Selection of funding schemes by a borrowing decision model: a Hong Kong case study

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In financial decision-making, a number of mathematical models have been developed for financial management in construction. However, optimizing both qualitative and quantitative factors and the semi-structured nature of construction finance optimization problems are key challenges in solving construction finance decisions. The selection of funding schemes by a modified construction loan acquisition model is solved by an adaptive genetic algorithm (AGA) approach. The basic objectives of the model are to optimize the loan and to minimize the interest payments for all projects. Multiple projects being undertaken by a medium-size construction firm in Hong Kong were used as a real case study to demonstrate the application of the model to the borrowing decision problems. A compromise monthly borrowing schedule was finally achieved. The results indicate that Small and Medium Enterprise (SME) Loan Guarantee Scheme (SGS) was first identified as the source of external financing. Selection of sources of funding can then be made to avoid the possibility of financial problems in the firm by classifying qualitative factors into external, interactive and internal types and taking additional qualitative factors including sovereignty, credit ability and networking into consideration. Thus a more accurate, objective and reliable borrowing decision can be provided for the decision-maker to analyse the financial options.

Keywords: Construction firm, genetic algorithm, loan and finance, optimization

Introduction

Comparing to the other industries in Hong Kong, the construction industry generally experience a greater number of bankruptcies since the financial crisis of 1997. One of the major reasons of this phenomenon is that the financial resources are insufficient, or even worse, fail to persuade creditors (Kaka and Price, 1993; Russell, 2003). Financial decisions are some of the most important decisions in the investment management process. They involve the capital structure, leasing or borrow-and-buy decision as well as dividend decisions. The primary objective of financial management in a firm is to set and manage the capital budgeting practices employed (Padberg and Wilczak, 1999). Firms with low gearing should borrow to finance their investment schedules because interest payable on borrowing is tax deductible (Pike and Dobbins, 1986). Myers (1977) showed that firms with excessive debts might gain efficient investments opportunities as creditors are the main recipients of the cash flow in the future. However, to ensure the survival of the firm, borrowing needs to be carefully monitored and evaluated and over-borrowing should not be approved. Construction managers should also have an optimal borrowing schedule and strategies for emergencies to eliminate the financial distress suffered by the shareholders. Thus the possibility of bankruptcy can be minimized or avoided. In financial decision-making, a number of mathematical models have been developed for construction financial management problems (Hanssmann, 1968; Jennergren, 1990).

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However, less than 30% of respondents (building contractors) claim that their firms used financial modeling tools for managing their projects (Lam et al., 2001c, 2002). As many businesses have grown significantly and the environment has become more unpredictable, a borrowing decision model is required to provide the decision-maker with a tool kit for analysing his financial decisions.

In the study described in this paper, it aims at optimizing multi-project loans acquisition in medium-size construction firms in Hong Kong that represents 45% of the industry (Group B contractors for contracts of value up to $50 million in the list of approved contractors for public works) at maximum efficiency. This will enable these arrangements for acquiring funds from outsiders to be made well in advance to avoid the possibility of financial problems incurred in the firm. It has the following three objectives:

1. To investigate the coupling of a qualitative factor with fuzziness to a deterministic quantitative variable;
2. To improve a multi-objective decision model for multi-project borrowing decisions, undertaken in the fuzzy environment of a medium-size building contractor in a Hong Kong case study; and
3. To select suitable funding schemes by the improved borrowing optimization model with an adaptive genetic algorithm (AGA) approach.

The paper starts with a literature review on the application of artificial intelligence in finance. In particular, a borrowing decision model is then presented followed by its evaluation by a case study. The borrowing decision model developed by Lam et al. (1998) was improved by classifying the qualitative factors into external, interactive and internal types and taking additional qualitative factors including sovereignty, credit ability and networking into consideration. An AGA approach was then applied to the search for sources of funding schemes and a compromise solution of the monthly borrowing schedule of medium-size projects. Finally, results of the monthly borrowing schedule and the AGA are illustrated and analysed.

### Previous studies

#### Artificial intelligence in finance

There has been considerable progress in bridging the gap between finance and mathematics. Fuzzy engineering principles could be useful to enhance the scope of financial analysis (Chorafas, 1995). Boussabaine and Elhag (1999) introduced the fuzzy logic method to the cash flow outline. Most real world decision problems combine both qualitative and quantitative variables. According to Kenley and Wilson (1986), individual variations between projects are actually caused by a multiplicity of factors, such as economic and political climate, managerial structure, and personality conflicts. Conventional mathematics that combines qualitative and quantitative variables has difficulty in modelling real world construction problems. Having studied financial modelling for many years, Lam et al. (1998) found that three factors (loans, interest payments and other qualitative factors) could be optimized in a fuzzy content after fuzzy multiple-objective linear programming had been introduced into the development of the model due to the fact that the data applied in constraints of this type were unclear. They suggested that variables such as the period cash requirement, allowable-tax-shield, capital gearing ratio and borrowing limit restricted the borrowing model to a certain extent. In the pre-construction phase of recent projects, this optimization model provided a useful way to review the optimization factors, variables, constraints and other qualitative factors. However, assumptions have been made to the absence of difficulty in acquiring funds from the credit institutions and thus the input variables of qualitative factors were not well-defined.

Optimization techniques such as heuristic method (Son and Skibniewski, 1999), fuzzy multi-objective linear programming (Lam and Runeson, 1999), fuzzy neural network techniques (Bhokha and Ogunlana, 1999) and genetic algorithms (GAs) (Lam et al., 2001b) have been commonly used in the construction field to solve construction management problems. Besides, there has been a lot of improvements in the optimization techniques which include the integration of fuzzy reasoning technique and fuzzy optimization method (Lam et al., 2001d), using novel recurrent-neural-network (Lam et al., 2001a) and the adoption of AGA (Lam et al., 2001b) to solve the non-linearity problem. However, optimizing both qualitative and quantitative factors and the semi-structured nature of construction finance optimization problem are still key challenges in solving construction finance decisions. Because such an optimization process involves fuzzy variables and multiple-objective decisions, the objectives are in a linguistic form (variables) and the constraints are fuzzy in nature for most of the decision-making process.

