Exploiting hysteresis in position control: the magnetic shape memory push-push actuator

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Abstract

This paper discusses in details a control design approach for hysteretic systems with dynamics. The framework is quite general to embrace several unconventional actuators. In particular, we here propose a method which allows the exploitation of the particular hysteresis characteristic of a specific class of actuators based on magnetic shape memory alloys, namely the so-called push-push actuators. The exploitation of the hysteresis leads to the reduction of the energy losses for some positioning tasks, and can lead to a more compact design of the actuator and of its driving electronics. Moreover, heating phenomena (which are very influent for magnetic shape memory alloys) are reduced. The validity of the proposed method is verified by several experimental results performed on a prototype of push-push actuator.

1 Introduction

All smart materials, such as piezoelectric ceramics, magnetostrictive rods, thermal or magnetic shape memory alloys, exhibit hysteretic phenomena between the input and the output quantities. Those phenomena are often coupled with dynamic phenomena when the material is used inside an actuator system, e.g., a positioning actuator or a vibration damper [1]. Thus, control of smart-material based actuators usually requires the actuator to be characterized in terms of a hysteretic and a dynamic part. The designed controller should take both phenomena into considerations to ensure stability of the closed loop and to guarantee a required performance of the actuator. Most control methods rely on the adoption of an inverse hysteresis model [2], [3], whereas very few work concerns the design of linear controllers for such nonlinear hysteretic systems [4], [5].

In this paper, we discuss the control design for a magnetic shape memory push-push actuator (PPA) [6]. A theoretical framework for the design of linear controllers is introduced, based on [5]. As further contribution, we propose a modification of those controllers which is able to exploit advantageously the hysteresis characteristic of the PPA.

The paper is organized as follows. Section 2 describes the PPA in details; section 3 summarizes the controller design approach; section 4 illustrates several experimental results which confirm the validity of the proposed control approach.

2 The magnetic shape memory push-push actuator

MSM alloys are able to strain when excited by a magnetic field which acts perpendicular to the strain direction. To start motion, the field should overcome a threshold value which is related to the twinning stress of the alloy, a kind of internal friction hindering the strain. The twinning stress is the cause of the hysteresis in MSM alloys [7]. The PPA, makes use of two MSM elements, A and B, arranged antagonistically (as illustrated by the schema of Figure 1). The working principle is the following: when A elongates in the positive x direction, B contracts due to the loading exerted by A and vice-versa.

![Figure 1 Schema of the PPA](image)

The PPA has the following advantages: (I) the contraction and the elongation of the MSM elements are both driven by magnetic fields, and thus they are electrically controllable; (II) the actuator can exploit the twinning stress since it allows holding a position value with a minimum amount of electrical energy (assuming that the external load does not overcome the twinning stress value). Other design concepts which have similar advantages have been proposed, but they still need further developments to improve key features such as the maximum strain [8]. Hereafter, we refer to (II) as the energy-efficiency feature of the PPA for three reasons: (i) using a smaller value of the current while performing a given positioning task avoids that a certain amount of electrical energy is transformed into heat (Joule losses); (ii) the driving electronics of the PPA and the PPA itself can be designed for a small DC current, i.e., they can be much more compact, leading to an increased volumetric efficiency of the PPA; (iii) reducing the heating avoids problems related to temperature effects in MSM alloys.
Figure 2 shows a picture of the experimental set-up of PPA considered in this paper. The two MSM elements are not visible because they are located in the middle of the two magnetic circuits which provide the elongation fields (field A and field B) through the elements; the push-rod is connected to the extremities of both elements and provides the interface of the device, and carries a reflector part that enables a displacement measurement by means of a triangulation laser. The displacement of the push rod is related to the strain of the active elements. The two MSM elements used in the PPA are NiMnGa single crystalline elements of 3×5×15 mm³ each, which individually produce a magnetic field induced strain of about 6% at zero external load. The excitation fields are controlled by the unipolar currents in the two coils. AC and DC currents in the range [0, 3.5] A are used. The PPA is equipped with a laser position sensor for displacement measures (resolution of 2 µm) and two current sources that provide the control currents. All the signals are acquired and elaborated by a dSPACE board working at 5000 Hz. The control signal $u$ generated by the algorithm running on the dSPACE is bipolar and within $\pm 3.5$ A. To drive the PPA, $u$ is separated into two positive currents: $i_a(t) = u(t)$ if $u(t) \geq 0$, $i_b(t) = 0$ if $u(t) < 0$; and $i_a(t) = u(t)$ if $u(t) \leq 0$, $i_b(t) = 0$ if $u(t) \geq 0$. The signals $i_a$ and $i_b$ become the currents for element A and for element B.

