Hybrid PN-ZP-DMT Scheme for Spectrum-Efficient Optical Communications and its Application to SI-POF

Linning Peng, Maryline Hélard, Sylvain Haese, Ming Liu, and Jean-François Hélard

Abstract— In this paper, a novel discrete multi-tone (DMT) scheme is proposed for optical communications. Instead of the traditional cyclic prefix (CP), a hybrid structure of pseudo-noise (PN) sequence and zero-padding (ZP) sequence is proposed to serve as guard interval for the DMT transmission in order to achieve higher spectrum efficiency. The proposed PN-ZP-DMT scheme directly reuses PN sequence for channel estimation. The proposed PN-ZP-DMT scheme is then applied to plastic optical fiber (POF) transmission systems. Compared to the classical DMT blocks based channel estimation and time-frequency pilots based channel estimation, the PN sequence based channel estimation improves both performance and spectrum efficiency in POF. The optimal selection of PN sequence length is investigated. Simulation results show that the PN sequence based channel estimation can approach the optimal performance. Moreover, some practical features of PN-ZP-DMT transmission over POF such as system complexity and influence of time synchronization errors are studied. Finally, a PN-ZP-DMT transmission with 1.49 Gbps net rate is implemented in the low-cost 50 meters step index-POF system which eventually proves the effectiveness of the proposal.

Index Terms—DMT, OFDM, PN-ZP-DMT, POF, VLC, channel estimation, short-range, optical communications.

I. INTRODUCTION

Recen[tly, orthogonal frequency-division multiplexing (OFDM) has attracted more and more research interests for optical communications [1]. The OFDM can provide flexible and spectrum-efficient transmissions for optical networking [2]. Discrete multi-tone (DMT), a variant of OFDM, has been considered as an advanced modulation scheme for high-speed optical systems, especially in the field of short-range communications [3][4].

Compared to the single carrier modulation systems, DMT/OFDM systems have the advantage of high robustness to inter-symbol interferences (ISI). Owning to the insertion of the cyclic prefix (CP) at the beginning of each DMT/OFDM symbol, the ISI after the transmission over channel can be easily canceled at the receiver [5]. Additionally, the equalization process of the DMT/OFDM systems is relatively simpler than with single carrier modulation systems. With the help of channel estimation, the received signal can be simply equalized by 1-tap equalization for each DMT/OFDM subcarrier in the frequency domain. Furthermore, in the DMT systems, bit-loading techniques are usually employed in order to approach the channel capacity [4] [6]. Therefore, the overall performance of these systems is highly dependent on the accuracy of the channel estimation.

In wireless communications, in order to estimate the channel response at the receiver, pilots are usually inserted in the OFDM symbols at the transmitter side [5]. The strategy of frequency-domain pilot deployment has been investigated in [7]. According to different channel conditions, two kinds of pilot deployment strategies could be considered. The first one consists in using all the subcarriers of some specific OFDM symbols as pilots and commonly referred to as ‘block type’ pilot. This block type pilot strategy is usually used for the static or slowly varying channel conditions. The other pilot deployment strategy is to insert pilot tones in each OFDM symbol, which builds a time-frequency pilots grid for channel estimation. This is a typical implementation in wireless communications where the channel transfer function changes rapidly [7][8]. Once the pilots are received, channel estimation can be processed using least square (LS) or minimum mean square error (MMSE) criterions [9]. The MMSE estimator assumes a priori knowledge of noise variance and channel covariance, while the LS estimation only relies on the channel coefficients. Moreover, the MMSE estimator is more complex than LS estimator [9]. The LS estimator is proved to be both simple and adequate in high signal to noise ratio (SNR) scenarios [9]. However, for low SNR cases, the trade-off between estimator complexity and performance should be considered [8][9].

In recent years, a novel OFDM waveform, commonly referred to as time domain synchronous (TDS)-OFDM, has been proposed [10]. Pseudo-noise (PN) sequences, instead of the classical CPs, are used as the guard intervals. The PN sequences are known at the receiver side and reused as training sequences for channel estimation and synchronization [11][12]. Therefore, there is no need to employ frequency-domain pilots, which can improve the spectrum efficiency. The TDS-OFDM waveform has been adopted in the Chinese digital television/terrestrial multimedia broadcasting (DTMB) system [13][14]. However, its application in optical fiber systems has not been considered in the literature.

Compared with wireless communication scenarios, the optical channel is more static [1], but commonly the optical
communications require higher channel estimation accuracies to support high data rates, especially for those employing bit-loading techniques [15].

In the optical OFDM systems, the block type pilot channel estimation is commonly used [16][17]. The application of zero padding (ZP)-OFDM in optical communications was recently investigated in [18]. The time-frequency pilots based channel estimation is also reported in [18]. Compared to the block based channel estimation, the time-frequency pilots based channel estimation can reduce the pilot occupation ratio depending on the chosen parameters. For example in [18] the pilot occupation ratio decreases from 9.16 % with block pilots to 2.48 % with time-frequency pilot arrangement.

Recently, high-speed transmission over plastic optical fiber (POF) has attracted many research interests [19-21]. In POF transmissions, the channel presents a low-pass frequency response [4][6][22] and the signal at low-frequency subcarriers has very high SNR. Therefore, high order modulations, such as 256-quadrature amplitude modulation (QAM-256), can be used according to the channel conditions [23][24]. In addition, it is worth noting that many other optical communications such as short-range optical communications over single-mode fiber (SMF) [25], multi-mode fiber (MMF) [26], and optical wireless with visible light communications (VLC) [27], also have similar channel characteristics as POF. Accurate channel estimation is particularly required when bit-loading is employed, since the better the estimated channel information, the greater the attainable throughput. Therefore, efficient pilot arrangement and channel estimation schemes are critical for the overall system spectrum efficiency. In the aforementioned CP-OFDM [16] and ZP-OFDM systems [18], the used modulation order is only quadrature phase-shift keying (QPSK) or QAM-16, which is not the case in the short-range optical DMT systems with very high order modulations and bit-loading. Therefore, the well studied channel estimation techniques in conventional optical OFDM systems are not accurate enough for the short-range optical DMT systems. In addition, a demonstration of experimental transmission over step index (SI-) POF employing a proposed PN-ZP-DMT scheme was reported in [28] showing the high efficiency of the new transmission scheme. However, no detail and optimization on the algorithm has been presented in the literature.

