Intra-Firm Bargaining and Wage Dynamics: A Model of Asymmetric Learning

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Abstract

This paper develops a bargaining model between employers and workers that is driven by asymmetric information between current employers and potential employers. Both the current employer and the worker have the same information regarding the worker’s productivity. This information is not available to outside firms which observe only wages. Existing literature that analyzes models in which workers are price takers finds that wages are attached to publicly observable characteristics such as job assignment. High and low ability workers in the same job earn the same wages. This result prompts a question: Why are high productivity workers not able to capture a larger portion of the surplus than less productive workers? I develop a bargaining model that addresses this question and analyze how the market learns about employed workers skills. This model generates a semi-separating equilibrium. More able workers compensate their employers by earning lower wages in the first period to elicit future offers from outside firms. Their wages depend on their actual productivity. Outside firms observe wages and infer these workers’ productivity. They then make offers equal to the worker’s productivity and the current employer matches the offers. Less able workers for whom it is too costly to reveal ability through wages earn a wage below their productivity in all periods. I then show that this model of bargaining can generate predictions consistent with several regularities in wage patterns of managers within firms. Existing literature which explains empirical findings on wage dynamics in internal labor markets mainly focuses on incentive models or models in which wages are determined in spot markets.
1 Introduction

This paper explores how the bargaining process between firms and workers affects wage dynamics within a firm. When a worker is first hired there is typically uncertainty regarding his qualifications. It is natural to think of the long-term employment relationship in organizations as one in which both the current employer and the worker get more precise information regarding the worker’s ability than other potential employers.\footnote{Greenwald[1986], Lazear[1986], Bernhardt and Scoones[1993], and Gibbons and Katz[1991] explore the implications of this asymmetric information on turnover patterns. Milgrom and Oster[1987] develop a model of discrimination.}

The existing literature that explores wage dynamics in firms when there is asymmetric information between incumbent and potential employers (e.g., Waldman[1984], Bernhardt[1995]) analyzes models in which workers are treated as price takers. In this literature the wage is the highest offer made by outside firms. Since outside firms observe only job assignment, wages are a function of publicly observable characteristics and jobs, but not actual ability. This allows employers to capture rents, as the outside offers are below the actual productivity of the worker. In this literature, all workers who look similar to outside firms earn the same wages. Among workers with similar observable characteristics, more able workers generate larger rents than less able workers; as a result, more able workers are more valuable to their employers. Bargaining over compensation with workers or a failure to reach an agreement can lead to delays in production or to a smaller amount of output produced; hence, bargaining with more productive workers is more costly for employers than bargaining with less productive workers. This intuition suggests that the more productive workers in a firm may earn higher wages even if outside firms cannot observe that they are more able. However, these patterns are not captured by the models above in which workers are price takers. I
develop an intra-firm bargaining model which analyzes this intuition. I then show that several of the stylized facts on wages of managers in firms can be explained by a model which focuses on bargaining.

In standard bargaining models (with symmetric information), when employer and worker bargain over wages and the worker has an outside option, there is no dynamic link between current wages and future outside offers and wages.\textsuperscript{2} Workers earn the outside option (which is the highest offer made by an outside firm) if the offer exceeds the bargaining outcome. My model demonstrates that in a dynamic setting, the link between current wages and outside options may be more complicated than the existing literature suggests. Surprisingly, more able workers may choose to accept lower wages than the initial market wage; this allows them to elicit higher offers in succeeding periods. When the employer and worker have better information about the worker’s productivity than outside firms, a wage that results from bargaining between the worker and employer that is different from outside firms’ offers may contain information regarding the worker’s productivity. Such information can then be used by outside firms, which observe wages, to infer the worker’s quality. Therefore, these wages affect the future payoffs of both the employer and worker. Hence, when the worker and employer bargain over wages they take into account this information externality.\textsuperscript{3}

In my model each worker is equally productive in all firms. Workers are het-

\textsuperscript{2}Shaked and Sutton[1984] analyze a one period bargaining model in which players have outside options. However, when the information is symmetric, each period can be analyzed separately. Stole and Zweibel [1996, 1993], analyze implications of intra-firm bargaining under non-binding contracts on organizational design. They analyze a static model in which the information is symmetric.

\textsuperscript{3}Chang and Wang[1996] examine a two-period model of asymmetric information and the implications on investment in human capital. In their model wages in the second period are determined by the Nash bargaining solution. However, since the second period is the last period there is no information externality, and the bargaining solution is the solution that arises in a static model.
erogeneous with respect to their productivity and their publicly observable characteristics (such as age and education). When a worker is first hired there is uncertainty regarding his skills. Early during the worker’s career, the employer and worker learn about the worker’s productivity. This information is not observed by outside firms. Workers cannot commit to more than a one period contract, and can renegotiate wages. Potential employers observe wages and may make offers to employed workers. The result is a semi-separating equilibrium that sorts workers into two groups: a high-productivity group whose wages reveal the workers’ actual productivity, and a low-productivity group whose wages depend only on publicly observable characteristics.

A high productivity worker from the first group earns low wages early in his career because the outcome of the bargaining process reveals his productivity. Observing the wage, outside firms offer the worker his marginal revenue product (MRP). The current employer then matches the outside offers. This “revealing” wage compensates the employer for the loss of future informational rents, and hence is lower than the wage that would arise in bargaining under symmetric information. Moreover, the worker may chose to earn a wage below the initial market wage. The compensation to the employer (through a low wage payment) increases with the size of the future loss of informational rents. Informational rents are increasing in the worker’s productivity and decreasing in the worker’s initial market wage.

Workers from the low productivity group earn wages equal to the offers made by outside firms. Hence, these workers’ ability is not reveal to the market. These workers earn wages that are lower than their marginal productivity. Each worker chooses between receiving a low revealing wage for a period, which is accompanied by wage growth in later periods, and a larger wage in the current period that is accompanied by no wage growth in later periods. The outside option binds
only when the bargaining outcome is lower than the outside option by a certain amount; that is, there is a gap between the revealing wage level for which the worker is indifferent between earning the outside option and earning a revealing wage. As a result the marginal worker who earns the outside option produces strictly more than the wage he earns.

In my model wage innovations within job levels occur without increases in productivity. They occur because outside firms learn with a time lag about workers’ abilities. If the equilibrium wage is a revealing wage, then in succeeding periods, outside firms offer the worker his MRP. Baker, Gibbs, and Holmstrom [1994] (BGH) analyze personnel records of managerial employees from a single large firm. Their findings suggest that wage innovations within job levels are important to understanding wage dynamics in firms. They find that promotion premia explain only a modest portion of the differences in average wages across job levels. They also find a substantial variation in wage levels and wage growth within job levels.

There are several sources of variation in wage levels and wage growth in my model. Heterogeneity in workers’ abilities generates variation in wages and wage growth within job levels. More interestingly, equally productive workers in the same job level may earn different wages, and experience different wage growth. Further, the model predicts a negative correlation between wage levels and wage growth for equally productive workers. If two workers are equally productive, one may earn higher wages if they have higher outside options due to differences in publicly observable characteristics. Low initial outside offers and having more years before retirement imply that the worker’s benefit from revealing ability is larger. Therefore, such workers are more likely to choose to earn lower wages (and elicit high outside offers in succeeding periods) than workers who currently have high outside offers or fewer years before retirement. A related result is found empirically by BGH. They find that among workers who have the same performance rating, the
expected percentage wage increase for workers in the same job level with the same
tenure is negatively related to the current wage. They interpret it as a company
attempt to compress the wage dispersion within job levels.

Other papers that explain regularities found by BGH are Gibbons and Wald-
man[1999], and Kwon[2001]. Gibbons and Waldman develop a model of symmetric
learning in which wages are determined in spot markets. Their paper focuses on
regularities related to promotions which I do not analyze. They also explain why
wages are not downward rigid and why there is variation in wages and wage growth
within job levels. Kwon[2001] develops a model of moral hazard, and considers
incentives. His paper also focuses on explaining regularities related to promotions.

I address issues of efficiency in job assignment and promotion by extending
my model such that each firm offers more than one job. In contrast to previous
models of asymmetric learning, workers are assigned to the job in which they are
most productive. Previous research (e.g., Waldman[1984], Bernhardt[1995]) found
that when employers have better information than outside firms, outside firms use
job assignment information to infer an employed worker's qualifications. Workers
who are promoted are believed to be of high quality and receive attractive wage
offers from outside firms. The wage offers are the expected MRP conditional on
job assignment. In equilibrium, employers promote workers only if the increase in
productivity exceeds the increase in wage that is required to retain the worker. As
a result, some workers who have a comparative advantage in high level jobs are
not promoted. In the equilibrium of this model, however, outside firms use wages
rather than job assignment to infer ability.

