Low Variance Adaptive Filter for Cancelling Motion Artifact in Wearable Photoplethysmogram Sensor Signals

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Abstract—The photoplethysmogram (PPG) is an extremely useful wearable sensing medical diagnostic tool. However, the PPG signal becomes highly corrupted and unusable when the sensor wearer is in motion. This paper investigates how confidently Widrow’s Adaptive Noise Cancellation can eliminate motion artifact and recover a motion corrupted PPG signal for a wearer engaged in jogging motions. It has previously been shown that Widrow’s Adaptive Noise Cancellation can recover a motion corrupted PPG signal for certain data sets by using a collocated accelerometer to measure the corrupting motion. However, wearer motion is band limited, and provides little information for estimating motion-to-PPG noise transfer dynamics. This means that, without proper care, recovery results can be unreliable. In the present work, both Finite Impulse Response (FIR) and Laguerre series black box transfer dynamics models are evaluated for how confidently they can be identified. Model confidence is quantified in terms of variance of the transfer dynamics estimate at the motion frequencies. For typical jogging motion, it is found that standard deviation of the FIR model transfer dynamics is 30% of the mean value at the motion input frequency. The standard deviation of the Laguerre model transfer dynamics is only 1%. Time domain data shows how a Laguerre model outperforms a FIR model in accordance with the computed model variance.

I. INTRODUCTION

Wearable medical sensors are expected to revolutionize home health monitoring and sports medicine [1]. The photoplethysmogram (PPG) sensor, in particular, contains rich information about heart pulsation and blood oxygen saturation as well as breath rate [2], and [3] has developed a PPG based sensor for estimating arterial blood pressure. However, the physiologic information found in a PPG signal may be so significantly corrupted by non-physiologic artifacts that the measured signal can no longer be used for any medical assessment.

In the case of motion induced artifact, the motion may be measured directly using an accelerometer. In an ideal illustrative scenario, the system, or transfer, dynamics relating input acceleration to the measured artifact would be known. The transfer dynamics could be inverted and used with the input acceleration to exactly cancel the artifact, leaving only the physiologic signal of interest.

Unfortunately, in physiologic systems like the PPG sensor, the motion-to-artifact transfer dynamics have not been well described by a physically based model and are an unknown function of time. Time variations arise from bodily changes, such as local tissue changes, or from sensor attachment changes. Time varying dynamics means that the transfer dynamics identification technique must be adaptable in time. Since transfer dynamics change instantaneously in the case of a sensor shift, the identification must be able to rapidly adapt to the new dynamics.

Widrow’s Adaptive Noise Cancellation (ANC) [4] is very appealing for application to cancelling artifact in the PPG because it is capable of removing in-band disturbances and can be rapidly tuned to current sensing conditions if there is sufficient motion excitation. This method requires a motion reference measurement and assumes that the artifact is uncorrelated with the physiologic signal in the PPG measurement. Accelerometers are a natural choice for motion reference and are convenient because they are already used for daily monitoring of the elderly, rehabilitation patients, and Chronic Obstructive Pulmonary Disease patients.

Widrow’s ANC has been utilized with accelerometer motion reference by the authors’ group [5], [6] and others [7] for motion artifact reduction in PPG sensors, as well as ECG sensors [8],[9]. In each of these cases, artifact channel dynamics have not been well described by a physically inspired model, and a black-box finite impulse response (FIR) model was utilized. Focusing on the PPG sensor, [5] found a phase lag between the input motion and the induced artifact. After compensating for the perceived phase lag, [5] found that a filter order of 10~100 FIR parameters at 1kHz could be utilized. Also, [7] found that ~30 parameters at 200Hz were required without compensating for a phase lag. Finally, in [6], the authors experimentally found the motion-to-artifact transfer dynamics to contain a slow dominant pole. The authors introduced a Laguerre series model compression to describe the slow system with many fewer parameters than were required using an FIR basis.

All of these implementations have utilized black box transfer dynamics models that are estimated based on a motion reference input. This is convenient because the algorithm provides the best possible estimate of the transfer dynamics based on current physical conditions and current motion, allowing the algorithm to locally linearly approximate the actual sensor system dynamics, which may be nonlinear. However, the model parameters can be estimated using only the information in the motion input. If the input is not sufficiently complex, then there will be...
The objective of the adaptive filter is to minimize the estimate of the physiologic signal, leading to unreliable PPG signal recovery.