In this paper, a detailed investigation of the PN-ZP-DMT scheme is presented. The PN sequence based channel estimation is introduced for the DMT transmission over optical fiber and compared with classical channel estimation methods for optical OFDM systems. Moreover, a hybrid PN-ZP-DMT scheme combined with PN and ZP sequences is proposed in order to further improve the spectrum efficiency and save the transmission power. The proposed hybrid PN-ZP-DMT scheme can significantly reduce the overhead caused by pilots and improve the accuracy of the channel estimation. The contributions of this paper are as follows: 1) The principle of PN sequence based channel estimation is firstly introduced for optical OFDM systems; 2) The system complexity, spectrum efficiency and performance of the PN-ZP-DMT scheme are investigated and compared to the classical CP-DMT schemes; 3) The time synchronization and its error influence to PN-ZP-DMT systems are discussed; 4) The optimal PN length selection for a specified 50 m SI-POF systems is investigated.

The remainder of this paper is organized as follows. In Section II, the classical CP-DMT, ZP-DMT and the proposed PN-DMT and hybrid PN-ZP-DMT systems are introduced. In Section III, the block and time-frequency pilots (TFP) based channel estimation, the PN sequence based channel estimation and the improved PN sequence based channel estimation are investigated. In Section IV, the system complexity of different channel estimation methods are analyzed. In Section V, the time synchronization and the influence of time synchronization error for PN-ZP-DMT schemes are discussed. Section VI presents the Gaussian low-pass filter channel model for 50 m SI-POF transmission system. Finally, the simulated and experimental results are presented in Section VII. Section II to V could be applied to all optical systems using OFDM. Section VI and VII are dedicated to investigations for the SI-POF transmission system.

II. MULTI-CARRIER MODULATION SYSTEM SCHEMES

A. CP-Assisted DMT System

The classical CP-DMT systems use the CP to protect the DMT symbol from the ISI. The CP-DMT waveform is depicted in Fig. 1. The $k^{th}$ CP-DMT symbol is shown as $S^k = [0, S^k_1, \cdots S^k_{N-1}]^T$, where $S^k_i$ is QAM mapped symbol for the $i^{th}$ subcarrier. $N$ is the number of active subcarriers. The modulated DMT symbol is presented as:

$$S^k = [S^k_0, \cdots S^k_{2N-1}]^T = F_{2N}^H \left[ S^k_T, \bar{S}^k_T \right]^T, \quad (1)$$

where $F_{2N}^H$ is the 2N points inverse fast Fourier transform (IFFT) for DMT modulation, $F_{2N}^H(a,b) = \frac{1}{\sqrt{2N}} e^{j\pi \frac{2\pi ab}{2N}}$. $\bar{S}^k$ is the conjugation process in order to follow Hermitian symmetry property. Therefore only real value samples are generated. The CP is

$$c^k_{cp} = \left[ S^k_{2N-L_{cp}}, \cdots S^k_{2N-1} \right]^T, \quad (2)$$

where $L_{cp}$ is the length of CP. When $L_{cp}$ is larger than the channel length $L$, the useful part of the received CP-DMT symbol is not corrupted by the ISI caused by the channel memory. Finally, the modulated $k^{th}$ CP-DMT symbol can be represented as:

$$c^k = \left[ c^k_{cp}, S^k \right]^T. \quad (3)$$

Given the channel frequency response (CFR) $H^k = [H^k_0, H^k_1, \cdots H^k_{N-1}]^T$, the received $k^{th}$ CP-DMT symbol at $i^{th}$
subcarrier \( R^k_i \) can be given as:

\[
R^k_i = H^{k}_i S^k_i + W^k_i,
\]

where \( W^k_i \) is the additive noise at \( i^{th} \) subcarrier of the \( k^{th} \) CP-DMT symbol. Then, with the help of frequency domain 1-tap equalizer and LS equalization for each subcarrier, the equalized symbol \( S^k_i \) is restored as:

\[
S^k_i = H^{-1}_i \cdot (H^k_i S^k_i + W^k_i) = S^k_i + e^k_i,
\]

where \( e^k_i \) is the combined noise and channel estimation error distortion at \( i^{th} \) subcarrier, \( H^{-1}_i \) is the estimated 1-tap inverse CFR at \( i^{th} \) subcarrier.

### B. ZP-Assisted DMT System

Similar to ZP-OFDM in [18], the ZP-DMT waveform is shown in Fig. 2. Compared to the classical CP-DMT system, CP symbols are replaced by a series of zeros with ZP-DMT systems. The modulated \( k^{th} \) ZP-DMT symbol can be represented as:

\[
Z^k = \begin{bmatrix} z_{zp}^k & s^k \end{bmatrix}^T,
\]

where \( z_{zp}^k = \left[0_{L_{zp} \times 1}\right]^T \), \( L_{zp} \) is the length of the zeros, \( s^k \) is the modulated DMT symbols. As shown in Fig. 2(b), the received DMT symbols are distorted due to the multi-path channel. The received \( k^{th} \) ZP-DMT symbol is written as:

\[
p^k = \bar{h}^{k-1} \ast z_{zp}^{k-1} + \bar{h}^k \ast z^k + w^k,
\]

where \( \ast \) is linear convolution process, \( \bar{h}^k = [h^k_0, h^k_1, \ldots, h^k_{L-1}]^T \) is the channel impulse response (CIR) at the time of \( k^{th} \) ZP-DMT symbol, \( w \) is the noise, \( \bar{h}^{k-1} \ast z_{zp}^{k-1} \) is the ISI caused by the previous ZP-DMT symbol. When ZP length \( L_{zp} \) is larger than the channel delay spread \( L \), the received time-domain \( n^{th} \) sample of the \( k^{th} \) ZP-DMT symbol can be expressed as:

\[
r^k_n = \sum_{l=0}^{L-1} h^k_l z^k_{n-l} + w^k_n, \quad n > L_{zp},
\]