A key difference between this model and the models in the papers mentioned
above is that I incorporate offer matching. When employers have better infor-

4In Gibbons and Waldman information about the worker’s ability is gradually revealed
to all firms in the market simultaneously.
mation regarding their workers’ productivity than outside firms they can choose whether to match the outside offer, and they retain workers only if their MRP exceeds the offer made by outside firms. As a result, outside firms can only raid employed workers with a wage that exceeds the worker’s MRP; that is, outside firms suffer from the winner’s curse. Therefore, outside firms do not make offers to workers who are promoted. Since promotion itself doesn’t require a wage increase to the worker, employers promote workers efficiently.

This paper is related to the literature on asymmetric information dynamic bargaining. In this literature the asymmetric information is typically between the parties that bargain (e.g., Kennan[2001]). The most closely related paper on asymmetric information between current and potential employers is Ricart I Costa [1988]. The paper analyzes a model of asymmetric information in which outside firms cannot observe the worker’s productivity. In that paper, wages depend on productivity even though ability is unobserved by outside firms. However, the paper analyzes contracts in which wages can be contingent on output. Therefore, even if outside firms cannot observe output they can offer screening contracts in which wages depend on output. In contrast, I focus in this paper on situations in which contracts cannot be contingent on output and analyze how the market learns about workers’ productivity.

The paper is organized as follows. Section 2 describes the model. Section 3 contains equilibrium analysis, section 4 provides sensitivity analysis, section 5 analyzes wage dynamics, section 6 explores efficiency implications, and section 7 concludes.

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5Ricart I Costa assumes that workers are risk averse. If workers are risk neutral and contingent contracts are feasible, then wages are similar to wages in the symmetric information case; workers earn the marginal productivity.
2 The model

This section describes the economy, and the model.

*The economy:* There is one good in the economy, and its price is normalized to one. Each period risk neutral firms compete over risk neutral workers. Firms and workers have a common discount factor $\delta$. The output of each firm is the sum of the workers’ labor inputs.

Workers work for three periods and then retire. Each worker is equally productive in all firms. Workers are heterogeneous with respect to their productivity. A worker is characterized by ability parameter: $\theta > 0$. The productivity parameter is drawn from a known distribution with continuous density $f(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} > 0$. Workers have no disutility from effort and have a reservation utility of zero; a worker’s utility function is the discounted sum of wages.

When inexperienced workers enter the labor market, firms cannot determine the actual productivity of a worker ($\theta$ is unknown to workers and firms). I assume that in order to assign workers to tasks or jobs, the firm needs to know $\theta$ and that if the worker is randomly assigned to tasks the expected productivity is zero.\(^6\) Hence, if a firm hires an inexperienced worker, it takes the firm one period to train the worker and learn the worker’s productivity and no output is produced during that period. The only cost of training a worker is the wage that the firm pays the worker. Output produced is not observed by outside firms, and is unverifiable by a court. Workers cannot commit to more than a one period contract but firms can commit to long terms contracts. If a worker quits or is fired, he cannot return to work for the firm.

\(^6\)This technology assumption simplifies the analysis and can be replaced by weaker assumptions. For example, I can assume that if a worker is assigned to the "wrong" task he produces a fraction of the output he can produce in the "right" task provided that the fraction is small enough.
2.1 Timing and information structure of the game

I describe below the order in which workers and firms move, the information available to them in each stage, and the payoffs (see figures 1-4).

First period (figure 1): At the beginning of the first period, all the firms observe the distribution of $\theta$ and neither firms nor workers observe the worker's productivity. Firms offer workers a long term contract which specifies wages for periods one, two and three. Firms do not commit to employment.\(^7\)

During this period, the worker and employer privately learn the worker's productivity, $\theta$; outside firms cannot observe the worker's productivity.

At the end of the first period, the employer pays the worker the wage for the training period.

Second period: At the beginning of the second period (see figure 2), only the employer and worker know $\theta$ (outside firms observe past wages only).

Outside firms make offers and the employer makes a counteroffer. The worker can stay with the current employer or accept an outside offer. If the worker joins an outside firm (see figure 3) he works for the wage he accepted. If the worker rejects an outside offer, he cannot decide to join this firm later during this period.\(^8\)

If a worker stays with the current employer (see figure 4), he can produce immediately or renegotiate the wage. In case of renegotiation (description of the renegotiation stage is below), if the employer and worker agree upon a wage, the worker produces.

At the end of each period the worker earns the wage agreed upon. If there was no production the worker earns nothing.

\(^7\)Assuming that firms can commit to future employment will not change the equilibrium outcome.

\(^8\)The assumptions that if a worker joins another firms he works for the wage agreed and doesn’t bargain simplifies the analysis and avoid the analysis of bargaining in which the worker has better information than the new employer.
Third Period: The timing is similar to the timing in the second period (similar to the timing in figures 2, 3, and 4). The difference is that employment history includes the second period wage (and choice of a firm). At the beginning of the period, outside firms observe wages and turnover history; the date the wage contract was signed is verifiable. Based on this information, outside forms make wage offers for the third period. The current employer and worker know $\theta$.

If the worker received offers from outside firms, the current employer makes a counteroffer.

The worker either accepts an outside offer, or stays with the current employer. If the worker accepts an outside offer, he produces in the new firm. If the worker stays with the employing firm, he either produces and does not negotiate the wage or negotiates the wage and produces if agreement is reached. At the end of the period, if agreement hasn’t been reached, no payments are made. If the worker didn’t negotiate, he earns the wage that the employer offered at the beginning of the period. If the worker negotiated the wage, he earns the agreed upon wage. If the worker accepted an outside offer he is paid the agreed upon wage.

Bargaining: The equilibrium qualitative outcome holds for many different bargaining procedures, I choose this bargaining game in order to simplify the analysis. There are two production periods (periods two and three) in which the employer and worker can bargain over wages. It takes one period to produce, and production periods are indexed by $s$. Before production starts the employer and worker can bargain over the wage. In each production period, there are two rounds of offers with no discounting between the two rounds $t = 1, 2$. If agreement hasn’t been reached at the first or the second bargaining round, nothing is produced in that production period. In each bargaining round the employer and worker flip a coin to determine who makes the offer in that round only (i.e, each player makes an

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9I discuss this assumption in section 4.
offer with probability $\frac{1}{2}$ in each round). If the employer makes an offer, the worker can choose to accept the offer, reject it and wait until the next round, or reject the offer and quit. If the worker makes the offer, the employer either accepts the offer or rejects it and waits until the next round.10

3 Equilibrium Analysis

Next, I solve for a perfect Bayesian equilibrium. I characterize the equilibrium wage in the second and third production periods. The equilibrium outcome in period one is discussed in section 3.4.

Definition 1: A “revealing wage” is a wage which depends directly on the worker’s MRP, $\theta$ (i.e., the wage is an invertible function of $\theta$).

Definition 2: A “nonrevealing wage” is a wage which depends only on publicly observable characteristics.

Proposition 1 (Equilibrium outcome in the second and third period)

A) No turnover occurs in equilibrium.

B) There is a threshold ability level $\theta^*$. Workers with productivity above $\theta^*$ earn revealing wage. Workers with ability below $\theta^*$, earn nonrevealing wage.

$$\theta^* = 2\theta$$ (1)

C) Revealing wages:

workers with ability above $\theta^*$ renegotiate the wage in the second period.

- Agreement is reached in the first bargaining round of the second period

- The second period "Revealing wage" is:

10 For example, assume that in the second production period the worker makes the first offer (occurs with probability $\frac{1}{2}$). If the employer rejects it then in the second bargaining round the employer and worker flip a coin again and each player makes the last offer with probability $\frac{1}{2}$. 

12
\[ w_2 = \frac{\theta}{2}[1 - \delta] \]  
 (2)

- The third period wage is the MRP:

\[ w_3 = \theta \]

D) "Non-revealing wage":

firms offer workers when they are first hired a wage of \( \theta \) for periods two and three. Workers with ability below \( \theta^* \) do not renegotiate with the employer and earn:

\[ w_2 = w_3 = \theta \]

No turnover occurs in equilibrium because all workers are equally productive in all firms.\(^\text{11}\) The equilibrium is semi-separating; there is an ability threshold level which sorts the workers into two groups. More able workers earn a wage in the second period that reveals \( \theta \), and after the second period earn a wage equal to their MRP. Workers with ability below the threshold level earn the wage which is specified in the initial contract, and their ability cannot be inferred directly from the wage; these workers earns this wage until retirement. Figure 5 describes the second period wage as a function of \( \theta \), when \( \theta = 1, \delta = 0.6 \). The ability threshold level is 2 (\( \theta^* = 2\theta \)). Below \( \theta^* \) workers earn nonrevealing wages (\( \theta \)), and above it they earn revealing wages. Figure 6 shows the second and third period wages as a function of productivity.

To motivate the equilibrium, consider a similar game with different bargaining rules. In particular, consider a bargaining game in which workers always make the last offers. In this case, wages of workers are the MRP. Alternatively, if the

\(^{11}\text{One way to generate turnover is to introduce firm-employee match component to the worker's productivity. However, the purpose of the paper is to analyze wage signaling and the main features of the equilibrium remain when workers are not equally in all firms.}\)
bargaining rule is such that employers always make the last offers in all periods, wages do not depend on actual productivity. Instead, wages are equal to the wage specified in the initial contract ($\theta$). In this model, in which bargaining is a symmetric process, wages which are an outcome of the bargaining depend directly on ability but the wage is lower than the wage that arises in symmetric information bargaining ($\bar{\theta}$).

