Well temperature testing—an extension of Slider’s method*

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Abstract
A new technique has been developed for determination of the formation thermal conductivity, skin factor and contact thermal resistance for boreholes where the temperature recovery process after drilling operations is not completed. Slider suggested a technique for analysing transient pressure tests when conditions are not constant. We extend Slider’s method for transient temperature well tests. It assumes that the volumetric heat capacity of formations is known, and the instantaneous heater’s wall temperature and time data are available for a cylindrical probe with a constant heat flow rate placed in a borehole. A semi-analytical equation is used to approximate the dimensionless wall temperature of the heater. A simulated example is presented to demonstrate the data processing procedure.

Keywords: well testing, Slider’s method, thermal conductivity, contact resistance

Introduction
The similar analytical form of Darcy’s and Fourier’s laws, which describe the transient flow of an incompressible fluid in a porous medium and the heat conduction in solids, respectively, leads to a correspondence between the thermal and hydraulic parameters. It is also important to show that the product of porosity and total compressibility can be obtained from multi-well tests (interference testing). Fortunately, the analogous thermal parameter—the product of density and specific heat—varies within narrow limits and can be determined from cuttings (Kappelmeyer and Haenel 1974, Somerton 1992). Below we will also show that an additional parameter, the well’s contact thermal resistance, can be expressed through the skin factor and the formation thermal conductivity. Thus, the same analytical solutions of the diffusion equation (at corresponding initial and boundary conditions) can be utilized for the determination of the hydraulic and thermal parameters.

However, this approach can be used only for large dimensionless times (when the solution of the diffusion equation can be expressed by an exponential integral). Generally, the mathematical model of pressure well tests is based on the assumption that the borehole is an infinitely long linear source with a constant flow rate in an infinite, homogeneous reservoir. In this case, the well-known solution of the diffusion equation is an exponential integral. In thermal measurements the borehole (or the cylindrical heater) cannot be considered as an infinitely long linear source of heat. This is due to low thermal diffusivity of rocks in comparison with the hydraulic diffusivity and corresponding low values of dimensionless time. Kutasov (2003) showed that the convergence of solutions of the diffusion equation for cylindrical and linear sources occurs at dimensionless time of about 1000. A semi-theoretical equation was suggested by Kutasov (1987) to approximate the dimensionless heat flow rate from a cylindrical source with constant bore-face temperature. This equation was used to process pressure and temperature data from well tests and to develop a technique for determining the formation permeability, skin factor, thermal conductivity and thermal resistance of the borehole (Kutasov 1998, Kutasov and Kagan 2003a, 2003b, Kutasov and Eppelbaum 2005).

Recently Eppelbaum and Kutasov (2006) proposed a new technique, based on semi-analytical equation for the dimensionless temperature at the wall of an infinite long cylindrical source with a constant heat flow rate (Kutasov 2003). This method allows us to determine the formation thermal conductivity, the contact thermal resistance and

formation temperature. The utilization of this method requires that (prior to the test) the thermal recovery is practically completed. However, the drilling process greatly alters the temperature of rocks immediately surrounding the well. The temperature change is affected by the duration of drilling fluid circulation, the temperature difference between the reservoir and the drilling fluid, the well radius, the thermal diffusivity of the reservoir and the drilling technique. Thus, the determination of formation temperature (with a specified absolute accuracy) at any depth requires a lengthy period of shut-in time.

The objective of this paper is to suggest a similar technique for in situ evaluation of thermal conductivity and thermal resistance (expressed as skin factor) for boreholes where thermal recovery is not completed (short shut-in periods). Here we must note that Sass et al. (1981) suggested in situ determination of heat flow in boreholes by the use of cable power supplying. We will consider a long cylindrical electrical heater (cylindrical heater is an analogue of radius of borehole with skin in pressure well testing) with a large length/diameter ratio. Mufti (1971) demonstrated that for practical purposes a cylindrical heater whose length is five times or more its diameter could be treated as an infinite cylindrical source of heat. Thus, the temperature field in and around the borehole is a function of time, radial distance, rock thermal diffusivity and borehole thermal resistance. The 'effective radius' concept must be introduced to evaluate the effect of the contact thermal resistance on the heat flowing into the formation (for applying the proposed procedure is necessary to select a comparatively homogeneous interval of geological strata). For validating the proposed method we use an 'exact' solution (numerical) and generate synthetic data for a test. Then the semi-analytical equation was used to process the results and to compare the obtained and assumed input parameters. The final step in the validation of the proposed technique is to conduct a number of field tests and to compare the results of these tests with those obtained by other methods. A simulated example is presented to demonstrate the data processing procedure for determination of formation thermal conductivity, skin factor and contact thermal resistance.

