# Delay-Optimal Dynamic Mode Selection and Resource Allocation in Device-to-Device Communications—Part II: Practical Algorithm 

Lei Lei, Member, IEEE, Yiru Kuang, Nan Cheng, Student Member, IEEE, Xuemin (Sherman) Shen, Fellow, IEEE, Zhangdui Zhong, and Chuang Lin, Senior Member, IEEE


#### Abstract

In Part I of this paper ("Delay-Optimal Dynamic Mode Selection and Resource Allocation in Device-to-Device Communications-Part I: Optimal Policy"), we investigated dynamic mode selection and subchannel allocation for an orthogonal frequency-division multiple access (OFDMA) cellular network with device-to-device (D2D) communications to minimize the average end-to-end delay performance under the dropping probability constraint. We formulated the optimal resource control problem into an infinite-horizon average-reward constrained Markov decision process (CMDP), and the optimal control policy derived in Part I using the brute-force offline value iteration algorithm based on the reduced-state equivalent Bellman equation still faces the well-known curse-of-dimensionality problem, which limits its practical application in realistic scenarios with multiple D2D users and cellular users. In Part II of this paper, we use linear value approximation techniques to further reduce the state space. Moreover, an online stochastic learning algorithm with two timescales is applied to update the value functions and Lagrangian multipliers (LMs) based on the real-time observations of channel state information (CSI) and queue state information (QSI). The combined online stochastic learning solution converges almost surely to a global optimal solution under some realistic conditions. Simulation results show that the proposed approach achieves nearly the same performance as the offline value iteration algorithm and outperforms the conventional CSI-only scheme and through-put-optimal scheme in a stability sense.


Index Terms-Device-to-device (D2D) communication, mode selection, online stochastic learning, resource allocation.

## I. Introduction

IN Part I of this paper [1], we introduced the problem of optimal dynamic mode selection and resource allocation to minimize the average end-to-end delay under the constraint of packet-dropping probability for network-assisted device-to-

[^0]device (D2D) communications [2]-[4] with bursty traffic. We considered an orthogonal frequency-division multiple access (OFDMA) system with one base station (BS), multiple D2D user equipment (UE) pairs, and cellular UEs (CUEs) with uplink or downlink transmission. Compared with the resource control problem in traditional cellular networks, there are a number of unique issues to address to obtain resource optimization in D2D communications, such as 1) route selection between the one-hop route of D2D link (direct over-the-air link) in D2D mode and the two-hop route of cellular links in Cellular Mode, 2) resource allocation for D2D links and cellular links with resource reuse, and 3) joint uplink and downlink resource optimization for the end-to-end performance of the two-hop route when a pair of D2D UEs works in the cellular mode. To characterize the aforementioned issues, we first developed a queuing model whose underlying system state dynamics evolves as a controlled Markov chain, where the system state includes the joint queue state of the queues at the UEs for uplink transmission and the queues at the BS for downlink transmission, as well as the joint channel state of all the D2D links, cellular uplinks, and cellular downlinks. Specifically, we introduced two important concepts to characterize the unique features of D2D communications. The first concept is the radio resource group (RRG), which defines a group of links that may reuse radio resources. Therefore, the channel state of a link is a tuple including its adaptive modulation and coding (AMC) states in all the RRGs that this link belongs to. The second concept is the link constraint set of a queue to characterize the set of servers for the queue in different routes. Based on the queuing model, the delay-optimal resource control over frequency-selective fading channels with the AMC scheme in the physical layer is formulated as an infinite-horizon averagereward constrained Markov decision process (CMDP) [6], [7]. To formulate the CMDP model, the transition kernel of the controlled Markov chain was derived, which takes into account the coupling relationship between the uplink and downlink resource allocation. Moreover, closed-form expressions for end-to-end performance metrics, such as average delay and dropping probability, were given as functions of steady-state probabilities of the controlled Markov chain, based on which the cost function of CMDP model was given. We utilized the Lagrangian approach, to turn the CMDP problem into an unconstrained Markov decision process (MDP) problem, and established the strong duality result over the space of randomized policy. Moreover, we further proved the existence of
an optimal policy, which is either a deterministic policy or a mix of two deterministic policies, equivalent to choosing independently one of two deterministic policies at each epoch by the toss of a (biased) coin. To solve the unconstrained MDP problem, we derived an equivalent Bellman equation with reduced state space. We showed by simulations that the optimal policy derived by the brute-force offline value iteration algorithm based on the equivalent Bellman equation achieves significant gain compared to various baselines such as the conventional channel state information (CSI)-only control and the throughput-optimal control (MaxWeight algorithm).

It is worth noting that the complexity of the brute-force offline value iteration algorithm based on the reduced-state equivalent Bellman equation still grows exponentially with the number of users in the network, limiting its application in practical scenarios. In fact, it is well known that there is no simple solution for the infinite-horizon average-reward MDP problem that delay-aware resource control belongs to, because the bruteforce value iterations or policy iterations could not lead to any viable solution due to the curse of dimensionality [8]-[11]. Moreover, our problem for network-assisted D2D communications is further complicated due to the unique issues listed earlier. For example, the channel state transition probabilities, which are used to derive the conditional expectations of cost function and queue state transition probabilities in the equivalent Bellman equation, are very difficult to obtain when more than two links are allowed to reuse the same time-frequency resource.

In Part II of this paper, we address the curse-ofdimensionality problem in solving the CMDP formulated in Part I so that a practical algorithm with acceptable computational complexity and signaling overhead can be derived. To reduce the complexity, we obtain a delay-optimal solution using approximate dynamic programming and online stochastic learning. Specifically, we approximate the value function in the equivalent Bellman equation by a sum of per-queue value functions. The per-queue value functions are estimated and learned using an online stochastic learning algorithm based on the realtime observations of the CSI and queue state information (QSI), eliminating the need of deriving the channel state transition probabilities. Moreover, the Lagrangian multipliers (LMs) for the constraint optimization problem are updated simultaneously with the value functions over different timescales. The optimal dynamic-mode-selection and resource-allocation actions can be determined by an algorithm that has a similar structure with the MaxWeight algorithm in the Lyapunov stability approach, with the weight determined by the per-queue value functions instead of the queue lengths. We prove the almost-sure convergence of the proposed algorithm. We also show by simulations that our proposed scheme achieves significant gain compared to various baselines such as the conventional CSI-only control and the throughput-optimal control (MaxWeight algorithm). Together with Part I, this pair of works provide a general framework for the dynamic constrained optimization of mode selection and resource allocation in D2D communications under the bursty traffic model, where the general form of the optimal policy and a practical algorithm with simple structure and near-optimal performance are given.

This paper is organized as follows: We recall the general network model for network-assisted D2D communications and the MDP problem formulation for dynamic mode selection and resource allocation in Section II. In Section III, we derive a lowcomplexity learning algorithm, which updates the per-queue value functions based on real-time observations of CSI and QSI, as well as a resource-allocation algorithm with similar structure as the MaxWeight algorithm. In Section IV, we discuss the performance simulations. Finally, we summarize the main results in Section V.

## II. Network Model and Previous Results

## A. Network Model

Consider a frequency-division duplex (FDD) OFDMA cellular network with D2D communications capability, where there are $D$ D2D UE pairs, $C_{\mathrm{u}}$ cellular UEs (CUEs) with uplink communications, and $C_{\mathrm{d}}$ CUEs with downlink communications in a single cell. A D2D UE pair consists of a source D2D UE (src. DUE) and a destination D2D UE (dest. DUE) within direct over-the-air communications range with each other, which is formed through the various neighbor/peer/service discovery mechanisms proposed in literature. The whole uplink or downlink spectrum is divided into $N_{\mathrm{F}}$ equal-size subchannels. A subchannel in the uplink (respectively downlink) spectrum shall be referred to as uplink (respectively downlink) subchannel in the remainder of this paper. Moreover, we assume that D2D links share uplink resources with cellular uplinks. Time is slotted, and each time slot has an equal length.

The aforementioned OFDMA cellular network with D2D communications can be formulated as a general network model with a set $\mathcal{N}$ of nodes and a set $\mathcal{L}$ of transmission links. Define $\mathcal{N}:=\{0,1, \ldots, N\}$, where node 0 represents the BS, and nodes $1, \ldots, 2 D$ represent the DUEs, nodes $2 D+1, \ldots, 2 D+$ $C_{\mathrm{u}}$ represent the uplink CUEs, and nodes $2 D+C_{\mathrm{u}}+$ $1, \ldots, N=2 D+C_{\mathrm{u}}+C_{\mathrm{d}}$ represent the downlink CUEs. We use $i$ or $j$ to denote the index of a node within $\mathcal{N}$ (i.e., $i, j \in \mathcal{N}$ ) in the remainder of this paper. Each transmission link represents a communication channel for direct transmission from a given node $i$ to another node $j$, and is labeled by $(i, j)$ (where $i, j \in$ $\mathcal{N}$ ). All data that enter the network are associated with a particular connection, which defines the source and destination of the data. Let $\mathcal{C}_{\mathrm{D}}=\{1, \ldots, D\}, \mathcal{C}_{\mathrm{Cu}}=\left\{D+1, \ldots, D+C_{\mathrm{u}}\right\}$, and $\mathcal{C}_{\mathrm{Cd}}=\left\{D+C_{\mathrm{u}}+1, \ldots, D+C_{\mathrm{u}}+C_{\mathrm{d}}\right\}$ represent the set of D2D connections, cellular uplink connections, and cellular downlink connections, respectively. Define $\mathcal{C}:=\{1, \ldots, C\}=$ $\mathcal{C}_{\mathrm{D}} \cup \mathcal{C}_{\mathrm{Cu}} \bigcup \mathcal{C}_{\mathrm{Cd}}$ (with $C=D+C_{\mathrm{u}}+C_{\mathrm{d}}$ ) as the set of all connections in the network. We use $c$ to denote the index of a connection within $\mathcal{C}$ (i.e., $c \in \mathcal{C}$ ) in the remainder of this paper.

The data from connection $c$ are transmitted hop by hop along the route(s) of the connection to its destination node. Each node $i$ along the route(s) of connection $c$ maintains a queue $q_{i}^{(c)}$ for storing its data, except for the destination node, since the data are considered to exit the network once they reach the destination. Define $\Theta$ as the set of queues in the system. We assume that each queue has a finite capacity of $N_{\mathrm{Q}}<\infty$ (in number of bits or packets). The set of queues can be divided
into two nonoverlapping disjoint sets, i.e., uplink queues $\Theta_{u}$ and downlink queues $\Theta_{d}$, according to whether a queue is maintained by a UE or the BS. Define $\Theta_{\mathrm{Cu}}$ and $\Theta_{\mathrm{Cd}}$ as the set of queues for cellular uplink connections and cellular downlink connections, respectively, whereas define $\Theta_{D-u}$ and $\Theta_{D-d}$ as the set of uplink and downlink queues for D 2 D connections, respectively. Define the per-queue link constraint set of a queue $q_{i}^{(c)}$ as $\mathcal{L}_{i}^{(c)}$, where the data from a queue $q_{i}^{(c)}$ can only be transmitted via links in $\mathcal{L}_{i}^{(c)}$. Note that there is only one link in $\mathcal{L}_{i}^{(c)}$ for all the queues, except the uplink queues for D2D connections, i.e., $q_{i}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{u}}$, which can be served by either D2D link or cellular uplink.

## B. Physical Layer Model

We define an RRG $\mathcal{B}_{u}$ as the subset of links $(i, j) \in \mathcal{L}$ that can be scheduled for transmission simultaneously on any subchannel in a time slot. Let $\mathcal{U}$ represent the set of RRG indexes and $\mathcal{U}_{\mathrm{u}}$ and $\mathcal{U}_{\mathrm{d}}$ represent the subsets of RRG indexes for uplink and downlink subchannels, respectively. We use $u$ to denote the index of an RRG within $\mathcal{U}$ (i.e., $u \in \mathcal{U}$ ) in the remainder of this paper. For any link $(i, j) \in \mathcal{L}$, define $\mathcal{U}_{i j}:=$ $\left\{u \mid(i, j) \in \mathcal{B}_{u}, u \in \mathcal{U}\right\}$ as the index set of RRGs that contain link $(i, j)$.

Assume that the instantaneous channel gain comprising the path loss, shadowing, and fast-fading effects of the wireless channel from the transmitter of node $i \in \mathcal{N}$ to the receiver of node $j \in \mathcal{N}$ on any subchannel $m$ remains constant within a time slot and independent identically distributed (i.i.d.) between time slots, the value of which at time slot $t$ is denoted by $G_{i j, t}^{(m)}$. Let $p_{i j, t}^{(m)}$ be the transmission power of link $(i, j) \in \mathcal{L}$ on subchannel $m$ at time slot $t$. Assume that every scheduled link on a downlink subchannel (respectively uplink subchannel) always transmits at constant power $P_{\max }^{\mathrm{BS}} / N_{\mathrm{F}}$, (respectively $P_{\max }^{\mathrm{UE}} / N_{\mathrm{F}}$ ). The signal-to-interference-plus-noise ratio (SINR) value of a link $(i, j)$ on a subchannel $m$, when RRG $\mathcal{B}_{u}$ is scheduled on it at time slot $t$, can be derived as $\forall(i, j) \in \mathcal{B}_{u}$

$$
\begin{equation*}
\operatorname{SINR}_{i j, t}^{(m, u)}=\frac{p_{i j, t}^{(m)} G_{i j, t}^{(m)}}{N_{i j, t}^{(m)}+\sum_{\left(i^{\prime}, j^{\prime}\right) \in \mathcal{B}_{u} \backslash\{(i, j)\}} p_{i^{\prime} j^{\prime}, t}^{(m)} G_{i^{\prime} j, t}^{(m)}} \tag{1}
\end{equation*}
$$

where $N_{i j, t}^{(m)}$ denotes the noise power on subchannel $m$ at time slot $t$.