This work evaluates how confidently the transfer dynamics can be identified for a jogger wearing a ring-type PPG sensor [10]. It begins by introducing adaptive noise cancellation for the PPG and discussing why transfer dynamics variance is a useful means of assessing confidence in the estimate, then evaluates the variance for the FIR model used in [5] and for the Laguerre series model used in [6]. Finally, time domain data is shown to illustrate how improved model confidence improves algorithm performance.

II. ADAPTIVE NOISE CANCELLATION

A. Noise Cancellation Framework

The focus of this work is on motion artifact in a ring-type PPG sensor proposed and utilized by [10]. A ring PPG sensor is held in place at the base of a finger by a band. Figure 1a shows a tightly attached ring sensor with an attached accelerometer for motion reference utilized in all experiments in this work. Figure 1b shows a cross sectional view of the PPG physical function. The band of the ring sensor has an embedded photodetector (PD) and a photoemitter (an LED). The ring sensor measures volume fluctuations in the digital artery by emitting a constant intensity light from the LED into the finger and measuring the intensity absorbed at the PD. Figure 2 shows a basic block diagram for active noise cancellation discussed presently. Since the physical phenomena that introduce artifact into the PPG signal are highly complex, the best we can do is pose a black box corruption model. Based on experimentation, bulk motions induce artifact in the PPG signal only when motion occurs along the finger axis. Based on this observation, we model the axial acceleration as adding, through some unknown linear motion-to-artifact transfer dynamics, $H_{\hat{y}}(q)$, to the physiologic signal, $y_h(t)$, yielding the PPG measurement as shown in Fig. 2a. Note that $q$ is the time step operator. The adaptive noise cancellation part of the algorithm, Fig. 2b, measures the motion using a collocated MEMS accelerometer, as shown in Fig. 1a which is used with an estimate of the transfer dynamics, $\hat{H}(q)$, to estimate and subtract the motion induced artifact from the measured PPG signal, yielding an estimate of the physiologic signal.

![Figure 1](image1.png)

**Figure 1:** (a) ring PPG sensor with attached accelerometer used in all experiments in this paper; (b) cross-sectional drawing of the PPG ring and finger anatomy.

The objective of the adaptive filter is to minimize the mean squared error between actual motion artifact and its estimate: $E\left((w(t) - \hat{w}(t))^2\right)$. This can be achieved by considering the power in the output

$$E\left[\hat{y}^2\right] = E\left[(y_h + w - \hat{w})^2\right]$$

$$= E\left[y_h^2\right] + 2E[y_h w] - 2E[y_h \hat{w}] + E\left[(w - \hat{w})^2\right]$$

If the inputs, $y_h(t)$ and $a(t)$, are uncorrelated then $E[y_h w] = 0$ and we can select $E[y_h \hat{w}] = 0$ [4] so that

$$E\left[\hat{y}^2\right] = E\left[y_h^2\right] + E\left[(w - \hat{w})^2\right]$$

and since $E\left[y_h^2\right]$ is not involved in the parameter tuning, minimizing $E\left[(w - \hat{w})^2\right]$ is equivalent to minimizing the power in the output, $E\left[\hat{y}^2\right]$. To the authors’ knowledge, save for our work in [6], only Finite Impulse Response models have been previously used in Widrow’s ANC. If an FIR model with $n$ tunable parameters is selected, then the estimated motion artifact can be written as

$$\hat{w}(t) = \varphi^T(t) \hat{h}$$

where $\hat{h} = [h_1 \cdots h_n]^T$ and $\varphi(t) = [a(t)q^{-1} \cdots a(t)q^{-n}]^T$.

Using this model, Eq. (2) can be minimized by

$$E\left[\varphi(t) \varphi^T(t)\right] \hat{h} = E\left[y(t) \varphi(t)\right]$$

The expectations in Eq. (4) can be approximated by averaging over a recent data window of $N$ previous samples:

$$\frac{1}{N} \sum_{i=1}^{N} \varphi(t) \varphi^T(t) \hat{h} = \frac{1}{N} \sum_{i=1}^{N} y(t) \varphi(t)$$

which can be solved for $\hat{h}$. The quantity $R = E\left[\varphi(t) \varphi^T(t)\right]$ is the sample auto-covariance matrix and, as will be discussed in the next chapter, influences confidence in the estimated transfer dynamics. The window length, $N$, defines the adaptation rate. See [11] for more details on the least squares problem.

