Channel Stuffing with Short-Term Interest in Market Value

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We study how a manager’s short-term interest in the firm’s market value may motivate channel stuffing: shipping excess inventory to the downstream channel. Channel stuffing allows a manager to report sales in excess of demand in order to influence investors’ valuation of the firm. We apply an inventory model which highlights the potential role of inventory in the manager’s channel stuffing and the investors’ valuation strategies. Sales in our model are constrained by available inventory. Our model yields a semi-separating and semi-pooling equilibrium contingent on the initial inventory level: When the demand is lower than a threshold that depends on and is below the initial inventory level, the manager pads sales by a part of the excess inventory and releases the inflated sales report. The investors “correct” the reported sales and are able to infer perfectly the firm’s value. When the demand reaches or exceeds this threshold, the manager pads any excess inventory to the sales and reports the initial inventory is sold out, which censors large demand realizations. Then, the investors only infer the real demand is no less than the threshold and value the firm accordingly by expectation. Channel stuffing can influence the inventory decision, too. We find both over- and under-investment in the initial inventory can arise in our model. We discuss empirical and managerial implications of our findings.

Keywords: Channel Stuffing, Inventory Management, Market Value

1. Introduction

According to Meyer (1998), in 1997, the 200 largest U.S. firms allocated more than 13 percent of their outstanding shares for management incentive plans. Such compensation links managers’ interest to their firms’ market value. Since managers typically know more about the firm’s internal operating environment than external parties, they may try to influence the firm’s market value. In some cases, such efforts can deteriorate the firm’s real performance. For instance, as sales are important for assessing a firm’s value, managers may engage in channel stuffing: shipping excess inventory to the downstream channel to report more sales in the short term, even if doing so creates no real value and instead costs the firm in the long term.
U.S. Security Exchange Committee (SEC) has prosecuted firms and/or their managers for activities related to channel stuffing, thereby severely impacting firms’ long-term performance.\(^1\) Besides those cases investigated by SEC, empirical research has found some common “fiscal-year-end phenomena” in which firms’ reported sales surge at the end of fiscal years (Oyer 1998) while inventory dips (Lai 2008). As both Oyer and Lai discuss in their studies, managerial incentives may have led managers to pull future demand forward into the fiscal fourth quarters before reporting.

The accounting literature recognizes channel stuffing as a form of real earnings management, reflecting that firms and/or managers use distortions of real operations to manage earnings. Real earnings management differs from accrual earnings management in that the latter influences a firm’s financial reporting through adopting alternative accounting policies or estimation of discretionary accruals but has no direct cash flow implications (see, e.g., Roychowdhury 2006). According to Cohen et al. (2008), real earnings management increased significantly after the passage of Sarbanes-Oxley Act (SOX) in 2002; “firms switched [from accrual earnings management] to managing earnings using real methods, possibly because these techniques, while more costly, are likely to be harder to detect.” Therefore, understanding potential driving forces for real earnings management, such as channel stuffing, and the impacts on firms’ operations is meaningful.

In this paper, we study how a manager’s short-term interest in the firm’s market value may motivate channel stuffing.\(^2\) We base our study on a two-period inventory management model with stochastic and positively correlated demands and lost sales. We focus on a self-interested manager who runs the firm. For convenience, we use “investors” to represent all possible external parties that value the firm (and call their valuation the market value of the firm).\(^3\) Besides his interest in the firm’s real value at the end of the second period, the manager cares about the firm’s market value at the end of the first period. The investors incorporate all accessible information, including the

\(^1\) Administrative proceedings or complaints can be found on the SEC’s website (http://www.sec.gov/), which include firms and/or their managers of, for instance, The Coca-Cola Company, Bristol-Myers Squibb Company, McAfee, Inc., Lucent Technologies, Inc., Sunbeam Products, Inc., Virbac Corporation, and Clearone Communications, Inc.

\(^2\) Beyond our focus, channel stuffing can arise for many other reasons. For instance, managers may ship more inventory to the downstream channel if they receive bonuses that depend on sales or revenues, or more positively, if to install more inventory can sometimes even stimulate demand (see Balakrishnan et al. 2004).

\(^3\) Channel stuffing can arise in both public and private firms. Managers of private firms may care about (potential) investors’ valuation, too, for instance, if their firm is in the IPO process or if they are attracting strategic investors. Thus we do not differentiate between whether the focused firm is public or private.
manager’s sales report, to value the firm; however, the investors do not observe the real demand. In our model, a larger demand in the first period can lead to a higher firm value due to increased sales revenues and reduced holding costs in the first period, as well as a larger expected demand in the second period because of the positive correlation. Although the reported sales should represent the real demand, in order to obtain a higher market value of the firm, the manager may be tempted to pad leftover inventory to the sales by “stuffing the channel” in the first period.

Our model yields a semi-separating and semi-pooling equilibrium of channel stuffing and market valuation contingent on the amount of inventory the manager installs in the first period (thereafter, the initial inventory). When the demand realization in the first period is lower than a threshold that depends on and is below the initial inventory, a part of the leftover inventory is padded to the sales in such a way that the reported sales (i.e., real demand plus padded sales) follow a strictly increasing function of the demand realization. The investors can thus infer perfectly the real demand as well as the padded sales, and value the firm accordingly (i.e., semi-separating). When the demand realization reaches or exceeds this threshold, the manager pads any excess inventory to the sales and reports the initial inventory is sold out, which censors large demand realizations. The investors can no longer infer the real demand or the padded sales and therefore value the firm at its expected value with the belief that the demand realization is no less than that threshold (i.e., semi-pooling). The demand censoring leads to a steep (discontinuous) increase in the investors’ valuation of the firm when the reported sales reach the inventory level, which in turn motivates the manager to pad all leftover inventory in the semi-pooling part of the equilibrium.

Our analysis shows that given any amount of initial inventory, channel stuffing will be more substantial in our model if the manager has a larger short-term interest in the firm’s market value or if the firm has a higher sales price, a lower inventory purchasing cost, a higher holding cost, a higher correlation of the demands, or a lower channel stuffing cost. The amount of initial inventory influences channel stuffing, too. In our model, more initial inventory always leads to more channel stuffing in the semi-separating part of the equilibrium. In the semi-pooling part of the equilibrium, however, more initial inventory may lead to less channel stuffing. Finally, we find that the incentives
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for channel stuffing influence the manager’s inventory decision. The manager can either under- or over-invest in initial inventory, compared to the first best in which channel stuffing is excluded.

Hence inventory plays an important role in the equilibrium. The investors utilize the inventory information in addition to sales to infer the real demand. The manager, however, may utilize the inventory constraint by stuffing the channel with all the leftover inventory to disguise low demand. Our study not only sheds light on explorations of managers’ incentives for channel stuffing and investors’ valuation strategies but also reveals the potential impact of channel stuffing on firms’ operations. The real earnings management literature has not identified this role of inventory, a real component of operations, and the inventory literature has not explored the impacts of real earnings management on operations. We discuss empirical and managerial implications of our findings.

The remaining sections are organized as follows: The next section reviews the related literature. Section 3 describes the model. We analyze the firm’s real value and reveal the channel stuffing incentives in section 4. We explore the equilibrium as well as the impact of channel stuffing on operations in section 5. We discuss the implications of our findings and conclude in section 6.

2. Related Literature

The real earnings management literature motivates our study. In a recent survey study, Graham et al. (2005) show that managers may take real economic activities to maintain accounting appearances. In particular, 39.1 percent of survey participants indicated they would provide incentives for customers to buy more product to influence the reported performance. Roychowdhury (2006) draws a similar conclusion, finding that managers may distort real operations besides accrual manipulation to manage earnings.

More closely related to our study, Oyer (1998) shows the fiscal-year-end sales pattern: sales at the industry level of a large panel of manufacturing firms are higher at the end of the fiscal year and lower at the beginning than they are in the middle. Correspondingly, Lai (2008) reveals an opposite fiscal-year-end inventory pattern: inventory is 10 percent lower in the fourth fiscal quarter in a large panel of manufacturers, wholesalers, and retailers. The increases in sales sometimes can be attributed to changes in the underlying business factors, such as an economy upturn or business
seasonalities, whereas, such changes may influence firms’ service levels, for instance, firms may increase inventory if they expect higher demand. Concurrent sales surges and inventory dips could lead to the suspicion of the existence of internal forces that intentionally rush sales. Both Oyer and Lai discuss managerial incentives may have caused the observed fiscal-year-end effects.

When investors value a firm, they may take into account these incentives. Hendricks and Singhal (2009) find that when excess inventory is at the announcing firm’s customers, the stock market reaction is more negative than when the excess inventory is at the announcing firm: “inventory buildups at customers could suggest that the announcing firm may have engaged in ‘channel stuffing’ or ‘trade loading,’ which could raise concerns about earnings management and possibility of lawsuits and litigation.” Given that investors may factor possible channel stuffing into their valuation, managers may not always have incentives to stuff the channel. The above empirical findings motivate us to explore the equilibrium behavior of channel stuffing and market valuation.

Similar to our research, Stein (1989) studies real earnings management where a manager cares about the firm’s market value. The manager may pull future earnings forward to inflate current earnings and obtain a higher market value of the firm by, for instance, liquidating assets that are not yet ripe. Stein finds that in equilibrium, the manager inflates the current earnings through inefficient actions, and the investors recognize his incentive and value the firm accordingly. We extend such exploration to a more detailed operations problem, inventory management and channel stuffing. Our model places a “real” constraint on channel stuffing—the availability of inventory. Such a constraint and the exploration of its impact are absent in Stein’s model.

Our work also relates to several studies in operations management that address agency issues. Wang and Zipkin (2009) investigate managers’ sales and purchasing decisions in a supply-chain context with a buy-back contract. They reveal that the managers in the up- and downstream firms may both have incentives to over-sell and over-order, which leads to a stuffed channel, when their compensation contracts are not aligned with their firm’s net profit. In Chen et al. (2010), it is also shown that when the manager of the downstream firm is compensated based on revenue he may over-order, which interestingly may alleviate the supplier hold-up problem when capacity
investment needs to be sunk upfront, for instance, in the semiconductor industry. Analogous to our study, Debo et al. (2008) investigate a self-interested service expert who, with private information concerning the work amount for customers, may pad the service in order to increase revenue. Xu and Birge (2008) discuss how a bonus scheme linked to the firm’s operating income may impact the manager’s inventory and financing decisions, and Chen (2000, 2005) and Chen and Xiao (2009) explore optimal contracts to motivate a sales force. In contrast to this literature, our research introduced managerial incentives that are linked to firms’ market values and examines the equilibrium behavior of sales and inventory decisions.

Finally, in line with the recent empirical literature in operations management, our study explores the interactions of operations and market valuation. Fisher et al. (2002) show that sales growth positively affects a firm’s long-term stock returns, whereas Chen et al. (2005) and Hendricks and Singhal (2009) show that excess inventory often negatively affects stock returns. With real examples, Raman (2006) illustrates that the performance of inventory management, such as inventory turnover, can have a substantial impact on firms’ stock returns; correspondingly, Gaur et al. (2005) investigate inventory turnover and assert that gross margin, capital intensity, and sales surprise can largely explain the variation of this metric. More broadly, Hendricks and Singhal (2001) reveal that investors generally respond positively to a firm’s adoption of total quality management, whereas they show in Hendricks and Singhal (1997, 2005) that the announcement of a product introduction delay or supply chain disruption can cause a substantial drop in a firm’s stock price. The above literature explores how a firm’s operations impacts investors’ valuations. However, the latter can also influence a firm’s operations. Our study enriches this literature by providing meaningful implications of managers’ operations strategies with incentives linked to the investors’ valuations.