Not all workers choose to renegotiate. It is costly for a worker to reveal ability, because the employer loses future informational rents. The employer is compensated for that loss in the second period, and receives a one period transfer. The employer's profit is an increasing function of the worker's ability. The employer receives half of the current period output plus a discounted value of future payoffs.

The second period revealing wage can be below $\theta$ (see figure 5), and I show later that there are always workers who earn revealing wages which are below $\theta$.\footnote{The second period wages are non-monotonic in ability. The reason I can sustain such an equilibrium in which wages reflect the true productivity is that I assume that outside firm can verify the date in which the contract was signed. I discuss the issue of collusion between the employer and worker that arise due to the non-monotonicity of wages in section 4.} As a result, some workers do not reveal ability. These are workers for whom the additional present value of receiving $\theta$ for all subsequent periods does not exceed the cost required to reveal information. All these workers earn wages below their MRP.

### 3.1 Equilibrium Strategies

Next, I describe the equilibrium strategies and beliefs of each player. The wage history at the beginning of period $s + 1$ is denoted by $H_{s+1}$, $w_1, ..., w_s$. $w_s$ is the wage realization in period $s$. The employing firm is denoted by $f$, outside firms are denoted by $m$, and workers by $l$. Wage offer strategies are denoted by upper case
letter \( W \) and wage realization by lower case \( w \). I denote the wage offer strategy used by the employer in bargaining round \( t \) by \( W_t^l(\theta) \), and the worker’s strategy by \( W_t^l(\theta) \). Below is a description of the equilibrium strategies of each player.

**Outside firms strategies and beliefs:**

Outside firms make wage offers to employed workers at the beginning of the second and third periods. The offers are made based on beliefs about the worker’s productivity. The information available to outside firms each period are the worker’s employment history (wages and employer). Let \( H_s \) denote the employment history available to outside firms at the beginning of period \( s \). I denote the offers made by outside firms at the beginning of period \( s \) by \( W_m^m(H_s) \), and beliefs about the worker’s productivity by \( \hat{\theta}(H_s) \). I denote the lower support function of the beliefs by \( l(\hat{\theta}) \).

**Third period strategies:** At the beginning of the third period outside firms observe workers’ employment history in period two. The worker either works for the first period employer or works for a new employer. If the worker currently works for the first period employer he either earned the initial contract wage or renegotiated the wage. Below are the strategies and beliefs.

If the worker worked for the same employer in periods one and two, renegotiated the wage, and agreement is reached in the first bargaining round, the offers are:

\[
W_3^m = \begin{cases} 
\left[ \frac{2}{1-\delta} \right]w_2 & \text{if } w_2 > 0 \\
0 & \text{otherwise.}
\end{cases}
\] (4)

If agreement is reached in the second bargaining round

\[
W_3^m = \begin{cases} 
\left[ \frac{2}{1-\delta} \right]w_2 & \text{if } w_2 > 0 \\
0 & \text{otherwise.}
\end{cases}
\] (5)

If the worker stayed with the first period employer and didn’t renegotiate the initial contract, or changed employers at the end of the first period, outside firms
offer a contract of zero for the second period and $\theta$ for the third period\textsuperscript{13}

**Second Period strategies:** outside firms offer all workers a contract of $W_2^m = 0$, $W_3^m = \theta$.

**Beliefs:**

Outside firms’ beliefs depend upon the employment and wage history at the beginning of the period. $\hat{\theta}(H_s)$ is a function of the wage history observed by outside firms. Outside firms observe the wage realization. The employer and worker’s wage offers strategies $W_t$ are invertible functions of $\theta$, outside firms form an estimate based on the wage realization (contract date and knowledge of the bargaining process).

Given the employer and worker equilibrium strategies, if renegotiation occurs, and agreement is reached in the first bargaining round the wage is as described in proposition 1. Outside firms observe the wage realization $w_2$ and estimate $\theta$:

$$\hat{\theta} = \begin{cases} \left[ \frac{2}{1-\delta} \right] w_2 & \text{if } w_2 > 0 \\ E[\theta | \theta \leq \theta^*] & \text{otherwise} \end{cases} \tag{6}$$

If agreement is reached in the second round and the wage is non-zero the beliefs are

$$\hat{\theta} = \begin{cases} \left[ \frac{2}{1-\delta} \right] w_2 & \text{if } w_2 > 0 \\ E[\theta | \theta \leq \theta^*] & \text{otherwise} \end{cases} \tag{7}$$

The beliefs are the expected MRP of workers, conditional on past wages. If the wage is equal to the "initial contract wage" $w_2^m$ (on-the-equilibrium path), then on-the-equilibrium path the worker’s ability is below $\theta^*$ and above $\theta$.

If outside firms observe at the beginning of the third period a worker who switched employers at the end of period one, then the wage is not a function of

\textsuperscript{13}This is a result of the assumption that a firm needs information regarding the worker’s productivity in order to assign him to jobs correctly. Note that contingent contracts aren’t feasible and workers will not report correctly their productivity.
the worker’s actual productivity.\footnote{If a worker switched employer after the first period than the second period wage is the wage offered by an uninformed outside firm at the end of the first period. Note that this occurs off-the-equilibrium path and many beliefs can support this equilibrium outcome.} Similarly to the beliefs in period two, the beliefs are the unconditional expectations, $\hat{\theta} = E(\theta)$.

**Payoffs:** The expected payoffs from the offer in the third period are the probability to raid the worker times the expected productivity conditional on raiding the worker minus the wage. The payoffs for a firm who makes an offer in the second and third period are described below. $\theta_x$ is the expected productivity of a worker who accepted the offer $x$. $\theta_x$ is conditional on the second period employment history (I omit the notation of the history, $H_s$, to simplify the exposition). Let $N$ be the number of outside firms. Let $P[x]$ be the probability to raid a worker with an offer $x$

$$
\pi_3^m(x) = \begin{cases} \frac{1}{N} P[x](\theta_x - x) & \text{if } \theta = \theta_x \\
0, & \text{otherwise} \end{cases} \quad (8)
$$

In equilibrium workers accept an outside offer only if it exceeds their productivity, since otherwise the employer matches the offer.

If a firm raids a worker in the second period with an offer of $x, y$ ($x$ is the second period offer and $y$ is the third period offer), the profit is the expected output of the worker (who accepted the offer) in the second period (which is zero if the firm doesn’t know the worker’s productivity) and third period minus the second period wage, $x$, minus the third period wage. The second period expected profit is :

$$
E\pi_2^m(x, y) = \begin{cases} \frac{1}{N} P[x, y](\theta_{x,y} - x) & \text{if } \theta = \theta_x \\
0, & \text{otherwise} \end{cases} \quad (9)
$$

Note that the expected profit in the second period is $-x$. In the third period the firm know the worker’s productivity, the third period wage is the maximum between the bargaining outcome, the contract offer for the third period $y$ and the highest outside offer. If the highest offer exceeds the worker’s productivity the
worker accepts the outside offer.

\[ E_{\pi_3}(x, y) = \begin{cases} 
\theta_{x,y} - \max\{\theta_2, y\}, & \text{if } \max\{\theta_2, y\} \geq w_3^m \\
\max\{\theta_{xy} - w_3^m, 0\}, & \text{otherwise}
\end{cases} \] (10)

**Employer’s strategy:**

The employer maximizes the present value of profits. The employer matches any outside offer if the worker’s productivity exceeds the wage offered. If renegotiation occurs, the employer’s strategy describes wage offers in each production period in each round. The continuation payoff of the employer given the wage history in period \( s \) is denoted by \( \pi_s \). The wage offer made by the employer in the first round is \( W_{1}^{f} = \frac{\theta}{2} - \delta \pi_3 \). The employer accepts any wage offer made by the worker which is equal to, or higher than the wage offer specified above. Otherwise, the employer rejects the offer. At the beginning of each period, if an employee receives an offer from an outside firm \( W_{3}^{m} \), the employer offers the same wage if \( \theta \geq w_3^m \). Otherwise the employer doesn’t make a counteroffer. On-the-equilibrium path, workers with \( \theta \geq \theta^* \) receive offers \( W_{3}^{m} = \theta \), and the employer offers \( \theta \) as well.

**Worker’s strategy:**

The worker maximizes the present value of lifetime earnings. At the beginning of periods two and three, the worker can choose to reject the outside employers’ offer and stay with the current employer. If he stays with the current employer he can earn the wage that was offered initially by the employer or renegotiate the wage with the employer. Alternatively, the worker can choose to accept an outside firm’s offer and earn the wage that the firm offered. If the worker negotiates the wage with the employer, the worker’s strategy is the wage offers he makes in each production period in each round.

