Diagnosis of broken-bars fault in induction machines using higher order spectral analysis

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Abstract

Detection and identification of induction machine faults through the stator current signal using higher order spectra analysis is presented. This technique is known as motor current signature analysis (MCSA). This paper proposes two higher order spectra techniques, namely the power spectrum and the slices of bi-spectrum used for the analysis of induction machine stator current leading to the detection of electrical failures within the rotor cage. The method has been tested by using both healthy and broken rotor bars cases for an 18.5 kW-220 V/380 V-50 Hz-2 pair of poles induction motor under different load conditions. Experimental signals have been analyzed highlighting that bi-spectrum results show their superiority in the accurate detection of rotor broken bars. Even when the induction machine is rotating at a low level of shaft load (no-load condition), the rotor fault detection is efficient. We will also demonstrate through the analysis and experimental verification, that our proposed proposed-method has better detection performance in terms of receiver operation characteristics (ROC) curves and precision-recall graph.

1. Introduction

For several decades both correlation and power spectrum (PS) have been most significant tools for digital signal processing (DSP) applications in electrical machines diagnosis area. The information contained in the PS is sufficient for a full statistical description of Gaussian signals. However, there are several situations for which looking beyond the autocorrelation of a signal to extract the information regarding deviation from Gaussianity and the presence of phase relations is needed. Higher order spectra (HOS), also known as polyspectra, are spectral representations of higher order statistics, i.e., moments and cumulants of third order and beyond. HOS can detect deviations from linearity, stationarity or Gaussianity in any signal \cite{1,2}. Most of the electrical machines signals are non-linear, non-stationary and non-Gaussian in nature and therefore, it can be more interesting to analyze them with HOS compared to the use of second-order correlations and power spectra. In this paper, the application of HOS on stator current signals for different rotor broken bars fault severities and shaft load levels has been discussed.

So far, the large usage of induction machines (IM) is mainly due to their robustness, to their power efficiency and to their reduced manufacturing cost. Therefore, the necessity for the increasing reliability of electrical machines is now an important challenge and because of the progress made in engineering, rotating machinery is becoming faster, as well as being required to run for longer periods of time. By looking to these last factors, the early stage detection, localization, and analysis of faults are challenging tasks. The distribution of IM failures has been reported in many reliability survey papers \cite{3–8}. These main faults include: (a) stator faults, which define stator windings open or short-circuits and stator inter-turn faults; (b) rotor electrical faults, including rotor winding open or short-circuits for wound rotor machines or broken rotor bars (BRB) or cracked end ring for squirrel cage machines; (c) rotor mechanical faults such as bearing damages, static or dynamic eccentricities, bent shaft, misalignment, and any load-related abnormal phenomenon \cite{3}.

Issues of preventive maintenance, on-line machine fault detection, and condition monitoring are of increasing importance \cite{3–9}. During the last twenty years or so, there has been a substantial amount of research dealing towards new condition monitoring techniques for electrical machine and drives. New methods have been developed for this purpose \cite{5}. Monitoring machine line current and observing the behavior in time-domain and frequency-domain characteristics, from healthy to faulty condition, has been almost the first step of analysis. This technique is known as machine current signature analysis (MCSA).

The MCSA technique has been widely used for fault detection and diagnosis and it is considered as the most popular not only...
for electrical faults but also for mechanical faults. Moreover, it can easily detect machine faults by using only current sensors which are not really invasive. Other signals to be monitored could be stator voltages, instantaneous stator power, shaft vibration, stray flux, electromagnetic torque, speed, temperature, acoustic noise and much more [3–9]. Features extracted from one or more of these signals could be used as the fault diagnosis indexes.

This paper focuses on BRB fault in IM which represent about 10% of total industrial induction motor faults [3], by using both spectral and bi-spectral analysis of the stator current. MCSA has been extensively used to detect BRB and end ring faults in IM [9–11]. The side-band frequency components at \( (1 \pm 2k)fs \) have been used to detect such faults, \( s \) is the rotor slip and \( f_s \) is the fundamental supply frequency, and one integer \( k=1, 2, 3, \ldots \).