3. The Model

**Firm Operation.** We consider a firm run by a risk-neutral manager who makes all the operational decisions. The firm sells a product to a representative downstream party in two periods $t \in \{1, 2\}$. The price the firm charges the downstream party is exogenous and fixed at $p > 0$ per unit. The downstream party does not make strategic decisions. Without channel stuffing, the downstream
party always orders the exact amount it needs in each period, which we refer to as the real demand. Ex ante, this demand $\xi_t$ in period $t$ satisfies: $\xi_1 = \eta_1$ and $\xi_2 = a\xi_1 + \eta_2$, where $\eta_1$ and $\eta_2$ are independent random variables. We use $\eta_t$ and $\xi_t$ to represent random variables and use $\eta_t$ and $\xi_t$ to represent real variables or realizations of random variables. $\eta_t$ follows a continuous distribution function $F_t(\cdot)$ (density $f_t(\cdot)$) over $[0, \infty)$ with finite mean. $a$ is a constant that captures the correlation of the demands in the two periods. If $a = 0$, the demands are independent, whereas $a \neq 0$ reflects a positive (negative) correlation. For the business motivations in which we are interested, we focus on the case where $a > 0$.

The firm operates in a make-to-stock industry. In each period $t$, the firm needs to invest in inventory $q_t$ at a cost $0 < c < p$ per unit before the demand is realized. The firm has no replenishment opportunity within a period. Any unmet demand will be lost, which is appropriate in settings where an out-of-stock demand leads to a substitution among competing products. On the other hand, if the firm has leftover inventory in the first period, this inventory needs to be stored at a holding cost $0 < h < c$ per unit and can be used in the second period. The holding cost is incurred at the end of the first period. Consistent with the two-period setting, we assume that any remaining inventory at the end of the second period is salvaged at zero value without incurring a holding cost; then the firm is liquidated at its exact value. We consider no time discount. The information of this underlying business (all the parameters) is common knowledge. We have described a classical two-period inventory problem with correlated demands and lost sales. Without further specifications, the manager’s objective would be to determine the optimal inventory levels.

**Manager Utility.** We assume the manager is interested in the firm’s “market value” at the end of the first period (i.e., a short-term interest). The market value is the investors’ expectation of the firm’s “final real value,” which is the net of the realized revenues from the real sales and the costs in the two periods (the inventory investment and holding costs and, potentially, the channel stuffing cost). This short-term interest could come from a payoff the manager realizes in the short term and depends on the firm’s market value. The manager also cares about the firm’s final real value (i.e., a long-term interest). This long-term interest could come from, for instance, compensation
the manager receives in the end.

To model the manager’s incentive scheme, we apply a simple utility function (which has been similarly applied in the literature; see, e.g., Stein 1989, Liang and Wen 2007): the manager receives a short-term utility equal to the firm’s market value at the end of the first period multiplied by a weight $\beta \in (0, 1)$; the manager also receives a long-term utility equal to the firm’s real value at the end of the second period multiplied by a weight $1 - \beta$. Hence $\beta$ and $1 - \beta$ capture the relative importance of the firm’s short-term market value and long-term real value in the manager’s consideration; if $\beta$ goes to 1(0), the manager cares only about the former (the latter). More complex incentive schemes do exist in reality. The one we adopt captures, in the simplest way, the key feature in which we are interested. We assume $\beta$ is exogenous and this incentive scheme is publicly known; furthermore, the manager’s utility is not deducted from the firm’s value in our model (e.g., his compensation is not a significant cost of the firm or he is paid by shares). We will formulate later the firm’s real value and market value and the manager’s objective function.

**Channel Stuffing.** As his utility depends on the firm’s short-term market value, the manager may be tempted to pad sales in the first period to obtain a more favorable valuation. More precisely, we assume the manager may pad extra $x \geq 0$ units to the sales beyond the downstream party’s real demand $\xi_1$ in the first period, and the downstream party pays the firm the same price $p$ per unit. Since these $x$ units do not correspond to the real demand, the downstream party is simply “over-buying” them and will incur holding costs $hx$ in the first period of keeping them in inventory until the second period. We assume the downstream party’s inventory level is unobservable to the investors who value the focal firm. In the second period, the downstream party first uses these units to satisfy its real needs after $\xi_2$ is realized. If $\xi_2$ is more than $x$, the downstream party orders the extra units $\xi_2 - x$, satisfied from the firm’s inventory as much as possible; if $\xi_2$ is less than $x$, the downstream party salvages the leftover inventory $x - \xi_2$ at zero value.

We assume that if channel stuffing arises in the first period then in the second period the firm

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4 In practice, there exist various incentive schemes that have a positive $\beta$, such as equity-based compensations. Firms and also some research studies (see a survey paper, Core et al. 2003) argue that such schemes can align managers’ interest with the firm value (sometimes, they are provided due to cash constraint or tax motivations). In this study, we assume the incentive scheme is exogenous, and we explore the channel stuffing frictions that may arise in such a context.
will compensate the downstream party for the holding costs of the padded sales as well as the purchasing costs of those unsold units of the padded sales; that is, \( hx + p(x - \xi_2)^+ \). In addition, the firm compensates the downstream party for \( \gamma(x) \), which we refer to as a “channel stuffing cost” and that represents all other costs linked to channel stuffing operations (including additional handling, logistics, and incentives for the downstream party). For tractability, we assume this channel stuffing cost depends only on the padded sales, and \( \gamma(\cdot) \) is continuous over \([0, \infty)\) and satisfies \( \gamma(0) = 0, \gamma'(\cdot) > 0 \) and \( \gamma''(\cdot) > 0 \). By this setting, the downstream party is fully compensated and thus an economic driving force for “pulling forward” the future demand does not exist.

In our model, the manager cannot pad sales more than the available inventory. Thus the inventory after the real demand \( \xi_1 \) in period 1 is satisfied, \((q_1 - \xi_1)^+\), determines the maximum possible padding amount. The manager may pad any amount \( x \in [0, (q_1 - \xi_1)^+] \), which leads to total \( z = \min(\xi_1 + x, q_1) \) units of sales and leftover inventory \( q_1 - z \). We assume the investors know this structure of channel stuffing and the associated costs.

**Reporting.** The investors in our model do not directly observe the firm’s operations. Instead, the manager releases a report at the end of the first period. We assume that the report reveals the sales revenues \( pz \), the cost of goods sold \( cz \), the leftover inventory \( c(q_1 - z) \), and the holding costs \( h(q_1 - z) \). The report does not reveal information regarding the costs related to channel stuffing the firm will compensate the downstream party; those costs are incurred for the firm only in the second period, that is, after the report. As \( p \) and \( c \) are common knowledge, the investors will know the units of \( z \) and \( (q_1 - z) \) (but not \( \xi_1 \) or \( x \)). Consequently, they will also know \( q_1 \). For convenience, we use \( z \) and \( q_1 \) to represent the information the report reveals.

**Timeline.** Figure 1 details the timeline of the model. In period 1, the manager first invests in inventory \( q_1 \). Then he receives the real demand \( \xi_1 \) from the downstream party and decides how many units \( x \) to pad beyond \( \xi_1 \), which leads to total sales \( z = \min(\xi_1 + x, q_1) \). The manager releases a report and the investors value the firm. The manager’s short-term utility, which is based on the firm’s market value at that time, is then determined. In the second period, the downstream party is compensated by \( hx + \gamma(x) \) for the costs of the padded sales incurred in the first period. The
The investors value the firm; then the manager’s short-term utility is determined. Period 1 Period 2
The manager invests in initial inventory, \(q_1\), and then receives the real demand, \(\xi_1\).

The manager pads \(x\) extra units, which leads to total sales \(z = \min(\xi_1 + x, q_1)\).

The manager releases a report that reveals sales revenues \(pz\), cost of goods sold \(cz\), leftover inventory \(c(q_1 - z)\), and holding costs \(h(q_1 - z)\).

The downstream party pays \(pz\) to the firm, and incurs channel stuffing and holding costs, \(\gamma(x)\) and \(hx\).

The remaining inventory is salvaged, and the firm is liquidated at its real value, which determines the manager’s long-term utility.

**The Manager’s Problem.** The manager’s problem is composed of three decisions: the first-period inventory decision \(q_1\), the channel stuffing decision \(x\) and the second-period inventory decision \(q_2\). In the following, we state the manager’s problem starting from the second period.

Given the inventory decision \(q_1\), the realized demand \(\xi_1\) and the padded sales \(x\) in period 1, and the inventory replenishment decision \(q_2\) and the realized demand \(\xi_2\) in period 2, we define the firm’s final real value as its net cash flow at the end of period 2 (where \(0 \leq x \leq (q_1 - \xi_1)^+\)):

\[
v_2(x, \xi_1, \xi_2, q_1, q_2) = -cq_1 + p \min(\xi_1 + x, q_1) - h(q_1 - \xi_1 - x)^+ - cq_2 + p \min((\xi_2 - x)^+, (q_1 - \xi_1 - x)^+ + q_2) - (hx + \gamma(x) + p(x - \xi_2)^+).\]

By consolidating the terms with \(x\), we can write Equation (1) as:

\[
v_2(x, \xi_1, \xi_2, q_1, q_2) = -cq_1 + p \min(\xi_1, q_1) - h(q_1 - \xi_1)^+ - \gamma(x) - cq_2 + p \min(\xi_2, (q_1 - \xi_1)^+ + q_2).\]

Under the setting in our model, channel stuffing does not create any real value to the firm but costs the firm as captured by \(\gamma(x)\).
When making the inventory replenishment decision \( q_2 \) in period 2, the manager is to maximize the firm’s final real value, since at that moment, his utility fully depends on that value. Notice from Equation (2) that \( q_2 \) only appears in the second line, which represents the profit the firm can realize from the real demand in period 2. Therefore, given the potential demand satisfies \( \xi_2 = a\xi_1 + \eta_2 \), the manager makes the inventory replenishment decision by

\[
\max_{q_2 \in [0, \infty)} -cq_2 + \mathbb{E}_{q_2}[p \min(a\xi_1 + \eta_2, (q_1 - \xi_1)^+ + q_2)].
\]

Let \( q_2^* (\xi_1, q_1) \) denote his optimal inventory decision in period 2 and

\[
\Pi_2 (\xi_1, q_1) \equiv -cq_2^* (\xi_1, q_1) + \mathbb{E}_{\eta_2}[p \min(a\xi_1 + \eta_2, (q_1 - \xi_1)^+ + q_2^* (\xi_1, q_1))]
\]

denote the corresponding expected profit that can be realized from the second-period demand \( \xi_2 \).