**Slider’s method**

Slider’s method is a technique for analysing transient pressure (p) tests (e.g., Earlougher 1977, pp 27–9). Figure 1 schematically illustrates the shut-in pressure declining (solid line) before a drawdown tests start at time $t_1$. The dashed line represents the expected pressure behaviour with time. If a constant flow rate starts at time $t_1$, pressure decreases as shown by the solid line. The analysis of such drawdown behaviour requires: (1) the correct extrapolation of the shut-in pressure, (2) the estimation of the difference between observed pressure and the extrapolated pressure ($\Delta p_\text{calc}$), (3) plotting $\Delta p_\text{calc}$ versus $\log \Delta t$.

For extending this method to the temperature analysis it is necessary to estimate the rate $U$ of temperature change at $t = t_1$ and to determine the values of the temperature decline (‘correct extrapolation’ as in figure 1). Corrections finally should be introduced to the observed (while testing) well temperatures.

**Effective radius of the heater**

We will use effective radius concept to take into account the effect of probe casing and the contact thermal resistance on the heat flow rate. This approach is widely used in transient pressure and flow well testing (Earlougher 1977) to evaluate the effect of formation permeability changing around the borehole on the pressure at the borehole’s wall. First, we introduce skin factor $(s)$—a parameter which allows quantitatively to determine the effect of the well thermal resistance on the heat flow rate. In our case

$$s = \left(\frac{\lambda}{\lambda_{\text{ef}}} - 1\right) \ln \left(\frac{r_w}{r_h}\right), \quad r_w \neq r_h,$$

(1)

where $r_w$ is the well radius, $r_h$ is the radius of the heater, $\lambda$ is the rock thermal conductivity and $\lambda_{\text{ef}}$ is the effective thermal conductivity of the $r_w - r_h$ annulus (figure 2).
It is more convenient to express the skin factor through the apparent (effective) heater radius (Earlougher 1977)

\[ r_{ha} = r_h \exp(-s), \]  
(2)

where \( r_{ha} \) is the effective radius of the heater.

For an open (uncased) borehole the \( r_w - r_h \) annulus is filled with the drilling fluid (or air), plastic like mud cake coating the borehole. This results from the solids in the drilling fluid adhering to the wall of the hole. The \( r_w - r_h \) ring in a cased borehole is composed of drilling fluid, steel and cement. The skin factor can be estimated from a temperature drawdown test. The thermal contact resistance \( R = 1/\lambda e_f \) is easily calculated from equation (1):

\[ \lambda_{ef} = \frac{\lambda \ln \frac{z_2}{z_1}}{s + \ln \frac{z_2}{z_1}} \quad R = \frac{s + \ln \frac{z_2}{z_1}}{\lambda \ln \frac{z_2}{r_h}}. \]  
(3)

We should mention that the skin factor was introduced in petroleum engineering by Hawkins (1956) to account for the pressure drop in the zone (around the wellbore) of altered permeability. The skin factor is a composite parameter and it takes into account permeability of different layers by introducing the effective (equivalent) permeability. The reliability of estimation of the skin factor depends only on the quality of the field pressure and flow data (test design, type of instrumentation, data processing technique, an adequate physical and mathematical model). Similarly, for a temperature test, the skin factor takes into account the thermophysical properties of the materials (e.g., drilling fluids, steel, cement, etc) through the effective thermal conductivity of the wellbore heater.

**Rate of temperature decline**

We selected long-term temperature data from two wells to demonstrate the application of Slider’s method for analysing results of temperature test in deep wells (tables 1–3 and figure 3).

**Figure 3.** Temperature profiles in the Put River N-1 well. The shut-in times for curves 1, 2, 3, 4, 5 and 6 are 5, 34, 48, 66, 117 and 1071 days, respectively (Clow and Lachenbruch 1998).

For an open (uncased) borehole the shut-in temperature recovery for short shut-in times could be approximated by the Horner plot

\[ T_s = T^* + B \ln \frac{t + t_s - t_t}{t_s}, \]  
(4)

where \( T^* \) is the temperature trend extrapolated to infinite shut-in time, \( t_s \) is the shut-in temperature, \( t_t \) is the shut-in time, \( B \) is the coefficient and \( t_s \) is the thermal ‘disturbance’ period for a given depth.