Although, the MCSA gives efficient results when the motor operates under rated or high load conditions, some drawbacks have been observed when the load shaft is light. Since the side-band frequency components \( (1 \pm 2k)fs \) are very close to the grid frequency \( f_o \), a natural spectral leakage can hide frequency components characteristics of the fault [12]. In this case, the standard MCSA method fails to detect BRB faults. This problem can be solved by increasing the sampling frequency but load variations during sampling may decrease the quality of the spectrum [9]. Hence a trade-off must be performed between spectrum leakage and frequency resolution which is difficult to be reached when using the Fast Fourier transform (FFT). Consequently, MCSA method is strongly dependent on the rotor slip condition and it has not been applied to BRB faults under no-load condition.

Due to those limitations of FFT, in the literature other methods for spectral analysis become potential options to the detection of BRB such as the zoom-FFT (ZFFT) [9–11], in [11] a method based on the multiple signal classification (MUSIC) has been proposed to improve diagnosis of BRB fault in IM, by detecting a large number of frequencies in a given bandwidth. This method is called zoom-MUSIC, wavelet approaches [13], neural network [14]. In those techniques, the computational time and the accuracy in a particular frequency range are increased. However, as expected, the frequency resolution is still affected by the time acquisition period and the load effect (especially no-load condition) was not studied.

The analysis of faults at low slip is important in industrial applications and would provide the following benefits [12]:

- Improving quality control of new motors,
- Allowing no-load analysis of all type of motors,
- Preventing confusion of faults with load-induced current oscillation,
- Reducing the cost of fault analysis.

In this context, the MCSA method based on a Hilbert transform (HT) was used to detect BRB faults at very low rotor slip conditions [15]. The effectiveness of the method was confirmed by experimental data on an induction motor with one BRB fault. The performance of the method was not evaluated for different numbers of BRB. Besides this method requires high computation and large amount of data is required for the high frequency resolution. In another study [16], the HT is used to extract signal envelop and then the wavelet transform is applied to estimate fault index around 2\( fs \), frequency component which reflects the energy variation of the phase current related to the BRB fault. The main drawback of this approach is that no clear methodology is proposed for the wavelet decomposition.

Various DSP techniques [4–9] have been extensively used for feature extraction purposes and HOS has been one of them [17–24]. The bi-spectrum is the third order spectrum and it results in a frequency–frequency–amplitude relationship which shows coupling effects between signals at different frequencies [19]. This tool has already demonstrated its efficiency in many detection applications including machinery diagnosis for various mechanical faults [20]. Other applications include the usage of the bi-spectral analysis to detect the fatigue cracks in beams [21], stator inter-turn faults [22], and bearings [23]. It has been also used in the case of the coupling assessment between modes in a power generation system [19]. The application of HOS techniques in condition monitoring has been already reported [24,25] and it is clear that multi-dimensional HOS would contain more useful information than traditional two-dimensional spectral analysis for diagnostics purposes.

The next section will give a brief development of the basic bi-spectrum theory followed by a description of the experimental set-up. The third section will develop the PS and the bi-spectrum signal processing tools in order to characterize the stator current frequency components in presence of BRB. The last section will be dedicated to the analysis of experimental results, and ends with a comparison of the two higher order spectra techniques (PS and proposed BDS methods) by means of the ROC curves, before giving some conclusions on the implementation of this advanced DSP technique for electrical machines fault detection.

### 2. HOS analysis

#### 2.1. Basic definitions

In this section, the theoretical concepts of the bi-spectrum method will be recalled. Let us assume that: \( x(n), n=0, 1, \ldots, N-1 \) is a discrete current signal, zero-mean and locally stationary random process with a length \( N \). The autocorrelation function of a stationary process \( x(n) \) is defined by:

\[
R_{xx}(m) = E(x(n)x(n+m))
\]

where \( E(.) \) is the expected mean operator (or equivalently the average over a statistical set) and \( m \) is a discrete time-delay. The PS is formally defined as the Fourier transform (FT) of the autocorrelation sequence:

\[
P_{xx}(f) = \sum_{m=-\infty}^{+\infty} R_{xx}(m) e^{-j2\pi fm}
\]

where \( f \) denotes the frequency.