We assume that AMC is used, where the SINR values are divided into $K$ nonoverlapping consecutive regions [12]. For any $k \in\{1, \ldots, K\}$, if the $\operatorname{SINR}$ value $\operatorname{SINR}_{i j, t}^{(m, u)}$ of link $(i, j)$ falls within the $k$ th region $\left[\Gamma_{k-1}, \Gamma_{k}\right)$, the instantaneous data rate of link $(i, j)$ on subchannel $m$, when RRG $\mathcal{B}_{u}, \forall u \in \mathcal{U}_{i j}$ is scheduled, is a fixed value $R_{k}$, according to the selected modulation and coding scheme in this state. Obviously, $\Gamma_{0}=0$, and $\Gamma_{K}=\infty$. In addition, we have $R_{1}=0$, i.e., no packet is transmitted in channel state 1 to avoid the high transmission error probability. Define the CSI of link $(i, j)$ to be $\mathbf{H}_{i j, t}:=$ $\left\{H_{i j, t}^{(m, u)} \mid\left(m \in\left\{1, \ldots, N_{\mathrm{F}}\right\}, u \in \mathcal{U}_{i j}\right\}\right.$, where $H_{i j, t}^{(m, u)}$ denotes the channel state of link $(i, j)$ on subchannel $m$ when RRG $\mathcal{B}_{u}$ is scheduled. Specifically, $H_{i j, t}^{(m, u)}=k$ if $\operatorname{SINR}_{i j, t}^{(m, u)}$ is between $\left[\Gamma_{k-1}, \Gamma_{k}\right)$.

## C. Bursty Source Model, Queuing Dynamics, and Queuing Model

Let $A_{c, t}$ denote the amount of new connection $c$ data ${ }^{1}$ that exogenously arrives to its source node during time slot $t$. We assume that the data arrival process is i.i.d. over time slots following general distribution $f_{A}(n)$ with average arrival rate $\mathbf{E}\left[A_{c, t}\right]=\lambda_{c}$. Let $A_{i, t}^{(c)}$ denote the amount of data arrived to node $i$ for connection $c$ during time slot $t$. When $q_{i}^{(c)} \in$ $\Theta_{\mathrm{u}} \bigcup \Theta_{\mathrm{Cd}}$, node $i$ is the source node of connection $c$, and $A_{i, t}^{(c)}=A_{c, t}$. Otherwise, when $q_{0}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}}$, it is the secondhop queue of connection $c$, and $A_{0, t}^{(c)}$ depends on the data departure process of the corresponding uplink transmission on cellular uplink $((2 c-1), 0)$.

Let $Q_{i, t}^{(c)}$ denote the length of $q_{i}^{(c)}$ at the beginning of time slot $t$. Let $r_{i, t}^{(c)}$ be the instantaneous data rate of queue $q_{i}^{(c)}$ during time slot $t,{ }^{2}$ which is equal to the sum of the instantaneous data rate $r_{i j, t}$ of the scheduled link $(i, j) \in \mathcal{L}_{i}^{(c)}$ at time slot $t$. If $Q_{i, t}^{(c)}$ is less than $r_{i, t}^{(c)}$ during time slot $t$, padding bits shall be transmitted along with the data. However, the amount of useful data transmitted from $q_{i}^{(c)}$ during time slot or the throughput of $q_{i}^{(c)}$ is defined as

$$
\begin{equation*}
T_{i, t}^{(c)}=\min \left[Q_{i}^{(c)}, r_{i}^{(c)}\right] \tag{2}
\end{equation*}
$$

Moreover, the amount of useful data transmitted via link $(i, j)$ during time slot $t$ or the throughput of link $(i, j)$ is defined for any link within the link constraint set of queue $q_{i}^{(c)} \in$ $\Theta_{\mathrm{d}} \bigcup \Theta_{\mathrm{Cu}}$ as

$$
\begin{equation*}
T_{i j, t}=\min \left[Q_{i}^{(c)}, r_{i j, t}\right] \quad \forall(i, j) \in \mathcal{L}_{i}^{(c)} \tag{3}
\end{equation*}
$$

For any queue $q_{i}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{u}}$, we assume that the data in the queue are first assigned to link $(2 c-1,0)$, and then, the remaining data left in the queue (if any) shall be assigned to link $(2 c-1,2 c)$. According to the aforementioned data assignment rule, we have that $T_{(2 c-1) 0, t}$ obeys (3), while $\forall q_{(2 c-1)}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{u}}$

$$
\begin{equation*}
T_{(2 c-1)(2 c), t}=\min \left[Q_{i}^{(c)}-T_{(2 c-1) 0, t}, r_{(2 c-1)(2 c), t}\right] \tag{4}
\end{equation*}
$$

Arriving data are placed in the queue throughout the time slot $t$ and can only be transmitted during the next time slot $t+1$. If the queue length reached the buffer capacity $N_{\mathrm{Q}}$, the subsequent arriving data will be dropped. According to the aforementioned assumption, the queuing process evolves as follows:

$$
\begin{equation*}
Q_{i, t+1}^{(c)}=\min \left[N_{\mathrm{Q}}, \max \left[0, Q_{i, t}^{(c)}-r_{i, t}^{(c)}\right]+A_{i, t}^{(c)}\right] . \tag{5}
\end{equation*}
$$

The queuing model is illustrated in Fig. 2 of Part I of this work [1].

[^1]
## D. System State, Control Policy, and State Transition Probabilities

The global system state of the aforementioned queuing model at time slot $t$ can be characterized by the aggregation of the system CSI and system QSI, i.e., $\mathbf{S}_{t}=\left(\mathbf{H}_{t}, \mathbf{Q}_{t}\right)$. The system QSI is defined as $\mathbf{Q}_{t}:=\left\{Q_{i, t}^{(c)} \mid q_{i}^{(c)} \in \Theta\right\}$, which is a vector consisting of the lengths of all the queues at the beginning of time slot $t$. The system CSI is defined as $\mathbf{H}_{t}:=$ $\left\{\mathbf{H}_{i j, t} \mid(i, j) \in \mathcal{L}\right\}$, where $\mathbf{H}_{i j, t}$ denotes the channel state of link $(i, j)$ in time slot $t$, as defined in Section II-B.

In each time slot, an uplink (respectively downlink) subchannel can be allocated to at most one uplink (respectively, downlink) RRG for uplink (respectively, downlink) transmission. Let $m \in\left\{1, \ldots, N_{\mathrm{F}}\right\}$ denote the index of a subchannel, which can be either the $m$ th uplink subchannel or the $m$ th downlink subchannel. Let $x_{u, t}^{(m)} \in\{0,1\}$ denote the subchannel allocation for RRG $\mathcal{B}_{u}, u \in \mathcal{U}$ at time slot $t$, where $x_{u, t}^{(m)}=1$ if subchannel $m$ is allocated to RRG $\mathcal{B}_{u}$, and $x_{u, t}^{(m)}=0$, otherwise. We have the constraint that $\sum_{u \in \mathcal{U}_{u}} x_{u, t}^{(m)} \leq 1$ and $\sum_{u \in \mathcal{U}_{\mathrm{d}}} x_{u, t}^{(m)} \leq 1$ for any $m \in\left\{1, \ldots, N_{\mathrm{F}}\right\}$. We assume that an RRG is scheduled for transmission only when all its links have nonempty queues.

A queue $q_{i}^{(c)}$ is scheduled in time slot $t$ when at least one RRG $\mathcal{B}_{u}$ containing a link $(i, j)$ in its link constraint set $\mathcal{L}_{i}^{(c)}$ is scheduled on any subchannel. Note that, except for the uplink queues of D2D connections, the per-queue link constraint set of every queue contains only one link. When mode selection of a D2D connection $c$ is performed dynamically at each time slot, the problem becomes deciding whether to schedule the D2D link $(2 c-1,2 c)$ or the cellular uplink $(2 c-1,0)$ on a subchannel to serve the queue $q_{(2 c-1)}^{(c)}$, which is essentially a subchannel-allocation decision. Therefore, the delay-optimal dynamic-mode-selection and subchannel-allocation problem can be solved by only considering the design of the delayoptimal subchannel-allocation algorithm.

In each time slot, the resource controller observes the system state $\mathbf{S}_{t}$ and chooses a subchannel-allocation action from the set of allowable actions in the action space $\mathcal{A}_{\mathrm{x}}$. A subchannelallocation action $\mathbf{x}$ is defined as $\mathbf{x}:=\left\{x_{u}^{(m)} \in\{0,1\} \mid u \in\right.$ $\left.\mathcal{U}_{\mathrm{u}} \cup \mathcal{U}_{\mathrm{d}}, m \in\left\{1, \ldots, N_{\mathrm{F}}\right\}\right\} \in \mathcal{A}_{\mathrm{x}}$.

A control policy prescribes a procedure for action selection in each state at all decision epoches $t$. We consider stationary Markovian deterministic control policies. ${ }^{3}$ A deterministic control policy given by $\Omega$ is a mapping $\mathcal{S} \rightarrow \mathcal{A}_{\mathrm{x}}$ from the state space to the action space, which is given by $\Omega(\mathbf{S})=\mathbf{x} \in$ $\mathcal{A}_{\mathrm{x}} \forall \mathbf{S} \in \mathcal{S}$.

Note that the instantaneous data rate $r_{i, t}^{(c)}$ is impacted by the subchannel-allocation action at time slot $t$, i.e.,

$$
\begin{equation*}
r_{i, t}^{(c)}=\sum_{(i, j) \in \mathcal{L}_{i}^{(c)}} \sum_{m=1}^{N_{\mathrm{F}}} \sum_{u \in \mathcal{U}_{i j}} x_{u, t}^{(m)} R_{H_{i j, t}^{(m, u)}} \tag{6}
\end{equation*}
$$

[^2]The system behavior of the aforementioned queuing model can be represented by the controlled discrete-time Markov chain (DTMC) $\left\{\mathbf{S}_{t}\right\}_{t=0,1, \ldots}:=\left\{\left(\mathbf{H}_{t}, \mathbf{Q}_{t}\right)\right\}_{t=0,1, \ldots}$. Given a system state $\mathbf{S}_{t}$ and an action $\mathbf{x}$ at time slot $t$, the state transition probability of the DTMC is given by

$$
\begin{align*}
\operatorname{Pr} .\left\{\mathbf{S}_{t+1} \mid \mathbf{S}_{t}, \mathbf{x}\right\} & =\operatorname{Pr} .\left\{\mathbf{H}_{t+1} \mid \mathbf{H}_{t}\right\} \operatorname{Pr} .\left\{\mathbf{Q}_{t+1} \mid \mathbf{S}_{t}, \mathbf{x}\right\} \\
& =\operatorname{Pr} .\left\{\mathbf{H}_{t+1}\right\} \operatorname{Pr} .\left\{\mathbf{Q}_{t+1} \mid \mathbf{S}_{t}, \mathbf{x}\right\} . \tag{7}
\end{align*}
$$

According to (5), the conditional probability of $Q_{i, t+1}^{(c)}$ given the system state $\mathbf{S}_{t}$ and an action $\mathbf{x}$ can be derived as

$$
\begin{align*}
& \operatorname{Pr} .\left\{Q_{i, t+1}^{(c)} \mid \mathbf{S}_{t}, \mathbf{x}\right\}=\operatorname{Pr} .\left(A_{i, t}^{(c)}=n\right) \\
& \quad \text { if } Q_{i, t+1}^{(c)}=\min \left[N_{Q}, \max \left[0, Q_{i, t}^{(c)}-r_{i, t}^{(c)}\right]+n\right] \tag{8}
\end{align*}
$$

where
$\operatorname{Pr} .\left(A_{i, t}^{(c)}=n\right)= \begin{cases}f_{A}(n), & \text { if } q_{i}^{(c)} \in \Theta_{\mathrm{u}} \cup \Theta_{\mathrm{Cd}} \\ 1, & \text { if } q_{i}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}} \text { and } n=T_{(2 c-1) 0, t} \\ 0, & \text { if } q_{i}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}} \text { and } n \neq T_{(2 c-1) 0, t} .\end{cases}$
The queue state transition probability $\operatorname{Pr} .\left\{\mathbf{Q}_{t+1} \mid \mathbf{S}_{t}, \mathbf{x}\right\}$ can be derived as the product of Pr. $\left\{Q_{i, t+1}^{(c)} \mid \mathbf{S}_{t}, \mathbf{x}\right\}$ over all queues $q_{i}^{(c)} \in \Theta$ as

$$
\begin{equation*}
\operatorname{Pr} .\left\{\mathbf{Q}_{t+1} \mid \mathbf{S}_{t}, \mathbf{x}\right\}=\prod_{q_{i}^{(c)} \in \Theta} \operatorname{Pr} .\left\{Q_{i, t+1}^{(c)} \mid \mathbf{S}_{t}, \mathbf{x}\right\} . \tag{10}
\end{equation*}
$$

Remark 1 (Channel State Transition Probability): Note that we do not recall the derivation of channel state transition probability Pr. $\left\{\mathbf{H}_{t+1} \mid \mathbf{H}_{t}\right\}=\operatorname{Pr} .\left\{\mathbf{H}_{t+1}\right\}$ in (7) given in Part I. This is because, to derive the delay-optimal subchannel-allocation action in Section III, we shall utilize the i.i.d. assumption of the CSI process and the stochastic approximation method to simplify the optimization problem, so that $\operatorname{Pr} .\left\{\mathbf{H}_{t+1}\right\}$ does not need to be derived. However, we would like to point out that, if the i.i.d. assumption of the CSI process does not hold or if the objective is to determine the steady-state probabilities of the queuing model for performance evaluation, $\operatorname{Pr} .\left\{\mathbf{H}_{t+1} \mid \mathbf{H}_{t}\right\}$ needs to be derived.

## E. Optimization Problem Formulation

Our objective is to optimize the subchannel-allocation policy so as to minimize the average weighted sum delay of all the connections subject to dropping probability constraints.