![Figure 2](image2.png)

**Figure 2:** (a) Additive motion mixing through motion-to-artifact transfer dynamics and (b) adaptive motion artifact reduction framework for motion artifact reduction.
B. Laguerre Series Transfer Dynamics

The authors found, in [6], that the motion-to-artifact transfer dynamics exhibited a slow dominant pole. Thus, a Laguerre series adaptive filter, which is known for its ability to represent dominantly slow dynamics [12], was proposed as a way of reducing the number of parameters required in a FIR ANC implementation.

In discrete time, the general form of the Laguerre basis expansion is given as

\[ L_i(q, \alpha) = \frac{K}{q - \alpha} \left( 1 - \frac{\alpha}{q} \right)^{i-1} \]

where \( 0 < \alpha < 1 \) is the estimated discrete slow pole. Thus, the motion artifact can now be estimated using \( \hat{h} = [h_1 \ldots h_{n_{\text{lag}}}]^T \), and \( \varphi(t) = [a(t) L_i(q, \alpha) \cdots a(t) L_{n_{\text{lag}}}(q, \alpha)]^T \), where \( n_{\text{lag}} \) is the new model order which should be smaller than with the FIR basis.

C. Filter Order and Adaptation Rate

Previous work (see [5], [6], [7]) has found that the transfer dynamics impulse response has a duration ~0.15sec. The number of parameters required to capture the response is dependent on sampling rate, so the smallest possible sampling rate, without aliasing, should be chosen. A human pulse rate is maximally 3 Hz and an arterial pulse has 8-10 harmonics, while a jogger stride is not greater than 2 Hz and acceleration along the axis of the finger shows fewer than 5 harmonics for typical jogging. Thus, to satisfy the Nyquist criterion with some safety factor, a sampling rate of 100Hz is acceptable and used in all experiments in this work.

With the required sampling rate, a 0.15sec impulse response requires \( n = 15 \) FIR parameters. Also, the authors have found that if a Laguerre pole of \( \alpha = 0.94 \) is chosen, as few as \( n_{\text{lag}} = 3 \) Laguerre parameters are sufficient to represent the transfer dynamics.

The data window, \( N \), must be chosen small enough to accommodate time variation in the transfer dynamics. The microcirculation evolves with a time constant on the order of seconds. However, since the physical system is nonlinear, the local linear approximation to the system dynamics may vary almost instantly if the speed of jogging changes. Also, any slippage of the sensor on the skin will invoke a nearly instantaneous change in system dynamics. Thus, the data window must be chosen as small as possible. However, the time average in Eq. (5) must approximate the ensemble mean in Eq. (4), so at least a few periods of motion and heart beat must be captured within the data window. The jogging rate considered here will be on the order of 3Hz, so a data window of \( N = 400 \) should be sufficient.

III. Model Confidence

As many as 15 parameters are required to capture the impulse response of the transfer dynamics. However, a typical jogger motion is very simple, typically containing 95\% of its power in the first 3 harmonics. With so many parameters and so little information, a metric is required that illuminates the affects of model order, \( n \), data window, \( N \), and inputs \( y(t) \) and \( a(t) \) on our confidence in the recovered physiologic signal. More specifically, we will look at confidence in the estimated transfer dynamics \( \hat{H}(q) \).

For linear, unbiased models, a common performance metric is variance in the transfer dynamics

\[ \text{Var}(\hat{H}(e^{j\omega})) = E[(\hat{H}(e^{j\omega}) - E[\hat{H}(e^{j\omega})])^2] \]

where \( q = e^{j\omega} \). At frequencies where the transfer dynamics show little variance, we say we are confident in the estimate, and thus confident in the recovered physiologic signal.