In the receiver, the overlap-and-add (OLA) technique is employed to restore the cyclicity of the received ZP-DMT symbols [14]. The process of this OLA technique is expressed as:

\[
r^k_n = r^k_n + r^k_{n+2N}, \quad 0 \leq n < L_N
\]

Consequently, the noise power is enhanced with a factor \( N + L_{zp}/N \) as compared to CP-DMT. The DMT symbol can be easily recovered in the frequency domain as done with CP-DMT, as shown in Fig. 2(c). Given the CFR \( \bar{H}^k = [H^k_0, H^k_1, \ldots, H^k_{N-1}]^T \), after the OLA process and DMT demodulation \( F_{2N} \), the received \( k^{th} \) ZP-DMT symbol at \( i^{th} \) subcarrier \( R^k_i \) can be written as:

\[
R^k_i = H^k_i S^k_i e^{-\frac{i\pi n}{N}} + W^k_i.
\]

In addition, it is worth noting that since no signal is transmitted during the guard interval, the transmission power can be reduced with ZP-DMT systems.

### C. PN-Assisted DMT System

Fig. 3 gives an illustration of the PN-DMT waveforms. Being different from ZP-DMT, PN-DMT employs PN sequences, instead of zeros, between two consecutive DMT symbols. The modulated \( k^{th} \) PN-DMT symbol can be given as:

\[
p^k = \begin{bmatrix} p^k_{pn} & s^k \end{bmatrix}^T,
\]

where \( p^k_{pn} = [m^k_0, m^k_1, \ldots, m^k_{L_{pn}-1}]^T \), \( m^k_n \) is the \( n^{th} \) element of the PN sequence, \( L_{pn} \) is the length of the PN sequence, \( s^k \) is the modulated DMT symbols.

At the receiver, PN sequences and DMT symbols overlap due to the channel memory effect, as shown in Fig. 3(b). The received \( k^{th} \) PN-DMT symbol is expressed as:

\[
p^k = \bar{h}^{k-1} \ast \tilde{p}^{k-1} \ast \bar{h}^k \ast \tilde{p}^k + w^k
\]

where \( \tilde{p}^{k-1} \ast \tilde{p}^{k-1} \ast \bar{h}^{k-1} \ast \bar{h}^k \ast \tilde{p}^{k} \ast \tilde{p}^{k} \) is the ISI caused by the previous PN-DMT symbol. As the PN sequence is reused for the channel estimation, the ISI of the PN sequence should be eliminated before the channel estimation. Adding CP for the PN sequence can easily cancel the ISI [14]. Therefore the PN-DMT system with the CP of the PN sequence can be presented as:

\[
p^k_{pn} = [m^k_{L_{cn}+1}, m^k_{L_{cp}+1}, \ldots, m^k_{L_{pn}-1}]^T
\]

When \( L_{cp} \geq L \), the \( n^{th} \) sample of the \( k^{th} \) PN-DMT symbol can be presented as:

\[
r^k_n = \sum_{l=0}^{L-1} h^k_l p^k_{n-l} + w^k_n, \quad n \geq L_{cp}
\]
As the PN sequences are known by the receiver, once the channel coefficients \( \tilde{h}_k \) are properly estimated, it is easy to remove them from the received PN-DMT symbols. After removing PN, the PN-DMT symbols are turned into equivalent ZP-DMT symbols, as shown in Fig. 3(c). With the help of OLA process, the frequency data symbol follows the same expression as (10).

It is worth noting that the CIR \( h_k \) can be estimated by using the received PN sequence \( \tilde{r}_k^{\text{PN}} = \left[ \tilde{r}_{k,0}^{\text{PN}}, \tilde{r}_{k,1}^{\text{PN}}, \cdots, \tilde{r}_{k,p-1}^{\text{PN}} \right]^T \).

An efficient PN-correlation-based estimator will be introduced in Section III. In addition, an imperfect channel estimation causes an imperfect PN removal and results in residual interferences from PN. Therefore, an accurate channel estimation is crucial to optimize the performance of the system.

**D. Hybrid PN-ZP-DMT System**

For the PN sequence based channel estimation, the longer the PN sequence, the better the channel estimation quality [29]. However, the use of excessively long PN sequences decreases the overall system efficiency. As it is well known that the optical channel is relatively stable, it is reasonable to consider \( \tilde{H}_{k-1} = \tilde{H}_k \) for a certain duration. Therefore it is not necessary to perform channel estimation for each DMT symbol. The PN sequence based guard intervals can be employed only for the first \( N_p \) DMT symbols. Then zero is used for the consequent \( N_z \) DMT symbols. This yields a hybrid PN-ZP-DMT transmission scheme. Each super frame \( \tilde{s}^{\text{f}} \) is grouped with \( N_p \) PN-DMT symbols and \( N_z \) ZP-DMT symbols:

\[
\tilde{s}^{\text{f}} = [\tilde{s}^1, \cdots, \tilde{s}^{N_p}, \tilde{s}^{N_p+1}, \tilde{s}^{N_p+2}, \cdots, \tilde{s}^{N_p+N_z}].
\]

(15)

The coefficients \( \tilde{H}^{N_p-1}_k \) of the LS equalizer obtained from \( N_p \) PN-DMT symbols can be used to equalize following ZP-DMT symbols, i.e. \( \tilde{H}^{N_p-1}_k = \tilde{H}^{N_p+2-1}_k = \cdots = \tilde{H}^{N_p+N_z-1}_k \).

