Complementary Results on the Stability Bounds of Singularly Perturbed Systems

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Abstract—In this note, we will provide a new systematic approach to characterize and compute the stability bound of a singularly perturbed linear system. The approach is based on the feedback system representation of an additional matrix perturbation problem. The idea is to change the stability bound problem to the stability problem of an underlying feedback system. This approach allows multiple choices in formulating the underlying feedback system and thus has the potential of characterizing and computing the stability bound in a number of different ways. By formulating two kinds of different feedback systems, some existing and new results on the stability bounds are derived based on the feedback system approach. The new results complement the existing frequency-domain-based stability criteria and make the frequency-domain technique more applicable and useful to the stability bound problem. An example is provided to show the new stability criterion is effective and useful in determining the stability bound.

Index Terms—Feedback system, matrix perturbation, singularly perturbed systems, stability bounds.

I. INTRODUCTION

There is a number of linear systems which contain two significantly different dynamic modes: fast modes and slow modes. These kinds of linear systems usually can be described by the following equations:

\[
\dot{x} = A_1 x + A_{12} z + B_1 u \\
\epsilon \dot{z} = A_{21} x + A_{22} z + B_2 u \\
y = C_1 x + C_2 z 
\]  
(1)

where \( x \in \mathbb{R}^n \), \( z \in \mathbb{R}^m \), \( u \in \mathbb{R}^r \), and \( y \in \mathbb{R}^p \). \( \epsilon \) is a small positive number. A linear system with the above state-space equations is called the singularly perturbed system in the literature \[1\], while \( \epsilon \) is called the perturbation parameter. Setting \( \epsilon \) in (2) to zero, we can get the following reduced-order system:

\[
\dot{x} = A_0 x + B_0 u 
\]  
(4)

where \( A_0 \) and \( B_0 \) are defined by

\[
A_0 = A_{11} - A_{12} A_2^{-1} A_{21} \\
B_0 = B_1 - A_{12} A_2^{-1} B_2 
\]  
(5)

assuming \( A_2 \) is nonsingular.

The system described in (4) is called the zero-order model and is an approximation to the slow-varying subsystem of the whole system. An important property of the reduced-order system (4) is in the following connection of its stability with that of the original system given by (1) and (2) for the sufficiently small parameter \( \epsilon \).

Theorem 1: If the matrices \( A_0 \) and \( A_{22} \) are stable, then there exists a positive number \( \epsilon_0 \) such that for all \( 0 < \epsilon < \epsilon_0 \), the original system defined by (1) and (2) is also stable.

Theorem 1 is of important significance, but its application to the problem at hand needs accurate knowledge of the stability bound \( \epsilon_0 \). Characterization and computation of this stability bound has attracted a number of researchers for more than two decades \[2\]—\[9\]. Basically, there are two kinds of methods to characterize and compute \( \epsilon_0 \): frequency-domain transfer function based techniques and state-space model based techniques. Both these methods can provide the exact stability bound as shown in \[4\], \[6\], and \[7\]. Relatively, the state-space model based technique usually involves complex matrix operations and hence is difficult to understand and use. The advantage of the state-space model based technique is its ability to deal easily with systems of very high order.

This note will focus on the frequency-domain based method to derive the stability bound \( \epsilon_0 \). More specifically, this note is directly motivated by the results reported in \[4\], which provided a method to compute the exact stability bound based on the generalized Nyquist plot of a specific transfer function matrix. It is well known that the Nyquist stability criterion is very useful to analyze the stability of a closed-loop feedback system. However, in \[4\] no feedback system is explicitly formulated in spite of the fact that the stability criterion provided there is exactly the same as that for a feedback system. This naturally raises the following question: Can the stability criterion provided in \[4\] be derived by using some feedback analysis tool like the Nyquist criterion? Or in other words, can the problem of finding the stability bound of a singularly perturbed system be viewed as the stability problem of a closed-loop feedback system? To view the stability bound problem as a feedback stability problem has at least one advantage: there are many analytical tools available for analyzing the stability of a feedback system.