Consequently, at the end of period 1, the expectation of the firm’s final real value, incorporating the manager’s optimal inventory decision in period 2, follows:

\[
v_1 (x, \xi_1, q_1) \equiv \mathbb{E}_{\eta_2}[v_2(x, \xi_1, a\xi_1 + \eta_2, q_1, q_2^* (\xi_1, q_1))]
\]

\[
= -cq_1 + p \min(q_1, \xi_1) - h(q_1 - \xi_1)^+ - \gamma(x) + \Pi_2 (\xi_1, q_1).
\]

The manager makes the channel stuffing decision in period 1 after receiving the real demand \( \xi_1 \). He considers both the short- and long-term utilities as described in the beginning of this section. Suppose the manager assumes the firm’s market value at the end of period 1 for any reported sales \( z \in [0, q_1] \) and initial inventory \( q_1 \in [0, \infty) \) follows a function \( P^M(z, q_1) \). Then, given \( \xi_1 \) and \( q_1 \), the manager maximizes his expected total utility: \( \beta P^M (\xi_1 + x, q_1) + (1 - \beta) \mathbb{E}_{\eta_2}[v_2(x, \xi_1, a\xi_1 + \eta_2, q_1, q_2^* (\xi_1, q_1))] \). The second term is identical to \((1 - \beta) v_1 (x, \xi_1, q_1) \) by the definition of Equation (5). Thus, the manager solves

\[
\max_{x \in [0, (q_1 - \xi_1)\]} \beta P^M (\xi_1 + x, q_1) + (1 - \beta) v_1 (x, \xi_1, q_1)\].
\]

We focus on pure strategies in the manager’s padding action.\(^5\) Let \( x^M (\xi_1, q_1) \) denote the channel stuffing strategy the manager follows, solved from Equation (6).\(^5\)

\(^5\) In principle, when the manager is indifferent between several padding actions, equilibria involving randomizing by the manager can exist. As in our equilibrium characterization, almost everywhere the manager’s action is pure, we assume the manager plays a pure strategy in order to save space.
At the beginning of period 1, the manager invests in $q^*_1$ inventory by taking the subsequent decisions into consideration. In other words, the manager solves $q^*_1$ from

$$\max_{q_1 \in [0, \infty)} \mathbb{E}_{\eta_1} [\beta P^M (\eta_1 + x^M (\eta_1, q_1), q_1) + (1 - \beta) v_1 (x^M (\eta_1, q_1), \eta_1, q_1)].$$  

(7)

**The Investors’ Problem.** In our model, the investors are identical and risk neutral. They take all accessible information into account when valuing the firm and follow Bayes’ rule whenever possible. They know the manager makes decisions to maximize his own utility. Recall that the underlying inventory problem, the manager’s incentive scheme, and the structure of channel stuffing are common knowledge. Thus the investors know the manager’s inventory replenishment strategy in period 2, that is, the function $q^*_2 (\xi_1, q_1)$, and hence the investors also know the form of the function $v_1 (x, \xi_1, q_1)$ in Equation (5). However, the investors do not observe the real demand $\xi_1$ or the padded sales $x$ in period 1. Therefore, we introduce the investors’ beliefs as follows: given any demand $\xi_1$ and initial inventory level $q_1$ in period 1, they believe the manager pads the sales following a function, $x^I (\xi_1, q_1) \in [0, (q_1 - \xi_1)^+]$ (as we focus on pure padding strategies).

With this belief, define a set $D(z, q_1) = \{ \eta_1 \in [0, z] : z = \eta_1 + x^I (\eta_1, q_1) \}$, given the reported sales $z$ and initial inventory $q_1$ ($D(z, q_1)$ is contained in $[0, z]$ since both the real demand and the padded sales are non-negative in our model). When $D(z, q_1) \neq \emptyset$, by Bayes’ rule, the investors believe the real demand in period 1, $\eta_1 (= \xi_1)$, follows the updated distribution (and density) function $F_1 (\eta_1 | z, q_1)$ (and $f_1 (\eta_1 | z, q_1)$):

$$f_1 (\eta_1 | z, q_1) d\eta_1 = \frac{f_1 (\eta_1) d\eta_1}{\int_{\eta_1 \in D(z, q_1)} dF_1 (\eta_1)} \text{ for } \eta_1 \in D(z, q_1) \text{ and } f_1 (\eta_1 | z, q_1) = 0 \text{ otherwise}. \quad (8)$$

Hence, when $D(z, q_1) \neq \emptyset$, the investors value the firm at (i.e., the firm’s market value):

$$P^I (z, q_1) \equiv \mathbb{E}_{\eta_1} [v_1 (x^I (\eta_1, q_1), \eta_1, q_1) | \eta_1 \in D(z, q_1)] = \int_0^z v_1 (x^I (\eta_1, q_1), \eta_1, q_1) dF_1 (\eta_1 | z, q_1). \quad (9)$$

The above expectation is taken with respect to the posterior demand distribution in period 1 based on the investors’ beliefs, for the given reported sales and initial inventory level.

It is possible that $D(z, q_1) = \emptyset$ for some $z \in [0, q_1]$, for a given $q_1$; this scenario implies that sales with such an amount $z$ are never reported corresponding to the given initial inventory $q_1$. In this
scenario, Bayes’ rule based on the investors’ beliefs does not impose any restrictions on the updated demand distribution function. Nevertheless, the investors still need to specify the value of the firm if such a scenario arises. Suppose when $\mathcal{D}(z, q_1) = \emptyset$ the investors specify a set $\mathcal{O}(z, q_1) \subseteq [0, z]$ in which they believe the real demand $\xi_1$ is and value the firm at $P^I(z, q_1) \equiv \mathbb{E}_{\eta_1}[v_1(z - \eta_1, \eta_1, q_1) | \eta_1 \in \mathcal{O}(z, q_1)]$.

**Equilibrium Definition.** For a given initial inventory level $q_1$, we impose that, in equilibrium, the manager’s belief about the firm’s market value coincides with the investors’ true valuation strategy for all possible reported sales, and at the same time, the investors’ beliefs about the manager’s channel stuffing strategy coincide with the manager’s true channel stuffing strategy for all possible demand realizations. Therefore, we define the following equilibrium concept. Similar concepts have been defined in the literature (see, e.g., Stein 1989, Liang and Wen 2007).

**Definition 1.** Given $q_1$, a perfect Bayesian market equilibrium is reached if $x^M(\xi_1, q_1) = x^I(\xi_1, q_1)$ and $P^M(z, q_1) = P^I(z, q_1)$ for any $\xi_1 \in [0, \infty)$ and $z \in [0, q_1]$.

For simplicity, we drop the superscripts “$M$” and “$I$” and use the notations $x(\xi_1, q_1)$ and $P(z, q_1)$ to denote the equilibrium. Furthermore, we consistently refer to $v_1(x, \xi_1, q_1)$ as the firm’s “real value” and $P(z, q_1)$ as the firm’s “market value.” These two functions will play an important role in deriving the equilibrium.

**4. Firm Real Value and Channel Stuffing Incentives**

Before deriving the equilibrium channel stuffing and market valuation, we first analyze the firm’s real value at the end of period 1, $v_1(x, \xi_1, q_1)$. In subsection 4.1, we derive the manager’s inventory decision in period 2, on which $v_1(x, \xi_1, q_1)$ depends. We discuss the properties of $v_1(x, \xi_1, q_1)$ in subsection 4.2, and in subsection 4.3, we reveal two distinct incentives for channel stuffing in our model, which will be useful to understand the equilibrium analysis in section 5.

**4.1. Inventory Decision in Period 2**

Recall from section 3 that the manager’s inventory decision in period 2 does not depend on the manager’s channel stuffing decision or how the investors value the firm in period 1 (see Equation (3)); it only depends on the demand realization and the inventory decision in period 1, and it is
a simple newsvendor problem with the demand distribution in period 2 conditioned on the real demand in period 1. Let $\kappa \equiv F_2^{-1}\left(\frac{p-c}{p}\right)$ and $\tilde{\xi}(q_1) \equiv \frac{(q_1-\kappa)_+}{1+\alpha}$. We obtain the manager’s optimal inventory decision from Equation (3):

\[
q_2^*(\xi_1, q_1) = \begin{cases} 
0, & 0 \leq \xi_1 \leq \tilde{\xi}(q_1) \\
\kappa + a\xi_1 - (q_1 - \xi_1), & \tilde{\xi}(q_1) < \xi_1 < q_1 \\
\kappa + a\xi_1, & \xi_1 \geq q_1.
\end{cases}
\]

(10)

Given $q_2^*(\xi_1, q_1)$, we obtain the expected profit, $\Pi_2(\xi_1, q_1)$, that the firm can realize from the second-period demand based on Equation (4):

\[
\Pi_2(\xi_1, q_1) = \begin{cases} 
ap\xi_1 + E_{\eta_2} [p \min(\eta_2, q_1 - \xi_1 - a\xi_1)], & 0 \leq \xi_1 \leq \tilde{\xi}(q_1) \\
ap\xi_1 + E_{\eta_2} [p \min(\eta_2, \kappa)] - c(\kappa + a\xi_1 - (q_1 - \xi_1)), & \tilde{\xi}(q_1) < \xi_1 < q_1 \\
ap\xi_1 + E_{\eta_2} [p \min(\eta_2, \kappa)] - c(\kappa + a\xi_1), & \xi_1 \geq q_1.
\end{cases}
\]

(11)

The characterization of $q_2^*(\xi_1, q_1)$ and $\Pi_2(\xi_1, q_1)$ will allow us to analyze $v_1(x, \xi_1, q_1)$ in the next subsection.

4.2. Properties of $v_1(x, \xi_1, q_1)$

From Equation (5), it is clear that $\frac{\partial v_1(x, \xi_1, q_1)}{\partial x} = -\gamma'(x)$ for any $x \in [0, (q_1 - \xi_1)_+]$. Channel stuffing always destroys the firm’s real value in our model. Notice the marginal value loss increases in the amount of padded sales ($\gamma(x)$ is convex by assumption).

Now, we discuss how the firm’s real value depends on the real demand in period 1. This dependency will play an important role in studying the manager’s channel stuffing incentives. We derive the partial derivative of $v_1(x, \xi_1, q_1)$ with respect to $\xi_1$ from Equation (5):

\[
\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = \begin{cases} 
p + h + \frac{\partial \Pi_2(\xi_1, q_1)}{\partial \xi_1}, & 0 \leq \xi_1 \leq q_1 \\
h, & \xi_1 \geq q_1.
\end{cases}
\]

(12)

where

\[
\frac{\partial \Pi_2(\xi_1, q_1)}{\partial \xi_1} = \begin{cases} 
ap - (1 + a)p(1 - F_2(q_1 - (1 + a)\xi_1)), & 0 \leq \xi_1 \leq \tilde{\xi}(q_1) \\
ap - (1 + a)c, & \tilde{\xi}(q_1) < \xi_1 < q_1 \\
ap - ac, & \xi_1 \geq q_1.
\end{cases}
\]

(13)

Notice from Equations (12-13) that the partial derivative $\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1}$ is independent of $x$; moreover, Lemma 1 holds.

**Lemma 1.** (i) $\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1}$ is positive and decreasing in $\xi_1$.

(ii) $\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1}$ increases in $p$, $h$, and $q_1$ and decreases in $c$. 


Lemma 1(i) implies that an increase of the real demand $\xi_1$ in period 1 always improves the firm’s real value. A larger $\xi_1$ can lead to more sales while less holding costs in period 1 (as long as there is excess inventory); in addition, a larger $\xi_1$ may also increase $\Pi_2(\xi_1, q_1)$, the second-period profit, given the demands are positively correlated. However, the magnitude of the improvement decreases in $\xi_1$ (i.e., $v_1(x, \xi_1, q_1)$ is concave in $\xi_1$) since the leftover inventory, $q_1 - \xi_1$, in period 1 is decreasing as $\xi_1$ increases and thus the firm has to invest in more inventory in the second period or would have less inventory to satisfy the second-period demand. In the left panel of Figure 2, we illustrate $v_1(x, \xi_1, q_1)$ when $x = 0$, which strictly and concavely increases in $\xi_1$.

Lemma 1(ii) shows that if $p$, $h$, or $q_1$ is higher or $c$ is lower, the benefit of an increase of $\xi_1$ to the firm’s real value will become more substantial. Given a higher $p$ (or $h$), extra demand will lead to more substantial increment of the sales revenues (or decrement of the holding costs), whereas with a higher $q_1$ or lower $c$, the negative impact of reduced leftover inventory in period 1 due to extra demand will be weakened. Hence, for a given amount of initial inventory, extra demand in the first period will imply a larger increase in the firm’s final real value if the sales price or the holding cost becomes higher or the inventory purchasing cost becomes lower; extra demand in the first period will also imply a larger increase in the firm’s final real value if there is more initial inventory. These intuitive properties will be useful when we discuss the comparative statics of the equilibrium in subsection 5.3.

Given the firm’s real value increases in the real demand in period 1 even when the inventory is sold out, Lemma 2 shows that the expectation of the firm’s real value conditional on the information that $\xi_1$ is no less than $q_1$ would be strictly larger than the firm’s real value conditional on the information that $\xi_1$ is equal to $q_1$. The information that the demand realization $\xi_1 \geq q_1$ censors the upper tail of the demand distribution associated with larger firm values.