When \( N_p = 1 \), the structure of the proposed PN-ZP-DMT scheme is shown in Fig. 4. By optimizing the length of the PN sequence \( L_{\text{pn}} \), the number of PN-DMT symbols \( N_p \), as well as the length of the ZP \( L_{\text{zp}} \), the number of ZP-DMT symbols \( N_z \), the proposed hybrid PN-ZP-DMT scheme can maximize the system spectrum efficiency in POF transmissions.

**III. CHANNEL ESTIMATION FOR DMT**

In the aforementioned DMT systems, the received DMT symbols can be restored by the equalizer. In order to get the coefficients \( \tilde{H}^{N_p-1}_k \) of the equalizer, different estimators can be used.

**A. Block Pilot Based Estimation**

Most DMT-POF transmissions reported in the literature adopt block pilot based channel estimation. The pilot symbols are transmitted in the first several DMT symbols. The estimated CFR \( \tilde{H}^k \) can be obtained by the LS estimation:

\[
\tilde{H}^k = R^k / S^k.
\]

(16)

As the optical channel is slowly varying, pilot blocks are repeatedly sent after several data DMT symbols [28][30].

**B. Time-Frequency Pilots Based Estimation**

In the TFP based estimation, the pilots are spread in the time-frequency domain. The criterions of the interval between time and frequency domain pilots are shown as [31]:

\[
f_{\text{max}} \cdot T_s \cdot n_k \leq 1/2 \quad \text{and} \quad \tau_{\text{max}} \cdot \Delta f \cdot n_k \leq 1/2,
\]

(17)

where \( f_{\text{max}} \) is the maximum frequency shift, \( T_s \) is the total symbol duration, \( n_k \) is the pilot spacing in time, \( \tau_{\text{max}} \) is the channel delay spread, \( \Delta f \) is the frequency spacing between subcarriers, \( n_k \) is the pilots spacing in frequency.

As the CFR \( \tilde{H}^k \) can be estimated using pilots \( (i, k) \) of the pilots index) with the same process in (16), the whole CFR can be estimated by linear interpolation techniques [32]. 2-dimensional Wiener filtering can further improve the estimation performance with an additional complexity [14].

**C. PN Sequence Based Estimation**

In the TDS-OFDM systems, the received PN sequences can be reused for the channel estimation [10]. In this paper, the m-sequences [33] are selected as the PN sequences in the PN-DMT and PN-ZP-DMT systems due to their ease of generation and their associated low-cost channel estimation algorithm. The channel estimation can be simply obtained by performing time domain correlation of known and received PN sequences [29]. The circular autocorrelation of the m-sequence is known as [33]:

\[
CR_j = \frac{1}{L_{\text{pn}}} \sum_{l=0}^{L_{\text{pn}}-1} m_i m_{(j+l)_{L_{\text{pn}}}} = \begin{cases} 1 & j = 0, \\ -1/L_{\text{pn}} & \text{else}. \end{cases}
\]

(18)

where \((\cdot)^*\) is the complex conjugate and \([\cdot]_{L_{\text{pn}}} \) denotes modulo-\(L_{\text{pn}} \) operation. Taking the signal expression in (14), the cross-correlation of the \( k \)th received and known PN sequence is:

\[
CR^k_i = \frac{1}{L_{\text{pn}}} \sum_{l=0}^{L_{\text{pn}}-1} \left( \sum_{\ell=-1}^{L-1} h^k_{\ell \ell} p^{(l+\ell)_{L_{\text{zp}}}} + w^k_{(l+\ell)_{L_{\text{zp}}}} \right) \cdot m^k_{(l+1)_{L_{\text{pn}}}},
\]

(19)

where \( L \) is the channel length, \( L_{\text{pn}} \) is the PN sequence length.

We assume that the noise is Gaussian additive, the power of the PN sequence is 1. Taking into account the circular autocorrelation property in (18), the cross-correlation in (19) can be represented as:

\[
CR^k_{i_{\text{interference}}} = h^k_i - \frac{1}{L_{\text{pn}}} \sum_{l=0}^{L-1} h^k_l + \frac{w}{\text{Noise}}.
\]

(20)
where $CR_{l}^{k}$ is the estimated CIR of the $l^{th}$ channel delay, $h_{l}^{k}$ is the real CIR. The noise variance of $w$ equal to $\sigma^{2}/L_{pn}$ is related to $\sigma^{2}$, the normalized received noise variance and $L_{pn}$ the PN length. The interference and noise terms in (20) cause the residual error from PN after the equalization.

Finally, the estimated CFR $\tilde{H}^{k}$ can be obtained by:

$$\tilde{H}^{k} = F_{N} \cdot I_{N-L_{pn}} \cdot \bar{CR}_{L_{pn}}^{k},$$  

(21)

where $I_{N-L_{pn}}$ represents the interpolation process. The block diagram of the PN correlation based channel estimation is depicted in Fig. 5.

D. Improved PN Sequence Based Estimation

When the channel length $L$ is known by the receiver, the interference in (20) can be easily canceled. From (20), the sum of the $L$ estimated CIR is represented as:

$$\Delta_{CIR} = \sum_{l=0}^{L-1} CR_{l}^{k} = \frac{L_{pn}}{L_{pn}} - L + 1 \sum_{l=0}^{L-1} h_{l}^{k} + w',$$  

(22)

It is worth noting that $\Delta_{CIR}$ can be calculated as a constant value. With the following process,

$$\bar{CR}_{L_{pn}}^{k} = \frac{L_{pn}}{L_{pn}+1} \left( CR_{L_{pn}}^{k} + \frac{1}{L_{pn} - L + 1} \Delta_{CIR} \right),$$  

(23)

we can get that:

$$\bar{CR}_{L_{pn}}^{k} = h_{l}^{k} + \frac{w'}{\text{Noi}se},$$  

(24)

where only noise remains in the estimated CIR. The noise variance of $w'$ in (24) is calculated as:

$$\sigma^{2}' = \frac{L_{pn}^{2}}{L_{pn}^{2} - L_{pn}L + 3L_{pn} + L} \sigma^{2}.$$  

(25)

which is similar to the noise variance in (20). The simulation results in [29] show that this improved PN correlation based estimator can reach the Cramér-Rao bound of a training sequence based channel estimation.