In this note, we will provide a general and systematic approach to characterize and compute the stability bound of a singularly perturbed system. The novel approach consists of two steps. The first step is to present the system matrix of the singularly perturbed system as the sum of a stable matrix and a perturbation matrix. The second step is to convert the matrix perturbation problem to the stability problem of an underlying feedback system. By following these two steps, one can obtain a systematic method to handle the stability bound problem. Comparing with the approach of \[4\], our approach can provide multiple stability criteria that can be used to estimate the stability bound. In addition, the method in \[4\] to compute the stability bound is difficult to use for systems with a higher order fast model \((m > n)\). In this note, we will develop a new criterion to compute the stability bound which is especially useful for systems with a lower order slow model. In this way, our results on the stability bound will complement the corresponding results developed in \[4\] and will make the frequency-domain-based techniques more suitable and useful in the stability analysis of singularly perturbed systems.
II. STABILITY ANALYSIS BASED ON A FEEDBACK FORMULATION

A. Feedback Representation of Matrix Perturbation

In this section, we will briefly review the method to represent the additional perturbation of a stable matrix using a feedback system with a stable open-loop transfer function, originally proposed in [10]. As will be shown later, such a feedback formulation of the matrix perturbation problem is very useful in the analysis of the stability bound of a singularly perturbed system.

Assume that the matrix $E$ under consideration can be written in the following form:

$$ E = A + BDC $$

where the matrices $A$, $B$, $C$, and $D$ have compatible dimensions. As is shown in [10], $B$ and $C$ are given matrices defining the perturbation structure, while $D$ can be considered as the unknown perturbation matrix. Furthermore, $A$ is assumed to be stable and the problem is to determine the conditions on $B$, $C$, and $D$ such that $E$ is also stable.

Consider the feedback system shown in Fig. 1. One can see that the system matrix of the closed-loop system in Fig. 1 is equal to $E$. In this sense, we say that $E$ can be represented by the feedback configuration shown in Fig. 1. We introduce the following transfer function based on the state-space model in Fig. 1

$$ F(s) = C(sI - A)^{-1}B. $$

B. Stability Bound Analysis

Based on the singular perturbation model given in (1) and (2), define the following matrix:

$$ P = \begin{bmatrix} A_{11} & A_{12} \\ A_{21}/\epsilon & A_{22}/\epsilon \end{bmatrix}. $$

Assume that the matrices $A_{11}$ and $A_{22}$ are stable. The problem of finding the stability bound $\epsilon_0$ is to search the maximum positive number $\epsilon$ with which $P$ is stable.

The first step in the stability analysis is to decompose $P$ as the sum of two parts as follows:

$$ P = \bar{P}_0 + \Delta P $$

where $\bar{P}_0$ is a stable matrix if both $A_{11}$ and $A_{22}$ are stable.

On the other hand, assume the matrix $Q$ is similar to $P$. That is, $Q$ is defined by

$$ Q = T^{-1}PT $$

for some invertible $T$. Then, we can analyze the stability of $P$ in terms of $Q$. Decompose $Q$ in the same way as (10), we have

$$ Q = Q_0 + \Delta Q. $$

The second step is to formulate the feedback system illustrated in Fig. 1. This can be done either based on the decomposition (10), or based on the decomposition of the similar matrix given in (12). Therefore, there are two kinds of feedback systems which can be used in the stability analysis: one is constituted based on the original matrix $P$; the other is constituted based on a similar matrix $Q$ of $P$. On the other hand, the decomposition (10) or (12) may have multiple solutions depending on how $\bar{P}_0$ (or $Q_0$) is chosen. Therefore, there are many choices available to us in formulating the underlying feedback systems. With these different choices we can develop different stability criteria which may be used to characterize and compute the stability bound. In the following, we will formulate two kinds of the underlying feedback systems based on the decomposition of (10) and (12), respectively, and then establish corresponding stability criteria. Based on these stability criteria, we can derive the main results on the stability bound $\epsilon_0$ originally developed in [4] and [5]. Furthermore, we will also derive some new results on the stability bound, which appear as the complements to the major results in [4] and [5]. In particular, one of our new result is especially useful for the case where the slow model is of lower order.

