Mellenbergh, & van Heerden, 2003). Similarly, establishing cross-level isomorphism should be seen as part of a process of multilevel construct validation. While the importance and procedures of construct validation are well established at a single level of analysis, they have not been addressed adequately in the multilevel contexts (Chan, 1998). Further, for reasons we discussed earlier, existing multilevel construct validation procedures (e.g., Chen et al., 2004) do not emphasize psychometric isomorphism. Therefore, we devote this section to discussing (and revisiting) how measurement equivalence isomorphism relates to multilevel construct validation. According to the *Standards for Educational and Psychological Testing*, validity is defined as “the degree to which accumulated evidence and theory support specific interpretations of test scores entailed by proposed uses of a test” (AERA et al., 1999, p. 184), and validation is the process of accumulating such evidence through multiple sources. We focus on three major sources of validity evidence that have been highlighted within the multilevel context (Chen et al., 2004):
test content, relationships with other variables, and internal structure (AERA et al., 1999). Consistent with older conventions (e.g., APA et al., 1954; APA, 1985), one might refer to them as content validity, criterion-related (or convergent-discriminant) validity, and psychometric (or factorial) validity, respectively.

First, a validity argument can be made based on the relevance and representativeness of the construct indicators. In the multilevel context, it is important for researchers to present conceptual arguments for the use of specific items (or indicators) that index the construct at multiple levels; it is a necessary condition for psychometric isomorphism that the constructs share theoretical similarity across levels. The indicators used to reference lower-level constructs must share the same content domain and have equivalent wording as those used to reference higher-level constructs. Such cross-level similarity in item content serves as evidence for multilevel construct validity. We also note that, in order to determine whether the cross-level item similarity is necessary in the first place, one must consider a theoretical compositional process through which indicators reflect individual- versus higher-level constructs (see Chan, 1998; Also see our examples of self- versus team-efficacy in the second part of this article).

Second, validity can be inferred and evaluated based on the observed relationships between the test score with other variables – whether theoretically hypothesized patterns of relations between measures of the hypothesized construct and other constructs are borne out in reality. At a single level of analysis, it involves testing relations between the target construct and other constructs in the proposed nomological net (Cronbach & Meehl, 1955). Within a multilevel context, this was introduced as a specific form of isomorphism termed homology. By testing homology, the researcher’s question is: to what extent are the relations between constructs equivalent across levels (Chen, et al., 2005)? However, as we argued, before homology can be
established, it is necessary to demonstrate psychometric isomorphism because the dimensional structure at different levels may in fact differ. A lack of psychometric isomorphism leads to invalid tests of homology. For example, if there are different numbers of dimensions across levels, there is no basis for homology. Further, if the indicators load differently across levels, it would cast doubt on the accuracy of stricter forms of homology which require relationship values between constructs to correspond on absolute terms.

The last, yet the most pertinent to our current context, is the internal structure of constructs established through procedures such as FA or IRT. At a single level of analysis, confirmatory factor analysis is frequently applied as a way to evaluate factorial structure of construct measures. For multilevel constructs, it is often necessary to go beyond single-level factor analysis (either at the individual or collective level) to determine whether the proposed multilevel construct bears factorial resemblance between levels. In doing so, there are a number of methodological considerations that are closely intertwined (e.g., types of psychometric isomorphism to be tested, underlying composition models, and estimation procedures).

**METHODOLOGICAL CONSIDERATIONS IN PSYCHOMETRIC ISOMORPHISM**

**Types of Psychometric Isomorphism**

Analogous to measurement equivalence (Drasgow, 1984; Vandenberg & Lance, 2000), we present a typology of the different testable forms of isomorphism in Table 1 and discuss each of them in greater detail below.

**Configural isomorphism.** Configural isomorphism means that the factor structure of lower- and higher-level constructs are similar. This demonstrates that the same types of factors – as indexed by the same (or comparable) indicators – hold across levels. As argued earlier, conceptual isomorphism would require a demonstration of configural isomorphism. In the case
of strong configural isomorphism, the same number of factors and the pattern of zero and nonzero factor loadings are expected to hold across levels. When only the number of factors is the same between levels, and the loading patterns appear similar, without the equivalence in the zero-and-nonzero loading patterns, we define it as a case of weak configural isomorphism. This is akin to running exploratory factor analysis and defining the initial factor structure without implementing additional constraints (i.e., constraining loading patterns to simple structure).

The lack of weak configural isomorphism indicates that individual-level constructs are not generalizable to the group level. This shows that the structure of inter-group variability is fundamentally different from that of inter-individual variability, and that the key dimensions by which individuals are distinguished do not apply to distinguishing groups. This further suggests that a different conceptual definition may be required for the group-level phenomenon. For example, at the individual level, job satisfaction may form two distinct factors related to task satisfaction and coworker satisfaction. However, these factors may not differ at the country level but instead converge into a single factor of job satisfaction. This would demonstrate that countries are not distinguished by the same construct(s) at the individual level, and it would be invalid to compare countries along the dimensions of task and supervisor satisfaction in the same manner that is applicable to their individual-level counterparts.

It is possible in some cases that the factor solution of the same set of indicators across levels produces some (but not all) common factors across levels. For example, common cultural value dimensions appear to exist across individuals and cultures, but there are also factors that are unique to individuals and may not be borne out at the cultural level (Schwartz, 1992, 2006). In such instances, it may be said that there is partial configural isomorphism.
**Metric isomorphism.** Going beyond configural isomorphism, one may be also interested in testing *metric isomorphism*: whether the factor loadings or item discriminations are similar across levels. This is analogous to metric equivalence in multigroup confirmatory factor analysis, in which the aim is to test the similarity of factor loadings. Parallel to the logics of multigroup measurement equivalence testing, it is necessary to establish configural isomorphism before metric isomorphism can be examined. In the presence of partial configural isomorphism, it is possible to test whether loadings of the common factors between levels are similar.

The presence of metric isomorphism would suggest that the interpretation of the common factors is similar across levels. Also, an examination of metric isomorphism can reveal whether there are differences in the defining characteristics of constructs between levels. For instance, job satisfaction may be primarily defined by socio-emotional indicators at the individual level as demonstrated by relatively higher factor loadings. At the country level, socio-emotional aspects of job satisfaction may not be the defining feature that distinguishes countries.

In the psychometric literature, one can test whether factor loadings are strictly equivalent (i.e., $\lambda_{1,\text{lower-level}} = \lambda_{1,\text{higher-level}}$). This type of testing can be undertaken when common indicators are used between levels and a procedure for the simultaneous estimation of higher- and lower-order constructs is used (i.e., multilevel FA/IRT). We define this as *strong metric isomorphism*⁴. On the other hand, *weak metric isomorphism* is demonstrated when the relative ordering of loadings (not their absolute values) is equivalent between levels. Therefore, broadly speaking, metric isomorphism is tested in terms of relative congruence between the two sets of factor loadings across levels.