Problem 1: The delay-optimal subchannel-allocation design can be formulated as the constrained optimization problem

$$
\begin{align*}
& \min _{\Omega} \lim _{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{E}^{\Omega}\left[g_{0}\left(\mathbf{S}_{t}, \Omega\left(\mathbf{S}_{t}\right)\right)\right] \\
& \text { s.t. } \lim _{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{E}^{\Omega}\left[g_{c}\left(\mathbf{S}_{t}, \Omega\left(\mathbf{S}_{t}\right)\right)\right] \leq d_{\max } \quad \forall c \in \mathcal{C} \tag{11}
\end{align*}
$$

where

$$
\begin{align*}
& g_{0}(\mathbf{S}, \mathbf{x})=\frac{Q_{i}^{(c)}}{\lambda_{c}\left(1-d_{\max }\right)}  \tag{12}\\
& g_{c}(\mathbf{S}, \mathbf{x})= \begin{cases}1-\frac{T_{(c+D) 0}}{\lambda_{c}}, & \text { if } c \in \mathcal{C}_{\mathrm{Cu}} \\
1-\frac{T_{0}(c+D)}{\lambda_{c}}, & \text { if } c \in \mathcal{C}_{\mathrm{Cd}} \\
1-\frac{T_{(2 c-1)(2 c)+}+T_{0(2 c)}}{\lambda_{c}}, & \text { if } c \in \mathcal{C}_{\mathrm{D}} .\end{cases} \tag{13}
\end{align*}
$$

For any given nonnegative LMs $\boldsymbol{\eta}=\left\{\eta_{c} \mid c \in \mathcal{C}\right\}$, we define the Lagrangian function of problem 1 as

$$
\begin{equation*}
L(\Omega, \eta)=\mathbf{E}^{\pi(\Omega)}[g(\mathbf{S}, \Omega(\mathbf{S}))]+\sum_{c \in \mathcal{C}} \lambda_{c} \eta_{c}\left(1-d_{\max }\right)^{2} \tag{14}
\end{equation*}
$$

where

$$
\begin{align*}
g(\mathbf{S}, \Omega(\mathbf{S}))= & \sum_{c \in \mathcal{C}_{\mathrm{Cu}}}\left(\omega_{c} Q_{(c+D)}^{(c)}-\eta_{c}\left(1-d_{\max }\right) T_{(c+D) 0}\right) \\
& +\sum_{c \in \mathcal{C}_{\mathrm{Cd}}}\left(\omega_{c} Q_{0}^{(c)}-\eta_{c}\left(1-d_{\max }\right) T_{0(c+D)}\right) \\
& +\sum_{c \in \mathcal{C}_{\mathrm{D}}}\left(\omega_{c}\left(Q_{(2 c-1)}^{(c)}+Q_{0}^{(c)}\right)\right. \\
& \left.-\eta_{c}\left(1-d_{\max }\right)\left(T_{(2 c-1)(2 c)}+T_{0(2 c)}\right)\right) . \tag{15}
\end{align*}
$$

Therefore, problem 1 can be divided into the following two subproblems:

$$
\text { Subproblem 1-1 : } G(\boldsymbol{\eta})=\min _{\Omega} L(\Omega, \boldsymbol{\eta})
$$

Subproblem 1-2 : $G\left(\boldsymbol{\eta}^{*}\right)=\max _{\boldsymbol{\eta}} G(\boldsymbol{\eta})$.
where $G(\boldsymbol{\eta})$ is the corresponding Lagrange dual function.
Subproblem 1-1 with given LMs $\boldsymbol{\eta}^{*}$ can be solved by the equivalent Bellman equation as follows:

$$
\begin{align*}
\theta & +V\left(\mathbf{Q}^{(\grave{z})}\right)=\min _{\Omega\left(\mathbf{Q}^{(\grave{z})}\right)}\left\{g\left(\mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{Q}^{(\grave{z})}\right)\right)\right. \\
& \left.+\sum_{\mathbf{Q}^{(\grave{y}) \in \mathcal{Q}}} \operatorname{Pr} \cdot\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{Q}^{(\grave{z})}\right)\right] V\left(\mathbf{Q}^{(\grave{y})}\right)\right\} \forall \mathbf{Q}^{(\grave{z})} \in \mathcal{Q} \tag{16}
\end{align*}
$$

where $V\left(\mathbf{Q}^{(\grave{y})}\right)=\mathbf{E}_{\mathbf{H}}\left[V\left(\mathbf{H}, \mathbf{Q}^{(\grave{y})}\right) \mid \mathbf{Q}^{(\grave{y})}\right]=\sum_{\mathbf{H} \in \mathcal{H}} \operatorname{Pr} .[\mathbf{H}] V(\mathbf{H}$, $\left.\mathbf{Q}^{(\grave{y})}\right)$ is the conditional expectation of value function $V(\mathbf{S})$ taken over the channel state space $\mathcal{H}$ given the queue state $\mathbf{Q}^{(\grave{y})}$, while $g\left(\mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{Q}^{(\grave{z})}\right)\right)=\mathbf{E}_{\mathbf{H}}\left[g\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right) \mid \mathbf{Q}^{(\grave{z})}\right]$ and $\operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{Q}^{(\dot{z})}, \Omega\left(\mathbf{Q}^{(\dot{z})}\right)\right]=\mathbf{E}_{\mathbf{H}}\left[\operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega(\mathbf{H}\right.\right.$, $\left.\left.\left.\mathbf{Q}^{(\dot{z})}\right)\right] \mid \mathbf{Q}^{(\dot{z})}\right]$ are conditional expectations of cost function $g(\mathbf{H}$, $\left.\mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)$ and transition probability $\operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{H}\right.$, $\left.\mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right]$ taken over the channel state space $\mathcal{H}$ given the queue state $\mathbf{Q}^{(\grave{z})}$, respectively. $\Omega\left(\mathbf{Q}^{(\grave{z})}\right)=\left\{\Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right) \mid\right.$ $\forall \mathbf{H}\} \subseteq \mathcal{A}_{\mathbf{x}}$ is the partitioned actions of a policy $\Omega$ as the collection of $|\mathcal{H}|$ actions, where every action is mapped by policy $\Omega$ from a system state with given QSI $\mathbf{Q}^{(\grave{z})}$ and a different realization of CSI $\mathbf{H} \in \mathcal{H}$.

As a remark, note that equivalent Bellman equation (16) represents a series of fixed-point equations, where the numbers of equations are determined by the possible values of value functions $V\left(\mathbf{Q}^{(\dot{z})}\right)$, which is $|\mathcal{Q}|$. Therefore, we only need to solve $|\mathcal{Q}|$ instead of $|\mathcal{H}| \times|\mathcal{Q}|$ fixed-point equations with the reduced-state Bellman equation (16). To solve one such fixed-point equation using value iteration, the right-hand side (RHS) of (16) has to be minimized with given value functions $V\left(\mathbf{Q}^{(\grave{y})}\right)$. For this purpose, the RHS of (16) can be written as

$$
\begin{equation*}
\min _{\Omega\left(\mathbf{Q}^{(\grave{z})}\right)} \sum_{\mathbf{H} \in \mathcal{H}} \operatorname{Pr} .[\mathbf{H}] f\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right) \tag{17}
\end{equation*}
$$

where

$$
\begin{array}{r}
f\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)=g\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right) \\
\quad+\sum_{\mathbf{Q}^{(\grave{y}) \in \mathcal{Q}}} \operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right] V\left(\mathbf{Q}^{(\grave{y})}\right) \tag{18}
\end{array}
$$

Since (17) is a decoupled objective function w.r.t. different CSI realizations $\mathbf{H}$ with a given queue state $\mathbf{Q}^{(\grave{)})}$, we need to obtain $|\mathcal{H}|$ optimal actions to achieve the minimization objective in the RHS of equivalent Bellman equation (16), where every optimal action is w.r.t. a system state $\left(\mathbf{H}, \mathbf{Q}^{(\dot{z})}\right)$ with given $\mathbf{Q}^{(\grave{z})}$ and a different CSI realization $\mathbf{H} \in \mathcal{H}$ that minimizes the value of $f\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)$. This means that the control policy obtained by solving (16) is based on the system state $\mathbf{S}$ instead of only the queue state $\mathbf{Q}$.

Since the brute-force value iteration algorithm in Part I faces the curse-of-dimensionality problem, we will develop a solution with reduced complexity using linear value approximation and online stochastic learning in the next section.

## III. Optimal Solution by Approximate Markov Decision Process and Stochastic Learning

Here, we will first assume that the optimal LMs are given and focus on the solution of subproblem 1-1 in Section V-A and B. Then, in Section V-C, we use an online stochastic learning algorithm with two timescales to determine the optimal LMs.

## A. Linear Value-Function Approximation

Here, we use the linear value-function approximation method to further reduce the state space.

First, we define the per-queue cost function as

$$
\begin{align*}
& g_{i}^{(c)}\left(\mathbf{S}_{i}^{(c)}, \Omega(\mathbf{S})\right) \\
& \quad= \begin{cases}\omega_{c} Q_{2 c-1}^{(c)}-\eta_{c}\left(1-d_{\max }\right) T_{(2 c-1)(2 c)}, & \text { if } q_{2 c-1}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{u}} \\
\omega_{c} Q_{c+D}^{(c)}-\eta_{c}\left(1-d_{\max }\right) T_{(c+D) 0}, & \text { if } q_{c+D}^{(c)} \in \Theta_{\mathrm{Cu}} \\
\omega_{c} Q_{0}^{(c)}-\eta_{c}\left(1-d_{\max }\right) T_{0(2 c)}, & \text { if } q_{0}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}} \\
\omega_{c} Q_{0}^{(c)}-\eta_{c}\left(1-d_{\max }\right) T_{0(c+D)}, & \text { if } q_{0}^{(c)} \in \Theta_{\mathrm{Cd}} .\end{cases} \tag{19}
\end{align*}
$$

Thus, the overall cost function is given by $g(\mathbf{S}, \Omega(\mathbf{S}))=$ $\sum_{q_{i}^{(c)} \in \Theta} g_{i}^{(c)}\left(\mathbf{S}_{i}^{(c)}, \Omega(\mathbf{S})\right)$, according to (15). Moreover, define $g_{i}^{(c)}\left(Q_{i}^{(c)}, \Omega(\mathbf{Q})\right)=\mathbf{E}_{\mathbf{H}}\left[g\left(\mathbf{H}_{i}^{(c)}, Q_{i}^{(c)}, \Omega(\mathbf{H}, \mathbf{Q})\right) \mid \mathbf{Q}\right]$ as the conditional per-queue cost function, which is equal to the conditional expectation of the per-queue cost function $g\left(\mathbf{S}_{i}^{(c)}\right.$, $\Omega(\mathbf{S})$ ) taken over the channel state space $\mathcal{H}$ given the queue state $\mathbf{Q}$.

Next, the linear approximation architecture for the value function $V(\mathbf{Q})$ is given by

$$
\begin{align*}
V(\mathbf{Q}) & =V\left(\left\{Q_{i}^{(c)} \mid q_{i}^{(c)} \in \Theta\right\}\right) \approx \sum_{q_{i}^{(c)} \in \Theta} \sum_{q=0}^{N_{Q}} \mathbf{I}\left[Q_{i}^{(c)}=q\right] \widetilde{V}_{i}^{(c)}(q) \\
& =\widetilde{\mathbf{V}}^{T} \mathbf{F}(\mathbf{Q}) \quad \forall \mathbf{Q} \in \mathcal{Q} \tag{20}
\end{align*}
$$

where

$$
\begin{aligned}
\tilde{\mathbf{V}} & =\left[\tilde{\mathbf{V}}_{i}^{(c)} \mid q_{i}^{(c)} \in \Theta\right]^{T}, \quad \tilde{\mathbf{V}}_{i}^{(c)}=\left[\tilde{V}_{i}^{(c)}(0), \ldots, \tilde{V}_{i}^{(c)}\left(N_{\mathrm{Q}}\right)\right] \\
\mathbf{F}(\mathbf{Q}) & =\left[\mathbf{I}\left[Q_{i}^{(c)}=0\right], \ldots, \mathbf{I}\left[Q_{i}^{(c)}=N_{\mathrm{Q}}\right] \mid q_{i}^{(c)} \in \Theta\right]^{T} .
\end{aligned}
$$

Denote $\widetilde{V}_{i}^{(c)}(q), q \in\left\{0,1, \ldots, N_{\mathrm{Q}}\right\}$ as the per-queue value function and $V(\mathbf{Q}), \mathbf{Q} \in \mathcal{Q}\}$ the global value function in the remainder of this paper. Therefore, $\widetilde{\mathbf{V}}_{i}^{(c)}$ and $\widetilde{\mathbf{V}}$ are the perqueue value function vectors for queue $q_{i}^{(c)}$ and all the queues in the network, respectively. Similarly, define the global value function vector as

As a remark, note that the number of global value functions is $|\mathcal{Q}|=\left(N_{\mathrm{Q}}+1\right)^{|\Theta|}$ in total, which grows exponentially with the number of queues. On the other hand, the number of perqueue value functions is $\left(N_{\mathrm{Q}}+1\right) \times|\Theta|$ in total, which grows linearly with the number of queues. Therefore, we can represent the $\left(N_{\mathrm{Q}}+1\right)^{|\Theta|}$ global value functions with $\left(N_{\mathrm{Q}}+1\right) \times|\Theta|$ per-queue value functions by the linear approximation architecture.

From (16), the key issue in deriving the optimal control actions is to obtain the global value function vector $\mathbf{V}$. With linear value-function approximation, we only need to obtain the per-queue value function vector $\widetilde{\mathbf{V}}$. To illustrate the structure of our solution, we first assume that we could obtain the per-queue value functions via some means (e.g., via offline value iteration) and focus on deriving the optimal action under every system state to minimize the value of $f\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)$. Define $Q_{i}^{(c, \grave{y})} \in\left\{1, \ldots, N_{Q}\right\}$ as the local queue state of queue $q_{i}^{(c)}$ when the global queue state is $\mathbf{Q}^{(\grave{y})}$, i.e., $\mathbf{Q}^{(\grave{y})}=\left\{Q_{i}^{(c, \grave{y})} \mid q_{i}^{(c)} \in\right.$ $\Theta\}$. Therefore, according to (20), we have

$$
\begin{equation*}
V\left(\mathbf{Q}^{(\grave{y})}\right) \approx \sum_{q_{i}^{(c)} \in \Theta} \widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{y})}\right) \tag{21}
\end{equation*}
$$

The optimal control action is given by the following subproblem 1-1(a).