The displacement-current characteristic is highly hysteretic, as shown by Figure 3. The figure also emphasizes another important information. It can be seen that, once a positive displacement value is reached, the current can be reduced to a certain minimum value, denoted by $i_{a\text{min}} = 0.8$ A, without relevant changes of the displacement. The same occurs for negative displacement values, where the current can be brought to $i_{b\text{min}} = -0.8$ A. The thresholds $i_{a\text{min}}$ and $i_{b\text{min}}$ could differ for the several minor hysteresis loops. The values of $i_{a\text{min}}$ and $i_{b\text{min}}$ should be, theoretically, very low for a PPA. The fact that they are, in general, not zero and not symmetrical, is mainly due to the differences between the magneto-mechanical properties of the two MSM elements (for instance, differences in the stress-strain curves) and on some mounting issues. Figure 4 reports the behavior of the displacement in response to current pulses, and emphasizes the potential of the PPA in terms of energy savings. Each pulse brings the push-rod to a certain position value. But, as soon as the excitation is removed, the displacement slightly changes. These motions are called in [6] reversible motions since they only depend on the presence or absence of excitation. From the control viewpoint, the behavior shown in Figure 4 means that the PPA can hold a required displacement value with a small current, necessary only to avoid the small deviations represented by the reversible motions (coherently with Figure 3).

3 Controller design

In section 3.1, a procedure for designing PID controllers for hysteretic systems is considered. After, a modification of such PID controllers is proposed in order to exploit the energy-efficiency feature of the PPA.

3.1 Design framework

The PPA can be mathematically represented as the series of a hysteresis operator $\Gamma$, which describes the hysteretic, rate-independent phenomenon, and a linear dynamics.
\[\Sigma, \text{ which describes the frequency, rate-dependent behavior of the actuator and load (Figure 5).}\]

**Figure 5** The control loop

As in most smart-material-based positioning systems, we assume that the linear dynamics \(\Sigma\) can be approximated by a second-order system, described by the following differential equations,

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_1 x_1 + a_2 x_2 + b \Gamma [u], \\
y &= x_1
\end{align*}
\]

where \(a_1, a_2\) and \(b\) are coefficients depending on stiffness, damping and mass of the MSM elements and of the load together, and \(x_1\) and \(x_2\) are the displacement and velocity, respectively. We assume that \(\Sigma\) is a standard PID controller, such that the control action \(u\) is given by:

\[u = -K_p(y - r) - K_i \int_0^t (y - r) d\tau - K_d \dot{y},\]  

(2)

The error \(r - y\) is referred to as the tracking error. Let us define the errors \(e_0 = \int_0^t (y - r) d\tau, e_1 = y - r\) and \(e_2 = \dot{y} - \dot{r}\), and the new variables \(\xi_0 = e_0, \xi_1 = e_1\) and \(\xi_2 = \dot{e}_2\). Let us consider the case in which the reference \(r\) is constant. The evolution of such variables is given by

\[
\begin{align*}
\dot{\xi}_0 &= \xi_1 \\
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= a_1 \xi_1 + a_2 \xi_2 + b \frac{d}{dt} \Gamma [u].
\end{align*}
\]

(3)

The time-derivative of the hysteresis appears in (3). For a strictly monotone, globally Lipschitz continuous hysteresis operator \(\Gamma\), the following relationship holds [5]:

\[\frac{d}{dt} \Gamma [u](t) = \sigma(t) \frac{d}{dt} u(t),\]

(4)

with \(\sigma(t) \in [\sigma_-, \sigma_+].\) The variable \(\sigma\) can be interpreted as the local slope of the hysteresis curve and \(\sigma_+\) and \(\sigma_-\) are its minimum and maximum values, respectively. Using (4), (2) and (3) one obtains

\[\dot{\xi} = (\mathbf{A} + \sigma(t) \mathbf{B} k) \xi,\]

(5)

where \(\xi = [\xi_0, \xi_1, \xi_2]^{\top}\) and

\[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & a_1 & a_2
\end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix}
0 \\
0 \\
b
\end{bmatrix}, \quad k = -K_p.
\]

