REVISTA MEXICANA DE FÍSICA **49 SUPLEMENTO 4**, 54–59 AGOSTO 2003

A muon detector to be installed at the Pyramid of the Sun

R. Alfaro, E. Belmont-Moreno, A. Cervantes, V. Grabski, J.M. López-Robles,
L. Manzanilla, A. Martínez-Dávalos, M. Moreno, and A. Menchaca-Rocha*
Instituto de Física, Universidad Nacional Autónoma de México,
Apartado Postal 20-364, 01000, México D.F., México

Recibido el 3 de febrero de 2003; aceptado el 7 de abril de 2003

Is the Pyramid of the Sun at Teotihuacan a mausoleum, or just a ceremonial monument? A similar question inspired Luis Alvarez over 30 years ago to carry out his famous muon detection experiment at the Chephren Pyramid, in Giza. A fortunate similarity between this monument and the Pyramid of the Sun is a tunnel, running 8 m below the base and ending close to the symmetry axis, which allows us to emulate Alvarez in a search for possible hidden chambers in one of the largest pyramids in Latin America. Here we elaborate on what is known about this monument, on a description of the proposed detector design, and its expected performance based on simulations.

Keywords: Cosmic rays; muon detection; archeometry; Teotihuacan.

¿Es la pirámide del Sol de Teotihuacan un mausoleo, o es solamente un monumento ceremonial? Una pregunta similar inspiró a Luis Alvarez hace más de 30 años a llevar a cabo su famoso experimento de detección de muones en la pirámide de Kefrén, en Giza. Una similitud afortunada entre éste monumento y la pirámide del Sol es la existencia de un túnel que corre 8 m por debajo de la base y termina cerca del eje de simetría, que nos permitirá emular a Alvarez en la búsqueda de posibles cámaras ocultas en la que es una de las pirámides más grandes de Latinoamérica. En éste trabajo se discute lo que se sabe de ésta pirámide, se hace una descripción del diseño propuesto para nuestro detector y se muestran resultados de su desempeño en base a simulaciones.

Descriptores: Rayos cósmicos; detección de muones; arqueometría; Teotihuacan.

PACS: 29.90.+r, 96.46.Tv

1. Introduction

Once again high-energy physics makes an attempt to solve the mystery of ancient pyramids by applying the Alvarez [1] muon detection technique to the Mexican Pyramid of the Sun at Teotihuacan. In spite of its fame, little is known about this, one of the largest pyramids in America, or even about the people who built it 20 centuries ago. Early excavations showed no identifiable internal structures of the kind recently uncovered in the nearby Pyramid of the Moon, which are also a relatively common feature in other prehispanic monuments in Mesoamerica. Then, what was the purpose of building such a large structure? was it just a ceremonial monument? or could it be a mausoleum housing the remains of an important person? A revealing discovery made in the early 1970's (shortly after Alvarez experiment) was the existence of a tunnel running 8 meters under the Pyramid of the Sun, ending beneath its symmetry axis. Besides the archeological implications of such a remarkable find, it represents the unique advantage of providing a site to install an atmospheric-muon detector to search for possible ($> 1m^3$) cavities in the body of the pyramid. From now on we shall refer to this tunnel (in particular to its end point) as the "observation tunnel". Our purpose here is to describe the current status of such a project. Although the basic ideas behind this experiment are not far from those of Luis Alvarez and his team [1,2], there are important differences between the monuments at Teotihuacan and Giza, concerning the monument shape, size, and building materials. Thus new estimates and simulation work have been undertaken to determine the detector design changes required to carry out this experiment in Teotihuacan.