IV. SYSTEM COMPLEXITY

In this section, we analyze the system complexity of different DMT schemes with different channel estimation methods. The DMT schemes and the associated channel estimation methods are listed in Table I.

- **TABLE I**

<table>
<thead>
<tr>
<th>DMT Scheme</th>
<th>Channel Estimation Method</th>
</tr>
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<tbody>
<tr>
<td>CP-DMT</td>
<td>Blocks based &amp; Time-Frequency pilots based</td>
</tr>
<tr>
<td>ZP-DMT</td>
<td>Blocks based &amp; Time-Frequency pilots based</td>
</tr>
<tr>
<td>PN-DMT</td>
<td>PN sequence based</td>
</tr>
<tr>
<td>PN-ZP-DMT</td>
<td>PN sequence based</td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Channel Estimation</th>
<th>Computational Complexity</th>
<th>Multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMT blocks based</td>
<td>$O(2N \log 2N)$</td>
<td>10 240</td>
</tr>
<tr>
<td>TFP based (Linear Interpolation)</td>
<td>$O(N)$</td>
<td>512</td>
</tr>
<tr>
<td>TFP based (Wiener Filtering)</td>
<td>$O(N^2 + N)$</td>
<td>262 656</td>
</tr>
<tr>
<td>PN sequence based</td>
<td>$O(6L_{pn} \log L_{pn} + O(N \log N))$</td>
<td>16 848</td>
</tr>
<tr>
<td>Improved PN sequence based</td>
<td>$O(2L_{pn} \log L_{pn} + 4L \log L) + O(M \log N)$</td>
<td>9 328</td>
</tr>
</tbody>
</table>

1. $N$ is the number of active subcarriers, $L_{pn}$ is the PN sequence length, $L$ is the maximum channel delay.

2. Obtained with typical system configurations: $N = 512$, $L_{pn} = 255$, $L = 32$.

In classical CP-DMT systems, $2N$ point FFT is employed to demodulate the DMT symbols, requiring a complexity of $O(2N \log 2N)$.

In the ZP-DMT systems, OLA operation is proved to be a low complexity process, which only requires $L$ additions. The PN-DMT systems need an additional operation of PN sequence removal, which requires $L_{pn}$ additions. The OLA in PN-DMT systems requires $2L$ additions. Compared with the FFT process, the additional operations in both ZP-DMT and PN-DMT are negligible. Therefore, PN-DMT and PN-ZP-DMT have the same levels of computational complexities as the classical CP-DMT.

The DMT blocks based estimation requires additional DMT probing symbols to retrieve channel information. The channel estimation complexity is the operation complexity of DMT demodulation: $O(2N \log 2N)$.

In the TFP based estimation, the pilot information is obtained from the demodulated DMT symbols. Therefore no dedicated block probing symbols are required. When the linear interpolation is used in the TFP based estimation, the computation complexity is $O(N)$. When the Wiener filtering is used, the computation complexity is $O(N^2 + N)$ [34].

In the PN correlation based estimation, the cross-correlation based CIR estimation requires $O(L_{pn}^2)$ operations. The CFR can be obtained by interpolation and FFT, which requires $O(N \log N)$ operations. In order to eliminate the interference in (14), $O(2L_{pn}^2)$ operations are required by OLA. Therefore, the total computation complexity is $O(3L_{pn}^2) + O(N \log N)$. Considering that the correlation process can be implemented by FFT and IFFT, the computation complexity can be reduced to $O(6L_{pn} \log L_{pn}) + O(N \log N)$. As shown in the calculation, the computational complexity greatly depends on the PN length $L_{pn}$.

In the improved PN correlation based estimation, due to the knowledge of channel length $L$, the additional operation for the interference cancellation is $L$ times additions and...
multiplications, which is negligible compared to the whole operations. The number of operations for OLA is reduced to $O(2L^2)$, and is further reduced to $O(4L \log L)$ when FFT is used for correlation process. Therefore, the total computational complexity is $O(2L_{cn} \log L_{cn} + 4L \log L) + O(N \log N)$. However, in a practical system, some additional complexities of acquiring a channel length might be taken into account.

The computational complexities of different channel estimation methods are given in Table II. To give an intuitive overview, we present as an example the required multiplications of different methods with typical system configurations. As shown in the table, the TFP based channel estimation with linear interpolation owns the lowest complexity. When the Wiener filtering is used, the complexity of TFP based channel estimation will be significantly increased. It is worth noting that when the PN length $L_{cn}$ is less than the number $N$ of active subcarriers, the complexity of PN correlation based channel estimation can be similar to that of DMT block based channel estimation. Moreover, the improved PN correlation based channel estimation can be even simpler than the standard PN correlation based channel estimation.

V. SYSTEM TIME SYNCHRONIZATION

Using the $L_{cp}$ length CP of the PN sequence, the time synchronization of the PN-DMT system can be implemented by the ML estimator [35]:

$$\hat{\theta} = \arg \max \lambda(\theta),$$

where

$$\lambda(\theta) = 2 \sum_{n=0}^{\theta \oplus L - 1} r_n^k r_{n+L_{cn}}^k - \rho \sum_{n=\theta}^{\theta \oplus L - 1} \left( |r_n^k|^2 + |r_{n+L_{cn}}^k|^2 \right),$$

where

$$\rho = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_w^2}, \quad \sigma_r^2 = E(|s^k|^2).$$

When a time synchronization offset of $\varepsilon$ samples occurs, the PN-correlation based channel estimation can be presented as:

$$CR(\varepsilon)_l^k = \frac{1}{L_{cp}} \sum_{l=0}^{L_{cp}-1} \left( \sum_{i=0}^{\varepsilon - 1} h_i^k m_{i+\varepsilon}^k + w_{i+\varepsilon}^k \right),$$

where

$$CR(\varepsilon)_l^k = h_{l+\varepsilon}^k + w_{l+\varepsilon}^k, \quad \varepsilon \leq \theta \oplus L - L_{cp}.$$ (30)

