The Infrared Behavior of Propagators in Landau Gauge QCD

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Abstract

A closed system of equations for the propagators of Landau gauge QCD is obtained in a truncation scheme for their Dyson–Schwinger equations which implements the Slavnov–Taylor identities for the 3-point vertex functions while neglecting contributions from irreducible 4-point correlations. In the pure gauge theory without quarks, non-perturbative solutions for the gluon and ghost propagators are available in an approximation which allows for an analytic discussion of their behavior in the infrared: The gluon propagator vanishes for small spacelike momenta, whereas the ghost propagator is found to be infrared enhanced. The running coupling of the non-perturbative subtraction scheme approaches the finite value $\alpha_c \simeq 9.5$ at an infrared fixed point. The gluon propagator entails a violation of positivity for transverse gluon states implying their absence from a positive subspace expected for asymptotic hadronic states and thus confinement. Both propagators, obtained for gluons and ghosts in the present scheme, compare well with recent lattice calculations.

In the quenched approximation, the quark propagator describes dynamical chiral symmetry breaking well, although the corresponding interaction in the gap equation for the quark self-energy is infrared suppressed. First results of a simultaneous solution to the coupled system of gluons, ghosts and quarks indicate towards weak, and possibly negligible, vacuum polarisation effects of dynamical quarks on the gluon and ghost correlations in the infrared.

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1 Introduction

Regarding its phenomenological implications on hadron physics \(^1\) and on ultra-relativistic heavy ion collisions, the infrared behavior of QCD Green’s functions comprises the essential information to describe non-perturbative phenomena. This infrared behavior should not only reflect confinement and chiral symmetry breaking, but also the respective deconfinement and chiral transition(s) at finite temperature and density. In Landau gauge QCD at zero temperature and density, our knowledge of the elementary correlations, the gluon, ghost and quark propagators, in the infrared has recently made progress. Corresponding solutions of truncated Dyson–Schwinger equations (DSEs) \(^2, \ 3, \ 4, \ 5\) and presently available results from lattice simulations \(^6,\ 7,\ 8\) now seem to yield a quite coherent picture. Models for these propagators at finite temperature and density to describe the deconfinement and chiral transitions are currently being developed.\(^9\)

An important issue, remaining to be addressed, is the interrelation of the different models of confinement that arise in different gauges. Unavoidable gauge ambiguities in gauges such as the Polyakov or the maximal Abelian gauge seem to suggest pictures ranging from monopole condensation and the dual Meissner effect to the dominance of center vortices which might percolate at the deconfinement transition.\(^10\) The connection to the linear Coulomb or covariant gauges might be provided by the Gribov ambiguity which suggests the infrared dominance of the Coulomb field or the ghost correlations respectively.\(^11\) The deconfinement transition seems harder to understand, but the different realizations of the center symmetry should play an obvious role in these gauges also.

Recently, a complete truncation scheme for the 2–point Green’s functions of Landau gauge QCD, i.e., the gluon, ghost and quark propagators, was established employing the present knowledge on the structure of the 3–point vertex functions. This is possible by systematically neglecting all explicit contributions from 4–point vertices to the propagator DSEs as well as in the constructions of the 3–point vertices. The truncated set of DSEs for the gluon and ghost propagators of the pure gauge theory was solved simultaneously in a one–dimensional approximation.\(^2, \ 3\) The analytically extracted infrared behavior of the solutions contradicts earlier solutions to the gluon DSE which relied on neglecting ghost contributions completely.\(^12, \ 13\) While the gluon propagator of the coupled system vanishes for small spacelike momenta, the apparent contradiction with the earlier DSE studies can be understood from the observation that the previously neglected ghost propagator now assumes an infrared enhancement similar to what was then obtained for the gluon. These results, in particular, the infrared dominance of ghost correlations in Landau gauge, were
later confirmed qualitatively, by studies of a further truncated set of DSEs, neither to depend on the particular constructions of the 3–point vertices nor to rely on the one–dimensional approximation. A simultaneous solution to the propagator DSEs that includes the quark propagator in the same scheme is presently under way. In the quenched approximation, the solution to the gluon–ghost system gives rise to an effective interaction in the gap equation for the quark–self energy which is suppressed in the infrared as compared to a massless gluon exchange. Nevertheless, dynamical chiral symmetry breaking can be described quite well. This result is qualitatively different already from the existing models of chiral symmetry breaking which basically all use interactions equally strong or stronger than the massless gluon–pole in the infrared.

Preliminary results obtained for the coupled system of ghost, gluon and quark propagators indicate towards little influence of the dynamical quark–loops on the gluon and ghost propagators in the infrared. In particular, the infrared dominance of ghost correlations seems to remain unaffected.

2 The set of truncated gluon and ghost DSEs

Besides all elementary 2–point functions, i.e., the quark, ghost and gluon propagators, the DSE for the gluon propagator also involves the 3– and 4–point vertex functions which obey their own DSEs. These equations involve successively higher n–point functions and are neglected in the present scheme.

The ghost and the transverse gluon propagator, $D_{gh}(k) = -G(k^2)/k^2$ and $D^T_{gl}(k) = Z(k^2)/k^2$ respectively, are parameterized by two invariant functions $G$ and $Z$. In order to arrive at a closed set of equations for these functions, first, a Slavnov–Taylor identity (STI) for the ghost–gluon vertex was derived from the usual Becchi–Rouet–Stora invariance. This STI can be resolved to express the vertex in terms of $G$ and $Z$ additionally employing the hermiticity of ghosts in Landau gauge, and thus the symmetry of the vertex, while neglecting irreducible 4–ghost correlations herein, in accordance with the truncation of the propagator DSEs. Such constructions are generally not unique. Undetermined transverse terms in this vertex can be neglected also, however, as they correspond to a non–trivial ghost–gluon scattering kernel. These transverse terms will become important in going beyond the present truncation scheme. Interesting constraints on their form arise from next–to–leading order perturbative results. Without explicit 4–gluon vertices the present scheme is incomplete at 2–loop level anyway, however.