1) Formulating the Feedback System Based on the Original Matrix $P$: The first step is to decompose $P$ as (10) where $\bar{P}_0$ and $\Delta P$ are given by

$$ \bar{P}_0 = \begin{bmatrix} A_0 & 0_{n,m} \\ A_{21}/\epsilon & A_{22}/\epsilon \end{bmatrix}, $$

$$ \Delta P = \begin{bmatrix} A_{12}A_{21}^{-1}A_{22} & A_{12} \\ 0_{m,n} & 0_{m,m} \end{bmatrix}. $$

We see that $\bar{P}_0$ is stable under the assumption on $A_{10}$ and $A_{22}$. The perturbation matrix $\Delta P$ can be written as

$$ \Delta P = P_1I_mP_2 $$

where $P_1$ is an $(n + m) \times m$ matrix given by

$$ P_1 = \begin{bmatrix} A_{12} \\ 0_{m,m} \end{bmatrix}, $$

and $P_2$ is an $m \times (n + m)$ matrix given by

$$ P_2 = \begin{bmatrix} A_{21}^{-1}A_{21} & I_m \end{bmatrix}. $$

Then in the second step, based on (7), the matrix $P$ can be represented by the feedback system in Fig. 1 with $B = P_1, C = P_2, D = I_m$, and $F(s)$ is given by

$$ F(\epsilon, s) = P_2((I_n + m) - R_0)^{-1}P_1. $$

One can show that

$$ F(\epsilon, s) = \begin{bmatrix} A_{22}^{-1}A_{21}I_m & (sI_{n+m} - \bar{P}_0)^{-1}A_{12} \\ [I_n + (sI_m - A_{22}/\epsilon)^{-1}A_{22}/\epsilon] & A_{22}^{-1}A_{21}(sI_m - A_0)^{-1}A_{12} \end{bmatrix}. $$

Applying the formula

$$ I + (sI - A)^{-1}A = s(sI - A)^{-1} $$

to (19), we can get

$$ F(\epsilon, s) = \epsilon(sI_m - A_{22})^{-1}A_{22}^{-1}A_{21}(sI_m - A_0)^{-1}A_{12}. $$

One can see that $F(\epsilon, s)$ is an $m \times m$ stable transfer function matrix. Based on the generalized Nyquist theorem, we see that the feedback system in Fig. 1 or the matrix $P$ is stable if and only if

$$ \det[I_m - F(\epsilon, j\omega)] \neq 0 $$

for all $\omega$ [11].

Based on (22), one can see that the matrix $P$ is stable if $\epsilon$ satisfies

$$ \|F(\epsilon, s)\|_{\infty} < 1. $$

1In this note, $0_{ij}$ represents an $i \times j$ zero matrix.
Inequality (23) is one of the results given in [5]. Note that the stability bound obtained based on (23) may be conservative because (23) is a sufficient condition for the feedback system to be stable.

Based on (22), one can derive the main result in [4], which is stated in the following theorem.

**Theorem 2:** Under the same assumptions of Theorem 1, the matrix $P$ is stable if and only if

$$
det[I_m + eG(j\omega)] \neq 0 \quad \forall \omega > 0$$

(24)

where $G(s)$ is defined by

$$G(s) = sA_{22}^{-1}A_{21}(sI_n - A_0)^{-1}A_{12} - I_m].$$

(25)

**Proof:** One can see that for $\omega = 0$ and $\omega = \infty$, condition (22) is true. Therefore, it is only needed to check (22) with finite nonzero $\omega$. We have

$$
det[I - F(e, j\omega)] = \det\left\{ [A_{22}(e^{j\omega}I_m - A_{22})]^{-1} [A_{22}(e^{j\omega}I_m - A_{22}) - e^{j\omega}A_{21}(e^{j\omega}I_m - A_0)^{-1}A_{12}] \right\}

= \det\{ A_{22}^{-1} [I_m - e^{j\omega}A_{22}] \} \times (I_m - A_{22}^{-1}A_{21}(I_m - A_0). A_{12})]

(26)

For finite $\omega$, $1/\det(e^{j\omega}I_m - A_{22}) \neq 0$. Therefore, (26) reduces to the following inequality:

$$
det\left\{ I_m + e^{j\omega}A_{22}^{-1}A_{21}(I_m - A_0)A_{12} - I_m \right\} \neq 0

(27)

and (24) follows.

Theorem 2 is an excellent result because it enables us to compute the exact stability bound using the familiar Nyquist plot. Obviously, the Nyquist plot of a lower order system is easier to handle than that of a higher order system. In this sense, Theorem 2 is preferable in systems with lower order fast modes ($m < n$) because the dimension of $G(s)$ is equal to $m$. For the cases of $m > n$, it is desired to have a stability criterion based on a transfer function whose dimension is equal to $n$, the order of slow modes. As will be shown later, we can develop such a criterion by appropriately formulating the underlying feedback system.

So far, by choosing $P_0$ as (13), we have constructed an underlying feedback system and obtained the main result of [4] and some of the results in [5]. In the following, we will choose $P_0$ differently from (13) and derive a complementary result to (23).