The GAs, developed by Holland in 1960s and 1970s, simulate Darwin-type natural evolution (Holland, 1975). GAs are powerful and largely applicable stochastic search techniques, that can search large and complex spaces on the ideas from natural genetics of a population and natural selection (Davis, 1991) in a...
number of applications (Colin, 1994). These algorithms have been successfully applied to a number of areas of construction project management, such as construction resource scheduling problems (Alcaraz and Maroto, 2001); the time-cost trade-off problem (Feng and Liu, 1997); especially in the financial application in budget allocation, portfolio selection, bankruptcy prediction and credit evaluation (Shin and Lee, 2002). However, financial decision modelling is still ill-defined and the optimization of multi-project loan acquisition problems involving fuzziness and imprecision have not been modelled and solved using a mathematical approach.

Modelling of the borrowing decision

A multiple-objective borrowing model was developed to assist the decision-maker in deciding upon suitable sources of funding. The model as shown in the Appendix was based on a mathematical model developed by Lam et al. (1998). However, it was based on the assumptions that contractors have no problems in acquiring funds from financial institutions and that contractors only depend on self-assessments of financial capability in acquisition of funds. There was deficiency in the input of qualitative variables. A further study involving a novel and realistic mathematical model for the borrowing decision problem in fuzzy environments is proposed. Figure 1 shows the procedures for modelling the construction borrowing decision. The procedure consists of two phases: a) modelling phase and b) application phase. In the modelling phase, objective functions are set. The amount and source of finance are the variables in the study. Let \( X_{i,n} \) be the amount of \( i^{th} \) loan source in the \( n^{th} \) working month. Interest rates are the key factor that determines \( X_{i,n} \) in the views of lenders and borrowers. Let \( r_i \) be the interest rate for the \( i^{th} \) source. The interest rate encountered includes the prime lending rate of the lending institutions. The repayment term can be monthly, half-yearly or yearly depending on the agreement terms between the lender and borrower. The basic objectives of the model are thus to maximize or minimize the discounted loan and to minimize the discounted interest payments for all projects. To

![Figure 1](image-url)  
Modelling of the construction borrowing decision
determine whether a loan is maximized or minimized, the tax-shield and profit conditions of a firm are considered. If the existing tax-shield is greater than the allowable one and the amount of payable interest incurred through borrowing plus the capital allowable is smaller than the gross profit in a year, the loan is maximized.

Selection of funding schemes is based on the qualitative contributors rather than the quantitative contributors. The remaining objectives are to maximize or minimize qualitative factors. The type of qualitative factor determines whether it is maximize or minimized. In this study, the qualitative factor for the borrowing decision model is considered in terms of the chance of having an unfavourable source of finance. Thus the third objective is to minimize this risk. There are numerous factors such as the track record of the borrower, the style of the lender, the type of loan and agreement terms, etc. With the professional knowledge and experience of the experts in the top level management team, the factors under considerations are divided into three categories: external, interactive, and internal. Different weights are assigned by the experts to each category respectively. Since the estimation of parameters in the constraints (e.g. period cash requirements, tax-shields and the capital gearing ratio) are fuzzy in nature in the forecasting stage, a fuzzy reasoning technique is applied in the model. $X_{i,n}$ in the borrowing model that are then solved by the AGA in the application phase.

### Case study

#### Problem background

A medium-size building contracting firm had three ongoing projects and a fourth project that was newly granted. The four projects required similar resources in terms of labour, plant, materials and management expertise. The four projects differ in sizes, profit margins and resource-mix proportions. The payment certificate was issued 21 days after the measurement with further 21 days for the payment. In order to simplify the calculation, a two-month delay in the payment was applied in this case study. According to the conditions stated in the contracts, some 1.5% of the retention money was deducted from the interim payment each month. Table 1 shows the details of the case study projects.

External financing can be classified into short and long-term loans in terms of satisfying the specific needs of borrowers. Furthermore, term loans and overdrafts have been the main source of finance in the construction industry. Small and medium enterprises (SMEs) are the backbone of Hong Kong’s economy (HKSAR, 2002). In 1998, the HKSAR Government launched the SME Loan Guarantee Scheme (SGS) for SMEs to help them in acquiring the installations and equipment they need to enhance their productivity and competitiveness. To enhance the effectiveness of the scheme, the Government has from the end of March 2003, increased the maximum amount of loan guarantee. Totally, 48 banks have joined the SGS to provide SMEs with extra cash to acquire capital. SGS has become the other source for financing corporations. The choice between short and long-term loans is influenced very much by the fluctuating period cash requirement, as illustrated in Figure 2. A fixed cash requirement should be financed by long-term funds since it can be accounted for in the long-term corporate cash flow. Conversely, temporary increase in cash requirement, which fluctuates through the cash cycle of the corporation, should be financed by short-term funds.

Ten sources of finance, coming from six different financial institutions, were allowed in satisfying a contractor’s borrowing requirements as shown in Table 2. This is because different borrowing and lending rates for the financing decisions are realistic (Padberg and Wilczak, 1999). The interest rate that banks charge to their preferred customers is the prime rate (P). P equals 5% at the time of study. The first source provided an upper limit of 1 million with interest being the P plus 1.0%. Sources 2 and 3 were granted from the same financial institution, and involved a two-year, medium-term loan of 15 and 20 million respectively and interest calculated as the P plus 1.5%. The schedule was jointly agreed upon by the bank and the contractor to follow the schedule of corporate cash

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### Table 1  Details of the case study projects

<table>
<thead>
<tr>
<th>Project type</th>
<th>Building (public)</th>
<th>Building (public)</th>
<th>Building (public)</th>
<th>Building+civil engineering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract sum, HK$</td>
<td>6 851 624</td>
<td>18 403 937</td>
<td>10 190 654</td>
<td>14 691 942</td>
</tr>
<tr>
<td>Contract duration, month(s)</td>
<td>9</td>
<td>21</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>Contract start, month</td>
<td>10th</td>
<td>3rd</td>
<td>5th</td>
<td>14th</td>
</tr>
<tr>
<td>Contract finish, month</td>
<td>19th</td>
<td>22nd</td>
<td>20th</td>
<td>27th</td>
</tr>
<tr>
<td>Profit margin, %</td>
<td>17.5</td>
<td>20.0</td>
<td>18.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>
flow, such that 50% of the loan was withdrawn at the beginning, and 30% and 20% of the loan would be withdrawn at stages 2 and 3, respectively. Sources 3, 4 and 5 of finance were negotiated between the bankers and the contractor with terms and interest rates as shown in Table 3.