An equivalent definition can be given by:

\[
P_{xx}(f) = E(X(f)X^*(f)) = E(|X(f)|^2)
\]

where \( X^* \) denotes the complex conjugate of \( X \) and \( X(f) \) is the discrete Fourier transform (DFT) of \( x(n) \), given by:

\[
X(f) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi fn/N}
\]

where \( N \) is the number of samples.

The third–order spectrum, called the bi-spectrum \( B(f_1, f_2) \) is defined as the double DFT of the third order moment. It can be defined as:

\[
B(f_1, f_2) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} M^*_3(k,l)W(k,l)e^{-j2\pi f_1(k+f_1)}
\]

where \( W(k,l) \) is a two-dimensional window function used to reduce the variance of the bi-spectrum and \( M^*_3(k,l) \) is the third-order moment of the process \( x(n) \), given by:

\[
M^*_3(k,l) = E(x^*(n)x(n+k)x(n+l))
\]

where \( k \) and \( l \) are discrete time delays.

As equivalence, Eq. (5) can be expressed in terms of the FT of \( x(n) \) as:

\[
B(f_1, f_2) = E(|X(f_1)X(f_2)X^*(f_1+f_2)|).
\]
It can be noted that the bi-spectrum presents twelve symmetric regions [1,2]. Hence, the analysis can take into consideration only a single non-redundant region. Hereafter, \( B(f_1, f_2) \) will denote the bi-spectrum in the triangular region \( T \) defined by:

\[
T = (f_1, f_2) : 0 \leq f_2 \leq f_1 \leq 2f_s, \quad f_2 \leq -2f_1 + f_s, \quad \text{where } f_s \text{ is the sampling frequency.}
\]

In order to minimize the computational cost, the direct method [1,2] has been used to estimate the bi-spectrum of a stator current signal as described in the following paragraph.

### 2.2. Bi-spectrum estimation

In general, the expected values coming from Eqs. (3)–(7) need to be estimated from a finite quantity of available data. This may be achieved by dividing a signal into \( M \) overlapping segments with \( k \) as a subscript, \( k = 1, 2, 3, \ldots, M \). A windowing function has been applied to each segment and the FTs of all segments are averaged [1]. The aim is to reduce the variance of the estimate by increasing the number of records \( M \) [1,2].

The estimated bi-spectrum \( \hat{B}(f_1, f_2) \) is given by:

\[
\hat{B}(f_1, f_2) = \frac{1}{M} \sum_{k=1}^{M} X_k(f_1)X_k(f_2)X^*_k(f_1 + f_2) \approx E \{X(f_1)X(f_2)X^*(f_1 + f_2)\}.
\]

(8)

The conventional PS considers only the square of the magnitude of the Fourier coefficients and it cannot give information about phase coupling [1,2]. On the other hand, the bi-spectrum is complex and, therefore, it has magnitude and phase. The bi-spectrum or bi-coherence (normalized bi-spectrum) based on HOS emphasizes the triple products of the Fourier coefficients and it is really able to provide such information. However, the computational cost is higher and the 3D-plot or the contours plots are very complex to describe as compared with that of the one-dimensional [23,24]. In fact, it is impossible to compute the bi-spectrum from large data sets. On the other hand, by using smaller data sets, the estimation error will increase a lot [2]. The proposed method is directly computed in one-dimensional bi-spectrum diagonal slice (BDS) which condenses the 2D distribution into 1D curve. It requires less computation and it can be used in real time [22].

The BDS of a process \( x(n) \) can be obtained by setting \( f_1 = f_2 = f \) and it is defined as follows:

\[
\hat{B}_{BDS}(f_1, f_2) = \hat{B}(f_1) = \hat{B}(f) = \hat{D}(f) \approx E \{X^2(f)X^*(2f)\}
\]

(9)

### 3. BRB faults diagnosis based on PS and BDS

In this section, laboratory experiments have been used to verify the above HOS-based IM fault detection techniques. Two identical IM have been used to compare the results with one being healthy and the other one being faulty with rotor broken bars.