In certain instances, it is permissible and appropriate to test for weak metric isomorphism because strong metric isomorphism cannot be examined. Consider an example where team-
efficacy is indexed by indicators that ask individuals about their perceptions of team-efficacy and this is compared with individual ratings of self-efficacy. Despite comparable response statements, the referent is fundamentally different: questions ask about the team versus the self. In such cases, testing strong metric isomorphism is impossible because multilevel psychometric techniques that use simultaneous estimation are not applicable (this point is elaborated in the next sections); however, by separately estimating the lower and higher levels, one can obtain the correspondence for the relative ordering of factor loadings between levels.

**Composition Models and Psychometric Isomorphism**

Whether and how we should test configural and metric isomorphism depends on the underlying composition processes through which higher-level constructs are formed from lower-level data. In order to explicate this more clearly, we draw upon the framework of composition models delineated by Chan (1998). We propose that higher-level constructs are amenable to psychometrics tests of measurement equivalence isomorphism if: (1) indicators of lower- and higher-level constructs have comparable content; (2) the multilevel construct conforms to conventional measurement models, and (3) potentially fulfill other forms of isomorphism, specifically score similarity.

In dispersion models, a higher-level construct reflecting within-group variability does not have a lower-level counterpart. Process composition models present a formulation of higher-level constructs that unfold over time and are not subject to conventional forms of factorial validation. Additive models do not require within-group agreement (i.e., score similarity isomorphism). Therefore, psychometric isomorphism is not assumed in these three composition models. The remaining two composition models in Chan’s (1998) framework (i.e., direct consensus and referent-shift) have a strong conceptual basis for psychometric isomorphism.
For the **direct consensus** model, higher-level constructs are operationalized using an aggregate of individual-level scores within groups. In effect, the higher-level construct is a group-level analogue of the individual construct. In order for researchers to conceptualize and discuss group-level dimensions in a manner congruent with individual levels, configural isomorphism would be expected and therefore requires testing. For example, team-efficacy is conceptualized as a single dimension in a manner congruent with self-efficacy. A more stringent conception of isomorphism is also tenable for these composition models. As discussed earlier, metric isomorphism would imply that the indices which characterize the dimensions are the same between levels. In this context, metric isomorphism states that aggregated indicators have the same relation to the higher-level construct as indicators to the lower-level construct. A simplistic example is that the inter-item correlations at the individual-level are equivalent to the aggregated-level. In this case, we can test for metric isomorphism.

For a **referent-shift** model, individuals within a group make ratings on the properties of the higher-level unit. For instance, indicators on the scale are worded so that individuals are asked to give ratings on their perceptions of *team-efficacy* rather than perceptions of *self-efficacy*. Because individual scores reference higher-level units, one implicit assumption is that the true score for a higher-level unit is based on the aggregate of individual-level unit scores. This follows the classical test theory that the true score at the higher level is defined as the average observed score based on an infinite number of independent observations. In effect, lower-level unit ratings of a higher-level unit are unreliable indicators of a higher-level unit’s true score (estimated from the average of lower-level unit ratings); departures of lower-level unit ratings from a higher-level unit’s true score are construed as error. Based on this argument, using these
scores to index both lower- and higher-level level constructs to test for isomorphism may not be conceptually valid.

In this case, lower-level indicators index higher-level constructs but it cannot be used to reflect lower-level constructs. We propose that another set of indicators that are commensurate with the higher-level construct be used to test for psychometric isomorphism when a referent-shift composition model is used. For example, when team-efficacy is measured using a referent-shift model, psychometric isomorphism should be examined using individuals’ own ratings of self-efficacy along with the team-efficacy ratings made by the same individuals.

When measurement equivalence isomorphism is examined using two sets of indicators, configural isomorphism would be expected. Again, this is in line with the basic understanding that for a conceptual label to hold between levels, the constructs must have a similar dimensional structure. In this case, individual-level and higher-level constructs are indexed using indicators with different referents. Despite the different methods, we would still expect conceptual alignment. For example, if individual self-efficacy has one dimension, we would expect the referent-shift team-efficacy to have a single dimension as well so that they are theoretically commensurate. This is in line with Hofmann and Jones’ (2005) study, in which the dimensionality of referent-shift personality was compared to the dimensions of self-referent personality ratings in order to establish the configural isomorphism of collective personality.

Metric isomorphism can also be expected when descriptions of dimensions are defined in the same manner across levels, because the relative strength of indicator loadings reflects how the construct is operationalized and what aspects of the definition are emphasized. In most cases, there is no strong conceptual rationale for how the lower- and higher-level constructs should differ in their definitional emphasis. If the definitional emphases are the same between levels,
this would be empirically demonstrated with metric isomorphism in which indicators load on the dimensions in the same manner. This is essentially the same situation as a direct consensus model in which indicators with the same content across levels are expected to load on the dimensions in a similar manner. In the referent-shift case, indicators with the same content are still used but the referent differs between levels.

**Separate and Simultaneous Estimation in Psychometric Tests of Isomorphism**

It has been noted that whether to use *separate* or *simultaneous* estimations in examining multilevel factor structure is dependent on the nature of how collective constructs emerge from individual-level constructs (Chen et al., 2004). However, much less information is available on the similarities and differences of these two approaches. Therefore, we devote this section to delineating the conceptual and methodological differences between separate and simultaneous estimation methods in the context of psychometric isomorphism testing.

**Separate estimation.** Separate estimation assumes that the data are level-free. As such, individual-level constructs are thought to be derived without the influence of the higher-level counterpart taken into account; higher-level constructs are empirically realized in collectively aggregated behavior. Methodologically, separate estimation of lower-level and higher-level units does not fully disentangle within- and between-group variability (Muthén, 1994). For instance, consider that lower-level unit scores inher in both individual and group means; therefore, lower-level units are not independent observations, which result in biased estimates and ostensibly better fit (Muthén, 1994). On the other hand, factor analysis of higher-level unit scores does not account for the accuracy of scores based on number of lower-level observations; that is, the reliability of higher-level unit scores is dependent on the number of lower-level unit observations and needs to be accounted for. Further, factor analysis of higher-level unit scores may be less
accurate because the number of higher-level units is usually small resulting in poorer estimation; whereas simultaneous estimation may need only as few as 30 to 50 observations at the higher-level (L. K. Muthén & B. Muthén, 2007). In view of this, we recommend that separate estimation procedures be only undertaken with an acknowledgement of its limitations.

However, we suggest that the statistical (in)adequacies of this approach are contingent on the meaning and purpose of the lower-level scores. When lower-level scores are ratings of higher-level unit properties – such as in a referent-shift composition model – this approach is appropriate and does not encounter statistical drawbacks. We propose that the analysis of psychometric isomorphism for the case of referent-shift composition requires two types of variables – a referent-shift composition for higher-level units and a self-referent for lower-level (i.e., individual) units. For example, if one attempts to measure team-efficacy using a referent-shift approach, a comparison of factor structure across levels is conceptually viable when there are corresponding ratings of self-efficacy. With a referent-shift composition, the conceptual question one attempts to resolve becomes: Is the higher-level referent construct structurally consistent with the lower-level counterpart with “self” as the referent (e.g., is team-efficacy similarly structured as self-efficacy)? A comparison of factor structures across levels should be undertaken such that the results of higher-level factor analysis for referent-shift compositions is compared to lower-level counterpart variables where the self is the referent. Otherwise, the separate estimation procedure is generally not recommended for assessing the structural similarities between levels.