We concentrate in the relevant experimental parameters: sensitivity, resolution and efficiency, on the basis of what is known about the local muon spectrum, the pyramid's external geometry and internal structure, keeping in mind the Egyptian experiment. The first part deals with aspects which are difficult to take into account in a simulation, such as the uncertainties introduced by an irregular geometrical shape and the somewhat heterogeneous nature of its matter composition. Then we focus on the physics processes related to the passage of muons through matter, and on important aspects of the detector design, all of it through simulation using Geant4 [4]

2. Method

As a first guess, the experimental setup we consider (see Fig. 1) is similar to the one used by Alvarez *et al.* [1], as it satisfies the basic requirements of being simple, and low cost. Our design consists on a three-plane multiwire-chamber tracker and two scintillator counters for triggering purposes. To determine the important parameters of this design, one has to take into account the differences between the present situation and Egypt's experiment, for example a smaller and less dense pyramid as well as the different muon energy domain. These conditions affect the muon flux behavior and, consequently, the sensitivity. Thus estimations of sensitivity and expected resolution are very helpful for understanding the technique and optimizing it.

The Alvarez *et al.* [1] method consists of two important aspects: simulation and experiment. The first part requires the best possible knowledge of the pyramid's external dimen-

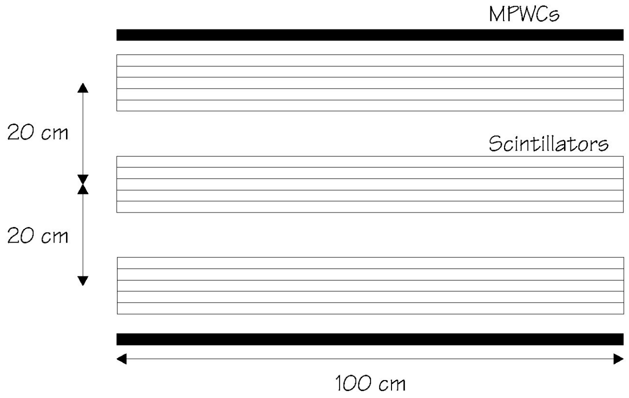

FIGURE 1. Experimental setup.

sions, its detailed geometrical shape, and a guess of its internal materials (elemental composition and density distribution), all of this assuming that the pyramid is solid (cavity-free). Then, the simulated muon distribution reproduced by a hypothetical detector, having the same structure as the real one, is subtracted from the experimental observations to search for possible differences. Significant deviations in a given direction indicate an appreciable matter density difference in the corresponding subtended volume.

Compared to the Egyptian pyramids, the Mexican one is more difficult to simulate as it lacks a simple geometrical shape. Moreover, from what is known, its fill is not as homogeneous as the one in Giza. Still, with the hypothesis that the ancient Teotihuacan architects didn't intend to play a trick on future generations, we assume that the mean composition and density distribution are similar to those found in early excavations by Noguera, and Gamio, who excavated a $2m$ high, $1m$ wide tunnel running in a near straight line across the $> 200m$ pyramid base, and 8 m above the observation tunnel. The Noguera-Gamio tunnel is also very valuable, as it represents a well located cavity to be used for calibration purposes. Another important problem is that the density ρ of mass inside the Mexican monument is more heterogeneous than in the Egyptian case, and its mean value is appreciably smaller than the density of rock. Hence, if the hidden chamber has rocky walls, this could compensate the effect of a gas-filled cavity on ΔM, the difference of matter with or without cavity along the muon path on which the method is based [1]. To quantify this, let us define the detected cavity size L_d as,

$$L_d = L_r - L_w \frac{\rho_w - \rho_p}{\rho_p} \tag{1}$$

where L_r represents the real cavity size, L_w is the total wall thickness, ρ_w and ρ_p are the wall and pyramid mean densities. Then, total wall-cavity compensation for a rocky wall ($\rho_w = 2.65$ g/cm^3) results when L_r/L_w=$(\rho_w - \rho_p)/\rho_p$=0.4.