**Third period:** if the worker stayed with the second period employer he can renegotiate. At the beginning of the third period the worker accepts an outside
offer if it exceeds the maximum between the employer’s offer and the expected bargaining outcome. I do not write explicitly the bargaining offers as they are similar to the offers in a symmetric information bargaining (since this is the last period). The prediction in the symmetric information bargaining is $\theta$ if agreement is reached in the first round.\footnote{Since there is no discounting between round, in the symmetric information case there is an equilibrium in which agreement is reached in the second round and the wage depend on who makes the final offer. However, the expected payoffs for each player is $\frac{\theta}{2}$.} That is, the worker accepts an outside offer if it exceeds the employer’s offer and the expected bargaining outcome

$$w^m_3 > \text{MAX}\{\frac{\theta}{2}, w^f_3\} \quad (11)$$

The second period: If the worker stayed with the current employer he can choose to renegotiate. Below is the equilibrium strategy if the worker renegotiates. I denote the continuation payoff of the employer in period three if he rejects an offer made by the worker in the last round of period two by $\pi^r_3(H^3)$ and the continuation payoff if the employer accepts the offer by $\pi^a_3(H^3)$. Note that these continuation payoffs depend upon the history at the beginning of the third period, $H^3$. The wage offer made by the worker in the first bargaining round (in case he makes the first offer) is $W^f_1 = \theta - \delta(\pi^r_3 - \pi^a_3)$. The worker’s offer, if he makes the last offer, is $W^f_2 = \theta - \delta(\pi^r_3 - \pi^a_3)$. If the employer makes the offer in the first round, the worker accepts any offer greater than or equal to $W^f_1 = \theta - \delta[\pi^r_3 - \pi^a_3]$; otherwise, he rejects the offer and waits to the next round. In the last round, the worker accepts any nonnegative wage offer. At the beginning of the second production period, the worker can either stay with the current employer or accept an outside offer. An outside firm’s offer is accepted if the discounted present value (second and third period wages) of accepting an outside offer exceeds the value of staying with the current employer.
3.2 Derivation of the bargaining outcome

In this section I prove proposition 1. I derive the bargaining outcome in the second production period, and show that the offers made by the worker and employer are optimal given the outside firms’ strategies and beliefs described above. I solve backwards, starting with the last production period, for any possible history. I first characterize revealing wages. That is, I find the equilibrium wages for the case in which workers negotiate the wage. I then show that outside firms strategies are optimal given the employer’s and worker’s strategies, and that the beliefs about the worker’s productivity, on-the-equilibrium-path satisfy Bayes’ rule. I find the conditions under which workers earn revealing wages, in section 3.3.

I next describe the equilibrium offers each player makes in each round in each production period, and show that they are optimal. I solve backwards to obtain the offers made in the first round. Solving for the outcome of renegotiation in each production period starting with the last production period is necessary to verify that workers that haven’t revealed their ability have no incentive to deviate and negotiate the wage.

Solving backward for solution of each production period is also used to estimate the continuation payoff of workers and employer when negotiation occurs in the second production period, if they deviate; the continuation payoff establishes threat points for the bargaining in the second period, and hence affects the wage. For workers who renegotiate the wage, ability is revealed after the second production period and the analysis of bargaining after the second period is off-the-equilibrium path.

It is important to note that in each bargaining round each player has an incentive to offer the other party the lowest share that the player is willing to accept. The condition required is that the beliefs about the worker’s productivity increase
in the worker’s wage; hence, the worker has incentive to offer the highest wage that the employer will accept. This is true in all periods for all bargaining rounds because outside firms’ beliefs strictly increase in the bargaining wage outcome; this implies that future payoffs increase with the current period wage. Any wage offer that is lower than this wage reduces the worker’s utility, and any higher wage offer is rejected by the employer.

The last production period:

In the last production period, if renegotiation occurs, the employer and worker’s threat points are both zero (there is no information externality since it’s the last period). Since the bargaining is symmetric, the prediction is that the expected payoff of each player is half of the surplus, \( \frac{\theta}{2} \).

If the employer makes the offer in the last round (round two), he offers \( W_f^2 = 0 \). If the worker makes the offer in the last round, he offers \( W_e^2 = \theta \). Since this is the last round in the last production period, the worker’s payoff if he quits is zero. If the employer rejects an offer he earns zero. Clearly, the strategies above are optimal.

In the first round, after a player is chosen to make the offer, he offers the other player the expected payoff that the player receives if he waits to the next round (there is no discounting between rounds). Hence, each player offers \( \frac{\theta}{2} \).

At the beginning of the third period outside firms make offers based on the second period wage. Since it is the last period the worker chooses the firm in which he will earn the highest wage, hence the strategy described in equation 11 is optimal.

Period two:

In the second period, in the last bargaining round, if the employer makes an offer, he offers the worker \( W_f^2 = 0 \). The worker accepts the offer, and receives no payment in the current period (the second period). On-the-equilibrium path, the
worker’s continuation payoff is $\frac{\delta}{2}\theta$. The continuation payoff if the worker rejects the offer and no production occurs is the same as the continuation payoff if he earns zero; this is because outside firms offers are similar in the cases in which the workers earns zero and when no production takes place (they offer zero). Therefore the worker is indifferent between accepting a wage of zero and rejecting it.

If the worker makes the offer in the final round, the highest wage offer that the employer accepts is an offer which makes the employer indifferent between accepting the offer and rejecting it (if the employer rejects the offer, no output is produced in the current production period). If the employer rejects the last offer, he earns nothing in that period. The worker’s offer maximizes his utility:

$$w_2^l = \arg\max\{w_2^l + \delta \max\left\{\frac{\theta}{2}, w_3^m(w_2)\right\}\}$$

The payoffs are increasing in the wage (as the third period wage is weakly increasing in the negotiation wage), hence the wage is the highest wage the employer accepts (the employer is indifferent between rejecting and accepting)

$$\theta - w_2^l + \delta \pi_3(w_2^l) = \delta \pi_3^r$$

(12)

Given the beliefs and third period offers, for any offer above $\theta - \frac{\delta}{2}\theta$ the employer’s profit if he accepts is zero. Therefore, in the last round the worker offers $W_2^l = \theta - \delta \pi_3^r$ and the employer accepts the offer. If the worker’s ability hasn’t been revealed, then in period three the employer and worker will bargain over wage and $\pi_3^r = \frac{\theta}{2}$.

In the first round of bargaining in this period, if the employer makes the offer he offers a wage such that the worker’s expected payoffs in both periods is equal to the payoff if the worker rejected the offer. Note that rejecting the offer means getting the expected payoff of the second round in which each player makes the offer with probability $\frac{1}{2}$. The worker’s expected utility in the second round (if
rejects the offer in round one):

$$\frac{1}{2}(\theta - \delta \pi^r + \delta \theta) + \frac{1}{2}\delta\left(\frac{\theta}{2}\right)$$  \hspace{1cm} (13)

That is, the offer

$$W_1^f + \delta w_2(w_1^f) = \frac{1}{2}(\theta - \delta \pi^r + \delta \theta) + \frac{1}{2}\delta\left(\frac{\theta}{2}\right)$$  \hspace{1cm} (14)

If the worker make the offer he offers the employer the expected profit from the next round (note that the sum of the expected payoffs in equilibrium is always \(\theta(1 + \delta)\):)

$$\frac{1}{2}\delta\pi^r + \frac{1}{2}\delta\left(\frac{\theta}{2}\right)$$  \hspace{1cm} (15)

on-the-equilibrium path \(W_1^f = \frac{\theta}{2} - \delta \pi^r\). If the worker makes the first offer: \(W_1^f = \frac{\theta}{2} - \delta \pi^r\).

If a worker entered negotiation in the second production period then, the solution to the bargaining is: \(w_2 = \frac{\theta}{2} - \delta \pi^r\), regardless of who makes the offer. On-the-equilibrium path the profit if the employer rejects the last offer is \(\pi^r = \frac{\theta}{2}\). Therefore, the second period bargaining wage is \(\frac{\theta}{2} - \delta \pi^r\).

Optimality of outside Firms’ Equilibrium Strategies

I prove next that outside firms’ strategies are optimal and that on-the-equilibrium-path the beliefs satisfy Bayes’ rule. If agreement is reached between the worker and the employer, outside firms’ offers are as described in equation 4,5. The beliefs about the worker’s productivity are as described in equations 6, 7.

I now show that given the equilibrium bargaining offers and beliefs, the outside firms strategies are optimal.

Since in equilibrium the beliefs are correct, that is, \(\hat{\theta} = \theta\), offering \(\hat{\theta}\) to workers who renegotiate yields zero profit. Any offer below it yields zero profit. The worker accepts any offer which exceeds his MRP, since it exceeds any offer that the employer makes or accepts; any offer above it yields negative payoff. The
beliefs are correct since it is optimal for each player to make the offers I specified above in each round of bargaining.