It is a reasonable assumption that the value of \( t_s \) is a linear function of the depth (\( z \)):

\[ t_s = t_{tot} \left( 1 - \frac{z}{H_t} \right), \]  
(5)

where \( z \) is the depth, \( t_{tot} \) is the total drilling time and \( H_t \) is the total well depth.

The values of \( T^* \) were computed by using values of shut-in temperature at \( t_t = t_{t1} \) and \( t_t = t_{t2} \) (table 2). To show that the Horner plot method cannot be applied in this case for determining formation temperatures we used the three point method (Kutasov and Eppelbaum 2003) to estimate the value of \( T_f \) (undisturbed temperature of formation)—please compare columns 5 and 7 in table 2. From equation (4) the rate of the temperature change is

\[ U = \frac{dT_f}{dt} = B \left( \frac{1}{t_{t1} + t_s} - \frac{1}{t_{t2} + t_s} \right). \]  
(6)

The calculated values of \( U \) are presented in table 2. Let now assume that after 128 days of shut-in time we carry out a temperature drawdown test, the absolute accuracy of field temperature measurements is 0.01 °C and the duration of the test is 10 h. Then, for the depth 121.6 m (the highest absolute value of \( U \)) the maximum correction to the observed temperature is \( \Delta T_s = 0.000628 \times 10 = 0.0063 \) (°C). It is evident that in this case we can consider that the initial formation temperature is 5.34 °C (table 2).

**Well Put River N-1**

We used values of transient temperature for two depths 670.56 and 701.04 m for shut-in time of 1071 days (table 3) to estimate the position of the 0 °C isotherm. The linear extrapolation gives that the base of permafrost is located at about 629 m. Let us now assume that at time \( t = t_s \) the phase transition (water–ice) in formation at a selected depth is completed,

**Table 1.** Well data and references.

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Well 192</th>
<th>PBF</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Kugpik 0–13</td>
<td>Put River N-1</td>
</tr>
<tr>
<td>Location</td>
<td>Lat: 68 52.8 N Long: 135 18.2 W</td>
<td>Lat: 70 19 07 N Long: 148 54 35 W</td>
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<tr>
<td>Hole depth (m)</td>
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<td>763</td>
</tr>
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<td>Drilling time (days)</td>
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<td>44</td>
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<tr>
<td>Number of logs</td>
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<td>9</td>
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<tr>
<td>Shut-in period (days)</td>
<td>35–2835</td>
<td>5–1071</td>
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</tbody>
</table>

**Well 192, Kugpik D-13**

For the 121.6–519.1 m section of this well (table 2) we assumed that the shut-in the temperature recovery for short shut-in times could be approximated by the Horner plot

\[ T_s = T^* + B \ln \frac{t + t_s - t_t}{t_s}, \]  
(4)

where \( T^* \) is the temperature trend extrapolated to infinite shut-in time, \( t_s \) is the shut-in temperature, \( t_t \) is the shut-in time, \( B \) is the coefficient and \( t_s \) is the thermal ‘disturbance’ period for a given depth.

It is a reasonable assumption that the value of \( t_s \) is a linear function of the depth (\( z \)):

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(5)

where \( z \) is the depth, \( t_{tot} \) is the total drilling time and \( H_t \) is the total well depth.

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\[ U = \frac{dT_f}{dt} = B \left( \frac{1}{t_{t1} + t_s} - \frac{1}{t_{t2} + t_s} \right). \]  
(6)

The calculated values of \( U \) are presented in table 2. Let now assume that after 128 days of shut-in time we carry out a temperature drawdown test, the absolute accuracy of field temperature measurements is 0.01 °C and the duration of the test is 10 h. Then, for the depth 121.6 m (the highest absolute value of \( U \)) the maximum correction to the observed temperature is \( \Delta T_s = 0.000628 \times 10 = 0.0063 \) (°C). It is evident that in this case we can consider that the initial formation temperature is 5.34 °C (table 2).
occurs in some temperature interval below 0°C. It is well known (Tsytovich 1975) that the freezing of the water recovery in sections of the well below the permafrost base. It is assumed that at \( t > t_s \) the cooling process is similar to that of temperature recovery in sections of the well below the permafrost base. It is well known (Tsytovich 1975) that the freezing of the water occurs in some temperature interval below 0°C (figure 4), i.e. the thermally disturbed formation has frozen. In this case at \( t > t_s \) the cooling process is similar to that of temperature recovery in sections of the well below the permafrost base. It is well known (Tsytovich 1975) that the freezing of the water occurs in some temperature interval below 0°C (figure 4).