2.1. Sensitivity

The amount of matter M traversed by a muon along its trajectory of length L inside the pyramid, may be estimated through the underground muon count N. The "sensitivity" ξ of this method may be defined as the ratio,

$$\xi = \frac{\Delta N}{(2N)^{1/2}} \tag{2}$$

where $\Delta N = N_1 - N_2$, using N_1, N_2 to denote the muon count in the cases where there is and there is not a cavity, respectively. The N value to be used in the denominator may approximated by the average number of counts $(N_1 + N_2)/2$ so that $(2N)^{1/2}$ represents the uncertainty of ΔN. For the muon flux F we use the distribution proposed by Bedewi [2] at sea level:

$$F(E, \Theta) = \kappa E^n cos^m(\Theta) \tag{3}$$

where E is the muon energy in GeV, Θ is the polar angle referred to the vertical direction, n and m are slowly varying functions the muon energy [2]. The proportionality constant κ in Eq. (3) is mostly a function of the location altitude and affects the observation time T, which is not important for our considerations here. For comparison with the Egyptian case, where Eq. (3) has been experimentally tested [2], we keep this somewhat restricted muon flux parameterization, even when more extended [3] ones are available. Thus, as a first approximation, in the region $E > 10$ GeV, we assume that Eq. (3) also reproduces the shape of the muon flux in Mexico City. Since less energetic muons are stopped by the pyramid mass, the low energy part of the muon spectrum, which is the most sensitive to the geomagnetic position and altitude, is also irrelevant. In fact, inside the detection tunnel the important muon energy region is $18 - 35$ GeV, where the n-value rages from -1.5 to -1.7 [2]. The equivalent interval for the Egyptian experiment was $n = -2.0$ to -2.1. To compare the sensitivity of the method for the two experiments we have made the following calculation of ξ for $n = -1.5$ to simulate the Mexico (using the sub index m) situation, and $n = -2.0$ to simulate the Egyptian (using the subindex e) situation. Thus, we obtain:

$$\frac{\xi_m}{\xi_e} \approx \frac{\kappa_m}{\kappa_e} \frac{(L_e \rho_e)^2}{(L_m \rho_m)^{3/2}} \left(\frac{N_e}{N_m}\right)^{1/2}. \tag{4}$$

Where L_i and ρ_i $(i = m, e)$ are the mean muon penetration and pyramid density in each case, and N_i is integral number of particles passing through the pyramid layer. Since the Egyptian pyramid is twice as tall as the Mexican one, our experiment is expected to be one order of magnitude more sensitive for the same cavity size X.

For our case ($n = -1.5$), the numerical integration of Eq. (3), and using Eq. (2), we find that the statistics necessary to obtain a given sensitivity may be estimated using:

$$N \approx 0.9 \left(\xi \frac{L}{X}\right)^2 \tag{5}$$

In the worst case, *i.e.*, when the maximum muon path-length is $L \approx 80$ m, the statistics necessary to detect a $X = 1$ m cave with a sensitivity $\xi = 3$ would be $\approx 5 \times 10^4$ counts.

The above estimations ignore multiple scattering, a process which will be discussed in a later section. Also, our arguments are based on a constant energy loss hypothesis, which is only valid for minimum ionizing particles. Higher energy muons ($E > 100$ GeV) loose energy through more complex mechanisms due to radiative processes, but their corresponding flux is sufficiently small to make their contribution insignificant.

2.2. Resolution

Another important aspect of the experiment is space resolution, which depends on the detector ability to reconstruct particle tracks. The resolution is quantified as the uncertainty associated to the reconstructed "entry point" of muons on the external boundary of the pyramid. Multiple scattering and detector reconstruction limitations transform this point into a finite size radial (*i.e.*, perpendicular to L) distribution known as the "point spread function". Roughly speaking, this function resembles a gaussian having a standard deviation σ_R which can be taken as an estimate of the resolution.