3.3 Characterization of types earning revealing wages

I solve for the conditions under which workers either negotiate wages, or earn a non-revealing wage, $\theta$, in each production period. Next, I prove part D in proposition 1. That is, I show that the ability cutoff level below which wages are non-revealing and above which wages are revealing is given by

$$\theta^* = 2\theta$$

Proof: I compare the value of receiving a revealing wage in the first period in the high level job (the second production period) plus the MRP in all following periods to the present value of receiving outside option $\theta$ in all periods. The ability threshold level is the level for which the two are equal.

$$\frac{\theta^*}{2}[1 - \delta] + \delta\theta^* = \theta + \delta\theta$$

(16)

It can be easily shown that all workers with $\theta < 2\theta$ always choose a nonrevealing wage, as the expected payoff if they bargain is less than $\theta(1 + \delta)$.

This completes the proof of proposition 1.

**Proposition 2** All workers who earn a revealing wage with MRP $\theta$ in the range of

$$\theta < \theta\left[\frac{2}{1 - \delta}\right]$$

(17)

earn wages that are below the outside option (that is $w_2 < \theta$).

The proof is obtained immediately by equating the second period revealing wage to $\theta$. 

24
When information is symmetric, bargaining with an outside option yields the result that the bargaining outcome is above the outside option. Therefore, workers with ability $\theta > \underline{\theta}$ receive the bargaining outcome rather than the outside option. In this model, workers with ability $\theta^* \geq \theta \geq \underline{\theta}$ earn $\underline{\theta}$.

### 3.4 First Period Equilibrium Wages

In this section I derive the first period contract that firms offer, and show that this contract is optimal. I Denote the initial offer for period two by $x$, and the initial offer for period three by $y$. Each worker chooses a contract that maximizes the expected lifetime earnings.

$$EU = w_1(x, y) + \delta E_\theta[w_2(\theta, x, y)] + \delta^2 E_\theta[w_3(\theta, x, y)]$$  \hspace{1cm} (18)

If no turnover occurs in equilibrium, then from zero profit condition the first period wage is

$$w_1 = \delta E_\theta(\theta - w_2(\theta, x, y)) + \delta^2 E_\theta(\theta - w_3(\theta, x, y))$$  \hspace{1cm} (19)

Therefore, the worker’s expected earnings if he stays employed in the current firm is $\delta E_\theta(\theta) + \delta^2 E_\theta(\theta)$. This is the expected surplus if no turnover occurs. As I showed earlier, if the second and third period wages in the initial contract are $\theta$, no turnover occurs in equilibrium. Clearly, the total expected surplus is maximized by this contract, and therefore, the worker’s expected utility is maximized. No contract that offers different wages can increase the total expected lifetime earnings.\(^{16}\)

At the beginning of the first period the information is symmetric and the market is ex-ante competitive. The equilibrium wage clears the market, and the expected discounted sum of profits is zero. The probability that a worker will earn

\(^{16}\)If firms commit to second period wages above $\underline{\theta}$, but not to employment, then some workers will may be dismissed; dismissals of workers can only (weakly) reduce total surplus produced.
a non-revealing wage in the second period is \( Pr(\theta \leq \theta^*) \). The probability that a worker will earn a revealing wage is \( Pr(\theta > \theta^*) \). The employer’s profit if a worker receives a revealing wage is \( \theta - w_2(\theta) \). Where \( w_2 \) is given by equation 2. The employer’s profit if the worker receives a non-revealing wage is \( (\theta - \bar{\theta})(1 + \delta) \).

**Proposition 3 (First period wage)**

\[
w_1(\bar{\theta}) = \delta \{ Pr(\theta \leq \theta \leq \theta^*) E[\theta - \theta|\theta < \theta^*](1 + \delta) + Pr(\theta > \theta^*) E[\theta - w_2(\theta)|\theta^* < \theta] \}
\]

**Proof:** The first period wage is obtained by imposing the zero profit condition:

\[
E(\pi)(\bar{\theta}) = \delta \{ Pr(\theta \leq \theta \leq \theta^*) E[\theta - \theta|\theta < \theta^*](1 + \delta) + Pr(\theta > \theta^*) E[\theta - w_2(\theta)|\theta^* < \theta] \} - w_1(\bar{\theta})
\]

Workers earn a wage that is larger than their MRP in the first production period (zero in this case). Expected life time earning of a worker when he enters the firm is the discounted sum of the expected MRP in all periods.

### 4 Robustness

This sections discusses how certain assumptions affect the equilibrium outcome.

The second period equilibrium wages are non-monotonic in ability. The outcome described in proposition 1 is an equilibrium outcome since there is no profitable single deviation. In general, however, a low ability worker who earns the outside option (\( \bar{\theta} \)) has incentive to collude with the employer and agree to be paid a lower wage in the second period. In the third period, outside firms would mistakenly believe that this is a high productivity worker and offer a high wage. Since the employer pays a lower wage in the second period collusion can be profitable to both the worker and the employer.
This scenario is ruled out in this model since the date and content of the contract are assumed to be verifiable by outside firms. If the wage agreement is signed after the beginning of the first period the employer has incentive to pay the worker a wage that reflects his actual ability, as the equilibrium bargaining wage is monotonic in ability. Hence, any such wage agreement has to be signed before the beginning of the period. However, outside firms will only interpret wages that resulted from renegotiation after the beginning of the production period and will not make offers based on wage contracts that are signed before the production period starts. Notice that there will not be an equilibrium in which wages are a signal and outside firms make offers unless outside firms can infer correctly the productivity.

4.1 Multiple periods

In the two period model, all workers can signal their productivity via wages. That is, workers can bargain over the wage and earn a portion of the surplus. Outside firms observe wages, infer the worker’s productivity, and in the following periods offer the worker his MRP. If a worker renegotiates the wage, the present value of output in both periods, \( \theta(1 + \delta) \), is divided equally between the worker and employer. Alternatively, a worker can choose not to bargain and earn a wage equal to the initial market wage, \( \theta \), in each period. Workers with lower productivity choose not to renegotiate as the present value of the outside offers exceeds the value of wages if they renegotiate.

17 In addition, if the employer and worker sign an agreement before the production period starts, and the worker’s ability can be inferred from the contract, the worker who can still switch employer at this stage, can use the contract to elicit high offer for the current period. In this case the employer makes no profits and hence will have no incentive to sign such contract before the beginning of the period.

18 The worker’s present value of wages is \( \frac{1}{2} \theta + \delta \theta \). The employer earns his share in the total output over both periods in the first period, and zero in the second period.
In the multiple periods model, one might expect that workers will renegotiate and "reveal" their productivity to the market immediately after they learn their productivity (in the second period) since there are more periods left to earn the full productivity. Moreover, employers should "allow" workers to reveal their productivity via wages early, as they are compensated for loss of future informational rents.

This intuition above is correct, and all workers who earn revealing wages in the three period model earn revealing wages in the $S$ period model. However, in a model with more than three periods, workers cannot always signal productivity via wages in the second period. The feature that enables workers to "signal" productivity via wages is that the bargaining outcome increases in the worker’s productivity. That is, high productivity workers earn larger wages if they renegotiate. This feature ensures that low productivity workers do not renegotiate and earn a wage similar to the wage that the higher type earns. The reason is that if there are more than two periods left, the negotiation outcome is non-increasing in productivity. In this case, outside firms will not use these wages as ability signals. The reason is that if negotiation wages decrease in productivity, low productivity workers and employers have incentive to agree on low bargaining wages. In this case the employer pays low first period wage and outside firms offer the low productivity workers a wage that exceeds their productivity. In the presence of firm-specific skills, uncertainty regarding offer arrival or an exogenous probability of quits, workers can signal productivity earlier since the expected loss of future informational rents is lower.

Next, I characterize the equilibrium for four periods ($S = 4$). Workers cannot reveal productivity in the second period if

$$1 - \delta - \delta^2 \leq 0$$  \hspace{1cm} (22)

28
I restrict the equilibrium analysis for the case

$$\theta \geq \theta(1 - \frac{\delta}{2} - \frac{\delta^2}{2})$$

(23)

Later, I discuss the equilibrium in the general case.

The game (timing and information) structure is similar to the two periods model. In the first period worker and employer learn the worker’s productivity (see figure 1). The second period up to the forth periods are similar to the second and third periods in the three period model. See figures 2, 3 and 4 for description of the game in periods 2 – 4.

**Proposition 4 (Equilibrium outcome \(S = 4\))**

1. The wages in period 2 for all workers is: \(\theta\)

2. ”Revealing wage”

   a) all workers with productivity \(\theta > \theta^*\) renegotiate the wage in period 3.

   b) Agreement is reached in the first round of the bargaining in the third period

   c) The bargaining wage is given by:

   $$w_{S-1} = \frac{\theta}{2}[1 - \delta]$$

   (24)

   d) The wage in period four is \(\theta\).

3. ”Non-revealing wage”:

   a) all workers with \(\theta \leq 2\theta\) earn \(w_j = \theta\), where \(j = 2, 3, 4\)

4. The threshold level is \(\theta^* = 2\theta\)

Proof of proposition 4 (and description of the strategies) is in Appendix A.