#### 3.1. BRB fault

It has been evaluated that rotor failures account about 10% of IM faults [3,4]. The objective of MCSA is to compute a frequency spectrum of the stator current and to recognize the frequency components which are distinctive of rotor faults. Eq. (10) gives the frequency components which are characteristic of BRB [3,4]:

\[
f_{\text{BRB}} = \left[ \left( \frac{k}{p} \right)(1-s) \pm s \right] f_s
\]

(10)

where \( f_s \) is the supply frequency, \( p \) is the number of pole pairs, \( k \) is an integer and \( s \) is the rotor slip. By considering the speed ripple effects, it was reported that other frequency components, which can be observed in the stator current spectrum, are given by [3,4]:

\[
f_{\text{BRB}} = (1 \pm 2ks)f_s
\]

(11)

The side-band frequency components at \((1 \pm 2)f_s\) have been used to detect such rotor faults. While the lower side-band is fault-related, the upper side-band is due to consequent speed oscillations. It has been shown that the sum of magnitudes of these two side-band frequency components is a good diagnostic index given by [4]:

\[
l_{db} = \left[ 20 \log_{10}\left( \frac{l_l}{I_f} \right) + 20 \log_{10}\left( \frac{l_u}{I_f} \right) \right]/2
\]

(12)

where \( l_l \), \( l_u \) and \( f \) are the amplitude of the lower, upper side-band frequency components and of the fundamental frequency of the stator current, respectively.

#### 3.2. Model of the BRB stator current

A simple model which characterizes the IM stator current, with electrical rotor asymmetries includes the so-called side-band frequencies and it can be expressed by [11]:

\[
i_d(t) = i_l \cos(\omega t - \varphi) + \sum_k i_{kr} \cos((\omega + \omega_{kr})t - \varphi_{kr})
\]

\[
+ \sum_k i_{kj} \cos((\omega + \omega_{kj})t - \varphi_{kj})
\]

(13)

with \( f_s \) as the fundamental frequency of the power grid.

\( i_l \) is the fundamental value of the stator current amplitude (index \( l \)), \( \omega \) is its angular frequency, \( \varphi \) is its main phase shift angle, \( \omega_{kr} = 2k\omega_{s} \), and \( i_{kr}, i_{kj} \) and \( \varphi_{kr}, \varphi_{kj} \) are the magnitude and the phase of the left(index \( k \) left) and right (index \( r \)) sidebands components, respectively.

As \( \omega_{kr} = 2k\omega_{s} \) and \( \omega = 2nf_s \), the expression (13) can be rewritten as:

\[
i_d(t) = i_l \cos(2nf_s t - \varphi) + \sum_k i_{kr} \cos(2nf_s t - \varphi_{kr})
\]

\[
+ \sum_k i_{kj} \cos(2nf_s t - \varphi_{kj})
\]

(14)

where \( f_{kr} = (1 - 2sf_s) \) and \( f_{kj} = (1 + 2sf_s) \).

This simplified expression of the stator current signal has been used extensively for IM fault condition monitoring. By checking the magnitude of the side-band frequency components through the spectrum computation, various faults such as rotor bar breakage can be detected with a high level of accuracy [19].

From Eq. (14), the FT can be computed:

\[
l_d(f) = \frac{i_l}{2} \delta(f - f_s) e^{j\omega t} + \frac{1}{2} \sum_k i_{kr} \delta(f - f_{kr}) e^{j\omega_{kr}}
\]

\[
+ \frac{1}{2} \sum_k i_{kj} \delta(f - f_{kj}) e^{j\omega_{kj}}
\]

(15)

where \( \delta(\cdot) \) represents the Dirac delta function.