Subproblem 1-1(a): For given per-queue value functions $\widetilde{\mathbf{V}}$ and LMs $\boldsymbol{\eta}$, find the optimal action $\Omega^{*}\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)$ for system state $\left\{\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right\}$ that minimizes the value of $f\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right.$, $\left.\Omega\left(\mathbf{H}, \mathbf{Q}^{(\dot{z})}\right)\right)$, which can be written (22) and (23), shown at the bottom of the page, where the post-action system state $Q_{i}^{(c, \grave{z})}\left(r_{i}^{(c)}, x\right)$ is defined by the following equation:

$$
Q_{i}^{(c, \dot{z})}\left(r_{i}^{(c)}, x\right)=\min \left[\max \left[0, Q_{i}^{(c, \dot{z})}-r_{i}^{(c)}\right]+x, N_{\mathrm{Q}}\right]
$$

$$
\begin{align*}
& \Omega^{*}\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right) \\
& =\arg \min _{\Omega} \sum_{q_{i}^{(c)} \in \Theta} g_{i}^{(c)}\left(\mathbf{H}_{i}^{(c)}, Q_{i}^{(c, \grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)+\sum_{\mathbf{Q}^{(\grave{y}) \in \mathcal{Q}}}\left\{\operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right] V\left(\mathbf{Q}^{(\grave{y})}\right)\right\} \\
& =\arg \min _{\Omega} \sum_{q_{i}^{(c)} \in \Theta} g_{i}^{(c)}\left(\mathbf{H}_{i}^{(c)}, Q_{i}^{(c, \grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)+\sum_{\mathbf{Q}^{(\grave{y})} \in \mathcal{Q}}\left\{\prod_{q_{i}^{(c)} \in \Theta} \operatorname{Pr}\left[Q_{i}^{(c, \grave{y})} \mid \mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right] \sum_{q_{i}^{(c)} \in \Theta} \tilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{y})}\right)\right\} \\
& =\arg \min _{\Omega} \sum_{q_{i}^{(c)} \in \Theta}\left(g_{i}^{(c)}\left(\mathbf{H}_{i}^{(c)}, Q_{i}^{(c, \grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)+\sum_{Q_{i}^{(c, \grave{y})}=1}^{N_{\mathbf{Q}}} \operatorname{Pr} .\left[Q_{i}^{(c, \grave{y})} \mid \mathbf{H}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right] \widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{y})}\right)\right)  \tag{22}\\
& =\arg \min _{\Omega} \sum_{q_{i}^{(c)} \in \Theta_{\mathrm{u}} \cup \Theta_{\mathrm{Cd}}} \underbrace{\left(g_{i}^{(c)}\left(\mathbf{H}_{i}^{(c)}, Q_{i}^{(c, \grave{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)\right)+\sum_{n} f_{A}(n) \widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}\left(r_{i}^{(c)}, n\right)\right)\right)}_{B_{i, c}^{\Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)}, q_{i}^{(c)} \in \Theta_{\mathrm{u}} \cup \Theta_{\mathrm{Cd}}}  \tag{23}\\
& +\sum_{q_{0}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}}} \underbrace{\left(g_{0}^{(c)}\left(\mathbf{H}_{0}^{(c)}, Q_{0}^{(c, \dot{z})}, \Omega\left(\mathbf{H}, \mathbf{Q}^{(\dot{z})}\right)\right)+\widetilde{V}_{0}^{(c)}\left(Q_{0}^{(c, \grave{z})}\left(r_{0}^{(c)}, T_{(2 c-1) 0}\right)\right)\right)}_{B_{0, c}^{\Omega\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)}, q_{0}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}}} \tag{24}
\end{align*}
$$

and when $Q_{i}^{(c, \grave{z})}=0$ and, thus, $r_{i}^{(c)}=0$, we define

$$
Q_{i}^{(c, z)}(x)=\min \left[x, N_{\mathrm{Q}}\right]
$$

Step (22) follows from the linear value approximation structure in (21). Step (24) holds because the arrival process for any queue $q_{i}^{(c)} \in \Theta_{\mathrm{u}} \cup \Theta_{\mathrm{Cd}}$ equals the arrival process for connection $c$, while the arrival process for any queue $q_{0}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}}$ depends on the departure process of the previous hop. Therefore, (8) and (9) are used to replace the local queue state transition probabilities on the left-hand side of the equality.

Remark 2 (Complexity of Solution Based on (24)): If we directly derive the optimal action based on (24), for every action $\mathbf{x} \in \mathcal{A}_{\mathbf{x}}$, we need to calculate the bid term $B_{i, c}^{(\mathbf{x})}$ for every $q_{i}^{(c)} \in$ $\Theta$ and derive the summation of bids over all queues. Finally, the action $\mathrm{x}^{*}$ with the minimum sum bid value is selected. The complexity of the aforementioned solution is $\mathcal{O}\left(\left|\mathcal{A}_{\mathrm{x}}\right|\right)$, which grows exponentially with the number of subchannels.

To deal with the exponentially increasing action space with the number of subchannels, we expand $\widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \dot{z})}\left(r_{i}^{(c)}, x\right)\right)$ in (24) using Taylor expansion as follows:

$$
\begin{equation*}
\widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}\left(r_{i}^{(c)}, x\right)\right)=\widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}\right)+\left(x-r_{i}^{(c)}\right)\left(\widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}\right)\right)^{\prime} \tag{25}
\end{equation*}
$$

where

$$
\left(\widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}\right)\right)^{\prime} \approx \widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}+1\right) / 2-\widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{z})}-1\right) / 2
$$

Therefore, (24) is equivalent to (26), shown at the bottom of the page, where $B_{(i, c)}^{(m, u)}$ is expressed in (27), shown at the bottom of the page, and $\mathbf{I}_{1}=\mathbf{I}((2 c-1, j)=((2 c-1),(2 c)))$ and $\mathbf{I}_{2}=\mathbf{I}((2 c-1, j)=((2 c-1), 0))$ with $(2 c-1, j) \in \mathcal{B}_{u}$.

Recall that any uplink (respectively, downlink) subchannel $m \in\left\{1, \ldots, N_{\mathrm{F}}\right\}$ can be allocated to at most one uplink (respectively downlink) RRG. Moreover, note that the summation index $m$ in the first and second terms of (26) represent the index of uplink and downlink subchannels, respectively. Therefore, for every $m$ in the first or second term, at most one $x_{u^{*}}^{(m)}=1$,
while all the other $x_{u}^{(m)}=0$. Now, subproblem 1-1(a) becomes determining the largest $B^{(m, u)}$ for every uplink and downlink subchannel $m$, where for any $m=1, \ldots, N_{\mathrm{F}}$

$$
B^{(m, u)}= \begin{cases}\sum_{q_{c}^{(c)} \in \Theta_{\mathrm{u}}} B_{(i, c)}^{(m, u)}, & \text { if } u \in \mathcal{U}_{\mathrm{u}}  \tag{28}\\ B_{(0, c)}^{(m, u)}, & \text { if } u \in \mathcal{U}_{\mathrm{d}}\end{cases}
$$

Algorithm 1 (Solution to Subproblem 1-1(a)): Given perqueue value functions $\widetilde{\mathbf{V}}$ and LMs $\boldsymbol{\eta}$, based on the observed system state $\mathbf{S}_{t}$ at the beginning of time slot $t$, the optimal action for subproblem 1-1(a) is determined as

$$
\begin{align*}
& x_{u}^{(m)} \\
& = \begin{cases}1, & \text { if } u \in \mathcal{U}_{\mathrm{u}}, u=\arg \max _{u^{\prime}} B^{\left(m, u^{\prime}\right)} \forall u^{\prime} \in \mathcal{U}_{\mathrm{u}} \\
& \text { or } u \in \mathcal{U}_{\mathrm{d}}, u=\arg \max _{u^{\prime}} B^{\left(m, u^{\prime}\right)} \forall u^{\prime} \in \mathcal{U}_{\mathrm{d}} \\
0, & \text { otherwise },\end{cases} \\
& \forall m=1, \ldots, N_{\mathrm{F}} . \tag{29}
\end{align*}
$$

Remark 3 (Complexity of Algorithm 1): Every uplink queue needs to compute $\sum_{(i, j) \in \mathcal{L}_{i}^{(c)}}\left|\mathcal{U}_{i j}\right| \times N_{\mathrm{F}}$ bids, while every downlink queue needs to compute $N_{\mathrm{F}}$ bids. Moreover, the BS needs to find the minimum values of $\left|\mathcal{U}_{\mathrm{u}}\right| \times N_{\mathrm{F}}$ uplink $B^{(m, u)}$ values and of $\left|\mathcal{U}_{\mathrm{d}}\right| \times N_{\mathrm{F}}$ downlink $B^{(m, u)}$ values. Therefore, the overall computational complexity of Algorithm 1 is $\mathcal{O}\left(\left(\left|\mathcal{U}_{\mathrm{u}}\right|+\left|\mathcal{U}_{\mathrm{d}}\right|\right) \times N_{\mathrm{F}}\right)$ and only grows linearly with the number of subchannels.

Remark 4 (Structure of Algorithm 1): The subchannelallocation solution in (29) has a similar structure with the MaxWeight algorithm based on the Lyapunov stability approach [13]. When the MaxWeight algorithm is applied to our network model for D2D communications, the weight for each link $(i, j)$ at each time slot is defined as its differential backlog $W_{i j}=Q_{i, t}^{(c)}-Q_{j, t}^{(c)}$. Given the link weight, we can select an RRG $u^{*}$, with the maximum sum over all its links of the product of link weight $W_{i j}$ and instantaneous data rate $R_{H_{i j}^{\left(m, u^{*}\right)}}$. Compared with the MaxWeight algorithm, Algorithm 1 only differs in that the weight of link $(i, j)$ is determined by the difference in the derivatives of per-queue value functions and the LM instead of the difference in queue length.

$$
\begin{equation*}
\Omega^{*}\left(\mathbf{H}, \mathbf{Q}^{(\grave{z})}\right)=\arg \max _{\Omega}\left(\sum_{m=1}^{N_{\mathrm{F}}} \sum_{q_{i}^{(c)} \in \Theta_{\mathrm{u}}} \sum_{(i, j) \in \mathcal{L}_{i}^{(c)}} \sum_{u \in \mathcal{U}_{i j}} x_{u}^{(m)} B_{(i, c)}^{(m, u)}+\sum_{m=1}^{N_{\mathrm{F}}} \sum_{q_{0}^{(c)} \in \Theta_{\mathrm{d}}} \sum_{(0, j) \in \mathcal{L}_{0}^{(c)}} \sum_{u \in \mathcal{U}_{0 j}} x_{u}^{(m)} B_{(0, c)}^{(m, u)}\right) \tag{26}
\end{equation*}
$$

$$
B_{(i, c)}^{(m, u)}= \begin{cases}R_{H_{(2 c-1) j}^{(m, u)}}\left(\left(\widetilde{V}_{2 c-1}^{(c)}\left(Q_{2 c-1}^{(c, \dot{z})}\right)\right)^{\prime}+\eta_{c}\left(1-d_{\max }\right) \mathbf{I}_{1}-\left(\widetilde{V}_{0}^{(c)}\left(Q_{0}^{(c, \grave{z})}\right)\right)^{\prime} \mathbf{I}_{2}\right), & \text { if } q_{2 c-1}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{u}}  \tag{27}\\
R_{H_{(c+D) 0}^{(m, u)}\left(\left(\widetilde{V}_{c+D}^{(c)}\left(Q_{c+D}^{(c, \dot{z})}\right)\right)^{\prime}+\eta_{c}\left(1-d_{\max }\right)\right),} \begin{array}{ll}
\text { if } q_{c+D}^{(c)} \in \Theta_{\mathrm{Cu}} \\
R_{H_{0(2 c)}^{(m, u)}}^{\left(\left(\widetilde{V}_{0}^{(c)}\left(Q_{0}^{(c, \dot{z})}\right)\right)^{\prime}+\eta_{c}\left(1-d_{\max }\right)\right),} & \text { if } q_{0}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}} \\
R_{H_{0(c+D)}^{(m, u)}\left(\left(\widetilde{V}_{0}^{(c)}\left(Q_{0}^{(c, \grave{z})}\right)\right)^{\prime}+\eta_{c}\left(1-d_{\max }\right)\right),} & \text { if } q_{0}^{(c)} \in \Theta_{\mathrm{Cd}}
\end{array}\end{cases}
$$

In the previous discussion, we assume that the per-queue value function vector $\widetilde{\mathbf{V}}$ is already known in subproblem 1-1(a) and propose Algorithm 1 to derive the optimal control action under every system state. However, we still have to determine $\widetilde{\mathbf{V}}$, to solve subproblem 1-1. For this purpose, we let $\widetilde{V}_{i}^{(c)}(0)=$ $0, \forall q_{i}^{(c)} \in \Theta$. Therefore, according to the linear approximation architecture, among the $\left(N_{\mathrm{Q}}+1\right)^{|\Theta|}$ global value functions, there are $N_{\mathrm{Q}} \times|\Theta|$ global value functions that equal to the $N_{\mathrm{Q}} \times|\Theta|$ per-queue value functions $\left\{\widetilde{V}_{i}^{(c)}(q) \mid \forall q_{i}^{(c)} \in \Theta, q=\right.$ $\left.1, \ldots, N_{\mathrm{Q}}\right\}$. We refer the system states of these global value functions as representative states, and they share the same characteristics that only one queue is nonempty while the queue lengths of all the other queues are zero. The set of representative states $\mathcal{Q}_{\mathrm{R}}$ is defined as

$$
\mathcal{Q}_{\mathrm{R}}=\left\{\mathbf{Q}_{(i, c)}^{(q)} \mid \forall q_{i}^{(c)} \in \Theta, q=1, \ldots, N_{\mathrm{Q}}\right\}
$$

where $\mathbf{Q}_{(i, c)}^{(q)}=\left\{Q_{i}^{(c)}=q, Q_{i^{\prime}}^{\left(c^{\prime}\right)}=0 \mid q_{i^{\prime}}^{\left(c^{\prime}\right)} \in \Theta \backslash q_{i}^{(c)}\right\}$ denotes the global queue state with $Q_{i}^{(c)}=q \in\left\{1, \ldots, N_{Q}\right\}$ for queue $q_{i}^{(c)}$ and $Q_{i^{\prime}}^{\left(c^{\prime}\right)}=0$ for all the other queues $q_{i^{\prime}}^{\left(c^{\prime}\right)} \in \Theta \backslash q_{i}^{(c)}$.