Below is an intuitive discussion.

---

19I conjecture that a pure strategies equilibrium may not exist when the ability distribution has a wide support.

20For example, if \(\delta = 0.9\), than equation 23 implies that \(\theta \leq 0.145\theta\).
The solution is obtained by backward induction. The last two periods (third and forth period) solution in case ability hasn’t been "revealed" before period three, is similar to the solution in the three period model. This occurs on-the-equilibrium path. The equilibrium in the third and forth period is similar to the equilibrium in the two periods model as the solution is obtained by backward induction. I now show that in equilibrium, no worker can reveal productivity in the second period if $1 - \delta - \delta^2 \leq 0$. Put differently, the condition for revealing productivity is that the bargaining wage in the first bargaining round is increasing in productivity; the bargaining wage increases in $\theta$ when the wage is positive.

Beliefs in periods $s = 2, ..., s - 2$ are for any bargaining round agreement

$$\hat{\theta} = \begin{cases} \overline{\theta} & \text{if } w_2 > \theta \\ \underline{\theta} & \text{if } w_2 \leq \theta \end{cases} \quad (25)$$

Consider the final period $S$ (period 4). If ability hasn’t been inferred by period $S - 2$ (period 2), then the game in period $S - 1$ (period 3) is a two period bargaining game which similar to the original game. Now consider period two ($S - 2$). In the final bargaining round if the employer makes the last offer, he offers zero and the worker accept. If the worker makes the last offer. The employer rejects all offers if the value of accepting the offer is less than the value of rejecting. If the employer rejects, in case of workers who reveal productivity, the employer earns $\delta^2(1 + \delta)$. If the last offer is such that outside firms infer ability the employer makes zero profits over the last two periods, hence the worker has to compensate for that value via earning a low wage today. But the present value of half of the surplus of two periods, $\frac{\delta^2}{2}(1 + \delta)$, exceeds the output produced in the current period: $\frac{\delta^2}{2}(1 + \delta) > \theta$.

I next generalize the four period model under assumptions 23 and $1 - \delta - \delta^2 < 0$. The last three periods equilibrium outcome is similar to the three period case. The equilibrium outcome for period two up to period $S - 2$ is that workers cannot
"reveal" ability since the bargaining outcome has to be negative and therefore outside firms do not use these wages as a signal of ability. Given that, the highest wage that a worker can earn is $\theta$.

In the game all the information required about the workers productivity is acquired in one period and it never changes, that is, the worker will keep producing this amount in all periods. The reason that workers cannot reveal ability early is that the amount of compensation for loss of future information rent is large relative to output produced in one period (because the employer earns zero profit in all remaining periods). Note that the employer needs to be compensated for all periods as there is a positive probability that the employer will make the last offer in all periods. In fact, since the bargaining process is symmetric, in case bargaining occurs, the total value of gross surplus should be divided equally. But bargaining is over lifetime earnings hence each player should earn $\sum_{s=1}^{S} \delta^s \frac{\theta}{2}$. Since outside firms can interpret these wages, the employer should earn it in the first period since he earns zero profit after the bargaining period. In this case wages are negative. The problem with negative wages is that in this range, the wage is decreasing in the worker’s productivity and the bargaining outcome is no longer monotonic in productivity. As a result workers of low type can pay the employer and be considered high types. That is, bargaining wages will not reflect true productivity and hence in equilibrium outside firms will not interpret negative wage as signal. Therefore, there is no incentives to workers to earn these low wages.

Workers can reveal ability earlier, the smaller the loss of future informational rent for the employer which result from "revealing ability" to the market. There are few factors that reduce the amount of compensation to the employer besides low discount factor. First, if there is uncertainty regarding arrival of outside offers in each period the employer’s expected future payoff is larger in case the worker earns revealing wage. Another factor, is the possibility of random quits of workers.
that are exogenous to the model (for example workers move for personal reasons). In this case, the continuation payoff of the employer if he rejects the last offer made by the worker is smaller. Therefore, the worker ”compensation” to the employer is smaller and bargaining wages are larger. Another possibility is firm-specific skills. In the presence of firm-specific skills, the employer loses only the portion of the general skills in case the worker’s ability is revealed to the market, and therefore the compensation is only for the loss of future profits associated with general skills. The larger the effect of these factor, the earlier workers are able to negotiate wages and signal ability.

4.2 Job assignment and gradual learning

In the basic model, uncertainty regarding workers skills is resolved after one period, and productivity stays constant over time. In reality, workers perform different tasks during a career in a firm and performance in one job doesn’t fully predict performance in another job (although they may be correlated). In this section, I analyze the equilibrium outcome when there are several tasks in the firm, and learning about performance in different tasks occurs gradually. In traditional learning models in which employers observe a noisy signal every period and update beliefs. In this paper, employer and worker learn the productivity in tasks performed. They observe however, noisy signal regarding productivity in other tasks. Only when a worker is assigned to a task the employer and worker learn productivity (see Lazear (2002)).

Consider for simplicity a three period model. Assume that there is a hierarchy of two jobs in the firm. In the first job level, workers perform simple tasks and in

\footnote{See Gibbons and Farber (1993) Gibbons and Waldman (1999a), and Gibbons and Waldman (1999b) and Neal and Rosen (2000) for surveys which include discussions about learning model.}
the second job level two workers perform complex tasks. In the first period the employer and worker learn the worker’s productivity in job level one and observe a noisy signal regarding productivity in the second job. Assume that the employer and worker can observe whether or not the worker is qualified to work in job level two. In the second period the worker works in level one and the employer and worker receive an additional noisy signal regarding the worker’s productivity in the second job. The signal is unobserved by outside firms. Assume that if the worker is qualified his productivity in level two exceeds his productivity in level one, but until the worker actually works in the second job there is uncertainty regarding his actual productivity.

The timing of the game is as before with the following exceptions. First, in the first period, in addition to learning the worker’s productivity in the first job, the employer learns whether or not the worker is qualified to work in the second job. Secondly, at the end of the second period the employer decides on job assignment for the third period. The worker can either be assigned to job level two in the third period or keep working in level one. Outside firms observe at the beginning of the third period the second period wage and the worker’s current job assignment.

The equilibrium analysis for workers who are not promoted stays the same, and the wages are similar to equilibrium wages in proposition 1. Next, I characterize the equilibrium outcome for workers who are promoted. In section 6 I show that the promotion rule is efficient. That is, I show that all qualified workers are promoted to job level two in the second period.

**Equilibrium outcome:**

In the third period, the wage is the maximum between the outside offers and half of the expected output. Since this is the last period, the analysis is similar to
the analysis in the symmetric information bargaining case.

\[ w_3 = \text{MAX}\{W_3^M, \frac{\theta_2}{2}\} \]

Consider bargaining in period two for workers who will be assigned to job two in
the next period (I show later that all qualified workers will be promoted).

**Bargaining in period two- Last round**

The employer accepts the last offer if the value of accepting the offer, which
is the current payment and the expected profit in the third period (continuation
payoff, \( E\pi_a \), which depend on the bargaining outcome \( w_e^2 \)) is greater or equal to
the value of rejecting the offer, \( E[\pi_r] \).

\[
\theta_1 - w_e^2 + \delta E[\pi_a] \geq \delta E\pi_r
\]

The value of rejecting an offer is the minimum of the profit if the worker accepts
the market offer, and the bargaining outcome in case the worker renegotiates the
wage.

\[
E\pi_r = \max\{\frac{E\phi(\theta_2)}{2}, E(\theta_2) - w^m_3\}
\]

The worker makes the highest offer the employer accepts:

\[ w_e^2 = \theta_1 - E\pi_r \]

If the employer makes the last offer, he offers zero and the worker accepts.

**Bargaining in period two- First round**

In the first round an offer is accepted if neither player can earn more if he waits
to the last round in which each player makes the offer with probability \( \frac{1}{2} \). The
player who makes the offer, offers:

\[ w_e^2 = \frac{\theta_1}{2} - \delta[E\pi_r - E\pi_a] \]
Claim 1: wage signaling is feasible if

\[ \frac{\partial (w_2^s)}{\partial \theta_1} > 0 \quad (26) \]

If the bargaining outcome is declining in \( \theta_1 \), then outside firms do not use wages as a signal since low productivity workers have incentive to renegotiate and "pretend" to be high type workers (see further discussion in section 4.1).

Claim 2: if condition 26 is satisfied, among those who are qualified, workers with \( \theta_1 > 2\theta \) who are promoted earn revealing wages.

Proposition 5 (Revealing wage) For those promoted wages are

\[ w_2 = \frac{\theta_1}{2} + \frac{\delta}{2} p(\theta_2 > 2\theta_1) E[\theta_2|\theta_2 > 2\theta_1] + \delta(1-p(\theta_2 > 2\theta_1))(E[\theta_2|\theta_2 < 2\theta_1] - \theta_1) - \frac{\delta}{2} [\theta_2|\theta_2 > 2\theta] \]

(27)

Proof: see Appendix B.