Then

$$\Delta_{CIR}(\varepsilon) = \sum_{i=\varepsilon}^{L_{cp} - 1} h_i^k + w_{l+\varepsilon}^k, \quad \varepsilon \leq \theta \oplus L - L_{cp}.$$ (31)

The impact of the time synchronization sample offset $\varepsilon$ in the improved PN-correlation based estimation can be derived:

$$CR(\varepsilon)_l^k = \frac{h_{l+\varepsilon}^k}{L_{cp}} - \frac{L_{cp}}{L_{cp} + 1} \left( \frac{h_{l+\varepsilon}^k}{L_{cp} - 1} \sum_{i=\varepsilon}^{L_{cp} - 1} h_i^k + w_{l+\varepsilon}^k \right) + \frac{\sigma_r^2}{\sigma_r^2 + \sigma_w^2} \sum_{i=\varepsilon}^{L_{cp} - 1} h_i^k + w_{l+\varepsilon}^k,$$ (32)

which means that due to the interference, an estimation error floor of the channel estimation exists. From the derivation we can get the conclusion that time synchronization error does not affect the classical PN-correlation based estimation, but seriously affect the improved PN-correlation based estimation.

VI. EXPERIMENTAL SETUP FOR POF TRANSMISSION

In order to investigate the performance of the different channel estimation methods for the DMT transmissions over POF, a 50 m step-index (SI) POF (ESKA™ Mega, Mitsubishi) transmission system with low-cost resonant cavity light emitting diode (RCLED, FC300R-120™, Firecomms) and Si-PIN diode (FC1000D-120™, Firecomms) is introduced. The experimental setup of the 50 m SI-POF system is depicted in Fig. 6. The optical power fed into the SI-POF is +0.2 dBm, the received optical power is -9.4 dBm. The DMT signal is designed to have 512 subcarriers which is a good trade-off between performance and complexity [15]. A total available transmission bandwidth of 300 MHz is used to generate DMT probing symbols and estimate the SNR of each subcarrier.

In order to compare the performance gap of different channel
estimation methods to the perfect performance, we introduce a POF transmission model and carry out simulations. The measured POF transmission channel can be modeled as a Gaussian low-pass filter [22], with the following function:

\[ H_f = A \cdot e^{-\left(\frac{f_0}{f_{3dB}}\right)^2}, \quad f_0 = f_{3dB}/\sqrt{\ln(2)}, \]  

(33)

where \( H_f \) is the CFR at \( f \) Hz, \( A \) is the fiber loss, \( f_{3dB} \) is the 3 dB bandwidth of the channel frequency response. We assume that the noise in the SI-POF transmission channel is Gaussian additive. The measured results of 3 dB bandwidth and normalized noise power spectral density with the average signal power of 1 are shown as:

\[ f_{3dB} = 90 \text{ MHz}, \quad \sigma_w^2 = -113.7 \text{ dB/Hz}. \]  

(34)

Using the POF channel model given in (33) and channel parameters given in (34), the SNR for each subcarrier can be calculated. According to the linear approximation of the BER-SNR table based bit-loading, which was initially proposed in [6], the obtained SNR can be used to determine the modulation and power that are allocated for each subcarrier with a targeted BER of \( 1 \times 10^{-3} \). The SNR and allocated bits of each subcarrier in the simulation are depicted in Fig. 7(a). Accordingly, these results in the real measured transmission are depicted in Fig. 7(b). As a result, each DMT symbol carries 2461 bits in simulations with used subcarrier index from 2 to 433 and 2572 bits in real transmissions with used subcarrier index from 2 to 512.

VII. SIMULATED AND EXPERIMENTAL RESULTS

A. Simulation Parameters

As in the experimental setups, we use a 300 MHz transmission bandwidth and 512 subcarriers. Therefore the 1024-point IFFT is used to generate the DMT symbols with a sampling rate of 600 M samples per second. Then an oversampling factor of 4 compared to the used bandwidth is used in both simulations and experiments in order to make appropriate continuous-time waveforms in simulations [36] and avoid high frequency aliasing in experiments [37]. Therefore, the sampling frequency of the DAC is equal to 1.2 G samples per second. The duration of each useful part of DMT symbol \( t_g \) is 1706.7 ns with a duration of each sample equal to 0.83 ns. A POF transmission channel is modeled by a finite impulse response filter with 13 taps and 0.83 ns time resolution.

<table>
<thead>
<tr>
<th>Degree ( D )</th>
<th>Length ( L_{pn} )</th>
<th>Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>31</td>
<td>( x^2 + x^2 + 1 )</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>( x^6 + x + 1 )</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>( x^7 + x + 1 )</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>( x^8 + x^6 + x^2 + x + 1 )</td>
</tr>
<tr>
<td>9</td>
<td>511</td>
<td>( x^9 + x^4 + 1 )</td>
</tr>
<tr>
<td>10</td>
<td>1023</td>
<td>( x^{10} + x^3 + 1 )</td>
</tr>
</tbody>
</table>

Therefore, the maximum channel delay \( L \) is considered as 13 (equal to 10.8 ns). The CP length \( L_{cp} \) and the ZP length \( L_{zp} \) are set equal to the channel maximum delay \( L \).

The m-sequences with different length \( L_{pn} \) are generated. With a given degree \( D \), the length of the m-sequence is:

\[ L_{pn} = 2^D - 1. \]  

(35)

The m-sequence can be generated using maximal linear feedback shift registers represented by the polynomials [38]. Some of the primitive polynomials used for the simulations are listed in Table III.

Although the POF channel is quite stable, the clocking offsets of the digital-to-analog converter (DAC) at the transmitter and analog-to-digital converter (ADC) at the receiver cause the time synchronization offset after the transmission of several DMT symbols. In order to evaluate this offset, we choose to make channel estimation once every 20 DMT symbols, as in [16][28].