**Simultaneous estimation.** The use of multilevel measurement approaches has been recommended for modeling lower- and higher-level constructs simultaneously (Chen et al., 2004; Dyer et al., 2005). It matches the purposes of a direct consensus composition model, where
lower-level and higher-level unit scores share common meaning. Fitting a multilevel measurement model captures such covariability across levels. We propose that multilevel measurement models be applied primarily to scores in which self-ratings are indicators of lower-level constructs.

In the case of a direct consensus model, collective constructs emerge from patterns of interactions among individuals and come to exert influence on such behaviors over time (Morgeson & Hofmann, 1999). By this definition, collective constructs are fundamentally located at the level of the individual, yet inferred through systematic differences that occur in *individual actions* across higher-level units. Methodologically, simultaneous estimation assumes that behavioral indicators reflect both psychological and collective constructs. Therefore, simultaneous psychometric techniques are most appropriate for capturing the codetermination of lower- and higher-level constructs. It assumes that the collective constructs stem from individuals’ views and behaviors but are also distinct from the individual-level construct, and that item responses are a function of an individual’s standing on the individual-level construct and the individual’s membership on the higher-level unit (the location of the higher-level unit on the higher-level construct also affects item endorsement). For example, Tay and Diener (2011) used simultaneous estimation (i.e., multilevel IRT) to examine the ordering of individual need fulfillment (e.g., basic, safety, social, mastery, and autonomy needs) and the extent to which country effects (additionally) bear upon this ordering in the same manner.

**Procedures of Testing Psychometric Isomorphism**

Building on the previous section, here we propose analytic procedures for testing measurement equivalence isomorphism based on a review and integration of previous literatures (e.g., Cheung et al., 2006; Dedrick & Greenbaum, 2011; Dyer et al., 2005; Fischer et al., 2010;

When the collective construct uses referent-shift composition, we recommend the use of separate estimation. Separate estimation can be applied to a referent-shift composition for higher-level units and a self-referent for individual-level units. Weak configural isomorphism, strong configural isomorphism, and weak metric isomorphism represent increasing degrees of isomorphism that can be tested. We note that separate estimation can be applied to direct consensus composition constructs but simultaneous estimation is preferable. When the collective construct of interest uses direct consensus composition, we recommend using simultaneous estimation. In such cases, we suggest a three-step preliminary analysis as follows: (1) exploring factor structure of the construct at the individual level (i.e., ignoring the multilevel structure in the data), followed by (2) calculating the intraclass correlation for each item, and last, (3) comparing multilevel model(s) with level-free model(s) to ascertain whether a multilevel structure of the construct is feasible. With simultaneous estimation, we can test for strong configural isomorphism, and weak and strong metric isomorphism. Table 2 presents detailed analytic procedures for testing isomorphism with separate and simultaneous estimation methods.

**Multilevel Measurement Model Selection.** With simultaneous estimation, models with different configural arrangements between levels (i.e., varying numbers of factors) can be specified and fit statistics can then be used to find a well-fitting and parsimonious model. We suggest the following strategies be used for determining the best-fitting model. First, relative model-data fit can be assessed using several information criteria – the Bayesian information criterion (BIC; Schwarz, 1978) and consistent Akaike information criterion (CAIC; Bozdogan,
The differences in these information criteria are that the penalty terms differ for model complexity. We seek to look for models with the lowest information criteria which indicate model parsimony.

Second, absolute model-data fit is assessed using confirmatory factor analytic fit indices if a FA approach is used. With the use of multilevel confirmatory FA (e.g., Dedrick & Greenbaum, 2011), we can obtain typical fit statistics such as the confirmatory fit index (CFI) (Bentler, 1990; Bentler & Bonett, 1980), Tucker-Lewis index (TLI) (Tucker & Lewis, 1973), the root mean squared error of approximation (RMSEA), and the standardized root mean square residual (SRMR). For both CFI and TLI, values closer to 1.00 indicate good fit. Whereas for RMSEA and SRMR, lower values closer to zero indicate good fit. One difference between single-level and multilevel CFA, is that for multilevel CFA, within-group SRMR and between-group SRMR can be obtained (B. Muthén & L. K. Muthén, 2007). Both types of SRMRs are interpreted in the same manner with lower values closer to zero indicating good fit.

Absolute model-data fit can also be assessed using the bivariate residual statistic (BVR) which can be used for both CFA and IRT approaches (Tay, Diener, Drasgow, & Vermunt, 2011; Tay, Newman, & Vermunt, 2011; Tay, Vermunt, & Wang, 2013). This statistic is similar to the doubles adjusted chi-square statistic utilized in IRT model-data fit proposed by Drasgow et al. (1995). For any pair of variables, the observed and predicted two-way contingency matrices are compared. In general, values smaller than 3 indicate reasonable model-data fit. Where possible, we recommend the use of multiple model comparisons to facilitate researcher’s decision on the best-fitting model. In other words, one can specify several (non-) configural-isomorphic models and compare their relative fit simultaneously. For example, one can examine whether a 1-1
higher-level factor; 1 lower-level factor) configurally isomorphic factor model fits better than a configurally isomorphic 2-2 or a non-isomorphic 2-1 factor model.

**A Side Note on Confirmatory versus Exploratory Approaches**

Although EFA and CFA are commonly understood as exploratory and confirmatory approaches, respectively, this distinction often becomes fuzzy in practice. With regard to separate estimation, exploratory models can be used to test for weak configural isomorphism; confirmatory models are applied to test for strong configural isomorphism. It is possible for researchers to use confirmatory models in an exploratory manner where different confirmatory models are compared to identify the best fitting model. It is also possible that confirmatory model comparisons are undertaken to show that the postulated model is the best fitting model. Further, a researcher may choose not to compare multiple models if confirmation of a proposed model is sought by examining the absolute fit of the model. With regard to simultaneous estimation, there are no (to our knowledge) exploratory multilevel factor models where researchers can determine the number of factors to be retained via specific rule-of-thumb or through inspection. Instead, confirmatory models are applied in simultaneous estimation. But again, even with confirmatory models, researchers may still choose to take an exploratory approach (e.g., exploring the best model amongst many models) as opposed to confirmatory approaches (e.g., pitting 2-3 models based on theory, evaluating a single model derived from theory).

**ILLUSTRATIONS: TESTING PSYCHOMETRIC ISOMORPHISM WITH SIMULTANEOUS ESTIMATION**

Our illustration of proposed psychometric isomorphism testing procedures focuses on the use of simultaneous estimation, because separate estimation procedures are more straightforward.
The first illustration uses IRT for modeling dichotomous responses, and the second illustration uses FA for modeling polytomous responses, although we note that both methods can be applied to either response type. For both IRT and FA, we simulated configural isomorphism. In the IRT simulation, we simulated a one-factor model at the lower and higher level with no metric isomorphism. In the FA simulation, we simulated a two-factor model at the lower and higher level, with only one factor having metric isomorphism. We label the lower-level factors as individual-level factors as these responses are most commonly associated with individual responses. We also label the higher-level factors as a group factors as is commonly referred to in multilevel modeling.