**Lemma 2.** $\mathbb{E}_{\eta_1}[v_1(0, \eta_1, q_1) | \eta_1 \geq q_1] > v_1(0, q_1, q_1)$.

### 4.3. Incentives for Channel Stuffing

Armed with Lemmas 1 and 2, we now study a scenario in which the investors accept the sales report at face value (i.e., the investors believe that the reported sales have not been padded). Such
Figure 2 The firm’s real value without channel stuffing (left panel), the firm’s market value when the investors assume sales are never padded (middle panel), and the manager’s corresponding expected utility by channel stuffing given the real demand $\xi_1 = 4.25$ (right panel). The parameters are $\beta = 0.4$, $q_1 = 6.8$, $p = 10$, $c = 6$, $h = 1$, $a = 1.5$, and $\gamma(x) = 4x + 2x^2$, and $\eta_{1,2}$ follow a Gamma distribution with both mean and variance equal to 5.

an inference may not hold in equilibrium, yet, it helps in understanding the manager’s incentives in equilibrium (which we analyze in section 5). In such a scenario, the investors assume the real demand $\xi_1 = z$ if $z < q_1$ and assume $\xi_1$ is no less than $q_1$ (i.e., $\xi_1 \geq q_1$) if $z = q_1$. Then, with Equation (9), the firm’s market value would follow $P(z, q_1) = v_1(0, z, q_1)$ when $z < q_1$ and $P(z, q_1) = \mathbb{E}_{\eta_1}[v_1(0, \eta_1, q_1) | \eta_1 \geq q_1]$ when $z = q_1$, which first strictly increases in $z$ (Lemma 1(i)) and then jumps upward at $z = q_1$ (Lemma 2), as the middle panel of Figure 2 illustrates.

Suppose the manager’s belief is consistent with such a market valuation. We identify two distinct incentives that would motivate the manager to pad the sales when the real demand $\xi_1 < q_1$:

First, to maximize his utility (see Equation (6)), the manager will have an incentive to pad and report more sales if the marginal gain of his short-term utility, $\frac{\partial}{\partial z} P(\xi_1 + x, q_1) = \frac{\partial P(\xi_1 + x, q_1)}{\partial z}$, is larger than the marginal loss of his expected long-term utility, $(1 - \beta) \gamma'(x)$ (note $\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = -\gamma'(x)$). Notice that at $x = 0$, $\frac{\partial P(\xi_1 + x, q_1)}{\partial z} = \frac{\partial v_1(0, \xi_1, q_1)}{\partial \xi_1}$ that is positive (Lemma 1(i)) and thus padding will arise as long as $\frac{\partial v_1(0, \xi_1, q_1)}{\partial \xi_1}$ is larger than $(1 - \beta) \gamma'(0)$. This channel stuffing incentive will become weaker if the demand realization $\xi_1$ becomes larger, given $\frac{\partial v_1(0, \xi_1, q_1)}{\partial \xi_1}$ decreases in $\xi_1$ (Lemma 1(i)); in contrast, this incentive will become stronger for any given $\xi_1$ if the sales price $p$, the inventory holding cost $h$, or the initial inventory $q_1$ becomes higher, or the inventory purchasing cost $c$ becomes lower (Lemma 1(ii)).

Second, due to demand censoring, $P(z, q_1)$ jumps upward at $z = q_1$ (Lemma 2), which leads to a
jump in the manager’s utility function when $x$ goes to $q_1 - \xi_1$. Consequently, the manager will be tempted to pad all the remaining inventory when $\xi_1$ is close enough to $q_1$ so that the cost of doing so, $\gamma(q_1 - \xi_1)$, is not prohibitively large.\(^6\)

We illustrate the above two incentives in the right panel of Figure 2. In this example, the manager’s utility first increases in $x$, which implies that padding some sales will be in the manager’s interest; moreover, the manager’s utility increases discontinuously at $x = q_1 - \xi_1$ and achieves the maximum, revealing the gain of padding all the inventory.

Given these two distinct motivations for channel stuffing in our model, the investors in equilibrium will not take the sales report at face value but instead will “correct” it. We characterize such a correction in the next section.

5. **Analysis of Market Equilibrium**

We characterize the equilibrium of channel stuffing and market valuation with any given $q_1$ in subsection 5.1. Then, in subsections 5.2 and 5.3, we present conditions under which channel stuffing would not occur and examine the comparative statics of the equilibrium. We discuss the impact of channel stuffing on the initial inventory decision $q_1$ in subsection 5.4.

5.1. **Equilibrium Channel Stuffing and Market Valuation**

Recall from section 3 that the firm’s value function, $v_1(x, \xi_1, q_1)$, is derived from common knowledge and hence is known to the investors. However, the investors do not observe the realization of the real demand $\xi_1$ or the padded sales $x$ in the first period. Corresponding to the two channel stuffing incentives revealed in subsection 4.3, we identify a *semi-separating* and *semi-pooling* equilibrium that satisfies the equilibrium conditions in Definition 1. We say that an equilibrium is semi-separating and semi-pooling in our model if it has the following structure given an initial inventory level $q_1$: (1) When the demand realization is below a certain threshold, the manager pads the sales following a function such that the reported sales are monotone in the demand realization.\(^6\)

\(^6\) In practice, make-to-stock firms might be able to (partially) backlog excess demand. As a result, it is possible that managers may report and “pad” backlogs after (intentionally) selling out the inventory (notice however firms are not required to audit and report backlogs by GAAP). In such scenarios, the censoring effect and the semi-pooling part of the equilibrium we characterize in section 5 could be weaker since the backlog number if reported can reveal partial demand information. Since this study focuses on real earnings management while padding backlogs is more like accrual earnings management as there is generally no physical shipment, we would leave the exploration with the potential of reporting and padding backlogs for future research.
Thus the investors can perfectly infer the real demand as well as the padded sales and value the firm accurately (i.e., semi-separating). (2) When the demand realization reaches or exceeds this threshold, the manager pads any remaining inventory and reports sales equal to $q_1$. Then, the investors can only infer the real demand is no less than the threshold and therefore value the firm by expectation accordingly (i.e., semi-pooling).

Theorem 1 asserts that for any given initial inventory level $q_1$, a threshold $\hat{\xi}(q_1)$ exists to characterize such an equilibrium. In the semi-separating part of the equilibrium, $\xi_1 \in [0, \hat{\xi}(q_1))$, we characterize the manager’s channel stuffing strategy via a function $\varphi(\xi_1, q_1)$, which provides the amount of sales to pad for a given demand realization $\xi_1$, and the investors’ valuation strategy via a function $\phi(z, q_1)$, which provides the padded sales the investors infer from the reported sales $z$. These two functions are determined by Equation (15), the integral form of an ordinary differential equation, and the relationship,

$$\varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1) \quad \text{for each } \xi_1 \in [0, \hat{\xi}(q_1)).$$

The latter asserts that the investors perfectly infer the padded sales, $\varphi(\xi_1, q_1)$, from the reported sales, $z = \xi_1 + \varphi(\xi_1, q_1)$. In the semi-pooling part of the equilibrium, the padded sales are simply $(q_1 - \xi_1)^+$, which is all of the leftover inventory.

**Theorem 1.** Given $q_1$, a threshold $\hat{\xi}(q_1) < q_1$ exists that characterizes the semi-separating and semi-pooling equilibrium in which the investors value the firm by

$$P(z, q_1) = \begin{cases} v_1(\phi(z, q_1), z - \phi(z, q_1), q_1), & z \in [0, q_1) \\ E_{\eta_1} \left[ v_1((q_1 - \eta_1)^+, \eta_1, q_1) | \eta_1 \geq \hat{\xi}(q_1) \right], & z = q_1, \end{cases}$$

where $\phi(z, q_1)$ is a non-negative function in $z \in [0, q_1)$ that satisfies $\phi(0, q_1) = 0$ and for $z \in (0, q_1)$,

$$\phi(z, q_1) = \max \left( 0, z - \frac{1}{\beta} \int_0^z \frac{\gamma'(\phi(\tau, q_1))}{\vartheta_1(\phi(\tau, q_1), \tau - \phi(\tau, q_1), q_1)} + \gamma'(\phi(\tau, q_1)) \, d\tau \right);$$

(15)

and the manager pads the sales by

$$x(\xi_1, q_1) = \begin{cases} \varphi(\xi_1, q_1), & \xi_1 \in [0, \hat{\xi}(q_1)) \\ (q_1 - \xi_1)^+, & \xi_1 \geq \hat{\xi}(q_1), \end{cases}$$

(16)

where $\varphi(\xi_1, q_1)$ is the unique solution of $\varphi = \phi(\xi_1 + \varphi, q_1)$ and satisfies that $\xi_1 + \varphi(\xi_1, q_1)$ strictly increases in $\xi_1$ and $\xi_1 + \varphi(\xi_1, q_1) < q_1$ for $\xi_1 \in [0, \hat{\xi}(q_1))$. 
In the following, we explain the intuition behind the equilibrium characterization. First, we discuss the investors’ valuation strategy and the manager’s channel stuffing strategy according to the theorem. Next, we discuss the off-equilibrium path belief. Finally, we provide a numerical illustration of the equilibrium.

Investors’ Valuation Strategy. The investors’ valuation strategy follows from the semi-separating and semi-pooling structure of the equilibrium. When the reported sales $z < q_1$, the investors will infer the real demand $\xi_1$ from $z$. They apply the function $\phi(z, q_1)$: they “correct” $z$ to a level $z - \phi(z, q_1)$ as the real demand, with $\phi(z, q_1)$ considered as the amount of padded sales. Such a correction leads to the market value of the firm: $P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)$ for $z < q_1$ based on Equation (9) (in this case, Equation (9) is degenerate). When the reported sales $z = q_1$, the investors will not be able to infer the real demand. In this case, they “correct” the sales by a threshold $\tilde{\xi}(q_1) < q_1$: they believe the manager will pad the sales by $x = (q_1 - \xi_1)^+$ and report $z = q_1$ whenever $\xi_1 \geq \tilde{\xi}(q_1)$. Consequently, the market value of the firm follows $P(z, q_1) = \mathbb{E}_{\eta_1}[v_1((q_1 - \eta_1)^+, \eta_1, q_1) | \eta_1 \geq \tilde{\xi}(q_1)]$ when $z = q_1$ based on Equation (9).

We explain below the characterization of $\phi(z, q_1)$, the padded sales the investors infer when $z < q_1$ (we will explain later the characterization of the threshold $\tilde{\xi}(q_1)$).

First, in our model, the demand realization $\xi_1$ and the padded sales $x$ are both non-negative. Therefore, given any reported sales $z$, the inferred demand, $z - \phi(z, q_1)$, as well as the inferred padded sales, $\phi(z, q_1)$, will also be non-negative. This condition asserts $\phi(z, q_1)$ is bounded in the interval $[0, z]$. Hence, if $z = 0$, $\phi(0, q_1) = 0$; intuitively, if the reported sales are zero, the investors believe there are no padded sales. This acts as the initial condition of $\phi(z, q_1)$.