Table 2. Calculated rates of temperature decline (\( U \)), observed temperatures, values of \( T^* \) and formation temperature (\( T_f \)), well 192, Kugpik D-13.

<table>
<thead>
<tr>
<th>( z, m )</th>
<th>( T_1 (°C) )</th>
<th>( T_2 (°C) )</th>
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<th>( T_f (°C) )</th>
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<td>14.43</td>
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Table 3. Observed shut-in temperatures, well Put River N-1.

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<tr>
<th>( z, m )</th>
<th>( t_{s1} ) (days)</th>
<th>( t_{s2} ) (days)</th>
<th>( T_{s1} (°C) )</th>
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Table 4. Calculated rates of temperature decline (\( U \)), Put River N-1 well.

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<tr>
<th>( z, m )</th>
<th>( t_{s1} ) (days)</th>
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In practice, however, \( t_s \) cannot be determined. This can be realized only by conducting long-term repetitive temperature observations in deep wells. For the permafrost interval we assumed that at \( t_s > t_s \) and for short shut-in times the shut-in temperatures can be approximated by a simple equation

\[
T_s = T_{ep} + C \ln \frac{t_s}{t_{s1}} \tag{7}
\]

where \( C \) is a coefficient.

It was assumed that \( T_{ep} = 0 \) °C s and the values of \( C \) and \( t_s \) were estimated by using values of shut-in temperature at \( t_s = t_{s1} \) and \( t s = t_{s2} \). Then the rate of the temperature decline is

\[
U = \frac{dT_s}{dt_s} = C \frac{1}{t_{s2}} \tag{8}
\]

For depths \( z > 640 \) m equation (6) was used to estimate the values of \( U \). The rates of the temperature decline are presented in table 4.

A well as a cylindrical source

A long cylindrical electrical heater (large length/diameter ratio) with a constant heat flux is often used in laboratory for determining the thermal conductivity of samples of rocks.
In this case the transient temperature $T_w$ is a function of time, thermal conductivity, and volumetric heat capacity of formations. Analytical expression for $T_w$ is available only for large values of the dimensionless time $t_D$ expressed by

$$ t_D = \frac{\pi \lambda t}{ rw^2 c_p \rho} $$

where $\lambda$ is the thermal diffusivity of formations, $t$ is the time, $rw$ is the well radius, $c_p$ is the specific heat at constant pressure, and $\rho$ is the density.

To determine the temperature $T_w$ it is necessary to solve the diffusion equation under the following boundary and initial conditions:

$$ T(t = 0, r) = T_f; \quad r_w \leq r < \infty, \quad t > 0 \quad (10) $$

$$ \left( \frac{\partial T}{\partial r} \right) r = a = 0, \quad t > 0, \quad (11) $$

$$ T(t, r \rightarrow \infty) \rightarrow T_f; \quad t > 0, \quad (12) $$

where $T_f$ is the initial (formation) temperature, $r$ is the radial distance and $q$ is the heat flow rate per unit of length.

It is well known that in this case the diffusion equation has a solution in complex integral form (Van Everdingen and Hurst 1949, Carslaw and Jaeger 1959). Chatas (Lee 1982, pp 106–107) tabulated this integral for large values of the dimensionless time $t_D$.

A semi-theoretical equation of the wall dimensionless temperature ($T_D$) for a cylindrical source with a constant heat flow rate is (Kutasov 2003)

$$ T_D(t_D) = \ln \left[ 1 + \left( \frac{c - \sqrt{t_D}}{a + \sqrt{t_D}} \right) \sqrt{t_D} \right] $$

$$ a = 2.7010505, \quad c = 1.4986055 $$

$$ T_D(t_D) = \frac{2 \pi \lambda (T_w - T_f)}{q} $$

Kutasov (2003) compared the values of $T_D$ calculated from equation (13) and results of a numerical solution (‘Exact’ solution) by Chatas (Lee 1982, pp 106–107). The agreement between values of $T_D$ calculated by these two methods is very good. For this reason the principle of superposition can be used without any limitations.