Therefore, given the solution of subproblem 1-1(a), we still have to solve the following subproblem 1-1(b), to solve subproblem 1-1.

Subproblem 1-1(b): Derive the per-queue value functions $\widetilde{\mathbf{V}}$ that satisfy the following equivalent Bellman equation under every representative state $\mathbf{Q}_{(i, c)}^{(q)} \in \mathcal{Q}_{\mathrm{R}}$ :

$$
\begin{align*}
& \theta+\widetilde{V}_{i}^{(c)}(q)=\min _{\Omega}\left\{g_{i}^{(c)}\left(q, \Omega\left(\mathbf{Q}_{(i, c)}^{(q)}\right)\right)\right. \\
& \left.+\sum_{q_{i}^{(c)} \in \Theta Q_{i}^{(c, \grave{y})}=1} \sum^{N_{Q}} \operatorname{Pr}\left[Q_{i}^{(c, \grave{y})} \mid \mathbf{Q}_{(i, c)}^{(q)}, \Omega\left(\mathbf{Q}_{(i, c)}^{(q)}\right)\right] \widetilde{V}_{i}^{(c)}\left(Q_{i}^{(c, \grave{y})}\right)\right) \tag{30}
\end{align*}
$$

where (30) is derived by combining (23) with (16).
Remark 5 (Complexity Reduction Due to Linear ValueFunction Approximation): Due to linear value-function approximation, the following simplifications can be achieved in solving problem 1.

- Only $\left(N_{\mathrm{Q}}+1\right) \times|\Theta|$ per-queue value functions need to be stored instead of $\left(N_{Q}+1\right)^{|\Theta|}$ global value functions.
- To determine the optimal control action in subproblem 1-1(a) with given per-queue value functions, Algorithm 1 with a simple structure as the MaxWeight algorithm can be derived with the help of Taylor expansion.
- Only $N_{\mathrm{Q}} \times|\Theta|$ fixed-point equations in (30) instead of $\left(N_{\mathrm{Q}}+1\right)^{|\Theta|}$ fixed-point equations in (16) need to be determined to derive the per-queue value functions.


## B. Online Stochastic Learning

1) Solution to Problem 1: Instead of solving the equivalent Bellman equation on the representative states (30) using offline value iteration, we will estimate $\widetilde{\mathbf{V}}$ using the online stochastic learning algorithm here.


Fig. 1. Implementation flow of Algorithm 2 with online stochastic learning (solution to problem 1).

Remark 6 (Motivation of Online Stochastic Learning): The motivation of using an online stochastic learning algorithm to update the per-queue value functions iteratively instead of using an offline value iteration algorithm is that the former algorithm can solve the Bellman equation iteratively without the need of explicitly deriving the CSI probability distribution $\operatorname{Pr} .[\mathbf{H}]$ to calculate the "conditional cost" $g_{i}^{(c)}\left(q, \Omega\left(\mathbf{Q}_{(i, c)}^{(q)}\right)\right)$ and "conditional transition probability" $\operatorname{Pr} .\left[Q_{i}^{(c, \grave{y})} \mid \mathbf{Q}_{(i, c)}^{(q)}, \Omega\left(\mathbf{Q}_{(i, c)}^{(q)}\right)\right]$ in (30).

The online iterative algorithm (Algorithm 2) is given by the following, which simultaneously solves subproblem 1-1(b) in deriving per-queue value functions and subproblem 1-2 in deriving LMs $\boldsymbol{\eta}$. Since Algorithm 2 embeds Algorithm 1 to solve subproblem 1-1(a), it is the complete solution for problem 1.

Algorithm 2 (Solution to Problem 1): Fig. 1 illustrates the implementation flow of the overall solution with detailed steps as follows:

- Step $1_{\sim}$ (Initialization): The per-queue value function vector $\widetilde{\mathbf{V}}_{0}$ and $L M$ vector $\boldsymbol{\eta}_{0}$ are initialized. The subscript denotes the index of time slot.
- Step 2 (Calculate Control Action): Based on the ob$\widetilde{\sim}_{t}$ served system state $\mathbf{S}_{t}$ and the per-queue value functions $\widetilde{\mathbf{V}}_{t}$ at time slot $t$, the optimal control action $\mathbf{x}$ is calculated using Algorithm 1 at the beginning of time slot $t$.
- Step 3 (Update Per-Queue Value Functions and LMs): Based on the observed system state $\mathbf{S}_{t}$ and the optimal action $\mathbf{x}$, the instantaneous data rate and throughput of every queue $q_{i}^{(c)}$ and its associated links $(i, j) \in \mathcal{L}_{i}^{(c)}$ are known. Based on the aforementioned information, the
per-queue value functions $\widetilde{\mathbf{V}}_{t}$ and LMs $\boldsymbol{\eta}_{t}$ can be updated at the end of time slot $t$ to $\widetilde{\mathbf{V}}_{t+1}$ and $\boldsymbol{\eta}_{t+1}$ using the following update function:

$$
\begin{align*}
& \widetilde{V}_{i, t+1}^{(c)}(q)=\left\{\begin{array}{l}
\left(1-\epsilon_{\tau_{i}^{(c)}(q, t)}^{v}\right) \widetilde{V}_{i, t}^{(c)}(q)+\epsilon_{\tau_{i}^{(c)}(q, t)}^{v} \Delta \widetilde{V}_{i, t}^{(c)}(q) \\
\text { if } \mathbf{Q}_{t}=\mathbf{Q}_{(i, c)}^{(q)} \\
\widetilde{V}_{i, t}^{(c)}(q) \\
\text { if } \mathbf{Q}_{t} \neq \mathbf{Q}_{(i, c)}^{(q)}
\end{array}\right. \\
& \forall q_{i}^{(c)} \in \Theta, q=1, \ldots, N_{\mathrm{Q}} \tag{31}
\end{align*}
$$

where $\epsilon_{\tau_{i}^{(c)}(q, t)}^{v}=\sum_{t^{\prime}=0}^{t} \mathbf{I}\left[\mathbf{Q}_{t^{\prime}}=\mathbf{Q}_{(i, c)}^{(q)}\right]$, and $\Delta \widetilde{V}_{i, t}^{(c)}(q)$ is expressed in (32), shown at the bottom of the page. Moreover, the LMs $\eta_{c, t}$ of every connection $c$ can be updated at the end of time slot $t$ to $\eta_{c, t+1}$ using the following function:

$$
\begin{equation*}
\eta_{c, t+1}=\eta_{c, t}+\epsilon_{t}^{\eta} \Delta \eta_{c, t} \tag{33}
\end{equation*}
$$

where $\Delta \eta_{c, t}$ is expressed in (34), shown at the bottom of the page. In the aforementioned equations, $\left(\left\{\epsilon_{t}^{v}\right\},\left\{\epsilon_{t}^{\eta}\right\}\right)$ are the sequences of step sizes, which satisfy

$$
\begin{aligned}
& \sum_{t=0}^{\infty} \epsilon_{t}^{v}=\infty, \quad \epsilon_{t}^{v}>0, \lim _{t \rightarrow \infty} \epsilon_{t}^{v}=0 \\
& \sum_{t=0}^{\infty} \epsilon_{t}^{\eta}=\infty, \quad \epsilon_{t}^{\eta}>0, \lim _{t \rightarrow \infty} \epsilon_{t}^{\eta}=0 \\
& \sum_{t=0}^{\infty}\left[\left(\epsilon_{t}^{v}\right)^{2}+\left(\epsilon_{t}^{\eta}\right)^{2}\right]<\infty, \quad \text { and } \lim _{t \rightarrow \infty} \frac{\epsilon_{t}^{\eta}}{\epsilon_{t}^{v}}=0 .
\end{aligned}
$$

- Step 4 (Termination): If $\left\|\widetilde{\mathbf{V}}_{t+1}-\widetilde{\mathbf{V}}_{t}\right\|<\delta_{v}$ and $\| \boldsymbol{\eta}_{t+1}-$ $\boldsymbol{\eta}_{t} \|<\delta_{\eta}$, stop; otherwise, set $t:=t+1$ and go to step 2 .

Remark 7 (Complexity and Implementation Consideration of Algorithm 2): In Algorithm 2, we assume that it is centralized implemented by the BS. The BS needs to store $\left(N_{\mathrm{Q}}+1\right) \times|\Theta|$ per-queue value functions and $C \mathrm{LMs}$. The computational complexity of Algorithm 2 at each time slot is the sum of two parts: 1) the computational complexity of determining the optimal action according to Algorithm 1 , which is $\mathcal{O}\left(\left(\mathcal{U}_{\mathrm{u}}+\right.\right.$ $\left.\left.\mathcal{U}_{\mathrm{d}}\right) \times N_{\mathrm{F}}\right)$, as given in Remark 3; and 2) the computational
complexity of updating the per-queue value functions. Note that $\widetilde{V}_{i, t}(c)(q)$ is only updated to a different value at any time slot $t$, when the global queue state $\widetilde{\mathbf{Q}}_{t}$ is the representative state $\widetilde{\mathbf{Q}}_{i, c}^{(q)}$, according to (33). This implies that at most one per-queue value function shall be updated to a different value with computational complexity $\mathcal{O}\left(N_{\mathrm{Q}}+1\right)$ at any time slot, while all the other per-node value functions remain the same. Therefore, the overall computational complexity of Algorithm 2 is at most $\mathcal{O}\left(\left(N_{\mathrm{Q}}+1\right)+\left(\left|\mathcal{U}_{\mathrm{u}}\right|+\left|\mathcal{U}_{\mathrm{d}}\right|\right) \times N_{\mathrm{F}}\right)$ at each time slot, which grows linearly with the buffer capacity and the number of RRGs and subchannels. The memory requirement and computational complexity of Algorithm 2 are greatly reduced compared to those of the offline value iteration algorithm. Moreover, the structure of Algorithm 2 enables distributed implementation, where the per-queue value functions are distributively maintained at nodes that maintain the corresponding queues. The optimal action in Algorithm 1 can be derived using an auction mechanism, in which each src. DUE and uplink CUE sends bid values to the BS for the queues that it maintains. In this way, the computation task can be offloaded to the UEs from the BS. However, larger signaling overhead need to be involved compared to the centralized implementation.
2) Convergence Analysis: Here, we shall establish technical conditions for the almost-sure convergence of the online stochastic learning algorithm (see Algorithm 2). Recall that the purpose of Algorithm 2 is to iteratively derive the perqueue value function vector $\widetilde{\mathbf{V}}$ in subproblem 1-1(b) and LMs $\boldsymbol{\eta}$ in subproblem 1-2 so that problem 1 can be solved. Given $\boldsymbol{\eta}^{*}$, subproblem 1-1 is an unconstrained MDP problem; hence, learning algorithms in [14] apply to update $\widetilde{\mathbf{V}}$, which is done in Algorithm 2. However, the correct $\boldsymbol{\eta}^{*}$ needs to be derived. We do this by a gradient ascent in the dual (i.e., Lagrange multiplier) space in view of subproblem 1-2. Since we have two different step size sequences $\left\{\epsilon_{t}^{v}\right\},\left\{\epsilon_{t}^{\eta}\right\}$ and $\left\{\epsilon_{t}^{\eta}\right\}=\mathbf{o}\left(\left\{\epsilon_{t}^{v}\right\}\right)$, the LM's update is carried out simultaneously with the perqueue value function's update but over a slower timescale. Here, we are using the formalism of two-timescale stochastic approximation from [15]. During the per-queue value functions' update (timescale I), we have $\eta_{c, t+1}-\eta_{c, t}=\mathcal{O}\left(\left\{\epsilon_{t}^{\eta}\right\}\right)=$ $\mathbf{o}\left(\left\{\epsilon_{t}^{v}\right\}\right) \forall c \in \mathcal{C}$, and hence, the LMs appear to be quasi-static

$$
\Delta \widetilde{V}_{i, t}^{(c)}(q)= \begin{cases}\omega_{c} q+\eta_{c}\left(1-d_{\max }\right) \min \left[q, r_{i, t}^{(c)}\right] &  \tag{32}\\ \quad+\sum_{n} f_{A}(n)\left(\widetilde{V}_{i, t}^{(c)}\left(q\left(r_{i, t}^{(c)}, n\right)\right)-\widetilde{V}_{i, t}^{(c)}(q(n))\right), & \text { if } q_{i}^{(c)} \in \Theta_{\mathrm{Cu}} \cup \Theta_{\mathrm{Cd}} \\ \omega_{c} q+\eta_{c}\left(1-d_{\max }\right) \min \left[q, r_{(2 c-1)(2 c), t}\right] & \\ \quad+\sum_{n} f_{A}(n)\left(\widetilde{V}_{i, t}^{(c)}\left(q\left(r_{i, t}^{(c)}, n\right)\right)-\widetilde{V}_{i, t}^{(c)}(q(n))\right)+\widetilde{V}_{0, t}^{(c)}\left(T_{(2 c-1) 0, t}\right), & \text { if } q_{i}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{u}} \\ \omega_{c} q+\eta_{c}\left(1-d_{\max }\right) \min \left[q, r_{0, t}^{(c)}\right]+\widetilde{V}_{0, t}^{(c)}\left(q\left(r_{0, t}^{(c)}, 0\right)\right), & \text { if } q_{i}^{(c)} \in \Theta_{\mathrm{D}-\mathrm{d}}\end{cases}
$$

$$
\Delta \eta_{c, t}= \begin{cases}\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{(c+D) 0}\right), & \text { if } c \in \mathcal{C}_{\mathrm{Cu}}  \tag{34}\\ \left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{0(c+D)}\right), & \text { if } c \in \mathcal{C}_{\mathrm{Cd}} \\ \left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{(2 c-1)(2 c)}-T_{0(2 c)}\right), & \text { if } c \in \mathcal{C}_{\mathrm{D}}\end{cases}
$$

during the per-node value functions' update in (31) and (32). On the other hand, since the per-queue value functions will be updated much faster than the LMs due to $\epsilon_{t}^{\eta} / \epsilon_{t}^{v} \rightarrow 0$, during the LMs' update in (33) and (34) (timescale II), the "primal" minimization carried out by the learning algorithm for MDPs in (31) and (32) is seen as having essentially equilibrated. Therefore, we will give the convergence of per-queue value function over timescale I and LMs over timescale II in Lemma 2 and Lemma 3, respectively.