For researchers interested in the understanding the mathematical formulas and how the data were generated, we detail such information in the first part of each illustration section. For researchers only interested in the illustration of the recommended procedures, we have structured the paper so that skipping ahead to the “analysis” sections logically flows from the current section. All our analyses were conducted in Latent GOLD 4.5 (Vermunt & Magidson, 2008). At the request reviewers, we also included additional results using Mplus 5.2 (B. Muthén & L. K. Muthén, 2007) for confirmatory FA as it provides typical FA fit statistics that researchers are familiar with (e.g., CFI, TLI, RMSEA). Syntaxes for both Latent GOLD and Mplus analyses are available from the first author.

**Illustration 1: Item Response Theory Modeling for Dichotomous Responses**

Item response theory (IRT) is a commonly used statistical technique for measuring psychological constructs (Drasgow & Hulin, 1990). IRT takes into account measurement error (i.e., random error, uniqueness) — as with factor analysis — and it describes the mathematical relationship between the latent trait ($\theta$) and the probability of endorsing an observed category on
an item \(i\). In the case of dichotomously scored data, a 2-parameter logistic model (2PLM) may be utilized,

\[
P(y_{ij} = 1|\theta_j) = \frac{1}{1 + e^{-(a_i\theta_j + b_i)}}.
\]

where \(y_{ij}=1\) reflects and endorsement (‘1’) by individual \(j\) on item \(i\); \(a_i\) and \(b_i\) denote the item discrimination and location, respectively.

Multilevel extensions to standard IRT models have been proposed (Adams, Wilson, & Wu, 1997; Fox & Glas, 2001; Raudenbush & Sampson, 1999; Skrondal & Rabe-Hesketh, 2004; Varriale & Vermunt, 2012; Vermunt, 2008). These models take into account the hierarchical structure of the data, and the measurement model parameters (\(b_i\) and \(\theta_j\)) are allowed to vary randomly across hierarchical units \(k\) (1,…,\(K\)) so that the measurement model is

\[
P(y_{ijk} = 1|\theta_{jk}) = \frac{1}{1 + e^{-(a_i\theta_{jk} + b_{ik})}}.
\]

In our illustration, we use a structural model such that the item locations depend on the group-level factor \(\eta_k\).

\[
b_{ik} = \beta_i + \alpha_i\eta_k + e_{ik}.
\]

Equation (3) shows that the item intercept \(b_{ik}\) is random and varies around the mean intercept of \(\beta_i\). In addition, the individual responses vary as a function of the group-level factor \(\eta_k\) with the loading \(\alpha_i\). The individual- and higher-level latent factors – \(\theta_j\) and \(\eta_k\), respectively – are assumed to be multivariate normally distributed and mutually independent. For practical purposes, the variance of the error term \(e_{ik}\) is assumed to be zero; otherwise, separate group latent factors would be needed for each indicator.

**Data.** Dichotomous data for 10 items were generated using the item response theory model in equations (2) and (3). Realistic item parameters were obtained from the article published by Stark, Chernyshenko, and Drasgow (2006) in which they analyzed responses to the
Illinois Supervisor Satisfaction scale collected from 1,057 nonacademic workers. These item parameters were chosen because scale items were based on a key construct in organization science and they were based on fairly large a working sample. Further, the authors reported parameters applicable to both IRT and factor analyses which is needed for our purposes. The item parameters were reported in the factor analytic metric of item thresholds ($\gamma_i$) and lambda parameters ($\lambda_i$). We converted them to $a_i$ and $\beta_i$ parameters as shown in Table 3, where

$$a_i = \frac{1.702 \times \lambda_i}{\sqrt{1-\lambda_i^2}} \quad \text{and} \quad \beta_i = \frac{a_i \times \gamma_i}{\lambda_i}.$$ 

Because IRT is commonly applied to unidimensional data, we simulated unidimensional individual- and group-level data. In this case, configural isomorphism is fulfilled, but not metric isomorphism. We note that for unidimensional data, configural isomorphism entails both weak and strong configural isomorphism. To simulate these data, the group-level loading parameters $\alpha_i$ were generated using a normal distribution, $N(1.2, .3)$. The mean and standard deviation of the distribution was aimed to generate data that would have moderate ICC effect sizes of about .10 to .20 (Bliese, 2000). The rank-order correlation between the generated group level loading parameters and individual-level loading parameters was -.29. Using the item parameters, we simulated 100 group-level units with 10 observations within each unit. In total, there were 1000 observations. We chose to use 100 group-level units as we wanted to obtain accurate estimates of group level loading parameters for this illustration. It has been suggested that 30 to 50 group-level units are needed for multilevel factor analysis (L. K. Muthén & B. Muthén, 2007) but less is known if a larger number of group-level units are needed for testing measurement equivalence isomorphism. We revisit this issue in our discussion section.

**Analysis.** Following the procedures in Table 2, we first fit one- and two-factor models to the data ignoring the multilevel structure. For the two-factor model, we examined the case where
the first five indicators loaded on the first factor and the other five indicators loaded on the second factor. It is assumed here that the researcher seeks to confirm that this particular factor structure based on an *a priori* conceptual model. Therefore, we directly test for strong configural isomorphism. In other instances, researchers can explore the potential factor structure by running exploratory factor analysis on tetrachoric correlations ignoring the multilevel structure. After which, one can decide on how the loadings should be structured if there is ostensibly more than one factor.

We found that the one-factor model (Model 1) had better relative and absolute fit than the two-factor model (Model 2) as shown in Table 4. From the first two rows, we see that relative model-data fit statistics BIC and CAIC showed lower values for the one-factor model. In addition, the largest BVR and average BVR values were smaller for the one-factor model ($BVR_{\text{largest}} = 2.96; BVR_{\text{average}} = 0.65$) as compared to the two-factor model ($BVR_{\text{largest}} = 10.19; BVR_{\text{average}} = 2.13$). Therefore it is likely that a one-factor model holds at the individual level.

It is important to recognize however that this step of comparing factor structures without considering the multilevel structure is usually undertaken when the factor structure is unknown. It gives a guide as to the rough number of dimensions that may hold in the data, and what indicators load on to the specific dimensions. Nevertheless, because we ignored the multilevel structure, it is possible that not accounting for the between-group variance could lead to an ostensible one-factor model. In view of this, it is necessary to undertake a few more steps to determine if a multilevel model would fit the data better; and if so, how many factors are needed. This exploratory step can usually be skipped if researchers have a specific factor model they seek to test.
To determine whether we need a multilevel model, we examine the ICCs of the indicators. This step allows us to assess whether variances in the indicators are in part attributable to between group-level variability. Because these were dichotomous indicators, we used a multilevel binomial regression model to estimate the ICCs. The item ICCs shown in Table 3 ranged from .01 to .37. The average value was .14 which was moderate in size. This indicated that a multilevel factor model may be more appropriate. As an observation, the magnitude of the group-level loadings were highly correlated with the observed ICCs ($r = .89$). This shows that between-group variability is proportionately larger as group loadings increase.