(6)

The system (5) can be seen as a linear parameter varying system (LPV) affected by the time-varying variable \(\sigma\). For a LPV such as (5), the convergence rate of the state \(\xi\) to the origin is usually described by the decay rate \(\alpha\), defined as the biggest positive value such that \(\lim_{t \to \infty} \|\xi(t)\|_2 = 0\). However, the decay rate is then related only to the worse-case scenario of convergence of (5). A better estimation of the actual decay rate of (5) can be provided by considering the constant matrix

\[F = y(\mathbf{A} + \sigma \mathbf{B} k^t) + (1 - \gamma)(\mathbf{A} + \sigma \mathbf{B} k^t),\]

(7)

where \(\gamma \in [0,1]\) is a design parameter that weights the vertices \(\mathbf{A} + \sigma \mathbf{B} k^t\) and \(\mathbf{A} + \sigma \mathbf{B} k^t\) in the actual dynamics matrix \(F\). Let us define the actual decay rate \(\hat{\alpha}\) as the biggest positive value such that

\[F^T P + PF + 2 \hat{\alpha} P \leq 0\]

holds. In this way, it is possible to introduce the actual decay rate in the design procedure in a very straightforward way. Let us assume that a value \(\gamma\) has been chosen for design, and that a certain actual decay \(\hat{\alpha}\) is desired from the application requirements (e.g., in terms of speed of the closed-loop). Then, the controller gain \(k\) which provides an actual decay rate at least equal to \(\hat{\alpha}\) can be found by solving the following LMI problem in \(Q\) and \(Y\):

\[
\begin{align*}
Q \mathbf{A}^T + \mathbf{A} Q + Y \sigma \mathbf{B}^T + \sigma \mathbf{B} Y^T &< 0 \\
Q \mathbf{A}^T + \mathbf{A} Q + Y \sigma \mathbf{B}^T + \sigma \mathbf{B} Y^T &< 0 \\
Q \mathbf{A}^T + \mathbf{A} Q + Y \sigma \mathbf{B}^T + \sigma \mathbf{B} Y^T + 2 \hat{\alpha} Q &\leq 0
\end{align*}
\]

(9)

where \(Q = P^{-1}\) is a positive definite matrix, \(Y = Qk\) and \(\sigma = \gamma \sigma_- + (1 - \gamma) \sigma_+\). The solution of (9) provides \(Q\) and \(Y\), and the control gain vector can be recovered as \(k = Q^{-1} Y\). Note that the actual decay rate is an approximated performance measure which aims to describe an average behavior of the LPV (5).

### 3.2 Improving the energy-efficiency of controllers

In section 2 it has been mentioned that the PPA can exploit the hysteresis of the MSM elements to hold a position with a minimum amount of current, which depends on the properties of the MSM elements and on mounting tolerances of the overall actuator. In particular, it has been shown that in the PPA presented here, the bounds on the minimum currents required to hold a position are approximately \(i_{a}^{\min} = -0.8\) A and \(i_{a}^{\min} = 0.8\) A. In this section we propose a very simple modification of the controllers, in order to exploit the advantageous hysteresis shape.

To introduce the idea, let us refer to Figure 3. Let us assume that a positive displacement value is required and reached at time \(r^0\) with a current \(u(r^0) > i_{a}^{\max}\). Let us define \(z(t) = \int_0^t (y - r) d\tau\). It holds that

\[u(r^0) = -K_z z(r^0),\]

(10)
if the proportional and derivative actions are considered negligible with respect to the integral one about the steady state. From the analysis of the PPA hysteresis, we know that the current can be decreased to \( i_d^m \) without a significant change of the displacement. In other words, if \( u(t^*) = i_d^m \) the actuator will hold the same position. This means that the integral action \( z \) can be reset as follows:

\[
u(t^*) = i_d^m \Rightarrow z(t^*) = -\frac{i_d^m}{K_i}.
\]

Eq. (11) offers a way to reduce the control current while keeping the same tracking performance. For negative displacements, the integral action can be reset as follows:

\[
z(t^*) = \frac{i_d^m}{K_i}.
\]

Based on the reset laws (11) and (12), a control strategy with integrator reset (energy-efficient strategy) can be constructed in this way:

- a) wait a defined amount of waiting time (\( WT \)), i.e., the amount of time that the PPA needs to reach a desired position
- b) if the tracking error is within a specified tolerance, and if the current required to hold the position value is bigger than \( i_d^m \) or smaller than \( i_a^m \), reset the integral with (11) or (12), respectively.