SGS was used as sources 6 to 10 in this study. Two different banks were selected to provide the financial sources. Sources 6 to 8 were selected from the same bank with the same interest rate (P\(\text{\textsuperscript{2}}1.00\%\)). Sources 9 and 10 came from the other bank. Source 6 (Type 1 of SGS) was business installation and equipment loans assisting the company to buy business installations and equipment to enhance business efficiency. The loan is up to 6 million and tenor up to five years. Source 7 was associated working capital loan (Type 2 of SGS) to meet additional operational expenses arising from or in relation to the business installations and equipment.

**Table 2** Financial data for a contracting firm and the terms of loans from different sources

<table>
<thead>
<tr>
<th>Types</th>
<th>Term loans</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Loan terms</td>
<td>Overdraft</td>
<td>2 yrs</td>
</tr>
<tr>
<td>Upper limit</td>
<td>1m</td>
<td>15m</td>
</tr>
<tr>
<td>Lower limit</td>
<td>–</td>
<td>10m</td>
</tr>
<tr>
<td>Interest rate (=Prime rate +)</td>
<td>1%</td>
<td>1.5%</td>
</tr>
<tr>
<td>Stage 1</td>
<td>–</td>
<td>50%</td>
</tr>
<tr>
<td>Stage 2</td>
<td>–</td>
<td>30%</td>
</tr>
<tr>
<td>Stage 3</td>
<td>–</td>
<td>20%</td>
</tr>
<tr>
<td>Stage 4</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

**Note:** m in HK\$ million(s); – means no restrictions; prime rate=5%

**Figure 2** Overall cash flow
acquired/to be acquired under the SGS. The loan upper limit of source 7 is as high as 3 million. This kind of loan allows the choices of instalment loan and overdraft. Source 8 (Type 3 of SGS) was accounts receivable loan to meet the working capital needs arising from provision of credit terms to the applicant’s customers. For source 8, the contractor can obtain the loan amount of 3 million and its guarantee period is up to two years. About source 9 (Type 1 of SGS) and 10 (Type 2 of SGS), the interest rates are as low as P+2.5% and P+3% respectively. Also, the types of the loan terms are different. The former one is the instalment loan while the latter one is the unsecured overdraft.

An analysis of the financial position of the contracting firm showed that the capital structure was

Table 3  Input variables and items for qualitative factors in borrowing decision model

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Item for qualitative factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Weighting=0.3</td>
<td></td>
</tr>
<tr>
<td>Type of banker, IQV&lt;sub&gt;t&lt;/sub&gt; where t=1…3</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;1&lt;/sub&gt; Unpleasant: not easy to approach</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;2&lt;/sub&gt; Moderate: approachable</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;3&lt;/sub&gt; Pleasant: friendly with the customer</td>
<td></td>
</tr>
<tr>
<td>Sovereign, IQV&lt;sub&gt;u&lt;/sub&gt; where u= 1…3</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;21&lt;/sub&gt; Soft: soft government’s action(s) to prevent the repayment and/or the raising of funds from financial institutions</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;22&lt;/sub&gt; Mild: moderate government’s action(s) to prevent the repayment and/or the raising of funds from financial institutions</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;23&lt;/sub&gt; Strict: strict government’s action(s) to prevent the repayment and/or the raising of funds from financial institutions</td>
<td></td>
</tr>
<tr>
<td>Interactive Weighting=0.4</td>
<td></td>
</tr>
<tr>
<td>Relationship with the banker, IQV&lt;sub&gt;v&lt;/sub&gt; where v=1…5</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;31&lt;/sub&gt; Very low: less than 1 job in the past 5 years involving the same banker</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;32&lt;/sub&gt; Low: less than 3 and more than one job in the past 5 years involving the same banker</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;33&lt;/sub&gt; Medium: 3 to 5 jobs in the past 5 years involving the same banker</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;34&lt;/sub&gt; High: 5 to 10 jobs in the past 5 years involving the same banker</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;35&lt;/sub&gt; Very high: more than 10 jobs in the past 5 years involving the same banker</td>
<td></td>
</tr>
<tr>
<td>Long-term prospects, IQV&lt;sub&gt;w&lt;/sub&gt; where w=1…5</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;41&lt;/sub&gt; Very unfavourable: less than 7% chance of co-operation with the banker in the future 5 years</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;42&lt;/sub&gt; Unfavourable: less than 15% chance of co-operation with the banker in the future 5 years</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;43&lt;/sub&gt; Acceptable: 9 to 60% chance of co-operation with the banker in the future 5 years</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;44&lt;/sub&gt; Favourable: 40 to 80% chance of co-operation with the banker in the future 5 years</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;45&lt;/sub&gt; Most favourable: more than 60% chance of co-operation with the banker in the future 5 years</td>
<td></td>
</tr>
<tr>
<td>Past co-operation, IQV&lt;sub&gt;x&lt;/sub&gt; where x=1…5</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;51&lt;/sub&gt; Very poor: lack of communication and mistrust</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;52&lt;/sub&gt; Poor: limited communication and mistrust</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;53&lt;/sub&gt; Neutral: fair communication and normal</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;54&lt;/sub&gt; Good: sufficient communication and trust</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;55&lt;/sub&gt; Very good: excellent communication and trust</td>
<td></td>
</tr>
<tr>
<td>Credit ability, IQV&lt;sub&gt;y&lt;/sub&gt; where y=1…5</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;61&lt;/sub&gt; Very low debt capacity: over 30% of financial transaction delayed in the past 5 years with the counter party</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;62&lt;/sub&gt; Low debt capacity: 20–40% of financial transaction delayed in the past 5 years with the counter party</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;63&lt;/sub&gt; Medium debt capacity: 10–30% of financial transaction delayed in the past 5 years with the counter party</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;64&lt;/sub&gt; High debt capacity: below 20% of financial transaction delayed in the past 5 years with the counter party</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;65&lt;/sub&gt; Very high debt capacity: below 10% of financial transaction delayed in the past 5 years with the counter party</td>
<td></td>
</tr>
<tr>
<td>Networking, IQV&lt;sub&gt;z&lt;/sub&gt; where z=1…5</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;71&lt;/sub&gt; Highly inactive: very poor business performance</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;72&lt;/sub&gt; Inactive: poor business performance</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;73&lt;/sub&gt; Slight: moderate business performance</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;74&lt;/sub&gt; Active: good business performance</td>
<td></td>
</tr>
<tr>
<td>IQV&lt;sub&gt;75&lt;/sub&gt; Highly active: excellent business performance</td>
<td></td>
</tr>
</tbody>
</table>
approximately 0.5–0.6, and that the allowable tax-
shield (profit before tax) was extracted to be around 6
to 6.5 million from the financial statement of the firm.
Total assets of the firm were valued at approximately
550 million. Given the financial data of the firm, the
decision-makers’ task is then to examine the possibility
of borrowing funds from different financial sources.