By ignoring contributions at negative frequencies which fall outside the useful region of the bi-spectrum, Eq. (15) can be written as:

\[
l_d(f) = \frac{i_l}{2} \delta(f - f_s) e^{j\omega t} + \frac{1}{2} \sum_k i_{kr} \delta(f - f_{kr}) e^{j\omega_{kr}} + \frac{1}{2} \sum_k i_{kj} \delta(f - f_{kj}) e^{j\omega_{kj}}.
\]

(16)

If the expression (16) is substituted in (7), the bi-spectrum becomes:

\[
B(f_1, f_2) = l_d(f_1) l_d(f_2) l_d^*(f_1 + f_2)
\]
The time domain signals considered are the stator current and its FT which are corrected for the window length to give the frequency domain operation. By using FT values in Amperes (A) in Eq. (17), bi-spectrum values are normalized (divided by the bi-frequency domain operation. By using FT values in Amperes (A) in its FT which are corrected for the window length to give the current for three broken bars at rated load. For generating this 3.3. Numerical simulation

To evaluate the bi-spectrum performance, a numerical simulation has been performed. This signal is similar to the measured current for three broken bars at rated load. For generating this signal coming from Eq. (14) for k=1, a sampling frequency of 128 Hz, a data length by segment of 1024 and M=6 (total length 6144) have been used. In addition, a Hanning window has been applied to the data to reduce spectral leakages due to the selection of the parameters in signal generation and computation. In this way, this simulation will also help to determine the analysis parameters for experimental stator currents:

\[
i_{st}(t) = i_1 \cos(2\pi f_1 t - \phi_1) + i_{l1} \cos(2\pi f_{l1} t - \phi_{l1}) + i_{r1} \cos(2\pi f_{r1} t - \phi_{r1})
\]  

(18)

where \( i_1 = 0 \) dB = 1A, \( i_{l1} = 80.1 \) dB = 114.8 \( \mu \)A and \( i_{r1} = -78.8 \) dB = 98.9 \( \mu \)A, \( f_1 = (1-2) f_s = 47.8 \) Hz and \( f_{l1} = (1+2) f_s = 52.2 \) Hz, \( f_1 + f_{l1} = 2 f_s \) and random phases \( \phi, \phi_{l1}, \phi_{r1} \) with a uniform distribution between 0 and 2\( \pi \).

Fig. 1 shows the different simulation results of the time domain signal given by (18), its PS and its bi-spectrum. Hence, the analysis can take into consideration only a signal non-redundant bi-spectral region [1,2].
with $f_1 = f_2 = f$ and given by:

$$D(f) = \frac{i_3^2}{8} \delta(f - f_3)e^{j\phi_3} + \sum_{k} \frac{i^2_k}{8} \delta(f - f_{1,k})e^{j\phi_{1,k}} + \sum_{k} \frac{i^2_k}{8} \delta(f - f_{r,k})e^{j\phi_{r,k}}$$

(19)

### 3.4. Experimental set-up

The tested three-phase squirrel-cage IM (Fig. 2) is used in measuring stator current signals of two three-phase 18.5 kW IM. The characteristics of the two three phase induction motors (healthy and faulty rotors) used in our experiment are listed in Table 2. One IM is undamaged (0BRB) and it is defined as the reference condition whereas the other one is with a synthetic rotor fault with one broken rotor bar (1BRB) or three broken rotor bars (3BRB) at different load levels. The synthetic rotor fault was obtained by drilling a small hole of 3-mm diameter in all the rotor bar depth. The load has been simulated by a magnetic brake which is coupled between the two machines shafts used to create the desired shaft load of operation.

For experiments, one current sensor of 20 kHz frequency bandwidth is used. The analog signals are passed through a low-pass anti-aliasing filter with a cut-off frequency of 2 kHz. The current signals are collected at a sampling frequency of 16,384 Hz for all the experiments by means of a 12-bit A/D converter. The measurements were carried out for different load conditions starting from 20% up to 100% of the rated torque. In order to minimize the FFT leakage effect, the Hanning window has been applied. The further signal processing is performed by using the MATLAB® environment in order to generate power spectra and bi-spectra and the LabView® software for the data acquisition.

### 3.5. Data analysis

The PS and the bi-spectrum of one phase current have been computed by using a 1024-point DFT length, a Hanning smoothing window and no overlap. Moreover, computations have been performed by using the formulas (3) and (8). As a first approach
The PS of the IM stator current has been represented in the frequency bandwidth [0 Hz, 1000 Hz] for the healthy case, for 1BRB, and for 3BRB. As expected, many frequencies have been detected because of the large ratio of signal-to-noise (SNR) being around 150 dB. Then, the same cases have been investigated by using the bi-spectrum in the frequency bandwidths [0 Hz, 60 Hz] (Fig. 4). The total number of samples in each measurement was 
\[ N = T_{\text{acq}} f_e \] with a sampling frequency of 16,384 Hz (\( \Delta t = 61 \mu s \)).