Temperature drawdown well test

Let us assume that the initial formation temperature (prior to the test) is known. At least two measurements of wall temperature (at time $t = t_1$ and $t = t_2$) are needed to calculate the formation thermal conductivity, skin factor and thermal contact resistance. As is customary in petroleum engineering the effect of the skin factor can be expressed by introducing the dimensionless time $t_Da$ based on the apparent well radius $r_{ha}$.

Let

$$ m = \frac{q}{2 \pi \lambda}, \quad t_Da = \frac{\lambda t}{C \rho c_p r_{ha}^2} $$

$$ F(t_Da) = \ln \left[ 1 + \left( \frac{c - \sqrt{t_Da}}{a + \sqrt{t_Da}} \right) \sqrt{t_Da} \right]. $$

Then

$$ T_h = T_f + m F(t_Da) $$

and

$$ \gamma = \frac{T_{h1} - T_f}{T_{h2} - T_f} = \frac{F(t_Da1)}{F(t_Da2)} = \psi(t_Da1) $$

$$ t_Da1 = \frac{\lambda t_1}{C \rho c_p r_{ha}^2}, \quad t_Da2 = \frac{t_2}{t_1}. $$

If we assume that the absolute accuracy of the ratio $\gamma$ is $\varepsilon$, then solving the following equation we calculate the value of $t_Da1$:

$$ \gamma - \psi(t_Da1) = \varepsilon $$

and from equation

$$ T_h = T_f + m F(t_Da1) $$

we can calculate the value of $m$. Then the formation thermal conductivity can be determined

$$ \lambda = \frac{q}{2 \pi m} $$

and, finally the skin factor and the thermal contact resistance per unit of length can be estimated from equation (3). Let us assume that we plan to conduct two drawdown temperature
test in the well Put River N-1 at the depths of 91.44 and 670.56 m after 34 and 22 days of shut-in, respectively. In the first case the initial temperature is approximately 4.101 °C (table 3) but the observed temperatures \( T_{obs} \) should be corrected
\[
T_{corr} = T_{obs} - U.
\] (23)
For this test, to avoid thawing of frozen formation cooling of formations is desirable (using a cylindrical heat sink). For the second test the initial temperature is 5.948 °C (table 3) and the observed temperatures should be corrected (equation (23)) and an electrical heater (probe) can be used (see the simulated example below).

**Simulated example**

A metallic electrical heater is placed into well Put River N-1 (uncased) at the depth 670.56 m (figure 3). The test is conducted after 22 days of shut-in and the rate of temperature decline is \( U = -2.470 \times 10^{-3} °C \ h^{-1} \) (table 4). The heater generates a heat flow into the formation of 80.0 W m\(^{-2}\) and operated for 10 h. The transient heater’s wall temperature was recorded (table 5). The well radius \( r_w \) = 0.10 m, the radius of the probe \( r_h \) = 0.08 m. The \( r_w - r_h \) annulus consists of mud cake and drilling fluid. We assume that the effective thermal conductivity of the \( r_w - r_h \) annulus is \( \lambda_{eff} = 0.9741 \ W \ m^{-1} °C \) and thermal contact resistance is \( R = 1/\lambda_{ref} = 1.027 \ m°C \ W^{-1} \). The initial formation temperature \( T_f(0) \) is 5.948 °C. The formation is sandstone with \( \rho = 2300 \ kg \ m^{-3} \), \( \lambda = 2.000 \ W \ m^{-1} °C \) and \( c = 783 \ J \ kg^{-1} °C \). Using the table of Chataas (Lee 1982, pp 106–107) of \( T_D = f(T_D) \) we generated data for this simulated example. The input data were chosen to avoid interpolation of \( T_D \) values. The results after equations (3) and (16)–(22) are presented in table 5. The example shows that the basic equation (13) can be used to compute the thermal conductivity of formations and contact thermal resistance. Indeed, the assumed and calculated values of \( \lambda \) and \( R \) and are in a good agreement.

**Conclusions**

A new method of determination in situ thermal conductivity and thermal resistance in a borehole, where the temperature recovery is not completed (after drilling operations), is proposed. This method is based on a semi-analytical equation, which approximates the dimensionless wall temperature of infinitely long cylindrical probe with a constant heat flow rate placed into a borehole. It is shown that Slider’s method (used in petroleum engineering) can be utilized to analyse results of temperature well tests. Two temperature logs recorded at short shut-in times are required to use the suggested method.

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