For the DMT blocks based channel estimation, we initially transmit \( N_f \) DMT probing symbols. The probing symbols are repeatedly transmitted every 20 DMT symbols. In our simulations, two conditions of \( N_f = 1 \) and 5 are considered. We note ‘Block-1’ and ‘Block-5’ for the DMT block based channel estimations with \( N_f = 1 \) and 5, respectively.

For the TFP based channel estimation, the channel delay spread \( t_{max} \) is 10.8 ns and the subcarrier spacing \( \Delta f \) is around 586 KHz. According to (17), the pilots spacing in frequency \( n_L \) should be less than 79. When a \( \pm 10 \) ppm frequency offset is assumed between the oscillators of the DAC and ADC with a sampling rate at 600M samples per second, the resulting maximum frequency shift \( f_{max} \) is 12 KHz. Given \( t_g = 1706.7 \) ns and according to (17), the pilot spacing in time \( n_K \) is at least 24. In practical systems, frequency pilot deployment with a higher density should be considered in order to improve the channel estimation performance [9]. Therefore, in our work, \( n_L \) is selected to be 28 and 14, noted as ‘TFP-28’ and ‘TFP-14’ respectively, to show the performance with higher pilot densities than the minimum value given by the sampling theorem. In addition, \( n_K \) is selected to 20 in order to make an equal pilots symbol repetition interval to the DMT blocks based
channel estimation. As the Wiener filtering requires complicated computations, in order to compare the performance with a similar system complexity, only the linear interpolation is adopted for TFP based channel estimation.

For the PN-correlation based channel estimation, PN sequences with lengths of 1023, 511, 255, 127, 63 and 31 are considered. The time resolution of PN sequence equals to the system sampling rate, which is 0.83 ns. As the channel estimation is required over 20 symbols, it is clear that \( N_p + N_z = 20 \). Therefore PN-ZP-DMT scheme with \( N_p = 1, N_z = 19 \) is set for the simulations. We note ‘PN-1023’, ‘PN-511’, ‘PN-255’, ‘PN-127’, ‘PN-63’ and ‘PN-31’ as the PN sequence based channel estimations with different PN length \( L_{pn} \) equal to 1023, 511, 255, 127, 63 and 31, respectively.

With the aforementioned system setups, system spectrum efficiencies of DMT schemes with different channel estimation approaches are listed in Table IV.

B. Mean Square Error of Zero-Forcing Equalizer with Channel Estimation

In most wireless OFDM systems, the measurement of the MSE of CFR estimation is averaged over each OFDM symbol [14]. In POF transmission systems, the CFR are relatively stable compared to wireless communication systems. In addition, as the bit-loading is employed in most of DMT-POF systems, the subcarriers with higher SNR are allocated with higher order QAM and inherently require lower equalization error after the channel estimation. In addition, in optical OFDM systems, the 1-tap zero-forcing (ZF) equalizer is usually employed for the consideration of low complexity. However, it is well-known that the noise is enhanced after ZF equalization when the channel is faded (corresponding to the subcarriers with low SNR). Therefore, it is important to evaluate the accuracy of equalization factor which indicates the noise enhancement level after equalization, and consequently suggest the impact of channel estimation techniques on the system performance. We define the MSE of the equalization factor as:

\[
\text{MSE}_i^k = E\left\{(\hat{G}_i^k - G_i^k)^2\right\},
\]

(36)

where \( \hat{G}_i^k = \frac{1}{H_i^k} \) with \( \hat{H}_i^k \) the estimated CFR, and \( G_i^k = \frac{1}{H_i} \) with \( H_i^k \) the real CFR.

The simulation is realized with 100000 DMT symbols. After the transmission over the aforementioned Gaussian low-pass channel model, different channel estimation methods are compared to estimate the CFR. The \( \text{MSE}_i^k \) is calculated using (36) and channel estimation performance is evaluated.

The comparison of the \( \text{MSE}_i^k \) of different channel estimation methods is shown in Fig. 8. As shown in the figure, with block based, TFP based and improved PN sequence based channel estimations, the pilots at low frequencies own higher SNRs and offer better MSE performance. On the contrary, the pilots at high frequencies suffer more from the noise, which degrades the channel estimation performance.

The simulation results in Fig. 8 show that the ‘Block-1’ and ‘PN-1023’ have the worst MSE performance, which is around \( 1 \times 10^{-3} \) for the subcarriers with high SNRs. The MSE performance of DMT block based channel estimation can be improved with the help of increasing the number of blocks for channel estimation, as shown by the curve ‘Block-5’. The TFP based channel estimation ‘TFP-28’ can provide better performance than the DMT block based channel estimation ‘Block-1’. TFP based channel estimation with higher pilot densities can further improve the performance, which is shown by the curve ‘TFP-14’. In the case of PN sequence based channel estimation, the low performance and the increased MSE at low frequencies of ‘PN-1023’ is mainly due to the interference shown in (20). With the help of improved PN sequence based channel estimation, the MSE is significantly reduced, which is shown by the curve ‘PN-1023 Imp.’. The improved PN sequence based channel estimation with a PN length of 1023 has the best MSE performance among all of the aforementioned channel estimation methods. The effectiveness of the improved channel estimation will be proved later by the enhanced BER performance of the system.

It is worth noting that with improved PN sequence based channel estimation, the noise in the estimated CIR in (24) is related to the PN sequence length \( L_{pn} \). The longer the PN sequence, the better the channel estimation. However, as demonstrated in Table IV, the spectrum efficiency will be affected with excessively long PN sequence. Therefore, the
impact of the PN sequence length to the MSE of ZF equalizer is worth to be investigated.

As shown in Fig. 9, the longer the PN sequence length used for channel estimation, the better the MSE performance. A comparison between Fig. 8 and Fig. 9 shows that even with the shortest PN sequence length, the ‘PN-31 Imp.’ can also provide better MSE performance than classical DMT block based channel estimation ‘Block-1’.