Frequency spacing between successive samples of the DFT called the frequency resolution has been 
\[ \Delta f = 0.125 \text{ Hz} \] corresponding with an acquisition time 
\[ T_{\text{acq}} = 8 s \] at rated load (\( s = 2.4\% \)). All the frequency component magnitudes have been normalized and expressed in dB scale with respect to the fundamental frequency of the power grid set at 0 dB (1 pu).

There are some limitations on the amount of information which can be extracted from spectral analysis. A spectral peak at a particular frequency may come from several possible faults. A rotating component may produce a number of spectral peaks at harmonics of the rotational frequency and they can have a constant phase which cannot be detected by the PS. These PS limitations have led to use other diagnostics techniques such as the bi-spectrum to extract the possible non-linear characteristics of the signal. This concept will be discussed in the following section.

The data coming from the stator current measurement have been sampled at the frequency 16,384 Hz which leads to a data set with 
\[ N = 131,072 \] samples for each variable. The spectra of motor current signals at various loads (0%, 25%, 50%, 75%, and 100% of rated load) and at different numbers of broken bars (0BRB, 1BRB, and 3BRB), are shown in Fig. 5. Indicate that the amplitude of the \( (1 \pm 2s) f_e \) components are noticeable and increase with fault severity. Indeed, in the case of a healthy machine, apart the fundamental frequency of the power grid, it is expected that all the other frequency components will have small magnitudes compared to faulted cases.

4. Experimental results

In this section, both PS and BDS analysis will be used to distinguish the fault severity in an IM with BRB at any level of shaft load. In fact, it is very important to detect BRB at low load level since it is well known that the influence of any rotor fault is reduced when the shaft load is low due to the fact that the rotor currents are reduced. Therefore, at low load levels, the influence of rotor currents on the stator side is reduced and the detection becomes more difficult.

4.1. PS analysis
with the fault severity and the higher the load is the larger is the fault visibility. It can be observed that at no load when the slip is too small, the fault-related frequency components have overlaps with the fundamental frequency coming from the grid. Otherwise, as the motor becomes more heavily loaded, the rotational frequency slows and the slip frequency increases. The greater the slip, the greater is the frequency separation observed between \((1 \pm 2ks) f_s\) and the fundamental frequency. The lighter the load, the larger the ratio between line frequency amplitude and that of the \((1 \pm 2ks) f_s\) sideband; especially, as load moves below 50% of full load. From 50% to 100% load this effect is less significant.

4.2. Bi-spectrum analysis

In order to provide a higher resolution in the sampled signal, the data is divided into \(K\) segments \((K=4)\) according to the step of the bi-spectrum estimation. The overlapping was zero for two consecutive segments with two different fault conditions such as for healthy and for one BRB mode.

The three-dimensional graph of the bi-spectrum is difficult to be described and to be analyzed (Fig. 4). However, the BDS has been computed for both healthy and faulty modes (Fig. 6). It has been shown that the BDS method gives similar results compared to PS computed for the same cases at different shaft load levels. However, the detection is even smoother than in the case of PS and it is much more visible. This is due to the fact that the bi-spectrum can effectively filter out the Gaussian noise [9].

Comparing the BDS results plotted in Fig. 6, reveals that there are clear differences in the \((1 \pm 2ks) f_s\) sideband component values between the healthy and faulty at different load conditions. Unlike the PS-based method, the proposed BDS-method determines the BRB fault at absolute no-load condition.

The different magnitudes of frequency components related to BRB have been computed in the three previous cases (healthy, 1BRB, 3BRB) by using the formula (12) for the average value \(I_{av}\) (Fig. 7). It can be seen that even when the machine is operating at no load the healthy rotor can be clearly distinguished from the faulty case (this difference is indicated by dashed circles in Fig. 7).