As shown in the previous section, low energy muons are most sensitive for pyramid cavity detection. However, slower particles are also the most affected by multiple scattering. So, an improved resolution suggests the need to eliminate low energy particle contributions. Thus, apparently Luis Alvarez favored resolution over sensitivity by introducing a $1.2m$ thick iron absorber ($36tons$), and an extra triggering scintillator, at the bottom of his setup. One advantage of the Egypt experiment was the reasonable assumption that the internal part of the Giza Pyramids are built with the same homogeneous rock blocks found outside, in the Belzoni Chamber (where Alvarez placed his detector), and in the tunnel leading to it. The regular external shape of those monuments also simplified Alvarez simulation work. In Teotihuacan the situation is not as simple. The external part of the Pyramid of the Sun has a relatively thin (< 0.5 m) rock layer, most of which was placed there a century ago for weather protection, and an external topology which is more complicated than the Egyptian case. Furthermore, the Noguera-Gamio tunnel revealed that the pyramid's interior is likely to be an heterogeneous admixture of loam and sand, with scattered tuff fragments. In this section we estimate how those features affect the resolution of the experiment.

Although the external geometrical shape of the pyramid, and the position of the detection tunnel, define the muon path length L on every direction subtended by the detector, both aspects are known with some uncertainty. So far, the pyramid's external topology has been extracted from a level diagram obtained 30 years ago using aero-photographic techniques. This provides information every $15 - 30$ cm in the horizontal plain, and every 100 cm in the vertical axis. The topological diagram has been digitized obtaining a $\approx 750,000$-point description of the pyramid surface. However, such amount of information is difficult to exploit in a GEANT simulation, so the results presented in the next section were obtained using a simplified geometrical description (see Fig. 2). The ΔM determination is also affected by geometrical definition uncertainties. In particular, the location of detection tunnel is only known to within 1.0 m, and the external topology of the pyramid is known within an accuracy of ≈ 0.5 m. Since the typical path length is Ł ≈ 50 m, then $\Delta L/L \approx 1\%$.

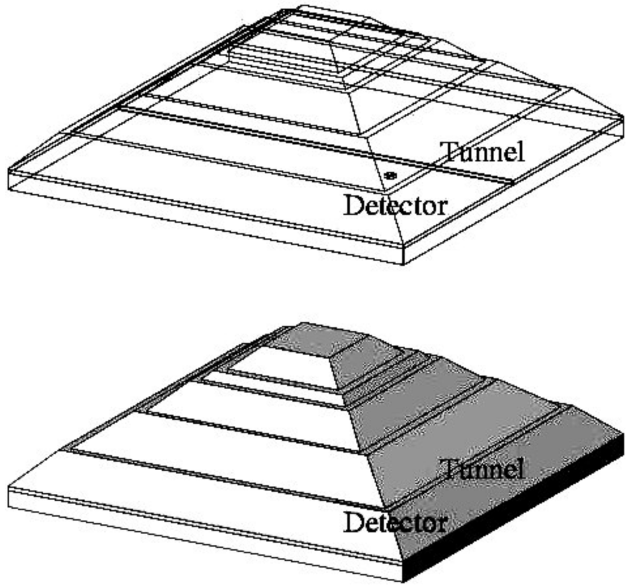

FIGURE 2. Pyramid internal (top) and external (bottom) model structure.

The simulation also requires a knowledge of the density and chemical composition of the materials contained in the pyramid. At this time, these can only be guessed from archeological documents, since the Noguera-Gamio tunnel was walled up 30 years ago to prevent collapse. While an elemental analysis based on soil samples taken inside those tunnels is underway, the following results are based on a standard [5] soil content. Our estimated mean material density originates in a description given by Millon [6] from 45 photographs taken in some regions along the Noguera-Gamio tunnel system before being walled up. From those photographs, each showing $\approx 3m^2$ tunnel sections, Millon published drawings with a code describing the space distribution of the most important filling materials such as sand, loam of various colors, adobe, volcanic tuff, scattered stones, etc. Thus, from a scan of those pictures, and standard geological information we arrive at the model density distribution per tunnel section shown in Fig. 3. Here the mean value of density is plotted as a function of the longitudinal coordinate running along the Noguera-Gamio tunnel. From Fig. 3 we conclude that the density distribution is expected to be fairly uniform, having an overall mean value of $\rho \approx 1.9g/cm^3$ and with standard deviation $\sigma_\rho = 0.14g/cm^3$ (dashed lines). The higher density in the first few meters correspond to a platform attached