Note that the auxiliary conditions in b) avoid resetting to \( i_d^m \) and \( i_a^m \) if the control current is already at a value either below \( i_d^m \) or beyond \( i_a^m \). The strategy in a) and b) has the goal of resetting the integrator only when really advantageous. Furthermore, note that the described energy-efficient strategy requires two design choices: the waiting time \( WT \), i.e., the time that the strategy must wait before activation, and the bounds \( i_d^m, i_a^m \).

4 Experimental results

4.1 Identification of the actuator

The design of PID controllers discussed in section 3 requires the identification of the parameters \( \sigma_1, \sigma_2 \) and the linear dynamics. The minimum and maximum slopes of the hysteresis curve can be estimated also graphically using the plot of Figure 3. The estimated values are \( \sigma_1 = 0.6 \mu m/A \) and \( \sigma_2 = 180 \mu m/A \). The linear dynamics is obtained based on the step response of the actuator. The obtained estimations are \( a_i = -b = -e^x \) and \( a_r = -1.9e^x \). The currents needed to hold a position are identified as \( i_d^m = -i_a^m = 0.8 A \).

4.2 Controllers

Five controllers (C1, C2, C3, C4, C5) are designed with respect to three desired actual decay rates, \( \ddot{z}^* \) \( = 1 \text{ s}^{-1} \) and \( \ddot{z}_1^* = 10 \text{ s}^{-1} \) and \( \ddot{z}_0^* = 50 \text{ s}^{-1} \), which should offer actual time constants of the closed-loop of \( \ddot{z}_i^* = 1 \text{ s} \), \( \ddot{z}_0^* = 0.1 \text{ s} \) and \( \ddot{z}_0^* = 0.02 \text{ s} \), respectively (the corresponding settling-times can be estimated as \( 4\ddot{z}_i^* = 4 \text{ s} \), \( 4\ddot{z}_1^* = 0.4 \text{ s} \) and \( 4\ddot{z}_0^* = 0.08 \text{ s} \). The design of the controller gain vector \( k \) is done by solving the LMI problem (9), which also involves the design choice of the parameter \( \gamma \). Such a choice is better discussed based on experiments. Moreover, most energy-efficient versions of the controllers share the design choice \( i_d^m = i_a^m = 0.8 A \) unless differently stated.

The controller C1 is designed with respect to \( \dot{z}_1^* \) and with the choice \( \gamma_{C1} = 0.5 \). The solution of the LMI (9) gives the following gains for C1, \( k_{C1} = [0.016, 2e^{-x}, 1e^{-x}]^T \). Figure 6 shows the tracking of a sequence of steps offered by C1. It can be noted that C1 is not able to track the steps having a duration of 2 s, because it has been designed for a settling time of about 4 s. The final step is however long enough to allow an accurate tracking. Figure 6 also compares C1 with its energy-efficient version C1e, entailing the reset strategy a)-b) with the choice \( WT_{C1e} = 4 \text{ s} \). It can be noted that C1e offers the same tracking on the initial steps. The last step emphasizes the important difference between C1 and C1e: the latter is able to reduce the current to a smaller value without affecting the tracking performance. To quantify the efficiency offered by C1e, let us introduce the index

\[
ISC = \int_0^T u(t) \, dt.
\]

where \( u \) is the current needed for the positioning task, and \( T \) is the time interval (in Figure 6 and in the following figures we have \( T = 28 \text{ s} \)). The ISC index is obviously related to the Joule losses in the electrical circuit, and to the heating and dimensioning of the driving electronics and the PPA. We can quantify the efficiency of C1e, denoted by \( Eff_{C1e} \), in this way:

\[
Eff_{C1e} = \frac{ISC_{C1e} - ISC_{C1e}^{\text{eff}}}{ISC_{C1}} \cdot 100 \text{\%}.
\]

Note that the value of \( Eff_{C1e} \) is strongly dependent on the particular displacement reference signal. In particular, the difference between C1 and C1e is determined by the duration of the last step, as can be appreciated in Figure 6. What we want to emphasize in this experiment is that the energy-efficient strategy reduces the amount of ISC that the controller uses for a given positioning task, and that in no case the efficient controller performs worse than the original controller.