The BDS technique discussed is characterized by a higher sensitivity than the PS, especially when the machine is operating at no-load.

In the next section, we present the detection performance evaluation results by comparing the PS-based method and our proposed BDS-based method using ROC curves, which make the results more convincing.

4.3. Detection performance evaluation: Need for ROC analysis

When comparing two or more diagnostic tests, ROC curves are often the only valid method of comparison [26]. An ROC curve is a detection performance evaluation methodology, and demonstrates how effectively a certain detector can separate two groups in a quantitative manner [26–28]. An ROC curve shows the trade-off between the probability of detection or true positives rate \((tpr)\), also called sensitivity and recall versus the probability of false alarm or false positives rate \((fpr)\). ROC curves are well described by Fawcett in [26]. The \(tpr\) and \(fpr\) are mathematically expressed in (20) and (21), respectively.

\[
tpr = \frac{\text{true positives}}{\text{true positives} + \text{false negatives}} \tag{20}
\]

\[
fpr = \frac{\text{false positives}}{\text{false positives} + \text{true negatives}} \tag{21}
\]
Additional term associated with ROC curves is,

\[ \text{specificity} = \frac{\text{true negatives}}{\text{false positives} + \text{true negatives}} = 1 - fpr \]  \hspace{1cm} (22)

\[ \text{sensitivity} = \text{recall} \]  \hspace{1cm} (23)

For each case (Healthy, 1BRB and 3BRB) and for each load level condition (0%, 25%, 50%, 75% and 100% of rated load), a series of 10 independent Monte-Carlo experiments are conducted. For each experiment, the probability of false alarm and the probability of detection are obtained by counting detection results at each sideband frequency components \((1 \pm 2\sigma_f)\), amplitude out of 150 independent Monte-Carlo experiments by both the PS-based standard method and our proposed BDS-based method.

Each point on the ROC curve corresponds to a specific pair of sensitivity and (1-specificity) and the complete curve gives an overview of the overall performance of a test. When comparing ROC curves of different tests, good curves lie closer to the top left corner and the worst case is a diagonal line (shown as a dashed line in Fig. 8).

Fig. 8 shows the ROC curves of the PS-based method and our BDS proposed method. The two curves are obviously not identical; detection algorithm based on BDS-method is better than PS-method, in the
The precision-recall (PR) graph \cite{28}. The PR measures the fraction classified as positive that are truly positive.

\[ tpr = \frac{TP}{TP + FN} \]

is almost not affected by the increasing \( tpr \). Another quantitative measure of the ROC curve performance is the precision-recall (PR) graph \cite{28}. The PR measures the fraction of positive examples that are correctly labeled. In PR space, one plots recall on the \(-y\)-axis and precision on the \(-y\)-axis. Recall is the same as \( tpr \), whereas precision measures that fraction of examples classified as positive that are truly positive.

\[
\text{Precision} = \frac{TP}{TP + FP}
\]

Fig. 9 shows the PS curves, which indicate that the BDS detector is a relatively good detector in terms of PR graph, and illustrates how the precision of both methods change as the relative recall \( tpr \) increases. The precision of the PS-based method drops sharply beyond the \( tpr=0.14 \), but the precision of the BDS-based method is almost not affected by the increasing \( tpr \).

5. Conclusion

In this paper, a short overview of the bi-spectral analysis has been presented and a method using the HOS to detect BRB faults has been presented. Data from real tests have been used to express the ability of the bi-spectrum and more exactly the DSB analysis and its associate phase for condition monitoring of three-phase IM with rotor faults at no-load. The form of bi-spectral slice has inside information about the phase and the non-linear system characteristic. Therefore, it can distinguish in between many operating conditions even with low influence in the PS.

The robustness of the proposed method is explored by running a series of independent Monte-Carlo experiments. The evaluation of results using ROC curves makes the results more convincing. By using the bi-spectral analysis for IM condition monitoring, the sensitivity of fault detection can be easily improved without changing the data acquisition. Alternatively, the Wigner bi-spectrum and the wavelet bi-spectrum can be used for condition monitoring in non-stationary cases met during transients and in many real applications.

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