Second, we explain Equation (15) in Theorem 1, which establishes $\phi(z, q_1)$. For any reported sales $z \in (0, q_1)$, the investors’ inferred demand, $z - \phi(z, q_1)$, can match the real demand the manager receives only if the manager indeed pads $x = \phi(z, q_1)$ when the real demand is $\xi_1 = z - \phi(z, q_1)$. For this scenario to arise, the marginal gain of the manager’s short-term utility based on the firm’s market value, $\beta \frac{\partial P(\xi_1 + x, q_1)}{\partial z}$, when $\xi_1 = z - \phi(z, q_1)$ must equal the marginal loss of the manager’s long-term utility based on the firm’s real value, $(1 - \beta)\gamma'(x)$, at $x = \phi(z, q_1)$; that is,
\[ \beta \frac{\partial P(z, q_1)}{\partial z} = (1 - \beta) \gamma'(\phi(z, q_1)). \] (17)

Intuitively, Equation (17) represents the first-order condition of the manager’s channel stuffing decision in the investors’ equilibrium beliefs.\(^7\) Given the form of the market value \( P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1) \) for \( z < q_1 \), Equation (17) can be organized to an ordinary differential equation in \( \phi(z, q_1) \),\(^8\) of which Equation (15) provides the integral form, after incorporating the lower boundary condition (i.e., \( \phi(z, q_1) \geq 0 \)). Notice from Equation (15) that the upper boundary condition (i.e., \( \phi(z, q_1) \leq z \)) of \( \phi(z, q_1) \) is always satisfied. Thus far, we have explained how the investors construct the function \( \phi(z, q_1) \) to value the firm when the reported sales \( z < q_1 \).

**Manager’s Channel Stuffing Strategy and the Threshold \( \hat{\xi}(q_1) \).** Suppose the manager holds a correct belief of the investors’ valuation strategy as described above. Then, for a demand realization \( \xi_1 \in [0, q_1) \), the manager will maximize his total utility by

\[
\max_{x \in [0, q_1 - \xi_1]} \left\{ \beta v_1(\phi(\xi_1 + x, q_1), \xi_1 + x - \phi(\xi_1 + x, q_1), q_1) + (1 - \beta) v_1(x, \xi_1, q_1), \quad x < q_1 - \xi_1 \right\} \beta E_{\eta_1}\left[v_1((q_1 - \theta_1)^+, \theta_1, q_1) | \theta_1 \geq \hat{\xi}(q_1)\right] + (1 - \beta) v_1(q_1 - \xi_1, \xi_1, q_1), \quad x = q_1 - \xi_1. \] (18)

As we show in the proof of Theorem 1 (see Appendix A), a unique threshold \( \hat{\xi}(q_1) < q_1 \) exists such that if the investors apply this threshold to value the firm when the reported sales \( z = q_1 \), the manager’s optimal strategy that solves Equation (18) is to pad by a proportion of the leftover inventory, \( x = \varphi(\xi_1, q_1) \), which uniquely solves \( \varphi = \phi(\xi_1 + \varphi, q_1) \), when \( \xi_1 \in [0, \hat{\xi}(q_1)) \), and to pad by all the remaining inventory, \( x = (q_1 - \xi_1)^+ \), when \( \xi_1 \geq \hat{\xi}(q_1) \).\(^9\) We illustrate in Figure 3 the manager’s channel stuffing decision for two different values of \( \xi_1 \); In the left panel, the demand realization \( \xi_1 \) is below the characterized threshold \( \hat{\xi}(q_1) \), and the manager’s optimal channel stuffing

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\(^7\) The first-order condition of the manager’s channel stuffing decision in the semi-separating part of the equilibrium is: \( \beta \frac{\partial \phi(\xi_1 + x, q_1)}{\partial x} = (1 - \beta) \gamma'(x) \). In equilibrium, the inferred demand and the inferred padded sales will match the real demand and the real padded sales; that is, \( \xi_1 = z - \phi(z, q_1) \) and \( x = \phi(z, q_1) \). Thus we can rewrite the first-order condition as a function of \( z \), \( \beta \frac{\partial \phi(\xi_1 + x, q_1)}{\partial x} = (1 - \beta) \gamma'(\phi(z, q_1)) \), which provides the intuition of Equation (17).

\(^8\) Expanding \( \frac{\partial P(z, q_1)}{\partial z} \) in Equation (17) yields \( \beta \left( \frac{\partial v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)}{\partial z} \phi(z, q_1) + \frac{\partial v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)}{\partial \phi(z, q_1)} \frac{\partial \phi(z, q_1)}{\partial z} \right) = (1 - \beta) \gamma'(\phi(z, q_1)) \). Substituting \( \frac{\partial v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)}{\partial \phi(z, q_1)} \frac{\partial \phi(z, q_1)}{\partial z} = -\gamma'(\phi(z, q_1)) \) and isolating \( \frac{\partial \phi(z, q_1)}{\partial z} \) leads to: \( \frac{\partial \phi(z, q_1)}{\partial z} = 1 - \beta \frac{\partial v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)}{\partial \phi(z, q_1)} \gamma'(\phi(z, q_1)). \)

\(^9\) We have assumed the manager pads \((q_1 - \xi_1)^+\) when he is indifferent between \((q_1 - \xi_1)^+\) and \( \varphi(\xi_1, q_1) \). In principle, the manager can also randomize between these two. At this place, a randomization strategy, however, will not change the investor’s valuation as the randomization occurs for a demand realization with mass zero (a singleton, \( \hat{\xi}(q_1) \), in the domain of a continuous variable).
strategy is $x = \phi(\xi_1, q_1)$. In the right panel, the demand realization $\xi_1$ is above the characterized threshold $\hat{\xi}(q_1)$, and the manager’s optimal channel stuffing strategy is $x = (q_1 - \xi_1)^+$. Hence, when $\xi_1 \in [0, \hat{\xi}(q_1))$, the padded sales the investors infer, $\phi(\xi_1 + \phi(\xi_1, q_1), q_1)$, from any reported sales, $z = \xi_1 + \phi(\xi_1, q_1)$, exactly match the real padded sales, $\phi(\xi_1, q_1)$; when $\xi_1 \geq \hat{\xi}(q_1)$, the manager’s channel stuffing strategy, $x = (q_1 - \xi_1)^+$, also corresponds with the investors’ beliefs. It follows that the equilibrium conditions in Definition 1 are satisfied.

**Off-Equilibrium Path.** Theorem 1 asserts that when the demand realization $\xi_1 < \hat{\xi}(q_1)$, the reported sales, $z = \xi_1 + \phi(\xi_1, q_1)$, strictly increase in $\xi_1$ and are always less than $q_1$; however, from $\xi_1 = \hat{\xi}(q_1)$, the manager starts always reporting $z = q_1$. Consequently, the reported sales jump at $\xi_1 = \hat{\xi}(q_1)$, which indicates those reports with $z \in \hat{\xi}(q_1) + \phi(\hat{\xi}(q_1), q_1)$ would not be observed in equilibrium. In Theorem 1, the investors’ beliefs over this off-equilibrium path follow the semi-separating part of the equilibrium; that is, if such a $z$ was reported, the investors would believe the sales had been padded by $\phi(z, q_1)$. Hence, the off-equilibrium belief that supports our equilibrium is: $O(z, q_1) = \{\xi_1 \in [0, z] : z = \xi_1 + \phi(\xi_1, q_1)\}$. It is a singleton and a subset of $[0, z]$. By this specification, the manager will not deviate from his strategy characterized in Theorem 1.
Figure 4  The equilibrium channel stuffing strategy and the corresponding market value of the firm for $\beta = 0.15$ (left panels), $\beta = 0.26$ (middle panels), and $\beta = 0.4$ (right panels). The other parameters are $q_1 = 6.8$, $p = 10$, $c = 6$, $h = 1$, $a = 1.5$, and $\gamma(x) = 4x + 2x^2$ and $\eta_{1,2}$ follow a Gamma distribution with both mean and variance equal to 5.

In the top left panel, the manager does not pad the sales in the semi-separating part of the equilibrium. In the top middle and right panels, the manager pads the sales in the semi-separating part of the equilibrium; however, his channel stuffing incentive is not strong in the middle panel, so the padded sales first increase and then decrease to zero again. In all these panels, the manager pads all the remaining inventory once the demand reaches the threshold $\hat{\xi}(q_1)$. In the bottom panels, the solid curves (the black dots) represent the market values in the semi-separating (semi-pooling) part of the equilibrium; the dashed parts represent the market values in the off-equilibrium paths, which are extensions of the solid curves.

**Equilibrium Illustration.** We provide an illustration in Figure 4 of the channel stuffing strategy $x(\xi_1, q_1)$ and the corresponding market valuation $P(z, q_1)$ in the identified equilibrium. The top panels show the channel stuffing strategy. In the left panel, we set a positive but small $\beta$. No sales are padded when $\xi_1 < \hat{\xi}(q_1)$. In the middle panel, we set an intermediate $\beta$, but the manager’s incentive is not strong. The padded sales first increase from zero then decrease and return to zero when $\xi_1$ becomes large. In the right panel, we set a large $\beta$, where the padded sales continuously increase in $\xi_1$ but with a decreasing slope. In all these panels, once $\xi_1$ reaches $\hat{\xi}(q_1)$, the manager
pads all the remaining inventory.

Notice that in the semi-separating part of the equilibrium, the padded sales (i.e., \( x(\xi_1, q_1) = \varphi(\xi_1, q_1) \)) may not necessarily always increase in the demand realization \( \xi_1 \). This is because the cost of padding additional sales increases (\( \gamma(\cdot) \) is convex) as the padded sales increase. Moreover, given the firm’s real value increases concavely in the first-period real demand as shown by Lemma 1(i) (which is due to the negative impact of reduced leftover inventory), the value of extra sales to the investors, even if the investors consider them real demand, will become less substantial as the total demand the investors infer increases. As a result, the manager’s incentive for channel stuffing may decrease when the real demand increases relative to the initial inventory. This observation, together with the characteristic of the semi-pooling part of the equilibrium, highlights the importance of investors analyzing jointly the reported sales and inventory levels.

The bottom panels illustrate the market valuation strategy. The solid curves represent the market value corresponding to the semi-separating part of the equilibrium where the manager follows \( \varphi(\xi_1, q_1) \) to pad the sales. The black dots represent the market value corresponding to the semi-pooling part of the equilibrium that is the average of the firm’s real value conditional on the belief that the real demand \( \xi_1 \) is no less than \( \hat{\xi}(q_1) \) and the manager follows \((q_1 - \xi_1)^+\) to pad the sales. The dashed curves capture the market value along the off-equilibrium path, corresponding to the jumps of the padding curves in the top panels. The market value contains a jump at \( z = q_1 \).

### 5.2. Nonoccurrence Conditions of Channel Stuffing

Our model shows that if the manager is not motivated by the firm’s short-term market value, he will not resort to channel stuffing.

**Proposition 1.** If \( \beta \) goes to zero, \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \) go to zero and \( \hat{\xi}(q_1) \) goes to \( q_1 \).

Proposition 2 shows conditions in which channel stuffing will not arise in the semi-separating part or the whole equilibrium, even if \( \beta > 0 \).

**Proposition 2.** Let \( \theta(q_1) \equiv \beta \frac{\partial v_1(0, 0, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \). When \( \theta(q_1) \leq 0 \), (i) \( \phi(z, q_1) = 0 \) and \( \varphi(\xi_1, q_1) = 0 \); (ii) if, at the same time, \( a \), the correlation factor, goes to zero, \( \hat{\xi}(q_1) \) goes to \( q_1 \).
We explain Proposition 2 as follows. Suppose the investors believe the manager does not pad the sales for any demand realization. Then the firm’s market value would follow \( P(z, q_1) = v_1(0, z, q_1) \) when \( z < q_1 \) and \( P(q_1, q_1) = \mathbb{E}_{\eta_1}[v_1(0, \eta_1, q_1) | \eta_1 \geq q_1] \). Given this valuation we see the following:

First, as \( v_1(0, \xi_1, q_1) \) concavely increases in \( \xi_1 \) (see Lemma 1(i)), the derivative \( \frac{\partial P(z, q_1)}{\partial z} \), when \( P(z, q_1) = v_1(0, z, q_1) \), is the largest at \( z = 0 \); that is, \( \frac{\partial P(0, q_1)}{\partial z} = \frac{\partial v_1(0, 0, q_1)}{\partial \xi_1} \). Therefore, under the condition of Proposition 2, \( \theta(q_1) \leq 0 \), or, equivalently, \( \beta \frac{\partial v_1(0, 0, q_1)}{\partial \xi_1} \leq (1 - \beta) \gamma'(0) \) when \( z < q_1 \). In words, the marginal return to padding a small amount of sales is no larger than the marginal cost of doing so at any \( z < q_1 \). This result implies that even if the investors value the firm based on the belief that sales are never padded for \( z < q_1 \), the manager’s total utility will decrease if he pads the sales to any level below \( q_1 \). Hence, when \( \theta(q_1) \leq 0 \), channel stuffing will not arise in the semi-separating part of the equilibrium (Proposition 2(i)).