The linguistic variables that deemed to be essential in
the decision-making process are classified as: (a)
external: type of banker \( IQV_{1t} \), sovereign \( IQV_{2u} \), (b)
Interactive: relationship with the banker, \( IQV_{2v} \), long
term prospects, \( IQV_{4w} \), past co-operation, \( IQV_{5x} \); and
(c) internal: credit ability, \( IQV_{6y} \), networking, \( IQV_{7z} \).
These variables are deemed to be the characteristic of
the qualitative factors, and are considered as the input
variables in the borrowing model. The details of the
input variables are given in Table 3. The newly added
variables, sovereign, credit ability and networking are
discussed in the Hong Kong case study in details.

Sovereignty (external factor)
A country’s political atmosphere has a differential and
significant impact on local and foreign firms. There are
many kinds of government policies such as expansion-
ary policies including increased government expenditure
and lower or redefinition of taxes, development
of bonds (Nazmi, 2002) and issuance of government
credit programme (Espinosa-Vega et al., 2002).
Expansionary policies stimulate economic growth
(Barro, 1989). The combination of expansionary policy
with creditor lending to provide loan is to launch a
credit programme or a finance scheme to increase long-
run production. The policy reaction will add to bank
reserves through open market purchases. Creditors are
willing to convert these excess reserves into loans with
lower borrowing rates for the lenders.

Sovereign as a variable is defined as the type of
government’s actions that prevent the repayment and/or
the raising of funds from financial institutions such that
if a soft financial scheme is launched by the government
for firms to assist them in acquiring capital to enhance
their productivity and competitiveness, the level of
sovereign is classified as soft. The stricter the govern-
ment action, the higher the level of sovereign available to
a company. A good sovereign needs to show that it
actually desires to reimburse debt in order to make it
different from a bad sovereign by releasing expansionary
policy, enduring a sovereign debt and even a recession.

Credit ability (internal factor)
Financial credibility indicates that creditors allocate a
relatively high chance to a firm that it will carry out the
promise to repay the loan within the predetermined
period. Credit ratings become more crucial in financial
markets and have a direct relationship with the debt
capacity granted to a firm. Debt capacity provides
valuable information in the stage of capital planning by
establishing margin on the quantity of the investment
projects (Hogan, 1985). It is a function of its ability
defined as the availability of cash flow as well as its
willingness to repay the loan (Jain, 1990).

The rate at which a firm can borrow is decided by its
ability to repay the loan. Therefore, the percentage of
financial transactions delayed in the past five years with
the counter party was adopted to quantify the credit
ability such that, if a firm has over 30% of financial
transactions delayed in the past five years with the
counter party, it is classified as having a very low debt
capacity and thus a low credit rating. The larger the
percentage of financial transactions delayed, the smal-
ner the debt of a company.

Networking (internal factor)
Networking is a subjective factor defined as the process
of sharing contracts among parties outside and inside in
a firm (Dubini and Aldrich, 1991). A network supports
the decision-makers by providing useful information
and resources and sometimes opportunities to a firm.
Successful entrepreneurs have large social networks
(Hansen, 1995). Lending institutions have complicated
models to predict financial lending risk and these
embrace a number of variables such as the dividend
payout. In the case study, the lending institution makes
decisions by considering whether the firm has a good
network to maintain its business, so that loan can be
repaid easily.

The levels of networking of a firm have positive
relationship with their business performance that could
be measured by the employment growth (Barkham et al.,
1996). The networking is associated with the level of
business performance. A marketing system was
introduced such that, if the marks of business
performance were less than 20, the networking of the
firm was classified as ‘highly inactive’. The higher the
business performance, the greater the networking
activity awarded to a company.

Professionals and experts in the construction firm
were invited to provide information to enable decision-
making criteria to be determined. Weights of 0.3, 0.4
and 0.3 are assigned to external, interactive and internal
factors respectively. Table 4 shows the input variables
of different sources of loans for the contracting firm.

Procedure to solve the problem
Figure 3 shows the procedure to solve the borrowing
optimization problem by AGA. It mainly consists of
two steps namely fuzzification of the qualitative factors (transformation phase) and an AGA (Initial, operation, decision and adaptive phase).

**Fuzzification of the qualitative factors**

The procedure of converting real data into membership values is known as fuzzification. The popular technique for fuzzification is either the $S$ function and the $\pi$ function that is a combination of two $S$ functions (Zadeh, 1975). After this procedure, the data of various input variables from experts is passed into the inference engine where a rule base is established.

Theoretically, $M$ rules are deemed necessary to cover the whole picture. However, the advantage of fuzzy reasoning is that even when all the rules are not present, we can still get a reasonable and usable result. The larger the rule base, the better the result. Weights assigned further enhance the reliability of the result. When a case ($x$) is evaluated, it is put into each rule in turn by the popular max-min operation and the final result is given by the centre-of-gravity method.