In specifying a multilevel IRT model, we need to examine models with different numbers of dimensions at each level of analysis. In doing so, we can also test for whether strong configural isomorphism holds. In this step, we fit configural isomorphic models (Models 3 and 4) and non-configural isomorphic models (Models 5 and 6) with the results shown in Table 4. We found that the configural isomorphic model with one individual factor and one group factor (i.e., Model 3) had the best relative fit with the lowest BIC and CAIC among all the competing models. Also, there was good absolute fit in that the BVR values were lower compared to the other models. This confirmed that strong configural isomorphism held in the data.

After finding strong configural isomorphism, we can proceed to test for metric isomorphism. There are two forms of metric isomorphism – weak and strong. We first tested for weak metric isomorphism via the rank-order correlation ($\hat{\alpha}_{i}$ and $\hat{\alpha}_{l}$) which was -.14, which was fairly close to our simulated value of -.29. The lack of positive correlation indicated that weak metric isomorphism did not hold between the levels. In general, we propose that if weak metric isomorphism does not hold, there may not be a need to test for strong metric isomorphism. Only for the illustrative purpose, however, we also examined whether strong metric isomorphism held.
To test for strong metric isomorphism, we constrained the loadings to be equivalent across levels and the results are shown in Model 7 of Table 4. We found that the strong metric isomorphism model fit worse than the strong configural isomorphism model (Model 3) as seen in the BIC, CAIC, and BVR values which were higher. Based on this analysis, we can conclude that although there is strong configural isomorphism between levels, metric isomorphism does not hold between levels.

**Illustration 2: Factor Analytic Modeling for Polytomous Responses**

The factor analytic model that was applied has similar features to the IRT model. The common factor analytic model can be described by

\[ y_{ij} = u_i + \lambda_i \theta_j + \epsilon_{ij} \]  

(4)

where \( u_i \) is the intercept and \( \epsilon_{ij} \) is the error term. The other terms \( y_{ij} \), \( \lambda_i \), and \( \theta_j \) are defined in the same manner in the previous section. In the multilevel FA model, we use a structural model such that the item intercepts depend on the group level factor \( \eta_k \),

\[ u_{ik} = \beta_i + \alpha_i \eta_k + e_{ik} \]  

(5)

This equation shows that the item intercept \( u_{ik} \) is random and varies around the mean intercept of \( \beta_i \), and the individual responses vary as a function of the group level factor \( \eta_k \) with the loading \( \alpha_i \). The individual and group level factors are assumed to be multivariate normally distributed and mutually independent.

**Data.** We generated data using item parameters from Stark, Chernyshenko, and Drasgow (2006) with a few modifications to suit our purposes. First, we set the item intercepts \( (\beta_i) \) to zero as is convention in many factor analytic simulations. This is because we are interested primarily in the similarities of the covariance structures at the individual and group level. Second, we sought to illustrate a two-factor model, thus the loadings \( (\lambda_{1i} \text{ and } \lambda_{2i}) \) were split between two
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factors as shown in Table 3. This follows the IRT analysis where a researcher seeks to confirm
the two-factor model with the first five indicators loading on the first factor and the remaining
indicators loading on the second factor. In this analysis, we simulated data that conformed to this
hypothesized model.

To illustrate metric isomorphism on only one of the factors, we halved the individual-
level loadings \( \lambda_{1i} \) and used them as group-level loadings \( \alpha_{1i} \) as shown in Table 3. This would
produce a rank-order correlation of 1. We had previously simulated different magnitudes of
group-level loadings and found that values around the ballpark of .40 produced observed ICCs in
the range of .10 to .20. For the second factor, we halved the individual-level loadings \( \lambda_{2i} \) but
reversed their order to produce a group-level factor that would not exhibit metric isomorphism.
The rank-order correlation between the loadings was -.95. Additionally, we simulated small
factor correlations at the individual and group level as .20 and .30, respectively. As with the IRT
illustration, we simulated 100 hierarchical units with 10 observations within each unit.

**Analysis.** We applied the stepwise procedures for testing multilevel measurement
equivalence outlined in Table 2. At the first step, we explore the data structure. We ignored the
multilevel structure and fit one- and two-factor models. Using the same procedure for the two-
factor model, we examined the case where the first five indicators loaded on the first factor and
the other five indicators loaded on the second factor. The results are shown in Table 5. We found
that the two-factor model (Model 2) fit substantially better than the one-factor model (Model 1).
Both the BIC, CAIC, and BVR values were lower for the two-factor model. Further, when we
examined the conventional fit statistics for FA, we found that the fit of the two-factor model was
superior (CFI = .996; TLI = .995; RMSEA = .016; SRMR\(_{\text{within}}\) = .022) whereas the one-factor
model fit poorly (CFI = .626; TLI = .519; RMSEA = .167; SRMR\(_{\text{within}}\) = .135). This was
consistent with the simulated data structure. Based on the results, a researcher would have initial evidence suggesting that a two-factor model may hold.

At the next step, we seek to determine whether a multilevel factor model is necessary. We note that ignoring the multilevel structure may lead to fit statistics that are inaccurate if between-group variability is substantial for the indicators. As shown in Table 3, the indicators ICCs were computed and we found that the average value was .21, and they ranged from .14 to .27. As expected, there was a strong positive relation between the ICCs and the magnitude of the group level loadings \( (r = .90) \). The moderate ICCs indicated that fitting a group factor would likely provide a better account for the data as ignoring the nested data structure may lead to a misspecified model.

In seeking to fit a multilevel factor model, we need to specify different numbers of dimensions at the individual and group level. In so doing, we can also test for strong configural isomorphism. In Table 5, we show the results of fitting the configural and non-configural isomorphic models (Models 3 to 6). We found that the configural isomorphic one-factor model (Model 3) and non-configural 1-2 isomorphic model (Model 5) clearly had very poor absolute fit. These models are therefore not considered and we focus on the remaining two models. A comparison of the configural 2-2 isomorphic factor model (Model 4) (BVR\textsubscript{largest} = 7.44; BVR\textsubscript{average} = 0.98; CFI = .993; TLI = .991; RMSEA = .020; SRMR\textsubscript{within} = .025; SRMR\textsubscript{between} = .059) and the non-configural 2-1 non-isomorphic factor model (Model 6) (BVR\textsubscript{largest} = 7.81; BVR\textsubscript{average} = 0.98; CFI = .956; TLI = .942; RMSEA = .049; SRMR\textsubscript{within} = .033; SRMR\textsubscript{between} = .329) showed that the former had better fit, especially when comparing the conventional FA fit statistics. In addition, the configural 2-2 isomorphic model (Model 4) had the lowest BIC and CAIC values among all the models compared (Models 1 to 6). Based on the analysis, we have
evidence showing that there are two dimensions in the data, and strong configural isomorphism holds across levels.

Given that we found strong configural isomorphism, we sought to determine if weak metric isomorphism held; that is, whether the magnitudes of loadings similar across levels. The estimated coefficients for the strong configural model are shown in Table 3. The rank-order correlation between the individual and group level factors for the first and second factor was 1.00 and -.90, respectively. This was consistent with the simulated values of 1.00 and -.95, respectively. This indicated that the first factor showed weak metric isomorphism but not the second factor.