Before Lemma 2 is given, we first give the relationship between the global value function vector $\mathbf{V}$ and the per-queue value function vector $\widetilde{\mathbf{V}}$ in matrix form, for ease of notation, as follows:

$$
\mathbf{V}=\mathbf{M} \tilde{\mathbf{V}} \quad \text { and } \quad \tilde{\mathbf{V}}=\mathbf{M}^{\dagger} \mathbf{V}
$$

where $\mathbf{M} \in \mathbb{R}^{|\mathcal{Q}| \times\left(N_{\mathrm{Q}}+1\right)|\Theta|}$ with the $\grave{z}$ th row $(\grave{z}=1, \ldots,|\mathcal{Q}|)$ equals to $\mathbf{F}^{T}\left(\mathbf{Q}^{(\dot{z})}\right)$. Therefore, the first equation aforementioned follows directly from (20). The second equation, on the other hand, uses the matrix $\mathbf{M}^{\dagger} \in \mathbb{R}^{\left(N_{\mathrm{Q}}+1\right)|\Theta| \times|\mathcal{Q}|}$ to select $N_{\mathrm{Q}}|\Theta|$ elements from $\mathbf{V}$, which correspond to the representative states. Specifically, $\mathbf{M}^{\dagger}$ has only one element of 1 in each row, while all the other elements equal 0 , and the position of 1 in the $\left(q+(i d(i, c)-1)\left(N_{\mathrm{Q}}+1\right)\right)$ th row $\left(q \in\left\{1, \ldots, N_{\mathrm{Q}}\right\}\right.$ and $i d(i, c) \in\{1, \ldots,|\Theta|\}$ is the index of queue $q_{i}^{(c)}$ within set $\Theta)$ corresponds to the position of the representative state $V\left(\mathbf{Q}_{i, c}^{(q)}\right)$ in the global queue state vector $\mathbf{V}$. Moreover, the position of 1 in the $\left(1+N_{\mathrm{Q}} \times i d(i, c)\right)$ th row corresponds to the position of global queue state with all queues being empty $V\left(\left\{Q_{i}^{(c)}=0 \mid q_{i}^{(c)} \in \Theta\right\}\right)$ in the global queue state vector $\mathbf{V}$.

Now, the vector form of the equivalent Bellman equation (30) under all the representative states can be written as

$$
\begin{equation*}
\theta \mathbf{e}+\tilde{\mathbf{V}}_{\infty}(\boldsymbol{\eta})=\mathbf{M}^{\dagger} \mathbf{T}\left(\boldsymbol{\eta}, \mathbf{M} \tilde{\mathbf{V}}_{\infty}(\boldsymbol{\eta})\right) \tag{35}
\end{equation*}
$$

where $\mathbf{e}$ is an $\left(N_{\mathrm{Q}}+1\right)|\Theta| \times 1$ vector with all elements equal to 1 . The mapping $\mathbf{T}$ is defined as

$$
\mathbf{T}(\boldsymbol{\eta}, \mathbf{V})=\min _{\Omega}[\mathbf{g}(\boldsymbol{\eta}, \Omega)+\mathbf{P}(\Omega) \mathbf{V}]
$$

where $\mathbf{g}(\boldsymbol{\eta}, \Omega)$ is the vector form of function $g(\mathbf{Q}, \Omega(\mathbf{Q}))$ defined in (16), and $\mathbf{P}(\Omega)$ is the matrix form of transition probability $\operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{Q}^{(\grave{i})}\right)\right]$ defined in (16).

Lemma 1 (Convergence of Per-Queue Value Function Learning Over Timescale I): Denote

$$
\begin{aligned}
& \mathbf{A}_{t-1}=\left(1-\epsilon_{t-1}^{v}\right) \mathbf{I}+\mathbf{M}^{\dagger} \mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \epsilon_{t-1} \\
& \mathbf{B}_{t-1}=\left(1-\epsilon_{t-1}^{v}\right) \mathbf{I}+\mathbf{M}^{\dagger} \mathbf{P}\left(\Omega_{t-1}\right) \mathbf{M} \epsilon_{t-1}
\end{aligned}
$$

where $\Omega_{t}$ is the unichain control policy at slot $t, \mathbf{P}\left(\Omega_{t}\right)$ is the transition matrix under the unichain system control policy, and I is the identity matrix. If for the entire sequence of control policies $\left\{\Omega_{t}\right\}$ there exists $\delta_{\beta}>0$ and some positive integer $\beta$, such that

$$
\begin{align*}
& {\left[\mathbf{A}_{\beta} \cdots \mathbf{A}_{1}\right]_{(k, \grave{I})} } \geq \delta_{\beta} \\
& {\left[\mathbf{B}_{\beta} \cdots \mathbf{B}_{1}\right]_{(k, \grave{I})} \geq \delta_{\beta} \quad \forall k } \tag{36}
\end{align*}
$$

where $[\cdot]_{(k, \grave{I})}$ denotes the element in the $k$ th row and the $\grave{I}$ th column(where $\grave{I}$ corresponds to the queue state $\mathbf{Q}^{(\grave{I})}$ that all queues areempty), and $\delta_{t}=\mathcal{O}\left(\epsilon_{t}^{v}\right)$, then the following statements are true.

1) The update of the per-queue value function vector will converge almost surely for any given initial parameter vector $\widetilde{\mathbf{V}}_{0}$ and $L M$ vector $\boldsymbol{\eta}$, i.e.,

$$
\lim _{t \rightarrow \infty} \tilde{\mathbf{V}}_{t}(\boldsymbol{\eta})=\tilde{\mathbf{V}}_{\infty}(\boldsymbol{\eta})
$$

2) The steady-state per-queue value function vector $\tilde{\mathbf{V}}_{\infty}$ satisfies (35).
Proof: See Appendix A.
Remark 8 (Interpretation of the Conditions in Lemma 2): Note that $\mathbf{A}_{t}$ and $\mathbf{B}_{t}$ are related to an equivalent transition matrix of the underlying Markov chain. Equation (36) simply means that the system state $\mathbf{S}^{I}$, representing any system state where all the queue lengths are zero, is accessible from all the system states after some finite number of transition steps. This is a very mild condition and is satisfied in most of the cases that we are interested.

Lemma 2 (Convergence of LMs' Update Over Timescale II): The iteration on the vector of LMs $\boldsymbol{\eta}$ converges almost surely to the set of maxima of $G(\boldsymbol{\eta})$. Supposing that the LMs converge to $\boldsymbol{\eta}^{*}$, then $\boldsymbol{\eta}^{*}$ satisfies the dropping probability constraints in problem 1.

Proof: See Appendix B.

## IV. Simulation Results

Here, we compare our proposed approximate MDP solution with online stochastic learning (see Algorithm 2) to the approximate MDP solution with offline value iteration based on the Bellman equation (30) and two other reference subchannelallocation algorithms. One is the CSI-only algorithm, in which the RRG selection is only adaptive to CSI and a subchannel is allocated to the RRG with the maximum sum over all its link transmission rates at every time slot. The other is the MaxWeight algorithm, which is adaptive to both CSI and QSI, as discussed in Remark 4. The offline value iteration algorithm can find the per-queue value functions and optimal policy that satisfy the Bellman equation (30) and therefore provide the performance upper bound. However, the CSI probability distribution Pr. $[\mathbf{H}]$ needs to be derived, which is very hard when there are more than two links in an RRG, as discussed in Part I. The simulation parameter setting is the same with that in Part I of this paper [1], except that the buffer size is set to be $N_{\mathrm{Q}}=10$ packets and there are $N_{\mathrm{F}}=10$ independent subchannels.

Fig. 2 shows the average weighted sum delay and the maximum dropping probability over all connections versus the mean arrive rate $\lambda$ for the simple network in Fig. 1 of Part I [1], where the number of connections $C=3$ (one D2D connection, one cellular uplink connection, and one cellular downlink connection). The dropping probability constraint is set to $d_{\max }=0.1$. It can be observed that the performance of the proposed approximate MDP solution with online learning is


Fig. 2. Average weighted sum delay and the maximum dropping probability over all connections versus the mean arrive rate $\lambda$ with $C=3\left(D=C_{u}=\right.$ $\left.C_{d}=1\right)$ and $d_{\max }=0.1$.
close to that of the approximate MDP solution using the value iteration algorithm, where both solutions have lower average delay than the two reference subchannel-allocation algorithms. Although the dropping probability of the MaxWeight algorithm is almost the same with both approximate MDP solutions in the light traffic load regime, it grows significantly higher than the approximate MDP solutions when $\lambda$ increases beyond 4 packets/slot. This is because the approximate MDP solutions will guarantee that the dropping probability is no larger than the constraint $d_{\text {max }}$, if this can be achieved by any policy under the given mean arrival rate $\lambda$. As a consequence, the proposed approximate MDP approach with online learning is an effective method to reduce the complexity and achieve optimal performance (with regard to the offline value iteration algorithm), while guaranteeing the dropping probability constraint.

Fig. 3 shows the average weighted sum delay and the maximum dropping probability over all connections versus the number of connections $C$ with $\lambda=1$ packets/slot. The dropping probability constraint is set to $d_{\max }=0.3$. Note that the performance of the approximate MDP solution with value iteration algorithm is not shown in Fig. 3. This is because the aforementioned policy cannot be derived when the number of connections becomes large, since there are some RRGs with more than two links, whose CSI probability distributions are hard to obtain. It is obvious that our proposed approximate MDP solution with online learning algorithm performs better in the average weighted sum delay and the maximum dropping probability than the two other reference algorithms. Among the three algorithms, the CSI-only algorithm performs the worst since it does not take the QSI into account. The dropping probability of the MaxWeight algorithm exceeds $d_{\max }=0.3$ with the increasing number of connections, while our proposed algorithm can always keep it under the constraint. When the connection number $C=30$, our simulation results show that, as compared to the MaxWeight algorithm, the average weighted sum delay and the maximum dropping probability achieved by


Fig. 3. Average weighted sum delay and the maximum dropping probability over all connections versus the number of connections $C$ with $\lambda=1$ packets/slot and $d_{\max }=0.3$. The number of connections is $C=3\left(D=C_{u}=\right.$ $\left.C_{d}=1\right), C=12\left(D=C_{u}=C_{d}=4\right), C=21\left(D=C_{u}=C_{d}=7\right)$, and $C=30\left(D=C_{u}=C_{d}=10\right)$, respectively.


Fig. 4. Convergence property of the proposed online stochastic learning algorithm and the brute-force offline value iteration algorithm with $C=3(D=$ $C_{u}=C_{d}=1$ ) and $\lambda=5$ packets/slot.
our proposed approximate MDP-online learning algorithm are decreased by $10 \%$ and $38 \%$, respectively.

Fig. 4 shows the convergence property of the proposed online stochastic learning algorithm and the brute-force offline value iteration algorithm. We plot a portion of per-queue value functions of the three connections versus the scheduling slot index at a mean arrive rate $\lambda=5$ packets/slot. It can be seen that the online stochastic learning algorithm converges fast, and after 1000 iterations, the values are close to the final converged results. The average weighted sum delay corresponding to the average per-queue value function at the 750th iteration is smaller than the other two reference algorithms. Moreover, it is clear that the value functions calculated online quickly approach the value functions calculated offline when the number of iterations grows.

## V. Conclusion

In this pair of papers, we considered a delay-optimal dynamic-mode-selection and resource-allocation algorithm under dropping probability constraints for network-assisted D2D communications with bursty traffic arrival, which is cast into an infinite-horizon average-reward CMDP in the first part of this work. In the second part of this work, we addressed the issue of exponential memory requirement and computational complexity by using linear value approximation techniques to reduce the state space. Moreover, an online stochastic learning algorithm with two timescales was adopted to update the value functions and LMs based on the real-time observations of CSI and QSI. The obtained solution has a simple structure with a computational complexity of $\mathcal{O}\left(\left(N_{\mathrm{Q}}+1\right)+\left(\left|\mathcal{U}_{\mathrm{u}}\right|+\left|\mathcal{U}_{\mathrm{d}}\right|\right) \times\right.$ $\left.N_{\mathrm{F}}\right)$, which grows linearly with the buffer capacity and the number of RRGs and subchannels. We proved that, under some mild conditions, the proposed approximate MDP and online stochastic learning solution converges almost surely (with probability 1) to a global optimal solution. Simulation results show that the proposed approach outperforms the conventional CSI-only scheme and throughput-optimal scheme (MaxWeight algorithm).