The matrix $P_0$ is chosen as

$$
P_0 = \begin{bmatrix} A_{12} & 0 \\ 0 & A_{22}/\varepsilon \end{bmatrix}.

(28)

Correspondingly, $\Delta P$ becomes

$$
\Delta P = \begin{bmatrix} A_{12}A_{22}^{-1}A_{21} & 0 \\ A_{22}^{-1}/\varepsilon & 0 \end{bmatrix}.

(29)

The matrix $\Delta P$ can be written as

$$
\Delta P = P_1 P_0 P_2

(30)

where $P_1$ and $P_2$ are defined by

$$
P_1 = \begin{bmatrix} A_{12}A_{22}^{-1} \\ I_m/\varepsilon \end{bmatrix}

= e \begin{bmatrix} sI_n - A_0 \end{bmatrix}^{-1} A_{12}(sI_m - A_{22})^{-1} A_{22}^{-1}.

(32)

We can obtain the corresponding transfer function

$$
F(e, s) = P_2 (sI_n + m - P_0)^{-1} P_1 = e \begin{bmatrix} sI_n - A_0 \end{bmatrix}^{-1} A_{12}(sI_m - A_{22})^{-1} A_{22}^{-1}.

(33)

Define $F_1(s)$ and $F_2(e, s)$ as

$$
F_1(s) = sA_{21}(sI_n - A_0)^{-1} A_{12}

F_2(e, s) = e(sI_m - A_{22})^{-1} A_{22}^{-1}

(34)

(35)

Then, $F(e, s)$ is given by

$$
F(e, s) = F_1(s)F_2(e, s).

(36)

Based on (36), we can get the following sufficient condition for $P$ to be stable:

$$
\|F_1(s)F_2(e, s)\|_\infty < 1.

(37)

While condition (23) can be written in terms of $F_1$ and $F_2$

$$
\|F_2(e, s)F_1(s)\|_\infty < 1.

(38)

Noting that generally $AB \neq BA$ for arbitrary matrices $A$ and $B$, inequalities (37) and (38) are not equivalent. Therefore, the stability bound obtained based on (37) may be less conservative than that obtained based on (38), and vice versa. In this sense, the result developed here complements the result of [5, Th. 1, (4b)].

2) Formulating the Feedback System Based on the Similar Matrix $Q$: In the following, we will formulate the feedback system based on some similar matrix of $P$. Introduce the following invertible matrix:

$$
T = \begin{bmatrix} I_n & 0 \\ 0 & -eA_{12}A_{22}^{-1} \end{bmatrix}.

(39)

Then, we can obtain a similar matrix of $P$ as follows:

$$
Q = TPT^{-1} = \begin{bmatrix} A_0 & eA_{12}A_{22}^{-1} \\ A_{21}/\varepsilon & A_{21}/A_{22}/\varepsilon + A_{22}/\varepsilon \end{bmatrix}.

(40)

Decompose $Q$ based on (12) and choose $Q_0$ and $\Delta Q$ as

$$
Q_0 = \begin{bmatrix} A_0 \\ A_{21}/\varepsilon \end{bmatrix}.

(41)

$$
\Delta Q = \begin{bmatrix} 0 & eA_{12}A_{22}^{-1} \\ 0 & A_{21}/A_{22}/\varepsilon \end{bmatrix} = Q_1 Q_2

(42)

where $Q_1$ and $Q_2$ are defined by

$$
Q_1 = \begin{bmatrix} eA_{0} \\ A_{21} \end{bmatrix}

(43)

$$
Q_2 = \begin{bmatrix} 0 & eA_{12}A_{22}^{-1} \\ 0 & A_{21}/A_{22}/\varepsilon \end{bmatrix}.

(44)

Then, we can formulate the feedback system in Fig. 1 with the corresponding transfer function given by

$$
F(e, s) = Q_2(sI_n + m - Q_0)^{-1} Q_1

= eA_{12}A_{22}^{-1}(sI_m - A_{22})^{-1} A_{21}(sI_n - A_0)^{-1}.

(45)

Unlike the transfer function given in (21), $F(e, s)$ given in (45) is an $n \times n$ matrix. Define

$$
F_1(s) = s(I_n - A_0)^{-1}

F_2(e, s) = A_{12}A_{22}^{-1}(sI_m - A_{22})^{-1} A_{21}.