Because only the first factor showed some degree of weak metric isomorphism, we proceeded to test for strong metric isomorphism in the first factor. We constrained the loadings to be equal in the first factor but not the second factor. As shown in Table 5, this model (Model 7) fit the data better than the strong configural isomorphic model (Model 4). In terms of the absolute fit statistics (i.e., BVR, CFI, TLI, RMSEA, SRMR), both models were very similar. However, the BIC and CAIC were lower when strong metric isomorphism was specified in the first factor. This demonstrated that this model was more parsimonious as compared to the strong configural isomorphic model. These findings are consistent with the simulated data, showing that while there is strong configural isomorphism, only the first factor demonstrated strong metric isomorphism but the second factor did not show metric isomorphism.

**DISCUSSION**

Multilevel theory and methods have become part and parcel of organization research. The scientific conception of multilevel constructs leads to, and is also facilitated by, the collection of empirical data and corresponding analytic procedures. With the ongoing interest in
understanding the nature and relationships between constructs across levels (e.g., Bliese et al., 2007; Chan, 1998; Chen et al., 2005; Kozlowski & Klein, 2000; Morgeson & Hofmann, 1999), we believe that the present study makes several significant contributions to the current body of multilevel research. First, our research carefully integrates different existing views of isomorphism, and brings much needed clarity to the meaning and importance of psychometric isomorphism. We have argued that the assessment of psychometric isomorphism is key requirement for ensuring that individual-level dimensions can be validly generalized to higher levels. Analogous to a framework for measurement equivalence research (Vandenberg & Lance, 2000), this study defines the relevant methodological and substantive issues related to psychometric isomorphism and delineates the types of isomorphism that are testable using both IRT and FA. It also represents a systematic attempt to integrate current multilevel psychometric methods and composition models. We hope that future research can use this framework for rigorously assessing multilevel constructs, whether various operational definitions of collective constructs yield a similar structure, and the extent to which psychometric isomorphism holds.

**Theoretical Implications**

**What does measurement equivalence across levels mean?** Tests of measurement equivalence isomorphism should primarily be viewed as a construct validation technique when we posit multilevel constructs which have similar content across levels, and where the higher-level construct is deemed to follow a composition or fuzzy composition process. As such, finding measurement equivalence isomorphism allows us to make theoretical inferences about the relationship of constructs across levels whereas non-isomorphism would prevent us from discussing constructs across levels in a similar manner. But what does it reveal about the phenomenon we are trying to measure when we do not find psychometric isomorphism? As we
emphasized earlier in this article, procedures for psychometric isomorphism are not primarily useful for examining differences in emergent processes across levels, because finding certain forms of *psychometric* isomorphism does not automatically lead us to a specific *substantive* conclusion. At the same time, if a researcher’s goal is to determine a specific type of substantive process that may lead to differences in isomorphism, then the proposed psychometric techniques may (and should) be used in conjunction with a rigorous research design which systematically rules out alternative processes. A similar example may be found in the context of multigroup measurement equivalence: Robert, Lee, and Chan’s (2006) sought to determine potential sources of measurement non-equivalence across groups (i.e., translation, culture, organization, and response context) by using multiple samples from the United States, Singapore, and Korea with differing levels of the four factors.

**Identifying possible sources of (non-)psychometric isomorphism.** It is possible that constructs that are theorized to be multilevel may in fact show different factor structures across levels of analysis due to *external factors (or “third variables”) that influence construct-indicator linkages*—that is, the arrows tying the constructs to their indicators. We note that this is distinct from construct emergence, where the focus is on how constructs come to be. More generally, we may also find that other forms of isomorphism (e.g., homology or within-group score similarity) are lacking as well.

It is important to consider how different factors may lead collective constructs to systematically differ from its lower-level counterpart. In doing so, we can design studies that allow us to identify different sources of non-isomorphism. To identify these sources, it is necessary to use at least two sets of data. For example, we would not expect psychometric isomorphism for groups that have the posited factors which lead to non-isomorphism as
compared to other groups that do not these factors; or we would expect less psychometric isomorphism for constructs that are likely influenced by specific factors compared to those constructs that are not influenced by such factors. We elaborate on what these potential factors might be. The factors that influence construct linkages can be categorized into two types: factors that influence the bottom-up process of construct emergence or factors that influence the top-down process of construct emergence.

A bottom-up process has been advocated by Morgeson and Hofmann (1999), whereby the interactions of lower-level units lead to the emergence of the collective construct. Psychometrically, non-isomorphism occurs when collective units systematically differ in ways distinct from how individuals view themselves. It is possible that initial differences in group composition lead to distinct interaction patterns within groups (Barry & Stewart, 1997; Pelled, Eisenhardt, & Xin, 1999). This may be further driven by processes of group differentiation, whereby groups attempt to emphasize unique and distinct aspects from other groups (Tajfel, 1982). Therefore, intra- and intergroup interactions lead to an emphasis on unique group characteristics apart from individual characteristics.

Another possibility is that the collective construct emerges from antecedent lower-level processes, giving rise to a different form at a higher level of analysis. In addressing the flow of culture, Hannerz (1992) states that there are different forms of externalization such that modes of thought are made public and accessible to others. Systematic patterns of interactions over time can result in externalized codification. For example, when a protocol proves to be successful in a team, there may be documentation of standard operating procedures to ensure that future members follow a similar script (Tay, Diener, et al., 2011). Therefore, reference to these documents can ensure continuity of mindset among group members and newly inducted
members. The embodiment of collective ideas across groups can lead to its differentiation from the psychological counterpart.

There are factors that can influence top-down processes to affect the degree of psychometric isomorphism between levels. Different structural features across groups are important. For instance, aspects of job satisfaction between groups may systematically differ because of the modes (e.g., online or face-to-face) through which team members communicate. Physical proximity may systematically strengthen the saliency of the relational aspects of job satisfaction in teams. Moreover, leadership across higher-level units is also important in changing the viewpoints of members and the way individuals interact between groups. Analysis of within- and between-group variability may manifest psychological differences and leadership differences respectively in the structure of individual- and higher-level constructs (e.g., Hofmann & Jones, 2005). At a macro-level (e.g., organizations and countries), some collective constructs may capture differences in policies and/or legislations. For example, countries can shape the job satisfaction of individuals in the country via policies regarding labor relations and social welfare. As such, the structure of job satisfaction across nations may reflect national differences in policies rather than individual determinants common across humanity. On the other hand, other types of collective constructs that are less influenced by such policies and/or legislations would reveal more psychometric isomorphism.

**Methodological Implications**

Foundational papers for understanding and conducting measurement equivalence research have been widely cited (e.g., Drasgow, 1984; Vandenberg & Lance, 2000) because the comparability of constructs across groups are, and continue to be, of great interest among researchers. Indeed, construct validation – more specifically, measurement equivalence –
between groups is fundamental for valid inferences of scores between groups. Although
measurement equivalence techniques are important for valid between-group comparisons, its
utility diminishes for multilevel data.