With the particular ghost–gluon vertex at hand, the construction of the 3–gluon vertex follows standard procedures. Its full Bose symmetry is manifest
and additionally possible transverse terms are subleading in the infrared as well as in the perturbative limit.

An angle approximation has to be employed in the numerical solution of the resulting system of coupled integral equations representing the truncated gluon–ghost DSEs. The infrared behavior can then be discussed analytically, the leading asymptotic form being \( G \sim (k^2)^{-\kappa} \) and \( Z \sim (k^2)^{2\kappa} \) with \( \kappa \simeq 0.92 \) for \( k^2 \to 0 \). This leading behavior of the gluon and ghost renormalization functions and thus of their propagators is entirely due to ghost contributions. Qualitatively similar results were obtained using a bare ghost–gluon vertex and neglecting the 3–gluon loop completely. Compared to the Mandelstam approximation without ghosts, in which the 3–gluon loop alone determines the infrared behavior of the gluon propagator and the running coupling in Landau gauge, this shows the importance of ghosts. In contrast to the infrared singular coupling obtained from the Mandelstam approximation, the result of the coupled gluon–ghost system implies an infrared stable fixed point in the running coupling of the non–perturbative subtraction scheme, defined by

\[
\alpha_S(s) = \frac{g^2}{4\pi} Z(s) G^2(s) \to \frac{16\pi}{9} \left( \frac{1}{\kappa} - \frac{1}{2} \right)^{-1} \approx 9.5 , \quad \text{for } s \to 0 .
\]

The Euclidean gluon propagator of the present scheme violates Osterwalder–Schrader reflection positivity. It thus precludes transverse gluon states to exist in the positive asymptotic Hilbert space of physical states. This is interpreted as a manifestation of gluon confinement.

3 Comparison to lattice results

The solutions to the coupled gluon–ghost DSEs compare quite well to recent lattice results. These use various lattice implementations of the Landau gauge condition to simulate the gluon and the ghost propagator. There are clear indications towards an infrared vanishing gluon propagator to be seen on the lattice also. Especially the ghost propagator, however, is in compelling agreement with the lattice data at low momenta. We therefore conclude that present lattice calculations confirm the infrared dominance of the ghost correlations in Landau gauge. The evidence for a violation of positivity for transverse gluons, as observed on the lattice, has also tremendously increased recently.

In order to compare the running coupling of the present scheme to lattice results as extracted from simulations of the 3–gluon and the quark–gluon vertex one has to account for the details in the definitions of the different schemes. The asymmetric schemes employed in most lattice calculations have thereby the potential problem that the infrared divergent ghost correlations
lead to artificial infrared suppressions in the momentum dependence of the asymmetrically defined couplings.

The ideal lattice calculation to compare to the present DSE coupling would be obtained from a pure QCD calculation of the ghost–gluon vertex in Landau gauge with a symmetric momentum subtraction scheme.

4 Quark Propagator

We have solved the quark DSE in quenched approximation. The quark–gluon vertex therein obeys its non–Abelian STI with the quark–gluon scattering kernel neglected. As a result, it explicitly contains a ghost renormalization function, \( \Gamma^\mu(p, q) = G(k^2) \Gamma^\mu_{CP}(p, q) \) where \( \Gamma^\mu_{CP} \) is the Curtis–Pennington vertex. In contrast to a naive Abelian approximation, this effectively leads to an infrared vanishing coupling \( \sim (k^2)^{2\kappa} \) in the quark DSE. The resulting interaction is peaked around a finite scale \( \mu \approx 200\text{MeV} \). In the infrared it is weaker than a massless gluon pole. This allows, e.g., to use the Landshoff–Nachtmann model for the pomeron in our approach. From the solution we estimate a pomeron intercept of approximately 2.7/GeV as compared to the typical phenomenological value of 2.0/GeV.

Besides this infrared suppressed interaction dynamical chiral symmetry breaking is manifest in the quenched approximation. Using a current mass of \( m(1\text{GeV}) = 6\text{MeV} \) we obtain a constituent mass of approximately 170 MeV. In the Pagels–Stokar approximation this yields approx. 50 MeV for the pion decay constant. These numbers are quite encouraging, especially, since with the momentum scale fixed from the running coupling at the Z–mass no parameters other than the current quark mass were adjusted.

Considering the quark loop in the gluon DSE one realizes that the quark loop will produce an infrared divergence which is, however, subleading as compared to the one generated by the ghost loop. In the latter there appear three ghost renormalization functions in the numerator and one in the denominator leading effectively to an infrared divergence of the order \( (k^2)^{-2\kappa} \). In the quark loop term there is only one factor \( G \), and thus a divergence of type \( (k^2)^{-\kappa} \) is anticipated. It can be shown, however, that after ultraviolet regularization and renormalization the coefficient of this infrared divergent term vanishes. Preliminary results indicate that the influence of the quark loop on the coupled system of gluons and ghosts is almost negligible, i.e. the propagators for gluons, ghosts and quarks are nearly indistinguishable from their quenched counterparts.
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8. Attilio Cucchieri, this conference.
9. See the contributions by Pieter Maris, Craig Roberts, Sebastian Schmidt, and Peter Tandy to this conference.
10. See, e.g., Hugo Reinhardt, this conference.
16. It was incorrectly asserted in Ref. 3 that undetermined transverse terms do not occur in this construction of the ghost–gluon vertex.