**Adaptive genetic algorithm (AGA)**

Basically, a GA undergoes the search process in four stages: initialization, selection, crossover, and mutation (Davis, 1991). The GA initializes with a random population of chromosomes, probability of crossover and mutation. The formulation of a GA requires the definition of a fitness function that assigns an overall performance value to each set of decision variable values, which is based on the realization of an objective function and an abuse of the constraints of different solutions. In the operation phase, Roulette Wheel Selection (RWS) is used to select the number of population from parents in which individuals are mapped to contiguous segments of a line, such that each individual's segment is equal in size to its fitness. Crossover and mutation operations are then applied to produce successively fitter chromosomes. The process is repeated until the desired number of individuals is obtained in the decision phase.

**Encoding**

The first step in establishing GA is encoding the problem in a chromosomal manner for subsequent evolitional processes. There are many different encoding methods such as binary strings, permutation codes, the recent slicing tree (Tam and Chan, 1998) and so forth. These techniques vary and depend on the nature of problem and the selection of good encoding system involves certain amount of art (Davis, 1991). After encoding, each chromosome represents a solution in term of linear string or bits and their genes are generated for crossover and mutation. As mentioned before, GA operates from a population instead of a single point and the initial population of chromosomes is necessary, in which they can be created randomly.

**Reproduction**

According to the rule of survival-of-the-fittest, the better chromosomes have greater chance to reproduce the offspring. Therefore, chromosomes with better fitness values in the population have higher probability in selection to the next generation. Weighted roulette wheel is one of the commonly used allocation methods in selection among the initial population. Continuing the process of the reproduction, the better chromosomes are selected and copied to the next generation and the ultimate generation will be full of the best chromosome from the initial population. It is noted that genetic change or improvement in the initial population does not occur under this operation. It is

<table>
<thead>
<tr>
<th>Types</th>
<th>Term loans</th>
<th>Source</th>
<th>SGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>1 2 3 4 5 6 7 8 9 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input variables</td>
<td>Type of banker, IQV$_{1s}$</td>
<td>26</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Sovereign, IQV$_{2s}$</td>
<td>.83</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Relationship with the banker, IQV$_{3s}$</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Long-term prospects, IQV$_{4s}$</td>
<td>67</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Past co-operation, IQV$_{5s}$</td>
<td>69</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Credit ability, IQV$_{6s}$</td>
<td>26</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Networking, IQV$_{7s}$</td>
<td>55</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Qualitative factor parameter</td>
<td>.497</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: – means the same bank as the previous one

**Table 4** Input variables of different sources of loans for a contracting firm
Figure 3  Procedure to solve the problem
the reason why crossover and mutation are introduced in GA.

Crossover
Chromosomes are selected and mated randomly in the population; genes are exchanged between the selected chromosomes (parent) in the crossover process. Under this process, new offspring is created and evaluations of their fitness value are carried out. The poor offspring is then discarded. As a result, new, better and quite different offspring is evolved with retaining the traits from the parent. Single point, edge recombination or partial string crossover are the common methods adopted in this exchanging process (See Figure 4).

Mutation
Similarly, evolution process may be trapped by the reproduction and crossover among the initial population as they may become overzealous and loss some potentially useful genetic material (Goldberg, 1989). Mutation can bring variations to the population by modifying the genes in an individual chromosome randomly so as to produce new offspring to enter the population. Mutation can be performed by swapping the entries in a chromosome randomly. Again, the fitness value of the new offspring is evaluated and good offspring is selected. As mentioned in initial study by Goldberg (1989), mutation plays a secondary role in GA so as to insure against premature loss of important notions. Also, it is guaranteed that the entries in the chromosome are never permanently fixed by mutation (Al-Tabtabai and Alex, 1999). The mechanisms of GA are surprisingly simple. Throughout some probabilistic settings, the search space is enhanced in an effective and efficient manner. According to the latest literature by Goldberg (2002), the innovation intuition is described below and in Figure 5:

Selection (Reproduction) + Mutation = Continual Improvement
Selection + Recombination (Crossover) = Innovation

Repeating these genetic operators, new generations are reproduced with continuous increasing their quality. Finally, the populations are replaced by new generation continuously and they converge until achieving the optimal solution.

Constant parameters are adopted in most applications of GAs. The parameters are decided on a set-and-test basis. Since GA has an inherently dynamic and adaptive nature, the change of the strategy parameter values during the optimization process is necessary by using (a) a rule, (b) feedback mechanism and (c) self-adaptive mechanism (Gen and Cheng, 2000) to reach

Figure 4  Crossover in GA

![Crossover in GA](image)

Figure 5  Mutation in GA

![Mutation in GA](image)
the evolutionary principle. In this study, the second method is used to determine the direction of the change in the strategy parameter. The GA optimization software, Evolver\textsuperscript{TM} (Palisade, 1998), is used as it provides a steady state (Cheung et al., 2002). Figure 2 illustrates an AGA to suit the dynamic nature of GAs and avoid the appearance of global optimization. An adaptive phase is included in the algorithm and effective in the presence of a new loan source. The mechanism of the phase is that when there is an appearance of new loan source, the parameters in the GA optimization process will be changed to fit the actual case instead of optimization all the known and unknown cases.

**Analysis of the results**

The optimal cash requirement curve from the overall cash flow model provides information that can be used to control risk and maximize profit. In the case study, the contractor planned to finance his projects according to an overall corporate borrowing strategy. The decision-maker decided that in order to minimize the financial risks of the firm, a more conservative decision would be made, by considering the optimal and early-period cash requirement curves as the allowed range of borrowing as shown in Figure 2. By doing this, the degree of risk associated with inadequate cash flows could be reduced.