The following development for \( \text{Var}(\hat{H}(e^{j\omega})) \) will be for a FIR model, but can be easily adapted to the Laguerre series model. For an FIR model, \( \hat{H}(e^{j\omega}) \) can be written as

\[ \hat{H}(e^{j\omega}) = W^T(e^{j\omega}) \hat{h} ; W(e^{j\omega}) = [e^{-j\omega} \ldots e^{-i\omega}] \]

from which

\[ \text{Var}(\hat{H}) = W^T(e^{j\omega}) E[(\hat{h} - \hat{h})(\hat{h} - \hat{h})^H] W(e^{j\omega}) \]

where \( E[(\hat{h} - \hat{h})(\hat{h} - \hat{h})^H] \) is the parameter estimation error covariance [11].

It can be shown that the parameter estimation error covariance is approximated by

\[ E[(\hat{h} - \hat{h})(\hat{h} - \hat{h})^H] \equiv R^{-1}E\left[\sum_{j=1}^{N} \varphi(j) \varphi^T(k) [y(j)][y(k)] \right] R^{-1} \]

Since \( y(t) \) is unavailable, \( \hat{y}(t, \hat{h}) \) or, if jogging in place, an uncorrupted PPG signal from the unmoving contralateral hand must be used in place of \( y(t) \).

Even though Eqs. (11) and (10) are correct and used for computed variance results in this work, it is instructive to consider a simplified expression for (10) that better illuminates how each of the parameters and inputs influence model variance.

It is shown in [11] that, if the physiologic signal is white Gaussian and the number of parameters and the data window become large, the transfer dynamics variance can be written as simply

\[ N, n >> 1 : \text{Var}(\hat{H}(e^{j\omega})) = \frac{n}{N} \frac{\Phi_{y_hy_h}(\omega)}{\Phi_{a_a}(\omega)} \]

where \( \Phi_{y_hy_h} \) and \( \Phi_{a_a} \) are the power spectra of the desired physiologic signal and acceleration, respectively. Note that Eq. (12) is valid only if the model is of a “shift” structure [11], which both FIR and Laguerre series models fall into.
For small $n$, Eq. (12) is not an accurate estimate [11]. However, it gives clear insight to how different system parameters affect variance. Variance increases with the number of tunable parameters, $n$, and decreases with the window length, $N$. Also, it is low at frequencies where there is high acceleration content and where there is little content in the physiologic signal.

From the perspective of algorithm design, we are only capable of choosing data window, $N$, and model order, $n$. The physiologic signal is not controllable and the wearer cannot be required to constantly provide complex excitation. It is clear then, that $n$ must be chosen as small as possible, and $N$ must be chosen as large as possible. However, as mentioned in section II.C, $N$ must be kept small and minimum $n$ depends on model structure. The next chapter will show how much variance the FIR and Laguerre series models present.

IV. RESULTS

Consider a typical jogger experiment, with 100Hz sampling rate and $N=400$, where the input power spectrum is shown in the lower subplots of Fig. 3. The transfer dynamics were estimated using a FIR model with $n=15$ and a Laguerre model with $n_{log}=3$. Using Eq. (10) to compute the variance and then plotting the nominal transfer function with its standard deviation, $\sigma$, yields Fig. 3a for the FIR model and Fig. 3b for the Laguerre model.

What is important to note from these results, is the model variance at the frequencies where most of the motion input power lies, because those are the frequencies at which the model must be accurate if the artifact estimate is to have low variance, which would mean a confident estimate of the physiologic signal. We can see from the results that with the FIR model and $n=15$, there is high variance: the standard deviation is 30% of the nominal transfer function magnitude at the frequency of greatest input power. In contrast, the Laguerre model, with $n_{log}=3$ parameters has less than 1% standard deviation at the same frequency.

To demonstrate how the variance is reflected in the ability of the algorithm to recover the desired physiologic signal, Fig. 4 compares the FIR (a) and Laguerre (b) estimates of the desired physiologic signal with a correct unmoving reference PPG.

V. CONCLUSION

The low variance of the Laguerre model relative to the FIR model is true not for just the experiment shown in Figs. 3 and 4, but for all nominal jogging experiments. Because the the FIR model always has high variance, its results are less reliable and it will often produce poor reconstructions as shown in Fig. 4a. Because the Laguerre model has so few parameters to tune and always has much lower variance, we are able to more confidently say that Laguerre reconstructed output is representative of the true physiologic signal.

REFERENCES