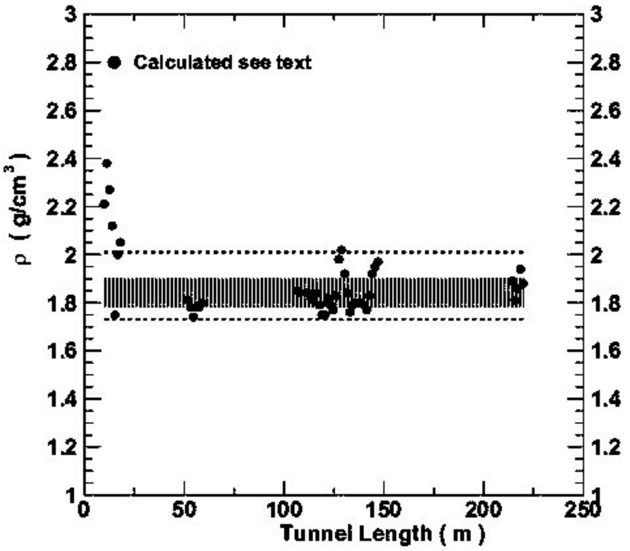

FIGURE 3. Longitudinal density distribution model from the data of Millon [6] of the Gamio-Noguera tunnel system.

to the pyramid in a late period, although still in pre Hispanic times. Removing the corresponding 4 points reduces σ_ρ to $0.08 g/cm^3$ (shaded region on Fig. 3). Based on this model we find that, even the uncorrected standard deviation does not represent a major problem if it were distributed randomly inside pyramid, because over a long L, the uncertainty in the mean value of the density depends on L, *i.e.*, $\sigma_\rho^L = \sigma_\rho/(L/S)^{1/2}$, where S represents the sampling length. In our case $S = 2$m, and L is typically 50m, so we expect that the contribution to ΔM due to σ_ρ would not be significant, i.e., $\sigma_\rho^L/\rho = \Delta M/M \leq 1\%$. Combining geometrical ($\Delta L/L$) and the density (σ_ρ^L/ρ) uncertainties, we estimate an overall uncertainty due to these two effects. The minimal cavity size which we should be able to detect depends on L according to the following empirical formula:

$$X_{min} \approx \left(\Delta L^2 + (\sigma_\rho^L/\rho)^2 SL\right)^{1/2} \approx (0.25 + 0.0035L)^{1/2}. \quad (6)$$

Based on the previous arguments, the simulations presented below assume a mean density homogeneously distributed inside the pyramid volume. This approximation, which is not so important for detector design, has the problem that cavities having rocky walls may not be differentiated since the detection method is sensitive to ΔM. On the other hand, if the cavity walls were too massive, or if there is a large monolithic rock (like a sculpture) inside the pyramid, then ΔM would be positive, hence distinguishable as a negative peak when subtracting the simulated muon yield from the experimental observation. The lack of a longitudinal coordinate dependency of the density shown in Fig. 3 supports the hypothesis of an isotropic filling.

Although the muon energy spectrum at Teotihuacan has not been measured, it is important to keep in mind that muons having less than 18 GeV do not penetrate the pyramid. The muon energy distribution at sea level is known [7] and we assume that the shape of the muon spectrum beyond $E = 18$ GeV (*i.e.*, above the energy region where geomagnetic effects are important) is reproduced by Eq. (3). Uncertainties in the altitude and geomagnetic location of Teotihuacan may result in Θ and L dependence in the subtraction from experimental data, but this should be easily identifiable during data analysis, thus we expect this to have little contributions to σ_R.