The monthly borrowing schedule from different sources of finance was found by applying the borrowing optimization model. Figure 6 and Table 5 show the results. Table 5 shows that financing started with sources 7 through 10 and later moved to sources 6 and 3 as work progresses. This means the favourable loan sources were mainly SGS (sources 6–10) and after they have been used up, the term loan (source 3) became the choice of the favorable loan. The lending scheme (SGS) is a government policy. With smaller input variables, ‘sovereign (0.17)’, SGS source of finance was first identified as the start of external financing. Sources 7 and 8 came from the same financial institution, with identical lending policy and characteristics. Source 7 was associated working capital loan (Type 2 of SGS) to meet additional operational expenses arising from or in relation to the business installations and equipment acquired / to be acquired under the SGS while source 8 (Type 3 of SGS) was accounts receivable loan to meet the working capital needs arising from provision of credit terms to the applicant’s customers. Sources 7 and 8 had the lowest interest rate (P-1%) and qualitative factor parameter (0.252) among both the SGS and term loans. The chance of obtaining an unfavourable source of finance was extremely small compared with the other sources. For sources 9 (Type 1 of SGS) and 10 (Type 2 of SGS) which were from the same financial institution, with the highest interest rate (P+3% and P+2.5% respectively) but the second lowest qualitative factor (0.263), the chance of obtaining an unfavourable source of finance was still small comparatively, such that the model was balanced between the two objective functions, and a compromise solution was achieved for the analysis.
The implementation of both type-2 SGS at the start of financing revealed a large additional operational expenses arising from business installations and required equipment under SGS. For source 6 (Type 1 of SGS), even though the lending term was the same as that of source 8, it was not chosen in the first instance due to its lengthy period of loan terms (5 years). For acquiring new or second-hand business installations and equipment, source 9 (Type 1 of SGS) was enough to meet the requirement at the starting stage. In the working month 10, the previous sources were nearly used up together with the introduction of a new project (project 1). The periodic cash requirement was about 0.8 millions. In order to meet the short-term financing, the remaining SGS loan, source 6, was enough and thus found to be the choice of external financing. In the working month 14 the last project (project 4) started. Due to its highest profit margin (25%), large amount of external financing was needed to propel the schedule upwards. For source 3, even though the qualitative factor was highest (0.846), the largest proportion of loans offered at the early stage and at comparatively low interest rate (P+1.5%) made it the optimized choice. Although overdraft source 1 had the lowest interest rate, its sum of loans was not enough to meet the requirement.

In Figure 6 the proposed monthly borrowing curve is located between the early and optimal period cash requirement curves. The monthly borrowing curve is however closer to the optimal periodic cash requirement curve than the early curve. Except in the working month 10 and 14, the introduction of project 1 and 4 respectively required additional resources to drive the project at a higher rate of progress as shown in Figure 4. With the optimal monthly borrowing schedule, the contractor could manage his cash efficiently and effectively, thus over or under-borrowings at the corporate level can be avoided. This analysis also provides useful complementary information during loan negotiations. The decision-maker can develop their own strategies for emergencies to eliminate the financial distress suffered by the shareholders. Because the financial risk inherent in financing projects can be identified in this study, it provides strong evidence for the beneficial effects of using financial data at both project and corporate levels, allowing the lender or creditor to have more confidence in supporting the lending. Also the possibility of getting an unfavourable source of finance and bankruptcy can be minimized or avoided.

The test on the GA followed the context of sequential decision-making shown in Figure 2. Since the tax-shield and profit conditions of the firm were different in the study, the overall goal of the three objectives was different that some were minimization and some were maximization. The GA optimization parameters such as the population size, the crossover rate and mutation rate varied throughout the optimization process. The values used ranged from 250 to 300, 0.5 to 0.8 and 0.1 to 0.25 respectively. The initial operation parameters were set such that population size was 300, crossover rate was 0.5 and mutation rate was 0.15. Table 6 shows the optimization results by the Evolver™ by the AGA. The time to find the near optimal solution ranged from one second to 17 minutes, 29 seconds with three seconds and 28

<table>
<thead>
<tr>
<th>Working month</th>
<th>Term loans</th>
<th>Source</th>
<th>SGS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>–</td>
<td>–</td>
<td>360988, 480751, 302271, 162725</td>
</tr>
<tr>
<td>5</td>
<td>–</td>
<td>–</td>
<td>741672, 650315, 371748, 466152</td>
</tr>
<tr>
<td>6</td>
<td>–</td>
<td>–</td>
<td>675094, 137740, 142307, 294325</td>
</tr>
<tr>
<td>7</td>
<td>–</td>
<td>–</td>
<td>698941, 762039, 311864, 276272</td>
</tr>
<tr>
<td>8</td>
<td>–</td>
<td>–</td>
<td>341617, 547585, 316030, 691997</td>
</tr>
<tr>
<td>9</td>
<td>–</td>
<td>–</td>
<td>180354, 420327, 706093, 342440</td>
</tr>
<tr>
<td>10</td>
<td>–</td>
<td>–</td>
<td>783816, 14734</td>
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<tr>
<td>11</td>
<td>–</td>
<td>–</td>
<td>29163, 65871</td>
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<tr>
<td>12</td>
<td>–</td>
<td>–</td>
<td>1800472</td>
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<tr>
<td>13</td>
<td>–</td>
<td>–</td>
<td>2177092</td>
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<td>–</td>
<td>648922, 1000232</td>
</tr>
<tr>
<td>15</td>
<td>–</td>
<td>–</td>
<td>1767399, 33072</td>
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<td>16</td>
<td>–</td>
<td>–</td>
<td>1729649, 70822</td>
</tr>
<tr>
<td>17</td>
<td>–</td>
<td>–</td>
<td>8281307, 889087</td>
</tr>
</tbody>
</table>

Note: – means the absence of external financing

Table 5 Monthly borrowing schedules from different sources

The values used for population size, crossover rate and mutation rate varied throughout the optimization process. The values used ranged from 250 to 300, 0.5 to 0.8 and 0.1 to 0.25 respectively. The initial operation parameters were set such that population size was 300, crossover rate was 0.5 and mutation rate was 0.15. Table 6 shows the optimization results by the Evolver™ by the AGA. The time to find the near optimal solution ranged from one second to 17 minutes, 29 seconds with three seconds and 28
In the first working month, no trials of input at the starting point led to large searching time. The searching times in the following working month (5 to 9) were very short (maximum about 1 min) after the initial sources of finance have been identified. The algorithm then provided feedback information to the borrowing model. In the working month 10, sources 7 and 8 were used up while sources 9 and 10 were still being used. Parameters were changed by the decision-makers who were on the top level management team of the firm with a decrease in the population (250) and increase in both the crossover rate (0.6) and mutation rate (0.25). The time to search the optimal value fluctuated vigorously at the first 500 trials and increased sharply to achieve the maximum solution. Afterwards, the curve was flattened until trial 6000. The final search time was nine seconds and occurred in trial number 814 as shown in Figure 7.