(46)

(47)
Then

\[ F(\epsilon, s) = F_2(\epsilon, s) F_1(s). \]  

(48)

Similar to (23), we can see that \( Q \) (or \( P \)) is stable if \( \epsilon \) satisfies

\[ \| F_2(\epsilon, s) F_1(s) \|_\infty < 1. \]  

(49)

Once again, (49) appears as a complementary criterion to the one\(^2\) in [5, Th. 1, (4a)].

Based on (45), we can furthermore obtain the following stability criterion.

**Theorem 3:** Under the same assumptions of Theorem 1, the matrix \( Q \) (or \( P \)) is stable if and only if

\[ \det \left[ I_n + H(\epsilon, j\omega) \right] \neq 0 \quad \forall \omega > 0 \]  

(50)

where \( H(\epsilon, s) \) is defined by

\[ H(\epsilon, s) = sA_0^{-1} \left[ A_{12} A_{22}^t \left( sI - A_{22}/\epsilon \right)^{-1} A_{21} - I_n \right]. \]  

(51)

**Proof:** Based on the generalized Nyquist criterion, one can see that \( Q \) is stable if and only if

\[ \det \left[ I_n - F(\epsilon, j\omega) \right] \neq 0 \quad \forall \omega > 0. \]  

(52)

We have

\[ I_n - F(\epsilon, j\omega) = \left[ \epsilon j\omega I_n - A_0 - j\omega A_{12} A_{22}^t \right] \times \left[ \epsilon j\omega I_n - A_{22}/\epsilon \right]^{-1} A_{21} \left( j\omega I_n - A_0 \right)^{-1}. \]  

(53)

Therefore, (52) becomes

\[ \det \left[ \epsilon j\omega I_n - A_0 - j\omega A_{12} A_{22}^t \left( j\omega I_n - A_{22}/\epsilon \right)^{-1} A_{21} \right] \neq 0. \]  

(54)

We also have

\[ \epsilon j\omega I_n - A_0 - j\omega A_{12} A_{22}^t \left( j\omega I_n - A_{22}/\epsilon \right)^{-1} A_{21} = -A_0 \left\{ I_n + j\omega A_1^{-1} \left[ A_{12} A_{22}^t \left( j\omega I_n - A_{22}/\epsilon \right)^{-1} \right. \right. \]  

\[ \times \left. \left. A_{21} - I_n \right\} \right. \right. \].  

(55)

Therefore, (54) can be changed to

\[ \det \left( I_n + j\omega A_1^{-1} A_{12} A_{22}^t \left( j\omega I_n - A_{22}/\epsilon \right)^{-1} A_{21} - I_n \right) \neq 0. \]  

(56)

and (50) follows.

The stability bound \( \epsilon_0 \) is the maximum number \( \epsilon \) such that (50) is satisfied. Unlike (24) the parameter \( \epsilon \) is embedded in \( H(\epsilon, s) \). Therefore, (50) is not convenient to use because one has to draw the Nyquist plot for the different values of \( \epsilon \). However, based on Theorem 3 we can develop the following useful result in which the parameter \( \epsilon \) is taken out from the associated transfer function.

**Corollary 1:** Under the same assumptions of Theorem 1, the matrix \( Q \) (or \( P \)) is stable if and only if

\[ \det \left( I_n - \epsilon H_1(j\omega) \right) \neq 0 \quad \forall \omega > 0 \]  

(57)

where \( H_1(s) \) is defined by

\[ H_1(s) = \frac{A_0}{s} + A_{12} A_{22}^t \left( sI_m - A_{22} \right)^{-1} A_{21}. \]  

(58)

**Proof:** Define \( \epsilon' = \epsilon s \). For \( \epsilon \neq 0 \) we have \( \epsilon' \neq 0 \). Then, \( H(\epsilon, s) \) can be written as

\[ H(\epsilon, s) = A_0^{-1} \left[ \epsilon s A_{12} A_{22}^t \left( sI_m - A_{22} \right)^{-1} A_{21} - s I_n \right] \]  

\[ = A_0^{-1} \left[ \epsilon' A_{12} A_{22}^t \left( s' I_m - A_{22} \right)^{-1} A_{21} - s' \epsilon I \right]. \]  

(59)