A number of aspects of the detector limit its ability to identify a muon and reconstruct its track in the system, such as electronics discrimination of small signals, and multiple-hit events (several particles crossing the system simultaneously). The ratio between the number of reconstructed tracks and the actual number of muons crossing the detector defines de detection efficiency. Our simulation indicates that 90% of the tracks can be unambiguously reconstructed using three $X - Y$ MWPC layers, to be compared with the two-layer spark-chamber aray used by Alvarez [1] yielding only a 60% efficiency. Thus, the setup proposed here, schematically represented in Fig. 1, while allowing a full rejection for multi-hit events, reduces collection time by 30%, relative to a two-layer MWPC system.

2.3. Results of Monte Carlo Simulation

The sensitivity and error in this method depend on including every process in the simulation. For this purpose we use the powerful simulation package Geant4 [4] which allows us to closely reproduce the relevant physical processes, in order to optimize the detector, and also represents a data analysis tool. All information of the muon flux was also included at the stage of data analysis, which was done by the use of the ROOT mathematical and graphical library [4]. Both in simulation and analysis we used HEP random generators [9]. The simulations were carried out on the 20-CPU cluster of the Physics Institute (IFUNAM) and the analysis of the corresponding simulated data on a Linux desktop computer.

For the resolution calculations the spatial geometry was build in such a way as to allow an easy change in the material thickness. The results of coordinate resolution simulations are presented on Figs. 4-6. On Fig. 4 we present the simulated point spread function. Note that in the peak region a gaussian distribution reproduces well the width. However, the simulated distribution has longer tails which tend to increase the RMS value slightly. Still, the real resolution is closer to the width of the fitted gaussian than the RMS.

The expected dependence of σ_R on the MWPC wire spacing is shown in Fig. 5 for two pyramid thickness, representing the minimum and maximum expected L values. As can be seen, σ_R is an increasing function of L, and of the wire spacing. The dependence of the mean σ_R on L for various possible detector configurations is shown in the top graph of Fig. 6. The empty circles correspond to a realistic, 5mm wire step, MWPC which represents our current choice, resulting from a compromise between cost and resolution. For comparison, full circles illustrate what would be expected from

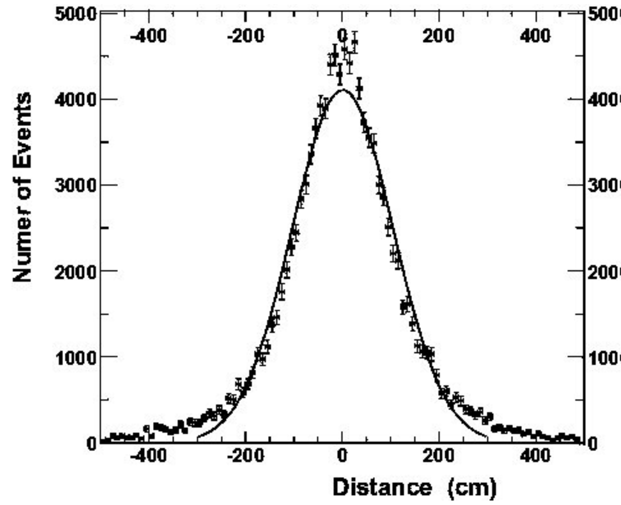

FIGURE 4. Spatial resolution (point spread function) at normal incidence.