In the working month 11, identification of new loan source, source 6, appeared and thus the operation parameters should be modified in the adaptive phase with an increase in both the population (300) and crossover rate (0.8) and a decrease in the mutation rate (0.1). The searching time was 21 seconds. In the working month 12, only one source 6 was still being used.

### Table 6  Optimization results by the AGA

<table>
<thead>
<tr>
<th>Working month</th>
<th>Overall goal</th>
<th>No. of trials</th>
<th>Original value</th>
<th>Best value found</th>
<th>Occurred on trial no.</th>
<th>Time to find this value</th>
<th>Termination condition</th>
<th>Total optimization Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Min.</td>
<td>222932</td>
<td>2635291.918</td>
<td>2605967.567</td>
<td>136762</td>
<td>0:17:29</td>
<td>No change until 252932 trials</td>
<td>0:28:55</td>
</tr>
<tr>
<td>5</td>
<td>Max.</td>
<td>7000</td>
<td>N/A</td>
<td>1692588.927</td>
<td>2129</td>
<td>0:00:43</td>
<td>No change until 10000 trials</td>
<td>0:01:26</td>
</tr>
<tr>
<td>6</td>
<td>Min.</td>
<td>11000</td>
<td>7722263.083</td>
<td>7722263.045</td>
<td>5014</td>
<td>0:00:40</td>
<td>No change until 11000 trials</td>
<td>0:01:01</td>
</tr>
<tr>
<td>7</td>
<td>Max.</td>
<td>2000</td>
<td>N/A</td>
<td>3444291.046</td>
<td>847</td>
<td>0:00:19</td>
<td>No change until 7500 trials</td>
<td>0:00:32</td>
</tr>
<tr>
<td>8</td>
<td>Min.</td>
<td>8500</td>
<td>N/A</td>
<td>15713215.150</td>
<td>3350</td>
<td>0:00:21</td>
<td>No change until 8500 trials</td>
<td>0:00:36</td>
</tr>
<tr>
<td>9</td>
<td>Max.</td>
<td>9500</td>
<td>2172828.045</td>
<td>2172968.330</td>
<td>2453</td>
<td>0:01:01</td>
<td>No change until 9500 trials</td>
<td>0:02:13</td>
</tr>
<tr>
<td>10</td>
<td>Max.</td>
<td>1000</td>
<td>N/A</td>
<td>1014254.725</td>
<td>814</td>
<td>0:00:09</td>
<td>No change until 6000 trials</td>
<td>0:00:10</td>
</tr>
<tr>
<td>11</td>
<td>Min.</td>
<td>20615</td>
<td>5380852.848</td>
<td>5350918.841</td>
<td>7</td>
<td>0:00:21</td>
<td>No change until 22867 trials</td>
<td>0:02:11</td>
</tr>
<tr>
<td>12</td>
<td>Min.</td>
<td>30</td>
<td>N/A</td>
<td>2726451.920</td>
<td>19</td>
<td>0:00:01</td>
<td>No change until 50 trials</td>
<td>0:00:02</td>
</tr>
<tr>
<td>13</td>
<td>Min.</td>
<td>60</td>
<td>N/A</td>
<td>5858910.107</td>
<td>23</td>
<td>0:00:01</td>
<td>No change until 80 trials</td>
<td>0:00:03</td>
</tr>
<tr>
<td>14</td>
<td>Max.</td>
<td>109850</td>
<td>1293401.712</td>
<td>1669808.348</td>
<td>78639</td>
<td>0:09:34</td>
<td>No change until 112827 trials</td>
<td>0:18:21</td>
</tr>
<tr>
<td>15</td>
<td>Max.</td>
<td>4542</td>
<td>N/A</td>
<td>3111162.626</td>
<td>235</td>
<td>0:00:08</td>
<td>No change until 8042 trials</td>
<td>0:00:59</td>
</tr>
<tr>
<td>16</td>
<td>Max.</td>
<td>1513</td>
<td>N/A</td>
<td>4533116.333</td>
<td>70</td>
<td>0:00:01</td>
<td>No change until 7263 trials</td>
<td>0:00:35</td>
</tr>
<tr>
<td>17</td>
<td>Min.</td>
<td>14000</td>
<td>N/A</td>
<td>31973585.300</td>
<td>8012</td>
<td>0:00:37</td>
<td>No change until 18000 trials</td>
<td>0:01:11</td>
</tr>
</tbody>
</table>

| Figure 7  Maximum solution in working month 10 |
used. Only one gene was presented. No offspring was produced. The mutation and crossover were set to zero. The searching time was just one second. In the working month 14, new loan source, source 3, was identified and operation parameters were changed with a decrease in the population (250) and an increase in both the crossover (0.7) and mutation rate (0.2). The searching time was large (9min 34s) as the other category of source of finance (term loans) was also identified.

Although Evolve™ divided the number of individual trials, the solutions obtained could be easily trapped in local optimum before reaching the global optimum. In this study, the adaptive phase in the algorithm improved the converging time of the GA. The searching ability of AGA in the borrowing optimization model was thus viable and thus prevented the possibility of trapping in local optimum.

It is noted that the input variables proposed here are restricted to the multiple projects of a medium-size construction company in Hong Kong. For small or international construction companies or for other special projects, more or less input variables and qualitative factor elements should be considered. The proposed system can deal with unlimited projects and variables. Besides, modelling future events makes the validation of the model difficult as the real world changes with time. Up-to-date information should be provided to the decision-makers. Nevertheless, the merit of the borrowing optimization model is that it actually provides a tailor-made system to determine the input variables for the multi-objective functions such that the qualitative and quantitative problem of borrowing decision-making problems can be solved.