With multilevel data, a researcher is usually less interested in determining the
measurement equivalence across many groups. For example, to our knowledge, few if any
multilevel textbook promulgates the examination of measurement equivalence between multiple
groups before undertaking random coefficients modeling. In most cases, the researcher is
interested in whether groups differ on meaningful dimensions common to individuals within
groups. This can be seen in the use of within-group score similarity indices (e.g., $r_{wg}$ and ICC) to
justify aggregation, reflecting interest in cross-level isomorphism (Bliese, 2000; Chan, 1998). In
this sense, horizontal equivalence of constructs (i.e., measurement equivalence) is of less
importance than the vertical equivalence (i.e., cross-level equivalence or isomorphism) of
constructs. It is important to establish that aggregated scores from the individual level delineate
the same dimensions at the higher level. Therefore, comparisons of individual constructs across
higher-level units require the use of validation techniques proposed in this paper (see also
Zyphur, et al., 2008). For example, the aggregation of individual values across nations can be
misleading because nations are not distinguished on the same types of dimensions that
individuals are (Fontaine & Fischer, 2011). In the case of cultural values (Schwartz, 1992, 2006),
a simple aggregation of all lower level dimensions to cultural levels may be invalid because not
all the constructs at the individual level hold at the cultural level. In view of this, we expect that
more researchers will use these techniques in making such inferences.

Future Research
While this paper seeks to establish a foundational basis for construct isomorphism in multilevel research, this area of research is in its infancy. Drawing from the developmental trajectory of measurement equivalence research, seminal articles on this topic (e.g., Drasgow, 1984) has promulgated research that has refined the procedure in the subsequent years. Not exhaustively, research to date has focused on a few issues: (a) the data requirements for measurement equivalence (Idaszak, Bottom, & Drasgow, 1988); (b) evaluating model-data fit indices (e.g., G. W. Cheung & Rensvold, 2002); (c) comparisons of IRT and FA procedures (e.g., Stark, et al., 2006); and (d) establishing broader and more flexible models for measurement equivalence (e.g., Tay, Newman, et al., 2011).

Drawing on this historical trend, we propose that research should start with determining the data requirements for psychometric isomorphism. How many data points are necessary for accurate recovery of individual and group level factors? It has been recommended in the past that there be a minimal range of 30 to 50 higher-level units for multilevel factor analysis (L. K. Muthén & B. Muthén, 2007). This may also depend on other factors such as the number of lower level units or the magnitudes of the indicator ICCs. At this juncture, if researchers have less than 30 higher-level units, we encourage them to collect more data if possible so that more rigorous conclusions about isomorphism can be made.

Other questions include, what is the optimal ratio of indicators to latent factors? In addition, how many indicators are needed to accurately evaluate metric isomorphism? For example, a reviewer pointed out that if a factor is defined by as few as 3 or 4 items, the congruence coefficients may not be as meaningful. And indeed, we would not be able to evaluate fit for a factor with 3 indicators because it is fully saturated. Clearly, more indicators are needed but the optimal number needs to be evaluated. Closely related to this, we advocate that fit indices
be examined to determine the criteria for model-data fit. What values of BVR would indicate reasonable model-data fit? Is it possible to extend the BVR statistic so that it can also detect model-data misfit at the higher level? What is the Type I error rate and power for detecting the right model using the proposed information criteria? Is the .50 criterion for metric isomorphism a reasonable one? Finally, to what extent does psychometric isomorphism need to hold in order for tests of homology and score similarity to be valid? Clearly, there are a number of unanswered questions that need to be systematically addressed in future research. We hope that this paper will help spur further research and theoretical discussions.
Endnotes

1. A quick trace of the etymological roots of the term isomorphism would suggest that it reflects similarity in form and/or function. The term isomorphism originated in the field of chemistry in the 1800s; it was used to describe a general law in which substances with similar chemical composition derive commensurate crystalline forms. Later, other fields of inquiry such as mathematics incorporated the term to denote similarity of mathematical properties in form and function. Eventually, organizational theory utilized the concept of isomorphism. For example, institutional isomorphism (e.g., DiMaggio & Powell, 1983; Meyer & Rowan, 1977) reflects processes in which organizations become more similar and homogenous over time. To our knowledge, the formal use of the term isomorphism in the multilevel context was borrowed from systems theory to refer to similarity or “formal identity” (Rousseau, 1985, p. 8).

2. Homology entails that isomorphic constructs have the same nomological network across levels. Arguably, if the basis of isomorphism is whether the linkages between constructs emerge in a similar manner or are qualitatively alike, we would be hard-pressed to find theoretical grounding for homology. Consider that a nomological link between two multilevel constructs features two higher-level constructs that emerge using different processes. In addition, the substantive interpretation behind the linking mechanism between constructs is radically different across levels. As an example, individual attitudes $\rightarrow$ behaviors through a series of biochemical processes; at the group-level, group attitudes $\rightarrow$ group behaviors through social-interactional dynamics. As a consequence, some scholars have chosen to use a pragmatic perspective to understand nomological links, such that we are merely concerned with whether the conceptual labels of inputs and outputs are similar across levels (Morgeson & Hofmann, 1999), and whether
the magnitude of the structural links are proportional (Chen, et al., 2005). This perspective is consistent with our proposition for measurement equivalence isomorphism.

3. There is usually a correspondence between composition process models and the way lower level indicators are combined. Composition process models assume linear combinations of lower level indicators so a higher-level construct is simply the aggregate. On the other hand, higher-level compilation constructs assume non-linear combinations. For instance, one may conceptualize team performance not merely as the aggregate of individual performance but dependent on the weakest team member, such as the case of mountain climbing (Kozlowski & Klein, 2000). Higher-level constructs that follow composition and fuzzy composition processes not only have a theoretical basis for isomorphism but have indicators and combination rules that are amenable to tests of psychometric isomorphism.

4. When strong metric isomorphism holds, the multilevel construct can further parsed as a unitary construct with variance partitioned to lower and higher levels (Skrondal & Rabe-Hesketh, 2004). In other words, a single latent construct underlies scale responses; variance in this construct can be partitioned into individual-level variance and group-level variance. This is because isomorphism to its fullest extent – defined by strong metric isomorphism – reflects commensurate constructs between levels. This model can be simplified into a single factor so that regardless of the levels-of-analysis, scores index the common construct in the same manner.

5. Although it is possible to empirically examine lower level referent-shift scores (e.g., van Mierlo, Vermunt, & Rutte, 2009), we argue that for the purposes of psychometric isomorphism, the scores of substantive interest belong primarily to higher-level units and dimensional analysis is applicable to higher-level units but not lower-level units. One might argue that perceptual differences between individuals about higher-level unit properties (e.g., teams or organizations)
are meaningful. We agree that they are meaningful, but only inasmuch as they reflect
disagreement about higher-level unit properties. However, they are less meaningful for a
construct of shared meaning at both levels because these scores do not index differences between
individuals on the dimension of interest. For example, assessing perceptions of team-efficacy
does not reflect individual-level efficacy. Differences in scores may reflect other individual
differences such as perceptual awareness or biases, but does not directly index self-efficacy,
which is the lower-level counterpart of team-efficacy.