Second, when the demand correlation factor \( a \) goes to zero, the value, \( \mathbb{E}_{\eta_1}[v_1(0, \eta_1, q_1) | \eta_1 \geq q_1] \), will go to \( v_1(0, q_1, q_1) \). This effect occurs because any extra demand after the inventory is sold out in period 1 will be lost and will also not affect the real demand in period 2 as \( a \) goes to zero. Consequently, when \( \theta(q_1) \leq 0 \), if, at the same time, the correlation factor \( a \) goes to zero, the firm’s market value \( P(z, q_1) \) will then be \( v_1(0, z, q_1) \) for all \( z \leq q_1 \), which implies the jump of the firm’s market value at \( z = q_1 \) will no longer exist. Now the manager will not be able to gain extra utility by intentionally reporting a sold-out outcome. Thus channel stuffing will not arise in the semi-pooling part of the equilibrium (Proposition 2(ii)). Notice the absence of channel stuffing in the semi-separating part of the equilibrium does not directly imply that the whole equilibrium contains no channel stuffing. When, due to demand correlation, \( \hat{\xi}(q_1) < q_1 \), channel stuffing may still occur in the semi-pooling part of the equilibrium.

Propositions 1 and 2 are useful in identifying an “environment” conducive to channel stuffing based on our model, that is, when \( \beta > 0 \) and at the same time, \( \theta(q_1) > 0 \) and/or \( a > 0 \). A positive \( \theta(q_1) \) indicates the marginal gain of the manager’s utility from padding the sales can be positive, and a positive \( a \) provides a sufficient condition for demand censoring, which leads to a jump in the firm’s market value when a sold-out report is released.
5.3. Comparative Statics

In the following, we discuss the comparative statics of the equilibrium. Since we do not obtain a closed-form solution for $\phi(z, q_1)$, $\varphi(\xi_1, q_1)$, or $\hat{\xi}(q_1)$, we supplement numerical results when an analytical assessment is intractable. Proposition 3 shows the impacts of the main parameters in our model on the equilibrium.

**Proposition 3.** Given $q_1$, if $\beta$, $h$, or $p$ increases, or $c$ decreases, then $\phi(z, q_1)$ and $\varphi(\xi_1, q_1)$ increase for any given $z$ and $\xi_1$ in the semi-separating part of the equilibrium, whereas $\hat{\xi}(q_1)$ decreases.

We explain Proposition 3 as follows. First, given an initial inventory level $q_1$, if $\beta$ is larger, the manager will care more about the firm’s market value and less about its real value, and thus, the manager will have more incentive to pad the sales (note that a smaller $\hat{\xi}(q_1)$ indicates a stronger channel stuffing incentive to pad all the remaining inventory). Second, according to Lemma 1(ii), if the sales price $p$ or the holding cost $h$ becomes larger, or the inventory purchasing cost $c$ becomes smaller, an extra amount of demand in period 1 would mean a larger increase of the firm’s real value ($\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1}$ increases in $p$ and $h$ and decreases in $c$). Hence the manager will be more tempted to engage in channel stuffing to report more sales: In equilibrium, $\phi(z, q_1)$ and $\varphi(\xi_1, q_1)$ will increase and the threshold $\hat{\xi}(q_1)$ will decrease, when $p$ or $h$ increases or $c$ decreases.

For the correlation factor $a$ and the channel stuffing cost function $\gamma(x)$, an analytical assessment of their impacts in the general situation is challenging. From our numerical analysis, we observe that an increase of $a$ will make $\phi(z, q_1)$ and $\varphi(\xi_1, q_1)$ increase and $\hat{\xi}(q_1)$ decrease; in contrast, if $\gamma(x)$ becomes larger for all $x$ then $\phi(z, q_1)$ and $\varphi(\xi_1, q_1)$ will decrease and $\hat{\xi}(q_1)$ will increase (see Figure EC.2 in Appendix B1 for a numerical illustration). These observations are intuitive: a larger correlation of the demands enhances the manager’s incentive to engage in channel stuffing to reveal a “better” future performance, whereas with a larger cost, channel stuffing destroys more of the firm’s real value, and thus the manager’s incentive to pad the sales decreases.

Besides the parameters discussed above, the manager’s incentive to pad the sales in our model also depends on the initial inventory level, $q_1$. 
Proposition 4. Keeping all parameters fixed, if $q_1$ increases then $\phi(z, q_1)$ and $\varphi(\xi_1, q_1)$ increase for any given $z$ and $\xi_1$ in the semi-separating part of the equilibrium.

The intuition of Proposition 4 is the following. As we have discussed at Lemma 1, a larger demand in period 1 has also some negative impact since less inventory will remain that can contribute to satisfying the real demand in period 2. This negative impact, however, will be weakened if the amount of initial inventory $q_1$ becomes larger, since the firm will expect more leftover inventory. Lemma 1(ii) has shown that with a larger $q_1$, an increase of the real demand in period 1 will increase the firm’s real value more ($\partial v_1(x, \xi_1, q_1) / \partial \xi_1$ increases in $q_1$). As a result, we find with a larger $q_1$, the manager will be more tempted to report more sales in the semi-separating part of the equilibrium.

The initial inventory influences the semi-pooling part of the equilibrium, too. Given the analytical challenge, we explore this impact numerically (numerical illustrations are provided in Appendix B1). First, we observe that the threshold $\hat{\xi}(q_1)$ increases in $q_1$. This observation is intuitive since when $q_1$ increases, the firm will expect more leftover inventory. It would be more costly to pad all the remaining inventory at $\hat{\xi}(q_1)$ if this threshold stayed the same. Second, we observe some scenarios where $\hat{\xi}(q_1)$ can increase more than $q_1$ and thus the interval, $[\hat{\xi}(q_1), q_1]$, may shrink when $q_1$ increases. In those scenarios, the magnitude of the jump in the firm’s market value when a sold-out report is released usually decreases when $q_1$ increases. This observation is interesting since it implies that the expected amount of channel stuffing in the semi-pooling part of the equilibrium may decrease if $q_1$ increases and it will also explain the intuition for why the manager may over-invest in the initial inventory, as discussed in the next subsection.

5.4. Impact on Initial Inventory Decision

We presented in section 3 that the manager makes the initial inventory decision following Equation (7). With the characterization of the equilibrium, we can reformulate Equation (7) to (see Appendix B2 for the derivation):

$$\max_{q_1 \in [0, \infty)} -cq_1 + E_{\eta_1} \left[ p \min(\eta_1, q_1) - h(q_1 - \eta_1) + v_2(\eta_1, q_1) \right] - E_{\eta_1} \left[ \gamma(x(\eta_1, q_1)) \right].$$

Notice that the manager’s objective function is a combination of the classical newsvendor terms and an extra negative term that reflects the expected channel stuffing cost. This is because the
investors in our model foresee the manager’s channel stuffing incentives and factor the expected channel stuffing cost into their valuation of the firm. As a result, the presence of channel stuffing incentives may distort the initial inventory investment, compared with the classical newsvendor solution (i.e., if the manager maximizes only the first part in Equation (19)).

In the following, we discuss such impacts based on our numerical analysis. Interestingly, we observe that both over- and under-investment may arise (see Figure EC.4 in Appendix B2). Over-investment occurs usually in scenarios where more initial inventory leads to less channel stuffing in the semi-pooling part of the equilibrium while in the semi-separating part channel stuffing does not increase significantly (e.g., \( \varphi(\xi_1, q_1) \) remains at zero when \( \beta \) is small), and thus the expected channel stuffing cost is reduced. In contrast, under-investment occurs often when \( \varphi(\xi_1, q_1) \) is positive and increases significantly when \( q_1 \) increases (e.g., when \( \beta \) is large). In such scenarios, an increase of \( q_1 \) leads to overall more channel stuffing, and hence the manager may choose to under-invest in \( q_1 \) to limit the expected channel stuffing cost.

6. Discussion and Conclusion

Our study sheds light on understanding channel stuffing with market-based interests. Firms often offer their managers equity-based incentives to align their interests with the firm value. Such compensation, however, may lead managers to focus on the short-term market value, as captured by the parameter \( \beta \) in our model, to stuff the channel. Our study reveals the frictions that channel stuffing can introduce (the direct channel stuffing cost and the inventory distortion). These frictions can be additional considerations for firms, beyond other concerns, when designing their compensation scheme. To compensate managers with fewer short-term equity-based incentives or to design corporate policies that would allow recovering the compensation (the so-called “claw back” policies; see, e.g., Morgenson 2007) may reduce the channel stuffing frictions.

Our study provides several predictions for future empirical research on channel stuffing. Our analytical and numerical analyses imply that conditional on the total available inventory in a period (i.e., the initial inventory level in our model), the amount of channel stuffing would be positively associated with a manager’s short-term interest in his firm’s market value, the sales price,
the inventory holding cost, and the demand correlation across time, whereas would be negatively associated with the inventory purchasing cost and the channel stuffing cost\textsuperscript{10}. Our results also imply that conditional on “sold-out” not being reported (i.e., the semi-separating part of the equilibrium in our model), the amount of channel stuffing would be positively associated with the inventory available in a period. These implications are interesting and also relatively straightforward for an empirical exploration.

To conclude, we discuss two key assumptions in our model, which may inspire further analytical investigation on channel stuffing. First, the compensation scheme we applied is linear and does not capture more complicated scenarios where managers are awarded nonlinear incentives. For instance, managers may receive bonuses and options with thresholds and they may have particularly more incentives to stuff the channel when the firm’s performance approaches to those thresholds. Second, in our model, the compensation scheme and the demand distributions are exogenous. A more detailed framework could be constructed to incorporate some “positive” effort that the manager can exert to improve the real demand and the decision of designing optimal compensation to motivate the manager to exert more positive effort while conduct less channel stuffing. We believe extending the above two assumptions can produce richer results.

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\textsuperscript{10} The channel stuffing cost could be higher if the downstream parties also need to report their performance to investors (and hence might be less willing to carry excess inventory); in such scenarios, we expect the channel stuffing magnitude to be lower.
References


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Online Appendix

Appendix A: Proofs

Proof of Lemma 1:

In this proof, we show that \( \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} \) is always positive and it decreases in \( \xi_1 \) and \( c \) and increases in \( p, h, \) and \( q_1 \).