The intuition is as follows. If a worker reveals productivity in the first job it will only affect wages in the second job if \( \theta_2 < 2\theta_1 \). Otherwise, the employer’s profit is unaffected. Hence, the worker earns half of the surplus produced in the current period and additional terms; these terms are the difference between the expected profit of the employer when ability in job one is revealed to the market, and the value of rejecting the offer and waiting to the last round, in which the employer makes offer with probability half.

Third period wage is:

\[ w_3 = \text{MAX}\{ \frac{E(\theta_2)}{2}, \theta_1 \} \]

Workers with \( \frac{E(\theta_2)}{2} > \theta_1 \) earn \( w_2 = \frac{\theta}{2} \) and \( w_3 = \frac{E(\theta_2)}{2} \).

Notice that revealing wage affect wages only in the case where

\[ E[\theta_2] < 2\theta_1 \]
Next, consider a four period model. If a worker is promoted in period three he works for two periods in job level two. This extensions allow to consider dynamics in which outside firms may learn gradually about workers productivity. That is, workers can reveal productivity in both levels over time. In the four period case, workers who are promoted work for two periods in level 2; uncertainty regarding the productivity in the second job is resolved after period three. Revealed productivity in job level may affect wages in level two for workers with ex-post productivity, \( \theta_2 < 2\theta_1 \). However, high ability workers in level two will reveal productivity and wages will be lower than \( \theta \) for one period. The pattern of wages in level two resembles the pattern in the one level model. That is, high ability workers earn a revealing wage and wages are relatively low for these workers. In the forth period they experience wage increase and are paid the marginal product. If ability is revealed is \( \theta_2 \). The third period revealing wage and its derivation are similar to the wage in the one level case:

\[
 w_3 = \frac{1 - \delta}{2} \theta_2
\]

### 5 Wage Dynamics

In this section I discuss empirical implications of the model. In particular, I show that many regularities in internal labor markets can be generated by a bargaining model. I focus on findings from Baker, Gibbs, and Holmstrom[1994a,b]. BGH analyze personnel data from a single firm over a twenty year time period. The information they have contains an employee i.d, job title, salary, and performance rating for each worker. All the workers are managerial employees.

I explain the following findings:

**Wage increase within job levels:**

**Corollary 1:** Workers who earn a revealing wage experience wage increases.
That is, after they earn the non-revealing wage they earn their marginal productivity. This wage increase is not associated with promotion or new information revealed to worker and current employer. Instead, this wage growth is caused by learning of outside firms about the worker’s productivity.

Corollary 2: wage increase upon promotion

BGH find that there is a substantial wage increase with promotion, but that the promotion premium explains only a modest portion of the average wage differences across job levels.

My model predicts that workers who earn revealing wages in the second period receive a wage increase in the third period because outside firms learn their productivity by observing the second period wage. In my model, the average wages increase for workers who receive revealing wages because outside firms learn about their productivity with a lag. The wage is \( \frac{E(\theta_2)}{2} \) if \( \frac{E(\theta_2)}{2} > 2w_2^{M} \). If the worker revealed ability in the first job the outside offer is \( \theta_1 \), if the worker earned nonrevealing wages in the first job the outside offer is \( \theta \). If the worker reveals productivity in level two the revealing wage \( (1-\delta)\theta \) in case of two periods in level two. The increase within level (average) is larger than increase in wage upon promotion for some distributions.

Variation in wage growth within job levels:

Among workers who earn raises and are similar with respect to publicly observable characteristics, more productive workers earn larger raises.

Similarly to symmetric information models, my model predicts that wages will vary due to differences in abilities (revealing wages). In contrast to symmetric information models, workers with the same productivity can receive revealing or non-revealing wages (due to differences in age and outside options) which generate differences in wages and wage growth.\(^{22}\)

\(^{22}\)See Medoff and Abraham[1980] for related evidence.
**Negative correlation of wage levels and wage innovations:**

BGH found that controlling for performance rating, workers with the same tenure in the same job level in higher salary quartiles earn lower proportional salary increases.

BGH suggested that this may be a result of a policy of the firm to reduce wage dispersion within job levels.

*Corollary 3:*

A) Workers with productivity $\theta$ below the threshold level, earn second period wage of $w_2 = \theta$, and do not receive raises.

B) Workers who earn revealing wages earn second period wages of $w_2(\theta)$, and in the third period receive a raise of $\theta - w_2$. In the case of worker with ability $\theta > 2\overline{\theta}$, the raise is $\left(\frac{1+\delta}{2}\right)\theta$

In my model, wage growth within levels depends on the worker’s productivity (increasing in the worker’s MRP), and the worker’s outside option $\overline{\theta}$.

Allowing for variation outside options, workers with high outside options (large $\overline{\theta}$), are more likely to receive nonrevealing wages (that is earn the outside option) and experience no wage increase, compared to equally productive workers with lower outside option. If the difference between the worker’s actual productivity and outside option is small, then the worker can receive a nonrevealing wage. In this case, the worker’s wage level can be high but his wage doesn’t increase.

My model predicts that given two equally productive workers, the worker with lower wages will earn larger raises. Consider equally productive workers (same $\theta$). In the second period, those with the highest pay are those who earn the nonrevealing wage: $\overline{\theta}$. They experience no increase in wage. BGH and Gibbons and Waldman interpret this effect as evidence of the existence of internal labor markets within the firm, and suggest that the firm has a policy to restrict raises in order to compress wage dispersion within levels.
Another finding is that real wage reductions are not rare. In my model, the entry level wage is equal to the expected discounted profits. Clearly, depending on the productivity realization, some workers will experience wage reduction at the second period.

Prediction power of publicly observable characteristics such as education declines over time. In my model entry level wages depend on publicly observable characteristics. Over time more worker’s have wages depend on actual productivity and therefore, wages reflect heterogeneity in ability which is unobserved by the econometrician. However, for workers who earn nonrevealing wages, ability isn’t reveal and wages can be predicted by publicly observable characteristics such as education, and past experience.

Wage dispersion of cohort of workers increases over time.

when workers are first enter the firm they vary by publicly observable. Over time, more talented workers negotiate wages. At first these workers earn relatively low wages as their wages compensate employers for loss of future informational rent. Put differently, workers invest in changing perceptions on market about their skills. After ability of some workers is revealed, some wages of high ability workers reflect full productivity and less talented workers do not experience wage growth. Learning about productivity in other tasks and promotions increase wage variance further.

6 Efficiency

I show next that the equilibrium is efficient. In particular, I show that informed employers do not waste resources in order to ”hide” talented workers from outside firm.

Waldman(1984) and Bernhardt(1995) analyze models of asymmetric learning
in which wages are determined by competitive bidding. In their models, when outside firms cannot directly observe employed workers' abilities and skills, they use signals such as education, training, past experience, and job assignment to infer an employed worker's qualifications. Waldman(1984) observed that when employers promote workers, outside firms infer that the worker is of high quality. Since the market is competitive, firms try to raid workers who were assigned to high level jobs and make a wage offer equal to the expected marginal revenue product (MRP), conditional on the job assignment. Hence, employers promote workers to high level jobs only if the increase in productivity exceeds the increase in the wage that is required to retain a worker in a high type job. As a result, some workers that have a comparative advantage in high type jobs are not promoted, and the equilibrium is inefficient.

Consider the same framework with the exception that each firm has two types of jobs. There are two reasons that the equilibrium in my model is efficient. The first reason is that, in contrast to the paper mentioned above, employer can match outside offers. If the employer can match outside offers then the equilibrium is efficient. A necessary condition for efficiency is that outside firms offer workers who are assigned to high level jobs the lower bound on their productivity, although the expected productivity is larger. When employers can match offers the winner’s curse phenomenon arises; that is, outside firms can only raid workers with a wage that exceeds the worker’s value. As a result, outside firms will offer a wage that is equal to the lowest MRP which outside firms infer, based on the worker’s job assignment and other publicly observed characteristics (e.g., job assignment, education, past experience). I discuss efficiency in details, in an asymmetric information model in which workers are price takers, in chapter one.

Unlike the models mentioned above, this model contains bargaining and promotions may change workers bargaining behavior. For example, workers with
relatively low productivity in one job who earns the outside option may negotiate wages in another job in which he may be more productive, if promoted. I will show that assignment of workers to jobs is always efficient and firms do not have incentives to distort signals, because in cases in which workers bargain employers are compensated for future loss of informational rents due to change in bargaining behavior in all cases in which promotion is efficient. Efficiency achieved if all workers are assigned to the job in which they are most productive.