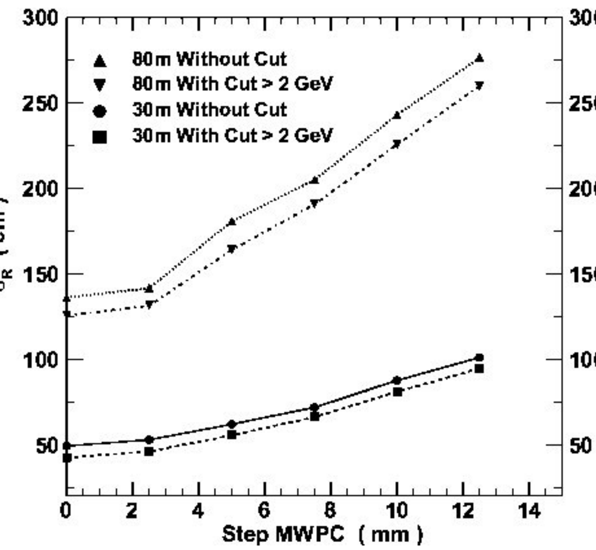

FIGURE 5. Resolution vs. wire spacing for the two extreme pyramid depths.

an ideal (infinitely small wiring spacing) MWPC. The full triangles represent a configuration combining an ideal MWPC with a 2 GeV absorber (like that of Alvarez) at the bottom, on what would represent a better configuration. Comparing the two ideal MWPC detector we find that the ideal Alvarez configuration only represents an $\leq 8\%$ improvement in resolution, while comparing the ideal and the realistic MWPC (the full and empty circles), represents a $\leq 12\%$ resolution-loss. Also presented in Fig. 6 are the expected mean and RMS values for the radial shift (middle plot) and the angle-straggling (bottom plot), both resulting from multiple scattering. From this figure we see that the main resolution-loss factor is a radial shift, since angle straggling is small and rapidly becomes independent of L. Finally, from Fig. 6 we estimate that the dependence σ_R on L is $\sigma_R(L) \approx 0.02L$

The existence of the Noguera-Gamio tunnel provides a natural calibration ground for the detection system. The sensitivity estimates given in Sec. 2.1 do not include the effect of multiple scattering. This becomes important when cavity size is comparable to the resolution. For example, the Noguera-Gamio tunnel has a ratio $X/L \approx 3 \times 10^{-2}$, so that in order to observe it with a $\xi = 3$, Eq. (5) requires $N = 10^4$ counts. At this tunnel's location (≈ 10 m) the expected resolution (top plot in Fig. 6) of 20 cm, is a factor of 5 less than the size of the tunnel, so the sensitivity is estimated using Eq. (5). The simulation shown in Fig. 7 agrees with this sensitivity estimate. In this figure, the muon yields from situations with and without this tunnel, with similar $N \approx 10^4$ muon statistics per (2-degree) angular bin, are subtracted and the result is plotted (solid histogram) as a function of the polar angle, while fixing the azimuthal angle on a plane perpendicular to the tunnel. The structure at ≈ 19 degrees illustrates the ex-

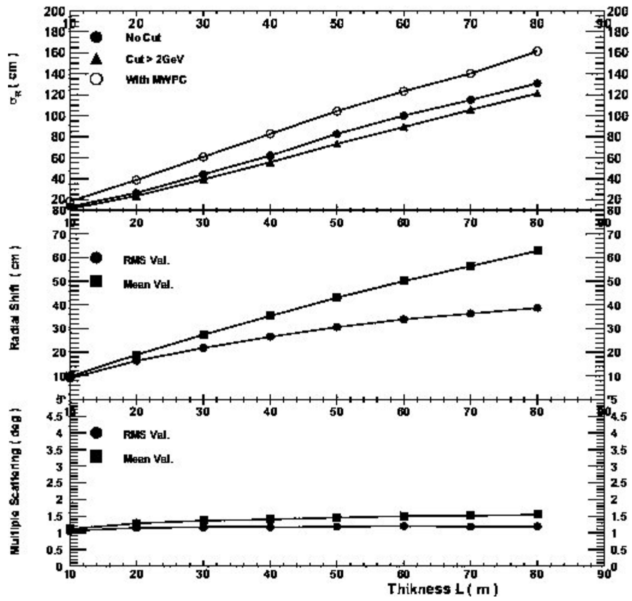

FIGURE 6. Pyramid-depth (L) dependence of resolution (top), radial shift (middle), and angle straggling (bottom).