We first solve the second-period inventory decision according to our model (see section 3):

\[
\Pi_2(\xi_1, q_1) = \max_{q_2 \in [0, \infty)} -cq_2 + pE_{q_2}[(a\xi_1 + \eta_2, (q_1 - \xi_1)^+ + q_2)].
\]  

(EC.1)

This is a classic newsvendor problem. Let \( \kappa \equiv F^{-1}_2(\frac{p-q}{p}) \) and \( \bar{\xi}(q_1) \equiv \frac{(q_1 - \kappa)^+}{k + a} \). Given the leftover inventory from period 1, \( (q_1 - \xi_1)^+ \), the optimal inventory decision in period 2 follows:

\[
q_2^*(\xi_1, q_1) = \begin{cases} 
0, & 0 \leq \xi_1 \leq \bar{\xi}(q_1) \\
\kappa + a\xi_1 - (q_1 - \xi_1), & \bar{\xi}(q_1) < \xi_1 < q_1 \\
\kappa + a\xi_1, & \xi_1 \geq q_1.
\end{cases}
\]  

(EC.2)

The firm’s expected real value at the end of period 1 follows:

\[
v_1(x, \xi_1, q_1) = p \min(q_1, \xi_1) - cq_1 - h(q_1 - \xi_1)^+ - \gamma(x) + \Pi_2(\xi_1, q_1).
\]  

(EC.3)

Taking the derivative of \( v_1(x, \xi_1, q_1) \):

\[
\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = \begin{cases} 
\frac{h + (1 + a)pF_2(q_1 - (1 + a)\xi_1)}{a(p - c)}, & 0 \leq \xi_1 \leq \bar{\xi}(q_1) \\
\frac{h + (1 + a)(p - c)}{\bar{\xi}(q_1) < \xi_1 < q_1} \\
 \frac{h + (1 + a)(p - c)}{\xi_1 \geq q_1}.
\end{cases}
\]  

(EC.4)

Notice that \( F_2(q_1 - (1 + a)\xi_1) \) decreases in \( \xi_1 \); moreover,

\[
\lim_{\xi_1 \to \bar{\xi}(q_1)^-} \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = \lim_{\xi_1 \to \bar{\xi}(q_1)^+} \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = h + (1 + a)(p - c).
\]  

(EC.5)

As a result, \( \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} \) is positive, continuous and decreasing in \( \xi_1 \) when \( \xi_1 \in [0, q_1) \). At \( \xi_1 = q_1 \), \( \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} \) drops from \( h + (1 + a)(p - c) \) to \( a(p - c) \). The latter is also positive. It is direct to observe from Equation (EC.4) that \( \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} \) increases in \( p, h \) and \( q_1 \) and decreases in \( c \). ■

Proof of Lemma 2: This proof is straightforward. Given \( \frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = a(p - c) \) when \( \xi_1 \geq q_1 \),

\[
\mathbb{E}_{\eta_1}[v_1(0, \eta_1, q_1)|\eta_1 \geq q_1] - v_1(0, q_1, q_1) = \int_{q_1}^{\infty} \left( v_1(0, \eta_1, q_1) - v_1(0, \eta_1, q_1) \right) \frac{f_1(\eta_1)}{F_1(q_1)} d\eta_1 > 0.
\]  

(EC.6)
**Important note:** In the following, we show two technical lemmas which will be useful for proving Theorem 1. Lemma EC.1 presents the properties of the function, \( \phi(z, q_1) \), as defined in Theorem 1. Lemma EC.2 shows that there is a unique solution of \( \varphi = \phi(\xi_1 + \varphi, q_1) \), defined as \( \varphi(\xi_1, q_1) \), and presents the properties of \( \varphi(\xi_1, q_1) \).

Recall from Theorem 1 that the function, \( \phi(z, q_1) \), represents the amount of padded sales the investors infer from the reported sales \( z \) when \( z < q_1 \). \( \phi(z, q_1) \) is defined as a function that satisfies: \( \phi(0, q_1) = 0 \) and for \( z \in (0, q_1) \),

\[
\phi(z, q_1) = \max\left(0, z - \frac{1}{\beta} \int_0^z \frac{\gamma'(\phi(\tau, q_1))}{\partial \xi_1(\phi(\tau, q_1), \tau - \phi(\tau, q_1); q_1)} d\tau + \gamma'(\phi(\tau, q_1))\right) \quad (EC.7)
\]

Notice that Equation (EC.7) is an integral form of an ordinary differential equation (ODE) with a non-negativity condition (imposed by a zero boundary constraint). An important property of the function \( \phi(z, q_1) \) is: \( z - \phi(z, q_1) \) strictly increases in \( z \), which has the implication that the demand the investors infer strictly increases in the reported sales. Correspondingly, the function \( \varphi(\xi_1, q_1) \) has the property: \( \xi_1 + \varphi(\xi_1, q_1) \) strictly increases in \( \xi_1 \), which indicates that the reported sales strictly increase in the demand realization.

To facilitate the proofs, we extend the domain of the function \( \phi(z, q_1) \) from \( z \in [0, q_1] \) to \( z \in [0, q_1] \). This extension will not change any other properties of \( \phi(z, q_1) \). Define

\[
\mathcal{G}(q_1) \equiv \{ z \in [0, q_1] : \phi(z, q_1) = 0 \text{ and } \beta \frac{\partial \phi_1(0, z, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \} \quad (EC.8)
\]

and \( z_N \equiv \min \mathcal{G}(q_1) \) when \( \mathcal{G}(q_1) \) is non-empty. As we will show below, \( z_N \), if it exists, represents such a threshold that the function \( \phi(z, q_1) \) reaches zero at \( z = z_N \) and remains at zero for \( z > z_N \). Correspondingly, we will show in Lemma EC.2 that if \( z_N \) exists then a threshold \( \xi_N(= z_N) \) exists such that the function \( \varphi(\xi_1, q_1) \) reaches zero at \( \xi_1 = \xi_N \) and remains at zero for \( \xi_1 > \xi_N \); vice versa, \( z_N \) and \( \xi_N \) will be useful for characterizing the boundary solutions of the equilibrium. Figure EC.1 illustrates the two functions, \( \varphi(\xi_1, q_1) \) and \( \phi(z, q_1) \), and the associated thresholds, \( z_N \) and \( \xi_N \).

Now we present Lemma EC.1.

**Lemma EC.1.** Given \( q_1 \):

(i) If \( z_N \) exists then \( \phi(z, q_1) > 0 \) when \( z \in (0, z_N) \) and \( \phi(z, q_1) = 0 \) when \( z \in [z_N, q_1] \); otherwise, \( \phi(z, q_1) > 0 \) for all \( z \in (0, q_1] \). \( z_N = 0 \) iff \( \beta \frac{\partial \phi_1(0, 0, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \).

(ii) Whenever \( \phi(z, q_1) > 0 \), \( \phi(z, q_1) \) follows

\[
\frac{\partial \phi(z, q_1)}{\partial z} = \frac{\beta \frac{\partial \phi_1(\phi(z, q_1), z - \phi(z, q_1); q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(\phi(z, q_1))}{\beta \frac{\partial \phi_1(\phi(z, q_1), z - \phi(z, q_1); q_1)}{\partial \xi_1} + \beta \gamma'(\phi(z, q_1))}; \quad (EC.9)
\]

if Equation (EC.9) does not hold at some \( z \) then \( z \geq z_N \), \( \phi(z, q_1) = 0 \), and \( \frac{\partial \phi(z, q_1)}{\partial z} = 0 \).

(iii) \( 0 \leq \phi(z, q_1) \leq z \), and \( z - \phi(z, q_1) \) strictly and continuously increases in \( z \) when \( z \in [0, q_1] \).

**Proof:**
Figure EC.1 Illustration of the two functions, \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \), and the thresholds, \( z_N \) and \( \xi_N \). Given \( q_1 \), the domains of \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \) are \( z \in [0, q_1] \) and \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), respectively. In this experiment, besides \( \beta \), the other parameters are: \( q_1 = 6.8, p = 10, c = 6, h = 1, a = 1.5, \gamma(x) = \gamma_1 x + \frac{1}{2} \gamma_2 x^2 \) where \( \gamma_1 = \gamma_2 = 4 \), and \( \eta_{1,2} \) follow a Gamma distribution with both mean and variance equal to 5. In the left panels, \( z_N = \xi_N = 0 \), and both \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \) remain at zero; in the middle panels, \( z_N = \xi_N \approx 3.9 \), and \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \) first increase from zero and then return to zero at \( z_N \) and \( \xi_N \); in the right panels, \( z_N \) and \( \xi_N \) do not exist and \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \) monotonically increase in \( z \) and \( \xi_1 \). In the left and middle panels, \( \phi(q_1, q_1) = 0 \) and thus the domain of \( \varphi(\xi_1, q_1) \) reduces to \( \xi_1 \in [0, q_1] \); in the right panels, \( \phi(q_1, q_1) > 0 \) (it is approximately 0.8) and thus the domain of \( \varphi(\xi_1, q_1) \) is \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \) (approximately \( [0, 0.6] \)).

**Overview:** We show point (i) in four steps. In **Step 1**, we prove that if \( z_N \) exists then \( \phi(z, q_1) = 0 \) when \( z \in [z_N, q_1] \). In **Step 2**, we prove that if \( z_N \) exists and \( z_N > 0 \) then \( \phi(z, q_1) > 0 \) when \( z \in (0, z_N) \). In **Step 3**, we prove that if \( z_N \) does not exist then \( \phi(z, q_1) > 0 \) for all \( z \in (0, q_1) \). Finally, in **Step 4**, we prove that \( z_N = 0 \) iff \( \beta \frac{\partial \gamma(0, 0, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \). Points (ii) and (iii) are straightforward.

(i) Following the overview, we show point (i) below in four steps.

- **Step 1**, if \( z_N \) exists then \( \phi(z, q_1) = 0 \) when \( z \in [z_N, q_1] \): Based on the definition of \( z_N \), \( \phi(z_N, q_1) = 0 \) and \( \beta \frac{\partial \gamma(0, z_N, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \). Recall from Equation (EC.4) that the derivative \( \frac{\partial \gamma_1(x, \xi_1, q_1)}{\partial \xi_1} \) is independent of \( x \), always positive and decreasing in \( \xi_1 \). Thus \( \beta \frac{\partial \gamma(0, x, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \) for any \( x \geq z_N \), which implies \( \frac{\beta \frac{\partial \gamma(0, x, q_1)}{\partial \xi_1}}{\beta \frac{\partial \gamma(0, x, q_1)}{\partial \xi_1} + \beta \gamma'(0)} \leq 0 \) for any \( x \geq z_N \) (the denom-
inator is always positive given both \( \frac{\partial v(0,z,q_1)}{\partial z} > 0 \) and \( \gamma'(0) > 0 \). When \( z \geq z_N \),
\[
\phi(z,q_1) = \max \left( 0, \phi(z_N,q_1) + \int_{z_N}^{z} \frac{\beta \partial v(\phi(z,q_1),\tau-\phi(z,q_1),q_1)}{\partial \tau} - (1 - \beta) \gamma'(\phi(\tau,q_1)) \, d\tau \right) \quad \text{(EC.10)}
\]
\[
= \max \left( 0, \int_{z_N}^{z} \frac{\beta \partial v(\phi(z,q_1),\tau-\phi(z,q_1),q_1)}{\partial \tau} - (1 - \beta) \gamma'(\phi(\tau,q_1)) \, d\tau \right). 
\]

Notice that if \( \phi(\tau,q_1) = 0 \), the integrand, \( \frac{\beta \partial v(\phi(z,q_1),\tau-\phi(z,q_1),q_1)}{\partial \tau} - (1 - \beta) \gamma'(\phi(\tau,q_1)) \), in the above equation reduces to \( \frac{\beta \partial v(0,z,q_1)}{\partial z} - (1 - \beta) \gamma'(0) \) which is non-positive for any \( \tau \geq z_N \). Hence, given \( \phi(z_N,q_1) = 0 \) and \( \frac{\beta \partial v(0,z,q_1)}{\partial z} - (1 - \beta) \gamma'(0) \leq 0, \phi(z_N + \varepsilon,q_1) \) will be zero for a small \( \varepsilon \). We can show \( \phi(z,q_1) = 0 \) for any \( z \geq z_N \) by iterating the above reasoning.