## Appendix A

## Proof of Lemma 2

Since each representative state is updated comparably often in the asynchronous learning algorithm, quoting the conclusion in [17], the convergence property of the asynchronous update and the synchronous update is the same. Therefore, we consider the convergence of the related synchronous version for simplicity in this proof. It is easy to see that the per-queue value function vector $\widetilde{\mathbf{V}}_{t}$ is bounded almost surely during the iterations of the algorithm. In the following, we first introduce and prove the following lemma on the convergence of learning noise.

Lemma 3: Define

$$
\mathbf{q}_{t}=\mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t}\right)+\mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{T}_{0}\left(\mathbf{M} \tilde{\mathbf{V}}_{t}\right) \mathbf{e}\right]
$$

where $\mathbf{T}_{0}(\mathbf{V})=\min _{\Omega}\left[\mathbf{g}_{\grave{I}}(\Omega)+\mathbf{P}_{\grave{I}}(\Omega) \mathbf{V}\right]$ denotes the mapping on the queue state $\mathbf{Q}^{(\grave{I})}$, where $\mathbf{g}_{\grave{I}}(\Omega)$ is the vector form of function $g\left(\mathbf{Q}^{(\grave{I})}, \Omega\left(\mathbf{Q}^{(\grave{I})}\right)\right)$, and $\mathbf{P}_{\grave{I}}(\Omega)$ is the matrix form of transition probability $\operatorname{Pr} .\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{Q}^{(\grave{I})}, \Omega\left(\mathbf{Q}^{(\grave{I})}\right)\right]$. When the number of iterations $t \geq j \rightarrow \infty$, the procedure of update can be written as follows with probability 1 :

$$
\tilde{\mathbf{V}}_{t+1}=\tilde{\mathbf{V}}_{j}+\sum_{i=j}^{t} \epsilon_{i}^{v} \mathbf{q}_{i}
$$

Proof: The synchronous update of per-queue value function vector can be written in the following vector form:
$\tilde{\mathbf{V}}_{t+1}=\tilde{\mathbf{V}}_{t}+\epsilon_{i}^{v} \mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t}\right)+\mathbf{J}_{t} \mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{T}_{0}\left(\mathbf{M} \tilde{\mathbf{V}}_{t}\right) \mathbf{e}\right]$
where the matrix $\mathbf{J}_{t}$ is the matrix form of queue state transition probability Pr. $\left[\mathbf{Q}^{(\grave{y})} \mid \mathbf{H}_{t(\grave{z})}, \mathbf{Q}^{(\grave{z})}, \Omega\left(\mathbf{H}_{t(\grave{z})}, \mathbf{Q}^{(\dot{z})}\right)\right]$ with given
$\mathbf{H}_{t(\grave{z})}$ in each row, which is the real-time observation of channel state at time slot $t(\grave{z})$ with queue state $\mathbf{Q}_{t(\grave{z})}=\mathbf{Q}^{(\grave{z})}$. Define

$$
\mathbf{Y}_{t}=\mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t}\right)+\mathbf{J}_{t} \mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{T}_{0}\left(\mathbf{M} \tilde{\mathbf{V}}_{t}\right) \mathbf{e}\right]
$$

and $\delta \mathbf{Z}_{t}=\mathbf{q}_{t}-\mathbf{Y}_{t}$ and $\mathbf{Z}_{t}=\sum_{i=j}^{t} \epsilon_{i}^{v} \delta \mathbf{Z}_{i}$. The online value function estimation can be rewritten as

$$
\begin{equation*}
\tilde{\mathbf{V}}_{t+1}=\tilde{\mathbf{V}}_{t}+\epsilon_{i}^{v} \mathbf{Y}_{t}=\tilde{\mathbf{V}}_{t}+\epsilon_{i}^{v} \mathbf{q}_{t}-\epsilon_{i}^{v} \delta \mathbf{Z}_{t}=\tilde{\mathbf{V}}_{j}+\sum_{i=j}^{t} \epsilon_{i}^{v} \mathbf{q}_{i}-\mathbf{Z}_{t} \tag{37}
\end{equation*}
$$

Our proof of Lemma 4 can be divided into the following steps:

1) Step 1: Letting $\mathcal{F}_{t}=\sigma\left(\mathbf{V}_{m}, m \leq t\right)$, it is easy to see that $\mathbf{E}\left[\delta \mathbf{Z}_{t} \mid \mathcal{F}_{t-1}\right]=0$. Thus, $\left\{\delta \mathbf{Z}_{t} \mid \forall t\right\}$ is a Martingale difference sequence, and $\left\{\mathbf{Z}_{t} \mid \forall t\right\}$ is a Martingale sequence. Moreover, $\mathbf{Y}_{t}$ is an unbiased estimation of $\mathbf{q}_{t}$, and the estimation noise is uncorrelated.
2) Step 2: According to the uncorrelated estimation error from step 1 , when $j \rightarrow \infty$, we have

$$
\begin{aligned}
\mathbf{E}\left[\left|\mathbf{Z}_{t}\right|^{2} \mid \mathcal{F}_{j-1}\right] & =\mathbf{E}\left[\left|\sum_{i=j}^{t} \epsilon_{i}^{v} \delta \mathbf{Z}_{i}\right|^{2} \mid \mathcal{F}_{j-1}\right] \\
& =\sum_{i=j}^{t} \mathbf{E}\left[\left|\epsilon_{i}^{v} \delta \mathbf{Z}_{i}\right|^{2} \mid \mathcal{F}_{j-1}\right] \\
& =\widetilde{\mathbf{Z}} \sum_{i=j}^{t}\left(\epsilon_{i}^{v}\right)^{2} \rightarrow 0
\end{aligned}
$$

where $\widetilde{\mathbf{Z}} \geq \max _{j \leq i \leq t} \mathbf{E}\left[\left|\delta \mathbf{Z}_{i}\right|^{2} \mid \mathcal{F}_{j-1}\right]$ is a bounded constant vector, and the convergence of $\widetilde{\mathbf{Z}} \sum_{i=j}^{t}\left(\epsilon_{i}^{v}\right)^{2}$ is from the definition of sequence $\left\{\epsilon_{i}^{v}\right\}$.
3) Step 3: From step $1,\left\{\delta \mathbf{Z}_{t} \mid \forall t\right\}$ is a Martingale sequence. Hence, according to the inequality of Martingale sequence, we have

$$
\operatorname{Pr} .\left[\sup _{j \leq i \leq t}\left|\mathbf{Z}_{i}\right| \geq \lambda \mid \mathcal{F}_{j-1}\right] \leq \frac{\mathbf{E}\left[\left|\mathbf{Z}_{t}\right|^{2} \mid \mathcal{F}_{j-1}\right]}{\lambda^{2}} \quad \forall \lambda>0
$$

From the conclusion of step 2, we have

$$
\lim _{j \rightarrow \infty} \operatorname{Pr} \cdot\left[\sup _{j \leq i \leq t}\left|\mathbf{Z}_{i}\right| \geq \lambda \mid \mathcal{F}_{j-1}\right]=0 \quad \forall \lambda>0
$$

Hence, from (37), we almost surely have $\widetilde{\mathbf{V}}_{t+1}=\widetilde{\mathbf{V}}_{j}+$ $\sum_{i=j}^{t} \epsilon_{i}^{v} \mathbf{q}_{i}$ when $j \rightarrow \infty$.

Moreover, the following lemma is about the limit of sequence $\left\{\mathbf{q}_{t}\right\}$.

Lemma 4: Supposing that the following two inequalities are true for $t=m, m+1, \ldots, m+n$ :

$$
\begin{align*}
& \mathbf{g}\left(\Omega_{t}\right)+\mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \tilde{\mathbf{V}}_{t} \leq \mathbf{g}\left(\Omega_{t-1}\right)+\mathbf{P}\left(\Omega_{t-1}\right) \mathbf{M} \tilde{\mathbf{V}}_{t}  \tag{38}\\
& \mathbf{g}\left(\Omega_{t-1}\right)+\mathbf{P}\left(\Omega_{t-1}\right) \mathbf{M} \tilde{\mathbf{V}}_{t-1} \leq \mathbf{g}\left(\Omega_{t}\right)+\mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \tilde{\mathbf{V}}_{t-1} \tag{39}
\end{align*}
$$

we then have

$$
\lim _{t \rightarrow+\infty} \mathbf{q}_{t}=0
$$

Proof: From (38) and (39), we have

$$
\begin{aligned}
\mathbf{q}_{t} & =\mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t}\right)+\mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{M} \tilde{\mathbf{V}}_{t}-\omega_{t} \mathbf{e}\right] \\
& \leq \mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t-1}\right)+\mathbf{P}\left(\Omega_{t-1}\right) \mathbf{M} \tilde{\mathbf{V}}_{t}-\mathbf{M} \tilde{\mathbf{V}}_{t}-\omega_{t} \mathbf{e}\right] \\
\mathbf{q}_{t-1} & =\mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t-1}\right)+\mathbf{P}\left(\Omega_{t-1}\right) \mathbf{M} \tilde{\mathbf{V}}_{t-1}-\mathbf{M} \tilde{\mathbf{V}}_{t-1}-\omega_{t-1} \mathbf{e}\right] \\
& \leq \mathbf{M}^{\dagger}\left[\mathbf{g}\left(\Omega_{t}\right)+\mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \tilde{\mathbf{V}}_{t-1}-\mathbf{M} \tilde{\mathbf{V}}_{t-1}-\omega_{t-1} \mathbf{e}\right]
\end{aligned}
$$

where $\omega_{t}=\mathbf{T}_{0}\left(\mathbf{M} \tilde{\mathbf{V}}_{t}\right)$. According to Lemma 4, we have

$$
\tilde{\mathbf{V}}_{t}=\tilde{\mathbf{V}}_{t-1}+\epsilon_{t-1}^{v} \mathbf{q}_{t-1}
$$

Therefore

$$
\begin{aligned}
\mathbf{q}_{t} & \geq\left[\left(1-\epsilon_{t-1}^{v}\right) \mathbf{I}+\mathbf{M}^{\dagger} \mathbf{P}\left(\Omega_{t}\right) \mathbf{M} \epsilon_{t-1}\right] \mathbf{q}_{t-1}+\omega_{t-1} \mathbf{e}-\omega_{t} \mathbf{e} \\
& =\mathbf{A}_{t-1} \mathbf{q}_{t-1}+\omega_{t-1} \mathbf{e}-\omega_{t} \mathbf{e} \\
\mathbf{q}_{t} & \leq\left[\left(1-\epsilon_{t-1}^{v}\right) \mathbf{I}+\mathbf{M}^{\dagger} \mathbf{P}\left(\Omega_{t-1}\right) \mathbf{M} \epsilon_{t-1}\right] \mathbf{q}_{t-1}+\omega_{t-1} \mathbf{e}-\omega_{t} \mathbf{e} \\
& =\mathbf{B}_{t-1} \mathbf{q}_{t-1}+\omega_{t-1} \mathbf{e}-\omega_{t} \mathbf{e}
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
& \mathbf{A}_{t-1} \cdots \mathbf{A}_{t-\beta} \mathbf{q}_{t-\beta}-C_{1} \mathbf{e} \leq \mathbf{q}_{t} \leq \mathbf{B}_{t-1} \cdots \mathbf{B}_{t-\beta} \mathbf{q}_{t-\beta}-C_{1} \mathbf{e} \\
& \quad \Rightarrow\left(1-\delta_{\beta}\right)\left(\min \mathbf{q}_{t-\beta}\right) \mathbf{e} \leq \mathbf{q}_{t}+C_{1} \mathbf{e} \leq\left(1-\delta_{\beta}\right)\left(\max \mathbf{q}_{t-\beta}\right) \mathbf{e} \\
& \quad \Rightarrow\left\{\begin{array}{l}
\min \mathbf{q}_{t}+C_{1} \geq\left(1-\delta_{\beta}\right) \min \mathbf{q}_{t-\beta} \\
\max \mathbf{q}_{t}+C_{1} \leq\left(1-\delta_{\beta}\right) \max \mathbf{q}_{t-\beta}
\end{array}\right. \\
& \quad \Rightarrow \max \mathbf{q}_{t}-\min \mathbf{q}_{t} \leq\left(1-\delta_{\beta}\right)\left(\max \mathbf{q}_{t-\beta}-\min \mathbf{q}_{t-\beta}\right) \\
& \quad \Rightarrow\left|\mathbf{q}_{t}^{k}\right| \leq \max \mathbf{q}_{t}-\min \mathbf{q}_{t} \leq C_{2}\left(1-\delta_{\beta}\right) \quad \forall k .
\end{aligned}
$$

Then, we have

$$
\begin{equation*}
0 \leq\left|\mathbf{q}_{m+n}^{k}\right| \leq C_{3} \prod_{i=0}^{\lfloor n / \beta\rfloor-1}\left(1-\delta_{m+i \beta}\right)=0 \quad \forall k \tag{40}
\end{equation*}
$$

where the first step is due to conditions on matrix sequence $\mathbf{A}_{t}$ and $\mathbf{B}_{t} ; \min \mathbf{q}_{t}$ and max $\mathbf{q}_{t}$ denote the minimum and maximum elements in $\mathbf{q}_{t}$, respectively; $\mathbf{q}_{t}^{k}$ denotes the $k$ th element of the vector $\mathbf{q}_{t} ;\left|\mathbf{q}_{t}^{k}\right| \leq \max \mathbf{q}_{t}-\min \mathbf{q}_{t}$ is due to $\min \mathbf{q}_{t} \leq 0$; and $C_{1}, C_{2}$, and $C_{3}$ are constants. According to the property of sequence $\left\{\epsilon_{t}^{v}\right\}$, we have

$$
\lim _{t \rightarrow+\infty} \prod_{i=0}^{\lfloor t / \beta\rfloor-1}\left(1-\epsilon_{i \beta}\right)=0
$$

In addition, noting that $\delta_{t}=\mathcal{O}\left(\epsilon_{t}^{v}\right)$, from (40), we have

$$
\lim _{t \rightarrow+\infty} \mathbf{q}_{t}^{k}=0 \quad \forall k
$$

Summarizing the aforementioned conclusions, we have

$$
\lim _{t \rightarrow+\infty} \mathbf{q}_{t}=0
$$

Therefore, (35) is straightforward when $\mathbf{q}_{t} \rightarrow 0$. This completes the proof.