The controller C2 is designed for \( \dot{z}_0^* \) and \( \gamma_{C2} = 0.5 \). The
solution of (9) gives the following gains vector $k_{c_3}=[0.21, 6e^{-3}, 6.5e^{-7}]$. The tracking performance is shown by Figure 7. Since C2 offers a settling time of about 0.4 s, all the steps are precisely tracked. The comparison with the energy efficient C2e ($WT_{C2e}=0.6$ s) reveals that C2e is able to reduce the current while keeping the same tracking precision. The efficiency of C2e is $Eff_{C2e}=45\%$ (calculated as in (14) with obvious changes of variables).

The controller C3 is designed for $\alpha_0^*$ (as C1) but with the choice $\gamma_{c_3}=0.1$. Such a choice guarantees that the desired decay rate $\alpha_0^*$ is satisfied by a larger portion of the trajectories of the LPV (5) but, on the other hand, it usually requires higher control gains. In fact, solution of (9) gives now $k_{c_3}=[0.11, 2.3e^{-4}, 1e^{-7}]$. The tracking performance is shown by Figure 8. The influence of $\gamma_{c_3}=0.1$ can be clearly appreciated, since almost all steps are tracked in a time interval which is much faster than 4 s. The parameter $\gamma$ determines how strong the requirement on the decay rate has to be on the average behavior of the system. When $\gamma \to 0$, then the requirement becomes very strict. Furthermore, the energy efficient C3e ($WT_{C3e}=2$ s, since C3 is faster than C1 thanks to $\gamma_{c_3}$) reduces the needed current and offers an efficiency $Eff_{C3e}=42\%$.

The controller C4 is designed for $\alpha_{00}^*$ (as C2) but with $\gamma_{c_4}=0.1$. We obtain $k_{c_4}=[0.99, 2e^{-3}, 1e^{-14}]$ (higher than $k_{c_2}$). The tracking is in Figure 9: all the steps are tracked and the energy efficient C4e ($WT_{C4e}=0.2$ s) gives $Eff_{C4e}=33\%$.

C5 is designed for $\alpha_{00}^*$, in order to offer a settling time of about 0.08 s, and with $\gamma_{c_3}=0.5$ to make the decay requirement not too strict and keep the controller gains small. The vector $k_{c_5}=[3.47, 7.3e^{-4}, 4e^{-6}]$ is obtained. The comparison with C5e in Figure 10, having $WT_{C5e}=0.1$ s reveals that the energy-efficient strategy improves very little the control effort. In fact, it has to be observed that the original controller is also able to reduce the current thanks to the presence of an overshoot in the tracking. Nevertheless, we have that $Eff_{C5e}=5\%$.

So far, all the energy-efficient controllers that we have presented had $i_{d}^m=-i_{\mu}^m=0.8$ A, which were chosen with respect to the experimental hysteresis in Figure 3. However, also $i_{d}^m$ and $i_{\mu}^m$ can be tuned/refined in order to improve the overall energy-efficiency. To demonstrate this idea, we present another energy-efficient modification of C5, called C5-2e, which has the same gains $k_{c_5}$ but different choices $i_{d}^m=-i_{\mu}^m=0.4$ A and $WT_{C5-2e}=0.02$ s with respect to C5e. The results are in Figure 11, and $Eff_{C5-2e}=62\%$. Thus, a better tuning of the energy-efficient strategy can greatly improve the efficiency of the controller.

5 Conclusion

This paper characterizes in detail a magnetic shape memory push-push actuator. Then, it presents a mathematical framework for the design of standard PID controllers with respect to a desired performance. A modification of the controllers that exploits the particular hysteresis of the push-pull is introduced. Such a modification improves several aspects: the Joule losses are reduced and consequently the actuator and the driving electronics can be designed in a much more compact way. The control design and the energy-efficient modification are validated by means of experiments, where it is shown that the proposed control approach can provide a reduction of energy losses of up to 60%.

6 Acknowledgment

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7 References


**Figure 6** Tracking performance of C1 and comparison with its energy-efficient version C1e

**Figure 7** Tracking performance of C2 and comparison with its energy-efficient version C2e

**Figure 8** Tracking performance of C3 and comparison with its energy-efficient version C3e

**Figure 9** Tracking performance of C4 and comparison with its energy-efficient version C4e

**Figure 10** Tracking performance of C5 and comparison with its energy-efficient version C5e

**Figure 11** Tracking performance of C5 and comparison with another energy-efficient version C5-2e