Consider the model with two job levels describe in section 4.2. Assume that while workers work in the first job the employer receives information on whether or not the worker is qualified to work in the second job level. The efficient promotion rule is to promote a worker if the expected productivity in the second job exceeds the productivity in the first level. Since the employer has private information regarding the productivity in the second level the worker can choose again whether or not to renegotiate the wage and reveal productivity through wages once the worker works in the second level. If the worker doesn’t renegotiate the wage in the second job, outside firms offer a contract of \{0, θ\} for periods two and three if they do not know the worker’s productivity in the first level (if the worker earned a nonrevealing wage in the first level) or the actual productivity if it has been revealed. The argument is that this is the lower bound on the worker’s productivity. The worker then chooses whether to reveal ability in the second level or not. The bargaining wages will be determined in a similar way to the bargaining outcome in the one level model. There are four possibilities: the worker doesn’t reveal ability in both jobs, the worker reveals ability in both jobs, the worker reveals ability in the first job but not in the second job, the worker reveals ability in the second job but not in the first job.

It is easy to see that the employer’s profit increases in the first three cases. If the worker earns a nonrevealing wage in both jobs the employer’s profit is the
productivity minus $\theta$, regardless of the job assignment. Clearly, the profit increases with the worker’s productivity. In the case in which the worker reveals ability in both jobs the employer’s profit increases with the worker’s productivity as well. The calculation is similar to comparing two worker with different productivity parameters who earn a revealing wage in the one job model. It is easy to see that the employer’s profit increases with the worker’s productivity. The only case that it is not obvious is the case in which the worker earns a revealing wage in the second job, and a non-revealing wage in the first job.

The intuition is as follows. Consider a worker who is more productive in the second job $E(\theta_2) > \theta$. There are the four cases.

1. Worker bargains and reveal $\theta$ regardless of promotion. $\theta > 2\theta$
2. Worker doesn’t reveal in either job. $E(\theta_2) < 2\theta$
3. Worker reveals if promoted and not if stays in level 1. $E(\theta_2) > 2\theta$ and $\theta < 2\theta$.

Expected profit if promotion occurs is (see equation 27)

$$\theta - w_2 + \delta \frac{E(\theta_2)}{2} = \frac{\theta}{2} + \frac{E\theta_2}{2}$$

Expected profit if not promoted is

$$(1 + \delta)(\theta - \theta)$$

Note that

$$\frac{E(\theta_2)}{2} > \frac{\theta}{2} > \theta - \theta$$

4. Worker reveals if not promoted and not if promoted to level 2. (this case cannot occur on-the-equilibrium path equilibrium)

Expected profit if promotion occurs is (see equation 2)

$$\theta_1 - \theta + \delta E(\theta_2) - \theta$$
Expected profit if not promoted is

$$(1 + \delta)(\theta_1 - \theta)$$

Note that

$$\frac{E(\theta_2)}{2} > \frac{\theta_1}{2} > \theta_1 - \theta$$

It is clear that in the first two case, since the outside option doesn’t change due to promotion itself and since the output produced id greater if the worker assigned to the right job that promotion is efficient.

Consider case 3.

If the worker earns a nonrevealing wage in the first job then the cost of revealing (which is exactly the employer’s profit in the revealing case) is larger than the benefit of revealing (which is exactly the employer’s profit in the nonrevealing case). That is, $\pi_1^{nr} < \pi_1^r$, where $\pi_1^{nr}$ is the employer’s profit in the nonrevealing case in the first job and $\pi_1^r$ is the employer’s profit in the revealing case. In the second job, however, the worker’s productivity is greater than the productivity in the first job and the worker chooses to receive a revealing wage. Therefore, in the second job $\pi_2^r < \pi_2^{nr}$. However, $\pi_2^r$ is increasing in the worker’s productivity, and hence, it is greater in the second job than in the first job $\pi_2^r > \pi_1^r$. Since in the first job $\pi_1^{nr} < \pi_1^r$, the employer’s profit is larger if he assigns the worker to the second job.
7 Conclusions

I develop a model of intra-firm bargaining in which the worker and current employer learn privately about the worker’s productivity. I show that several empirical regularities in internal labor markets can be explained by this bargaining model. The model generates a semi-separating equilibrium. Some workers’ wages depend directly on their ability and some workers wages are the outside option. Workers for whom wages depend directly on productivity earn their MRP in succeeding periods. Therefore, when they bargain over wages, these workers compensate their employers for loss of future informational rents. Since it is costly to reveal ability, workers who are less productive or have high outside options earn wages below their MRP. More productive workers or workers who have low outside options choose to compensate their employers by earning low wages in order to elicit future wage offers from outside firms. Since employers are compensated for the loss of their informational advantage, they do not waste resources and thus assign workers to task efficiently. This model captures a number of empirical regularities found in intra-firm wage data.
Appendix A: Proof of proposition 4

Outside firms strategies and beliefs: Beliefs in periods $s = 2, ..., s - 2$ are for any bargaining round agreement

$$
\hat{\theta} = \begin{cases} 
\bar{\theta} & \text{if } w_2 > \bar{\theta} \\
\theta & \text{if } \hat{\theta} \leq w_2
\end{cases}
$$ (28)

Agreement in the first rounds implies that wages are negative and decreasing in $\theta$, therefore, there is no separating equilibrium. However, if a worker bargains and makes an offer in the last round the wage is monotonic in $\theta$. Given the beliefs specified above, there is no incentive to bargain since wages below the outside option do not elicit high offers in the following periods and the employer rejects all offers above as he will earn zero profit and the wage is too high.

Third Period strategies: At the beginning of the third period outside firms observe employment history in period two. The worker either works for the first period employer or works for a new employer. If the worker works for the first period employer he either worked for the ”market wage” (outside option) or renegotiated the wage. Below are the strategies and beliefs.

If the worker worked for the same employer in periods one and two, and renegotiated the wage, then in equilibrium outside firms offer: $W_3^m = \theta^*$. If the worker stayed with the first period employer and earned the ”market wage” (didn’t negotiate) or changed employers at the end of the first period outside firms offer $W_3^m = 0$.

Second Period strategies: outside firms offer all workers $W_2^m = 0$, and commit to a third period wage of $\hat{\theta}$

Bargaining in period two: I show that given the beliefs and outside offers, no worker renegotiates in the second period.

Last bargaining round in the second period. $w_2^f = 0$ as before.
If the worker makes the last offer the employer rejects all offers for which
Consider the worker’s last offer. Only wage offer above \( \theta \) will increase wages in the following period, in which case the employer makes no profits.

That means that the employer rejects all offers if: 1. For workers with \( \theta > 2\theta_1 \):

\[
\theta - w_2^c \geq (\delta + \delta^2)\frac{\theta}{2}
\]

that is, if \( w_2^c > \theta(1 - \frac{\delta}{2} - \frac{\delta^2}{2}) \). If \( \theta \geq \theta(1 - \frac{\delta}{2} - \frac{\delta^2}{2}) \) that contradicts the assumption in 23.

2. For those with \( \theta < 2\theta_1 \) the employer reject all offers above \( \theta \) unless

\[
\theta - w_2^c \geq (\delta + \delta^2)(\theta - \theta_1)
\]

In this case the highest wage is \( MAX\{0, \theta - \delta(\theta - \theta) - \delta^2(\theta - \theta)\} \). If the wage is positive it is always below \( \theta \): \( \theta - \delta(\theta - \theta) - \delta^2(\theta - \theta) - \theta = (\theta - \theta)(1 - \delta - \delta^2) \).

But \( 1 - \delta - \delta^2 < 0 \). Hence, workers maximize utility if they offer \( \theta \). Given that, the expected wage is \( \frac{\theta}{2} \) and the bargaining wage cannot affect future payoffs. Therefore, there is no bargaining in equilibrium.

Q.E.D

**Appendix B: Proof of proposition 5**

the revealing wage in the first bargaining round makes the employer indifferent between accepting the offer and waiting for the last round. The first round bargaining wage is the value that equates the employer’s expected profit if he accepts the offer (equation 31) and the expected value of waiting to the last bargaining round (equation 32):

\[
\theta_1 - w_2 + \frac{\delta}{2} p(\theta_2 > 2\theta_1) E[\theta_2 | \theta_2 > 2\theta_1] \theta_1 + \delta(1 - p(\theta_2 > 2\theta_1))(E[\theta_2 | \theta_2 < 2\theta_1] - \theta_1)
\]

(31)
\[
\frac{1}{2} [\theta_1 + \frac{\delta}{2} p(\theta_2 > 2\theta)] E[\theta_2 | \theta_2 > 2\theta] + \frac{1}{2} \frac{\delta}{2} p(\theta_2 > 2\theta)] E[\theta_2 | \theta_2 > 2\theta] \quad (32)
\]
References


Figure 1: Period one-learning

hired, \( w_1, w_2, w_3 \) employer and worker learn \( \theta \) pay \( w_1 \)
Figure 2: end of period 1 / beginning of period 2

worker

outside offer counteroffer

stay

figure 4

turnover

figure 3
Figure 3: period 2-turnover

no production

begin period 2 \hspace{2cm} end period 2, pay $w^m_2$
Figure 4: period 2-work for first period employer

renegotiation

round 1

round 2

production

production deadline

no agreement

no production

end of period

no payment

pay market wage

payments
Figure 5: Second period wage as a function of ability $\theta = 1, \delta = 0.6$
Figure 6: The second and third period wages as a function of productivity. The case of $\theta = 1$