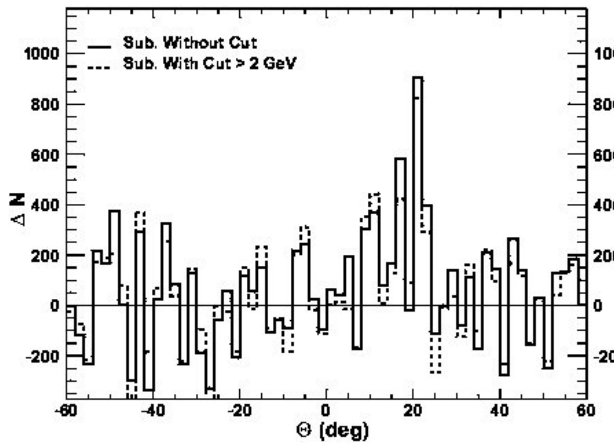

FIGURE 7. Simulated detector sensitivity to the Gamio-Noguera tunnel on polar coordinate with (dashed), and without (solid), low energy absorber.

pected 3σ deviation from the statistical errors. The mean angular inclination (19 degrees) reflects the fact that the detector location is not centered on the symmetry axis of the pyramid. This $N \approx 10^4$ statistics per angular bin, implies a 2-month observation time. The dashed histogram in this figure represents a detector configuration having a 2 GeV low energy absorber. This result could explain the final comment in the Alvarez *et al.* [1] paper, stating that the use of an absorber is, in fact, not necessasy.

2.4. Conclusions

The search for possible hidden chambers in the Pyramid of the Sun is possible using a simplified version of the Alvarez detector, without a low energy muon absorber. Within an homogeneous fill assumption, the proposed system is expected to provide a coordinate reconstruction resolution of $\sigma_R(L) \approx 0.02L$, and a minimum cavity size of $X_{min} \approx (0.25 + 0.0035L)^{1/2}$ We showed that a cavity having the cross sectional dimensions of the Noguera-Gamio tunnel can be easily located. The simulation has been used to illustrate the meaning of the famous last Alvarez words [1] concerning the muon absorber.

Acknowledgements

The help of undergraduate students E. Arrieta, A.D. Becerril, R. Núnez and E. Vázquez, the technical support of Carlos López, and the partial support of CONACYT (project G39091-E) and DGAPA-UNAM (project IN101501) are acknowledged.

∗. Correspondence address: IFUNAM, Apartado Postal 20-364, 01000 Mexico D.F., México. Tel: 525-6225060; fax: 525-6161535. e-mail: amen@servidor.unam.mx

1. L.W. Alvarez *et al.*, *Science* **167** (1970) 832.
2. F. El Bedewi, A. Goned, and A.H. Girgis, *J. Phys. A: Gen. Phys.* **5** (1972) 292.
3. T. Hebbeker and C. Timmermans, *Astroparticle Physics* **18** (2002) 107.
4. GEANT-4, http://geant4.web.cern.ch/geant4/G4UsersDocuments.
5. D.R. Lide, *CRC Handbook of Physics and Chemistry*, 72nd ed., (CRC Press, Boca Raton, 1991).
6. R. Millon, B. Drewitt, and J.A. Bennyhoff, T*rans. Amer. Phil. Soc.* **55** (1965) 6.
7. T.K. Gaisser and T. Stanev, *Phys. Rev. D* **66** (2002) 10001, on page 182.
8. http://geant4.web.cern.ch/geant4/G4UsersDocuments/ UsersGuides/PhysicsReferenceManual
9. http://proj-clhep.web.cern.ch/proj-clhep