Conclusion

In modelling financial decisions, optimizing both qualitative and quantitative factors and the semi-structured nature of construction finance optimization problem are key challenges in solving construction finance decisions. As shown in this paper, the application of GA in modelling financial decision is demonstrated (i.e. optimizing both quantitative and qualitative factors simultaneously) without making the usual simplifying assumptions as required by many conventional techniques. The optimization results showed that the introduction of an adaptive phase in GA increased the converging time and thus avoided being easily trapped in local minima before exploiting the global minimum. With the optimal monthly borrowing schedule, the amount of borrowing from external sources could be more realistic, thus reflecting the contractor’s financial position. The contractor could manage his cash efficiently and effectively and over or under-borrowings at the corporate level can be avoided. This analysis also provides useful complementary information during loan negotiations. The decision-maker can develop their own strategies for emergencies to eliminate the financial distress suffered by the shareholders. And the possibility of getting an unfavourable source of finance and bankruptcy can be minimized or avoided.

Acknowledgement

A part of the content was briefly presented at ARCOM 2005.

References


Funding schemes and a borrowing decision model


Appendix: The Mathematical Algorithm (Lam et al., 1998)

Objective functions

\[
\begin{align*}
\widetilde{\text{Max}} / \text{Min} & \quad f_1(X) = \sum_{i=1}^{I} \sum_{n=1}^{N} X_{i,n} e^{-m} \quad \text{[Loan]} \\
\widetilde{\text{Max}} & \quad f_2(X) = \sum_{i=1}^{I} \sum_{n=1}^{N} (1 + r_i) X_{i,n} e^{-m} \quad \text{[Interest payments]} \\
\widetilde{\text{Max}} / \text{Min} & \quad f_m(X) = \sum_{i=1}^{I} \sum_{n=1}^{N} \tilde{\omega}_j X_{i,n} \quad \text{*[External, interactive, and internal Qualitative factors]} \\
m = 3, 4, \ldots, I
\end{align*}
\]

Subject to the constraints

\[
g_1(X) = \sum_{i=1}^{I} \sum_{n=1}^{N} [X_{i,n} + (r_i) X_{i,n-1}] \geq \widetilde{Q}_n \quad \text{[Period cash requirement]} \\
\]

For \( \widetilde{Q}_n = \{ \sum_{j=1}^{F} \sum_{n=1}^{N} (\tilde{E}_{j,n} + \tilde{K}_{j,n} + \tilde{P}_{j,n} + \tilde{R}_{j,n-1}) + \sum_{n=1}^{N} \tilde{L}_n + \sum_{n=1}^{N} \tilde{M}_n, \tilde{P}_{i,n} + \sum_{n=1}^{N} \tilde{D}_n \} \)

\[
g_2(X) = \sum_{i=1}^{I} X_{i,n} e^{-m} \leq \tilde{S} \quad \text{[Allowable-tax-shield]} \\
g_3(X) = \sum_{i=1}^{I} \sum_{n=1}^{N} X_{i,n} \leq \Phi_n \quad \text{[Capital gearing ratio]} \\
g_4(X) = L_i \leq \sum_{n=1}^{N} X_{j,n} \leq U_i \quad \text{[Borrowing limit]} \\
X_{i,n} \geq 0, \quad i = 1, 2, 3, \ldots I, \quad n = 1, 2, 3, \ldots N, \quad j = 1, 2, 3 \ldots F
\]

* Proposed amendment
The symbols used are as follows:

\[
\begin{align*}
\text{\(f_1(X)\)} & \quad \text{Present value of the sum of loans} \\
\text{\(f_2(X)\)} & \quad \text{Present value of the sum of interest and loans} \\
\text{\(f_m(X)\)} & \quad \text{Any qualitative factors} \\
\text{\(m = 3, 4, \ldots, L\)} & \\
\text{\(X_{i,n}\)} & \quad \text{The amount of \(i\)th loan source in the \(n\)th working month} \\
\text{\(e^{-m}\)} & \quad \text{Discount factor} \\
\text{\(r_i\)} & \quad \text{Interest rate for the \(i\)th source} \\
\text{\(\omega_i\)} & \quad \text{Qualitative factor parameter of the \(i\)th loan source} \\
\text{\(Q_n\)} & \quad \text{Cash requirement in the \(n\)th month} \\
\text{\(E_{i,n}\)} & \quad \text{The \(i\)th project expenses in the \(n\)th month} \\
\text{\(K_{i,n}\)} & \quad \text{Material costs for \(i\)th project in the \(n\)th month} \\
\text{\(P_{i,n}\)} & \quad \text{Plant costs for \(i\)th project in the \(n\)th month} \\
\text{\(R_{i,n-1}\)} & \quad \text{Deduction of retention money for the \(i\)th project in the \(n-1\)th month} \\
\text{\(I_p\)} & \quad \text{Interest payment for previous borrowings} \\
\text{\(\Pi\)} & \quad \text{Interest payment for previous borrowings} \\
\text{\(\beta_n\)} & \quad \text{Tax at the end of the financial year} \\
\text{\(D_n\)} & \quad \text{Dividends to shareholders} \\
\text{\(W_j\)} & \quad \text{Initial working capital for project \(j\)th} \\
\text{\(M_{i,j,n}\)} & \quad \text{Interim payment of the \(i\)th project in the \(n\)th month} \\
\text{\(\Theta_i\)} & \quad \text{Released retention money of \(i\)th project (dependent on the agreement)} \\
\text{\(T_n\)} & \quad \text{Trade credit from supplier in the \(n\)th month} \\
\text{\(C_{n-1}\)} & \quad \text{Excess cash in the \(n-1\)th month} \\
\text{\(H_n\)} & \quad \text{Head office overhead in the \(n\)th month} \\
\text{\(PL_n\)} & \quad \text{Previous borrowings} \\
\text{\(S\)} & \quad \text{Allowable tax-shield in the \(n\)th month} \\
\text{\(\phi\)} & \quad \text{Capital gearing ratio} \\
\text{\(\omega_n\)} & \quad \text{Working capital in the \(n\)th month} \\
\text{\(U_i\)} & \quad \text{Upper limit of borrowing for the \(i\)th source} \\
\text{\(L_i\)} & \quad \text{Lower limit of borrowing for the \(i\)th source}
\end{align*}
\]

Subscript

\[
\begin{align*}
\text{\(i\)} & \quad \text{The \(i\)th loan source} \\
\text{\(j\)} & \quad \text{The \(j\)th project} \\
\text{\(n\)} & \quad \text{The \(n\)th month}
\end{align*}
\]