## Appendix B

## Proof of Lemma 3

Due to the separation of timescale, the primal update of the per-node value function converges to $\widetilde{\mathbf{V}}_{\infty}(\boldsymbol{\eta})$ w.r.t. current LM $\boldsymbol{\eta}$ [15]. By [18, Lemma 4.2], $G(\boldsymbol{\eta})$ is a concave and continuously differentiable, except at finitely many points where both right and left derivatives exist. Since the subchannel-allocation policy is discrete, we have $\Omega^{*}(\boldsymbol{\eta})=\Omega^{*}\left(\boldsymbol{\eta}+\triangle_{\eta}\right)$, i.e., $\nabla_{\eta}=$ $\left(\Omega^{*}\left(\boldsymbol{\eta}+\triangle_{\eta}\right)-\Omega^{*}(\boldsymbol{\eta})\right) / \triangle_{\eta}=0$. Therefore, $\partial G\left(\boldsymbol{\eta}_{t}\right) / \partial \boldsymbol{\eta}_{t}$ is expressed in the equation, shown at the bottom of the page, where $\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)=\arg \min _{\Omega} G\left(\boldsymbol{\eta}_{t}\right)$. Using the standard stochastic approximation theorem [14], the dynamics of the LM update equation in (33) can be represented by the following ordinary differential equation (ODE), shown at the bottom of the page. Therefore, we show that the aforementioned ODE can be expressed as $\boldsymbol{\eta}_{t}^{\prime}=\nabla G\left(\boldsymbol{\eta}_{t}\right)$. As a result, the aforementioned ODE will converge to $\nabla G\left(\boldsymbol{\eta}_{t}\right)=0$, which corresponds to (33). This completes the proof.

$$
\frac{\partial G\left(\boldsymbol{\eta}_{t}\right)}{\partial \boldsymbol{\eta}_{t}}= \begin{cases}\mathbf{E}^{\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)}\left[\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{(c+D) 0}\right)\right], & \text { if } c \in \mathcal{C}_{\mathrm{Cu}} \\ \mathbf{E}^{\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)\left[\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{0(c+D)}\right)\right],} & \text { if } c \in \mathcal{C}_{\mathrm{Cd}} \\ \mathbf{E}^{\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)}\left[\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{(2 c-1)(2 c)}-T_{0(2 c)}\right)\right], & \text { if } c \in \mathcal{C}_{\mathrm{D}}\end{cases}
$$

$$
\boldsymbol{\eta}_{t}^{\prime}= \begin{cases}\mathbf{E}^{\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)}\left[\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{(c+D) 0}\right)\right], & \text { if } c \in \mathcal{C}_{\mathrm{Cu}} \\ \mathbf{E}^{\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)}\left[\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{0(c+D)}\right)\right], & \text { if } c \in \mathcal{C}_{\mathrm{Cd}} \\ \mathbf{E}^{\Omega^{*}\left(\boldsymbol{\eta}_{t}\right)}\left[\left(1-d_{\max }\right)\left(\lambda_{c}\left(1-d_{\max }\right)-T_{(2 c-1)(2 c)}-T_{0(2 c)}\right)\right], & \text { if } c \in \mathcal{C}_{\mathrm{D}}\end{cases}
$$

## REFERENCES

[1] L. Lei et al., "Delay-optimal dynamic mode selection and resource allocation in device-to-device communications-Part I: Optimal policy" IEEE Trans. Veh. Technol., vol. 65, no. 5, pp. 3474-3490, May 2016.
[2] G. Fodor, et al., "Design aspects of network assisted device-to-device communications," IEEE Commun. Mag., vol. 50, no. 3, pp. 170-177, Mar. 2012.
[3] L. Lei, Z. Zhong, C. Lin, and X. Shen, "Operator controlled device-todevice communications in LTE-advanced networks," IEEE Wireless Mag., vol. 19, no. 3, pp. 96-104, Jun. 2012.
[4] L. Song, D. Niyato, Z. Han, and E. Hossain, Wireless Device-to-Device Communications and Networks. Cambridge, U.K.: Cambridge Univ. Press, 2015.
[5] H. Nishiyama, M. Ito, and N. Kato, "Relay-by-smartphone: Realizing multi-hop device-to-device communications," IEEE Wireless Commun., vol. 52, no. 4, pp. 56-65, Apr. 2014.
[6] M. L. Puterman, Markov Decision Processes: Discrete Stochastic Dynamic Programming. New York, NY, USA: Wiley, 2005.
[7] D. P. Bertsekas, Dynamic Programming and Optimal Control, 3rd ed. Belmont, MA, USA: Athena Scientific, 2007.
[8] Y. Cui, V. K. N. Lau, and R. Wang, "A survey on delay-aware resource control for wireless systems-large deviation theory, stochastic Lyapunov drift, and distributed stochastic learning," IEEE Trans. Inf. Theory, vol. 58, no. 3, pp. 1677-1701, Mar. 2012.
[9] R. Wang and V. K. Lau, "Delay-aware two-hop cooperative relay communications via approximate MDP and stochastic learning," IEEE Trans. Inf. Theory, vol. 59, no. 11, pp. 7645-7670, Nov. 2013.
[10] M. Moghaddari, E. Hossain, and L. B. Le, "Delay-optimal distributed scheduling in multi-user multi-relay cellular wireless networks," IEEE Trans. Wireless Commun., vol. 61, no. 4, pp. 1349-1360, Apr. 2013.
[11] G. Naddafzadeh-Shirazi, K. Peng-Yong, and T. Chen-Khong, "Distributed reinforcement learning frameworks for cooperative retransmission in wireless networks," IEEE Trans. Veh. Technol., vol. 59, no. 8, pp. 4157-4162, Oct. 2010.
[12] K. Zheng, F. Liu, L. Lei, C. Lin, and Y. Jiang, "Stochastic performance analysis of a wireless finite-state Markov channel," IEEE Trans. Wireless Comтип., vol. 12, no. 2, pp. 782-793, Feb. 2013.
[13] L. Georgiadis, M. J. Neely, and L. Tassiulas, "Resource allocation and cross-layer control in wireless networks," Found. Trends Netw., vol. 1, no. 1, pp. 1-144, 2006.
[14] H. J. Kushner and G. G. Yin, Stochastic Approximation and Optimization of Random Systems, 2nd ed. New York, NY, USA: Springer-Verlag, 2003.
[15] V. S. Borkar, "Stochastic approximation with two time scales," Syst. Control Lett., vol. 29, no. 5, pp. 291-294, Feb. 1997.
[16] Q. Liu, S. Zhou, and G. B. Giannakis, "Queueing with adaptive modulation and coding over wireless links: Cross-layer analysis and design," IEEE Trans. Wireless Commun., vol. 4, no. 3, pp. 1142-1153, May 2005.
[17] V. S. Borkar, "Asynchronous stochastic approximation," SIAM J. Control Optim., vol. 36, pp. 840-851, 1998.
[18] V. S. Borkar, "An actor-critic algorithm for constrained Markov decision processes," Syst. Control Lett., vol. 54, no. 3, pp. 207-213, Mar. 2005.


Lei Lei (M'13) received both the B.S. and Ph.D. degrees in telecommunications engineering from the Beijing University of Posts and Telecommunications, Beijing, China, in 2001 and 2006, respectively.

From July 2006 to March 2008, she was a Postdoctoral Fellow with the Computer Science Department, Tsinghua University, Beijing. From April 2008 to August 2011, she was with the Wireless Communications Department, China Mobile Research Institute. Since September 2011, she has been an Associate Professor with the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University. Her current research interests include performance evaluation, quality of service, and radio resource management in wireless communication networks.


Yiru Kuang received the B.S. degree from Beijing Jiaotong University, Beijing, China, in 2013. She is currently working toward the master's degree with the same university.

Her research interests are in wireless communications, particularly radio resource optimization and device-to-device communications.


Nan Cheng (S'13) received the B.S. and M.S. degrees from Tongji University, Shanghai, China, in 2009 and 2012, respectively. He is currently working toward the Ph.D. degree with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada.

Since 2012, he has been a Research Assistant with the Broadband Communication Research Group, Department of Electrical and Computer Engineering, University of Waterloo. His research interests include vehicular communication networks, cognitive radio networks, resource allocation in smart grid, and cellular traffic offloading.


Xuemin (Sherman) Shen (M'97-SM'02-F'09) received the B.Sc. degree from Dalian Maritime University (China), Dalian, China, in 1982 and the M.Sc. and Ph.D. degrees from Rutgers University, Newark, NJ, USA, in 1987 and 1990, respectively, all in electrical engineering.

He is a Professor and University Research Chair with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada, where he was the Associate Chair for Graduate Studies from 2004 to 2008. His research focuses on resource management in interconnected wireless/wired networks, wireless network security, social networks, smart grid, and vehicular ad hoc and sensor networks. He is a coauthor/editor of six books and has published more than 600 papers and book chapters on wireless communications and networks, control, and filtering.

Dr. Shen is a Registered Professional Engineer in Ontario, Canada. He is a Fellow of the Engineering Institute of Canada and the Canadian Academy of Engineering and a Distinguished Lecturer of the IEEE Vehicular Technology and Communications Societies (ComSoc). He is an elected member of the IEEE ComSoc Board of Governors and the Chair of the Distinguished Lecturers Selection Committee. He served as the Technical Program Committee Chair/Cochair for the IEEE International Conference on Computer Communications (Infocom'14) and the IEEE Vehicular technology Conference (VTC'10 Fall), the Symposia Chair for the IEEE International Conference on Communications (ICC'10), the Tutorial Chair for IEEE VTC'11 Spring and IEEE ICC'08, the Technical Program Committee Chair for the IEEE Global Communications Conference (Globecom'07), the General Cochair for the International Conference on Communications and Networking in China (Chinacom'07) and the International Conference on Quality of Service in Heterogeneous Wired/Wireless Networks (QShine'06), and the Chair for the IEEE ComSoc Technical Committee on Wireless Communications and on peer-to-peer Communications and Networking. He also serves/served as the Editor-in-Chief for the IEEE NETwORK, Peer-to-Peer Networking and Application, and IET Communications; a Founding Area Editor for the IEEE Transactions on Wireless Communications; an Associate Editor for the IEEE Transactions on Vehicular Technology, Computer Networks, the Association for Computing Machinery (ACM) Wireless Networks, etc.; and the Guest Editor for the IEEE Journal on Selected Areas in Communications, IEEE Wireless Communications, IEEE Communications Magazine, ACM Mobile Networks and Applications, etc. He received the Excellent Graduate Supervision Award in 2006 and the Outstanding Performance Award in 2004, 2007, and 2010 from the University of Waterloo; the Premier's Research Excellence Award in 2003 from the Province of Ontario; and the Distinguished Performance Award in 2002 and 2007 from the Faculty of Engineering, University of Waterloo.


Zhangdui Zhong received the B.S. and Ph.D. degrees from Northern Jiaotong University, Beijing, China in 1983 and 1988, respectively.

He is a Professor and Advisor of Ph.D. candidates with the Beijing Jiaotong University, Beijing, China, where he is currently the Dean of the School of Computer and Information Technology and a Chief Scientist of the State Key Laboratory of Rail Traffic Control and Safety. He is also a Director of the Innovative Research Team of the Ministry of Education and a Chief Scientist of the Ministry of Railways in China. His interests include wireless communications for railways, control theory and techniques for railways, and Global System for Mobile Communications-Railway (GSM-R) systems. His research has been widely used in railway engineering, such as for the Qinghai-Xizang railway, the Datong-Qinhuangdao heavy-haul railway, and many high-speed railway lines in China. He has authored/coauthored seven books, five invention patents, and over 200 scientific research papers in his research area.

Dr. Zhong is an Executive Council Member of the Radio Association of China and a Deputy Director of the Radio Association of Beijing. He has received the Mao Yisheng Scientific Award of China, the Zhan Tianyou Railway Honorary Award of China, and the Top 10 Science/Technology Achievements Award of Chinese Universities.


Chuang Lin (SM'04) received the Ph.D. degree in computer science from Tsinghua University, Beijing, China, in 1994.

He is a Professor with the Department of Computer Science and Technology, Tsinghua University. He is a Honorary Visiting Professor with the University of Bradford, Bradford, U.K. His current research interests include computer networks, performance evaluation, network security analysis, and Petri net theory and its applications. He has published more than 500 papers in research journals and IEEE conference proceedings in these areas, as well as five books.

Prof. Lin serves as the Technical Program Vice Chair for the 10th IEEE Workshop on Future Trends of Distributed Computing Systems (FTDCS 2004); the General Chair for the ACM SIGCOMM Asia Workshop 2005 and the 2010 IEEE International Workshop on Quality of Service (IWQoS 2010); an Associate Editor for the IEEE Transactions on Vehicular Technology; and the Area Editor for the Journal of Computer Networks.


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    L. Lei, Y. Kuang, and Z. Zhong are with the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China (e-mail: leil@bjtu.edu.cn).
    N. Cheng and X. Shen are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada.
    C. Lin is with the Department of Computer Science and Technology, Tsinghua University, Beijing 100084, China.

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[^1]:    ${ }^{1}$ The data can take units of bits or packets. The latter is appropriate when all the packets have fixed length.
    ${ }^{2}$ The instantaneous data rate can take units of bits/slot or packets/slot. The latter is appropriate when all the packets have fixed length and when the achievable data rates are constrained to integral multiples of the packet size.

[^2]:    ${ }^{3}$ In Part I, we have proven that the optimal policy is either a deterministic policy or a mix of two deterministic policies. In Part II, we focus only on the deterministic policy to facilitate implementation.