C. Simulation of BER Performance with Different Channel Estimation Methods

In this part, we evaluate the performance of different DMT transmission systems over 50 m SI-POF using the channel model in (33) and the allocated bits shown in Fig. 7(a). The raw transmission rate without system overhead cost is around 1.44 Gbps.

Different channel estimation methods are adopted to evaluate the BER performance after equalization. The LS estimation with ZF equalization is adopted to equalize the received DMT signal. The performance with perfect channel information is also evaluated to give a reference. In perfect channel information based channel estimation, the simulated CFR results in (33) are directly used for channel estimation results.

Monte-Carlo simulations are carried out to evaluate the BER performance. The simulation results with perfect channel information ‘Perfect Ch. Inf.’, DMT block based channel estimation ‘Block-5’, TFP based channel estimation ‘TFP-14’ and PN sequence based channel ‘PN-1023 Imp.’ are depicted in Fig. 10. As shown in the figure, the classical ‘Block-1’ has the worst BER performance, which is around $1.0 \times 10^{-2}$. The ‘Block-5’ can offer a BER performance of around $1.9 \times 10^{-3}$, which approaches the ‘Perfect Ch. Inf.’. However, as shown in Table IV, the spectrum efficiency of ‘Block-5’ is very low. The ‘TFP-14’ provides a similar BER performance as ‘Block-1’, which is around $8.1 \times 10^{-3}$. It is worth noting that the BER performance with ‘PN-1023 Imp.’ is very close to ‘Perfect Ch. Inf.’. Therefore, taking into account the system performance and the spectrum efficiency, the improved PN sequence based channel estimation is the best solution for the 50 m SI-POF system.

Consequently, the BER performance of the improved PN sequence based channel estimation with different PN lengths are shown in Fig. 11. As shown in the figure, the longer the PN sequence, the better the BER performance. It is worth noting that with the maximum channel delay $\tau_{\text{max}}$ of 10.8 ns and frequency spacing between subcarriers $\Delta f$ of 586 KHz, the DMT systems with PN lengths of 1023, 511 and 255 provide similar BER performance which is less than $1.5 \times 10^{-3}$ for all subcarriers. When the PN length is shorter than 63, the BER performance dramatically degrades due to inaccurate channel estimation. As the DMT symbol length is 2048 samples, it can be concluded that the PN sequence with a length of around 1/8 DMT symbol length is the optimal setup for the 50 m SI-POF system with a best trade-off between system performance and spectrum efficiency.

D. BER Performance of Real SI-POF Transmissions with Different Channel Estimation Methods

In this part, we evaluate the BER performance of real SI-POF transmissions with different channel estimation methods. DMT signal is generated with the allocated bits according to the measured real CFR, which is shown in Fig. 7(b). The raw transmission rate without system overhead cost is around 1.51 Gbps.

The electrical and optical setups are as the same as the ones in Section VI. The DAC works with a sampling rate of 1.2 G samples per second and the ADC works with a sampling rate of 5 G samples per second. The measured channel maximal delay $L$ in the real 50 m SI-POF is 14 samples, which is equal to 11.7 ns. Therefore, the CP and ZP lengths are set to 16 samples. The PN sequence length is selected as 255 according to the simulation results in the previous part.

In practical transmissions, 1600 DMT symbols with different channel estimation methods such as ‘Block-1’, ‘Block-5’, ‘TFP-28’, ‘TFP-14’, ‘PN-255’ and ‘PN-255 Imp.’ are generated and respectively transmitted over 50 m SI-POF. The measured BER performance of DMT transmission with different channel estimation methods are depicted in Fig. 12. As shown in Fig. 12(a) and Fig. 12(b), in classical DMT block
In this paper, a novel DMT transmission scheme with a hybrid structure of PN sequence and ZP sequence, namely as PN-ZP-DMT, is proposed for optical transmissions. It is theoretically described for all optical transmissions and applied by simulations and experiments over a 50 m SI-POF transmission. Four channel estimation methods including the DMT blocks based channel estimation, the time-frequency pilots based channel estimation, the PN sequence based channel estimation and the improved PN sequence based channel estimation are investigated for the DMT-POF system. MSE performance, system complexity, system spectrum efficiency and influence of time synchronization errors are analyzed for the proposed PN-ZP-DMT scheme. Simulation results show that the proposed PN-ZP-DMT scheme with the improved PN sequence based channel estimation outperforms all other channel estimation methods in terms of channel estimation MSE with an acceptable complexity. Furthermore, the PN-ZP-DMT scheme has an equivalent system complexity but offers higher spectrum efficiency compared to the classical CP-DMT schemes.

The BER performances are evaluated employing the bit-loading for the DMT system. Compared to the classical DMT blocks based channel estimation and the time-frequency pilots based channel estimation, the improved PN sequence based channel estimation can offer performance close to the perfect channel information case. In addition, the comparison of the BER performance with different PN sequence lengths shows that the longer the PN sequence, the better the system performance. Meanwhile, the PN sequence with a length of 1/8 DMT symbols length is the optimal selection for the 50 m SI-POF transmission system with a best trade-off between system performance and spectrum efficiency.

Finally, real experimental transmissions with different channel estimation methods are implemented to verify the simulation results. In practical 50 m SI-POF transmissions, the PN-ZP-DMT scheme with the improved PN sequence based channel estimation outperforms other solution. A net transmission rate of 1.49 Gbps with a total BER at $5 \times 10^{-4}$ is achieved by the PN-ZP-DMT transmission scheme.

In general, the proposed PN-ZP-DMT scheme can significantly enhance the DMT transmission rate in terms of channel estimation accuracy and spectrum efficiency. The experimental implementation in POF transmission systems shows that this proposed technique could be a promising transmission scheme and channel estimation method for short-range optical communications employing DMT/OFDM, such as the short-range SMF, MMF transmissions and optical wireless with VLC.

REFERENCES