Thus, we have

\[ I_n + H(\epsilon, s) = I_n - \epsilon' A_0^{-1} + \epsilon' A_0^{-1} A_{12} A_{22}^t \left( s' I_m - A_{22} \right)^{-1} A_{21} \]  

\[ = \epsilon' A_0^{-1} \left\{ I_n - \epsilon' \left[ \frac{A_0}{s'} + A_{12} A_{22}^t \left( s' I_m - A_{22} \right)^{-1} A_{21} \right] \times (s' I_m - A_{22})^{-1} A_{21} \right\}. \]  

(60)

Therefore, (50) becomes

\[ \det \left\{ I_n - \epsilon' \left[ \frac{A_0}{s'} + A_{12} A_{22}^t \left( s' I_m - A_{22} \right)^{-1} A_{21} \right] \right\} \neq 0 \quad \forall \omega > 0. \]  

(61)

Corollary 1 provides a new method to compute the exact stability bound in terms of a transfer function matrix whose dimension is equal to the order of the slow modes. It complements the frequency-domain based method developed in [4] and makes it suitable and applicable to both cases of the lower order fast modes and the lower order slow modes. The transfer function \( H_1(s) \) plays the same role as that of \( G(s) \) in Theorem 2. One should notice the following differences between \( G(s) \) and \( H_1(s) \): 1) \( G(s) \) is improper while \( H_1(s) \) is strictly proper; and 2) \( G(s) \) has a zero at the origin while \( H_1(s) \) has a pole at the origin.

One can compute the stability bound \( \epsilon_0 \) based on the generalized Nyquist plot of \( H_1(s) \) as described in [4]. Assume that some of the Nyquist plots of the eigenvalues of \( H_1(s) \) intersect with the positive real axis at \((a_1, 0), (a_2, 0), \ldots, (a_t, 0)\), where \( a_1 \geq a_2 \geq \cdots \geq a_t > 0 \) and \( t \leq n \). The stability bound is given by

\[ \epsilon_0 = 1/a_1. \]  

(62)

If there is no intersection between the Nyquist plots and the positive real axis, then the stability bound \( \epsilon_0 \) is infinite.

In the following, we will provide an examples to illustrate the application of Corollary 1. The example is taken from [5]. Comparisons are also made between the result based on our new criterion (37) and the result based on criterion (38), which is originally proposed in [5]. It is shown that the bound determined by the proposed method is less conservative than the corresponding result in [5].

In the example, \( n = 2 \) and \( m = 2 \), and the matrices are given by

\[ A_{11} = \begin{bmatrix} -3 & 4 \\ 0 & -2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -3 & 4 \\ -1 & -2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 2 & 3 \\ 0 & -3 \end{bmatrix}. \]

In this example, the transfer function matrix \( H_1(s) \) has two eigenvalues. One can find that the Nyquist plot of one eigenvalue of \( H_1(s) \)
does not intersect with the positive real axis, and the Nyquist plot of the other eigenvalue does intersect with the positive real axis as shown in Fig. 2. Based on Fig. 2 one can determine that $\epsilon_0 = 1/1.02 = 0.98$, as what was obtained in [6] and [7] using their state-space model-based methods.

Using criterion (38) to estimate $\epsilon_0$, one can find at $\epsilon = 0.406$, $\| F_2(\epsilon, \omega)F_1(\omega) \|_\infty = 1$. Therefore, the stability bound is $\epsilon_0 \approx 0.406$, which obviously is conservative. On the other hand, if our new criterion (37) is used, one can find at $\epsilon = 0.68$, $\| F_1(\omega)F_2(\epsilon, \omega) \|_\infty = 1$ and therefore $\epsilon_0 \approx 0.68$. Thus, for this example, our new criterion (37) does provide improved estimate on $\epsilon_0$ over the available corresponding criterion in the literature [5].

III. CONCLUSION

A new systematic approach has been developed to characterize and compute the stability bound of a singularly perturbed linear system. An important feature of the new approach is its ability to change the stability bound problem of a matrix to the stability problem of a feedback system. It has been shown that there are multiple choices in constituting the underlying feedback system and the proposed approach has the potential to handle the stability bound problem in a number of different ways. By using the feedback based approach, some existing frequency-domain based results have been rederived in a systematic and unified way. New results on the stability bounds have also been developed, which complement the existing results. In particular, a new Nyquist plot based method has been proposed, which complements the major available frequency-domain based method and is especially useful for the system with lower order slow modes. An example has shown that the proposed Nyquist plot-based method is effective, and our sufficient stability criterion works better in some cases than the available one in the literature.

REFERENCES