6. We propose a value of .50 because after a preliminary simulation of perfect isomorphism (rank
order correlation = 1.00) using 5 indicators, the observed rank-order correlations ranged from .50
to 1.00.
References


<table>
<thead>
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<th>Between-groups measurement equivalence</th>
<th>Cross-level isomorphism</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Configural invariance:</strong></td>
<td>Pattern of zero and nonzero factor loadings holds between groups</td>
<td><strong>Weak configural isomorphism:</strong> Same number of dimensions holds between levels. Dimensions are generally shown to be indexed by similar indicators across levels without fixing loading patterns.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Strong configural isomorphism:</strong> Same number of dimensions and the pattern of zero and nonzero factor loadings holds between levels</td>
</tr>
<tr>
<td><strong>Metric invariance:</strong></td>
<td>Factor loadings are equivalent between groups</td>
<td><strong>Weak Metric isomorphism:</strong> Relative ordering of factor loadings / item discriminations holds between levels (evidenced by high congruence of the loadings between levels)</td>
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<td></td>
<td><strong>Strong Metric isomorphism:</strong> Magnitude of factor loadings / item discriminations holds between levels.</td>
</tr>
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<td><strong>Scalar invariance:</strong></td>
<td>Indicator thresholds are equivalent between groups</td>
<td>No current models for estimating item thresholds across levels.</td>
</tr>
<tr>
<td><strong>Invariance in uniqueness</strong></td>
<td></td>
<td>No statistical basis for testing across levels.</td>
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</table>
Table 2

Analytic procedure for testing isomorphism with separate and simultaneous estimation

<table>
<thead>
<tr>
<th>Preliminary Analysis</th>
<th>Procedure</th>
<th>Separate</th>
<th>Simultaneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Exploring factor structure</td>
<td>When the initial structure is unknown, one should conduct exploratory analysis treating all observations as independent; that is, ignoring the nested data structure. It should be recognized that the standard errors and fit statistics may be biased because cross-level dependencies are not modeled. The aim is to determine the number of factors and preliminary data structure. This step may be skipped if researchers are seeking to test a specific factor structure. Estimate the intraclass correlation (ICC) for each item. The ICCs represent the variance attributable to between-group. If the ICCs are small, there may not be a need to conduct a multilevel factor analysis and there is little basis for isomorphism. As a rule of thumb, ICCs greater than .05 indicate reasonable amount of variability across groups (Dyer, et al., 2005).</td>
<td>Step 1</td>
<td></td>
</tr>
<tr>
<td>Step 2: Indicator Intraclass correlation (ICC)</td>
<td>Determine whether a multilevel model fits better than a levels-free model (In practice, this step is usually combined with the test of strong configural isomorphism).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Step 3: Ascertaining multilevel structure</td>
<td>Factor retention techniques applied only to higher-level units for referent-shift composition and compared to number of factors for self-referent lower-level units. For lower-level data, there are two ways of obtaining the number of factors. One can run factor analysis (1) ignoring nested structure or (2) on the pooled within-groups covariance matrix. Dimensions are generally shown to be indexed by similar indicators across levels.</td>
<td></td>
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Testing Isomorphism

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Step 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak configural isomorphism: Same number of factors holds between levels.</td>
<td>Step 1</td>
</tr>
<tr>
<td>Are the numbers of descriptive dimensions equivalent across levels?</td>
<td>Step 1</td>
</tr>
</tbody>
</table>
without requiring specific loadings to be fixed at zero.

| Strong configural isomorphism: | Same number of factors holds across levels and the pattern of zero and nonzero factor loadings holds between levels |
| Does the factor structure of descriptive dimensions hold across levels? |

### Separate Estimation:
- Confirmatory factor analysis applied to higher-level units for referent-shift composition; compared to confirmatory factor analysis for self-referent lower-level units. Standard goodness-of-fit indices at each level are examined. For lower-level data, one can run confirmatory factor analysis (1) ignoring nested structure; (2) on the pooled within-groups covariance matrix; or (3) using multilevel confirmatory factor analysis.

### Simultaneous Estimation:
- One can examine the fit of a factor model with fixed zero and nonzero loadings across levels. This is applicable only when there is more than one factor.

| Weak metric isomorphism: | Relative ordering of factor loadings / item discriminations holds between levels. |
| Are descriptive dimensions characterized by construct indicators in a similar manner across levels? |

- Application of congruence coefficient between loadings obtained from higher- and lower-level units. Based on preliminary simulations, we recommend that the rank-order correlation between factor loadings larger than .50 indicates weak metric isomorphism.

| Strong metric isomorphism: | Magnitude of factor loadings / item discriminations holds between levels. This is a stronger assumption than weak metric isomorphism of having similar rank-ordering. The multilevel factor has variability divided into a within component and a between component; therefore we can dimensionalize individuals and groups in the same way. |
| Are descriptive dimensions dimensionalized in a |

- In FA or IRT, one would constrain the loadings to be equal across levels. This means that within-group and between group covariances are proportional across levels. The proportionality is the ratio of the factor standard deviations. A better fit for the constrained model (compared to the unconstrained model) would indicate strong metric isomorphism.
**Similar manner across levels?**

<table>
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<tr>
<th><strong>Special case: Weak/Strong metric isomorphism for comparable factors:</strong> Different numbers of factors hold across levels but metric isomorphism occurs between some factors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak metric isomorphism: For comparable factors, one may obtain the congruence coefficient between loadings from higher- and lower-level units.</td>
</tr>
<tr>
<td>Strong metric isomorphism: For comparable factors, one may constrain the loadings to be equal across levels. We recommend examining the fit of the constrained and constrained model. A better fit for the constrained model would indicate strong metric isomorphism for the comparable factors.</td>
</tr>
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</table>

**Is there a correspondence among construct indicators in some descriptive dimensions across levels?**

- *Special case:* Weak metric isomorphism: For comparable factors, one may obtain the congruence coefficient between loadings from higher- and lower-level units.
- *If needed,* one can test for weak metric isomorphism.
- *If needed,* one can test for both weak and/or strong metric isomorphism.

**Note.** Separate estimation applicable to referent shift and direct consensus composition models, although it is recommended primarily for referent shift models. Simultaneous estimation is primarily applicable to direct consensus composition models.
Table 3

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<th>$\gamma_i$</th>
<th>$\alpha_i$</th>
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<td>Npar</td>
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</table>

*Note.* BIC=Bayesian information criterion; CAIC=Consistent Akaike information criterion; Npar=Number of parameters; BVR=Bivariate residual statistic. The italicized model has the best fit.
<table>
<thead>
<tr>
<th>Model Types</th>
<th>Model #</th>
<th>Individual</th>
<th>Group</th>
<th>Log-likelihood</th>
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<th>CAIC</th>
<th>Npar</th>
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</table>

Note. BIC = Bayesian information criterion; CAIC = Consistent Akaike information criterion; Npar = Number of parameters; BVR = Bivariate residual; CFI = Confirmatory fit index; TLI = Tucker-Lewis index; RMSEA = Root mean square error of approximation; SRMR = Standardized root mean square residual. The italicized model has the best fit.