- **Step 2**, if \( z_N \) **exists and** \( z_N > 0 \) then \( \phi(z,q_1) > 0 \) when \( z \in (0, z_N) \): Note that if \( z_N > 0 \) then \( \beta \frac{\partial v(0,0,q_1)}{\partial z} - (1 - \beta) \gamma'(0) \) must be positive (otherwise, combining the initial condition \( \phi(0,q_1) = 0 \) with \( \beta \frac{\partial v(0,0,q_1)}{\partial z} - (1 - \beta) \gamma'(0) \leq 0 \), we would have \( z_N = 0 \) by definition). Hence, from Equation (EC.7), we can observe that \( \phi(z,q_1) \) will increase from zero when \( z \) increases from zero. Now suppose a \( z' \in (0,z_N) \) exists such that \( \phi(z,q_1) \) returns to zero at \( z = z' \); that is, \( \phi(z,q_1) > 0 \) when \( z \in (0, z') \) and \( \phi(z',q_1) = 0 \). Then, the left derivative of \( \phi(z,q_1) \) with respect to \( z \) must be non-positive at \( z = z' \). On the other hand, when \( \phi(z,q_1) > 0 \), according to Equation (EC.7)
\[
\phi(z,q_1) = \int_{0}^{z} \frac{\beta \partial v(\phi(z,q_1),\tau-\phi(z,q_1),q_1)}{\partial \tau} - (1 - \beta) \gamma'(\phi(\tau,q_1)) \, d\tau. \quad \text{(EC.11)}
\]

Therefore, when \( z \in (0, z') \), given \( \phi(z,q_1) > 0 \), \( \frac{\partial \phi(z,q_1)}{\partial z} > 0 \). As the denominator of \( \frac{\partial \phi(z,q_1)}{\partial z} \) is always positive, the numerator, \( \beta \frac{\partial v(\phi(z,q_1),\tau-\phi(z,q_1),q_1)}{\partial \tau} - (1 - \beta) \gamma'(\phi(z,q_1)) \), must be non-positive as \( z \) goes to \( z' \); that is, \( \beta \frac{\partial v(\phi(z,q_1),\tau-\phi(z,q_1),q_1)}{\partial \tau} - (1 - \beta) \gamma'(0) \leq 0 \). The latter would imply \( z' \in G(q_1) \), which contradicts with the assumption that \( z_N \) is the smallest element in \( G(q_1) \). Hence, if \( z_N \) exists and \( z_N > 0 \) then \( \phi(z,q_1) > 0 \) when \( z \in (0, z_N) \).

- **Step 3**, if \( z_N \) **does not exist** then \( \phi(z,q_1) > 0 \) for all \( z \in (0,q_1] \): If \( z_N \) does not exist, according to the definition of \( z_N \), we know \( \beta \frac{\partial v(0,0,q_1)}{\partial z} - (1 - \beta) \gamma'(0) > 0 \) must hold. Thus \( \phi(z,q_1) \) becomes positive as \( z \) increases from zero; moreover, \( \phi(z,q_1) \) will not return to zero at any \( z \in (0,q_1] \) (otherwise, it would imply the existence of \( z_N \)).
• **Step 4**, \( z_N = 0 \) if \( \beta \frac{\partial \phi(0,q_1)\phi(q_1)}{\partial q_1} - (1 - \beta) \gamma'(0) \leq 0 \); This result is straightforward from the definition of \( z_N \) and the initial condition, \( \phi(0,q_1) = 0 \), of the function, \( \phi(z,q_1) \).

(ii) According to Equation (EC.7), when \( \phi(z,q_1) > 0 \),

\[
\phi(z,q_1) = \int_0^z \frac{\beta \frac{\partial \phi(\tau,q_1)\phi(q_1)}{\partial q_1} - (1 - \beta) \gamma'(\phi(\tau,q_1))}{\beta \frac{\partial \phi(\tau,q_1)\phi(q_1)}{\partial q_1} + \beta \gamma'(\phi(\tau,q_1))} d\tau. \tag{EC.12}
\]

Taking the derivative of \( \phi(z,q_1) \) with respect to \( z \) yields Equation (EC.9). Notice directly from the definition of \( \phi(z,q_1) \) that if \( \frac{\partial \phi(z,q_1)}{\partial z} \) does not follow the ODE in Equation (EC.9) then we must have \( \phi(z,q_1) = 0 \) and \( \frac{\partial \phi(z,q_1)}{\partial z} = 0 \) since the function \( \phi(z,q_1) \) is guided by the ODE and the zero boundary condition; moreover, \( z_N \) must exist and \( z \geq z_N \) (otherwise, \( \phi(z,q_1) > 0 \) and \( \phi(z,q_1) \) would follow Equation (EC.9)).

(iii) To show the result in point (iii), we obtain directly from Equation (EC.7) that

\[
z - \phi(z,q_1) = \min \left( z, \int_0^z \frac{\gamma'(\phi(\tau,q_1))}{\beta \frac{\partial \phi(\tau,q_1)\phi(q_1)}{\partial q_1} + \gamma'(\phi(\tau,q_1))} d\tau \right). \tag{EC.13}
\]

Notice that the integrand in Equation (EC.13) is positive and finite. Therefore, \( z - \phi(z,q_1) \) is continuous and strictly increasing in \( z \in [0,q_1] \). By assumption, \( \phi(0,q_1) = 0 \) and \( \phi(z,q_1) \) is non-negative. Thus \( 0 \leq \phi(z,q_1) \leq z \). This completes the proof. \( \blacksquare \)

Now we present Lemma EC.2.

**Lemma EC.2.** Given \( q_1 \), when \( \xi_1 \in [0,q_1 - \phi(q_1,q_1)] \), there is a unique solution of \( \varphi = \phi(\xi_1 + \varphi,q_1) \), denoted by \( \varphi(\xi_1,q_1) \), which satisfies:

(i) \( \xi_1 + \varphi(\xi_1,q_1) \) strictly and continuously increases in \( \xi_1 \);

(ii) \( \varphi(0,q_1) = 0 \), \( \varphi(q_1 - \phi(q_1,q_1),q_1) = \phi(q_1,q_1) \), and \( 0 \leq \varphi(\xi_1,q_1) \leq q_1 - \xi_1 \);

(iii) A \( \xi_N \) exists such that \( \varphi(\xi_1,q_1) > 0 \) when \( \xi_1 \in (0,\xi_N) \) and \( \varphi(\xi_1,q_1) = 0 \) when \( \xi_1 \in [\xi_N,q_1 - \phi(q_1,q_1)] \) iff \( z_N \) exists, and \( \xi_N = z_N \); if \( z_N \) does not exist, \( \varphi(\xi_1,q_1) > 0 \) for all \( \xi_1 \in (0,q_1 - \phi(q_1,q_1)] \);

(iv) Whenever \( \varphi(\xi_1,q_1) > 0 \), \( \varphi(\xi_1,q_1) \) follows

\[
\frac{\partial \varphi(\xi_1,q_1)}{\partial \xi_1} = \frac{\beta \frac{\partial \phi(\xi_1,q_1)\phi(q_1)}{\partial q_1} - (1 - \beta) \gamma'(\phi(\xi_1,q_1))}{\gamma'(\phi(\xi_1,q_1)); \tag{EC.14}
\]

if Equation (EC.14) does not hold at some \( \xi_1 \) then \( \xi_1 \geq \xi_N, \varphi(\xi_1,q_1) = 0 \), and \( \frac{\partial \varphi(\xi_1,q_1)}{\partial \xi_1} = 0 \).
Proof:

**Overview:** This proof is organized into two steps. In [Step 1], we prove when \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), there is a unique solution of \( \varphi = \phi(\xi_1 + \varphi, q_1) \). In [Steps 2], we show the properties (i-iv) of the function \( \varphi(\xi_1, q_1) \) (i.e., points (i-iv)).

- **[Step 1]:** In this step, we show when \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), there is a unique solution of \( \varphi = \phi(\xi_1 + \varphi, q_1) \). From Lemma EC.1, we know \( z - \phi(z, q_1) = 0 \) when \( z = 0 \), and \( z - \phi(z, q_1) \) is continuous and strictly increasing in \( z \) when \( z \in [0, q_1] \). \( z - \phi(z, q_1) \) reaches the largest when \( z = q_1 \), which is \( q_1 - \phi(q_1, q_1) \). Hence, for each \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), a unique \( z \in [0, q_1] \) satisfies: \( \xi_1 = z - \phi(z, q_1) \). We denote the solution of \( \xi_1 = z - \phi(z, q_1) \) by \( z(\xi_1, q_1) \). Notice \( \xi_1 = z - \phi(z, q_1) \) is identical to \( z - \xi_1 = \phi(\xi_1 + (z - \xi_1), q_1) \). Therefore, for each \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), there is a unique \( z - \xi_1 \in [0, \max\{\phi(z, q_1) : 0 \leq z \leq q_1 \}] \) that satisfies: \( z - \xi_1 = \phi(\xi_1 + (z - \xi_1), q_1) \). Replacing \( z - \xi_1 \) by \( \varphi \), we obtain that for each \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), a unique \( \varphi \in [0, \max\{\phi(z, q_1) : 0 \leq z \leq q_1 \}] \) exists that satisfies: \( \varphi = \phi(\xi_1 + \varphi, q_1) \). We denote this solution by \( \varphi(\xi_1, q_1) \), which has the domain of \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \).

- **[Step 2]:** In the following, we prove the four properties of the function \( \varphi(\xi_1, q_1) \) (i.e., points (i-iv)) in sequence:

**[Step 2(i)]:** For any \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \), given \( z(\xi_1, q_1) \) is the unique solution of \( z - \xi_1 = \phi(\xi_1 + (z - \xi_1), q_1) \) and \( \varphi(\xi_1, q_1) \) is the unique solution \( \varphi = \phi(\xi_1 + \varphi, q_1) \), we must have \( z(\xi_1, q_1) - \xi_1 = \varphi(\xi_1, q_1) \), or identically, \( \xi_1 + \varphi(\xi_1, q_1) = z(\xi_1, q_1) \). Given \( z - \phi(z, q_1) \) strictly increases in \( z \), the solution \( z(\xi_1, q_1) \) that solves \( \xi_1 = z - \phi(z, q_1) \) strictly increases in \( \xi_1 \). Hence \( \xi_1 + \varphi(\xi_1, q_1) \) which is equivalent to \( z(\xi_1, q_1) \) is continuous and strictly increasing in \( \xi_1 \).

**[Step 2(ii)]:** When \( \xi_1 = 0 \), \( z(\xi_1, q_1) = 0 \) is the unique solution of \( \xi_1 = z - \phi(z, q_1) \). Thus \( \xi_1 + \varphi(\xi_1, q_1) = z(\xi_1, q_1) = 0 \) when \( \xi_1 = 0 \). When \( \xi_1 = q_1 - \phi(q_1, q_1) \), \( z(\xi_1, q_1) = q_1 \) is the unique solution of \( \xi_1 = z - \phi(z, q_1) \). Thus \( \xi_1 + \varphi(\xi_1, q_1) = z(\xi_1, q_1) = q_1 \) when \( \xi_1 = q_1 - \phi(q_1, q_1) \). These two results indicate that \( \xi_1 + \varphi(\xi_1, q_1) \) is zero when \( \xi_1 = 0 \) and reaches \( q_1 \) when \( \xi_1 = q_1 - \phi(q_1, q_1) \). Hence, given \( \xi_1 + \varphi(\xi_1, q_1) \) strictly increases in \( \xi_1 \), we have \( 0 \leq \xi_1 + \varphi(\xi_1, q_1) \leq q_1 \), or identically, \( 0 \leq \varphi(\xi_1, q_1) \leq q_1 - \xi_1 \) when \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \).
Proof of Theorem 1:

[Step 2(iii)]: Given $\xi_1 + \varphi(\xi_1, q_1)$ strictly increases in $\xi_1$, if $z_N$ exists then a unique $\xi_N$ exists such that $\xi_N + \varphi(\xi_N, q_1) = z_N$. Given $\phi(z_N, q_1) = 0$, $\phi(\xi_N + \varphi(\xi_N, q_1), q_1) = 0$. Therefore, $\varphi(\xi_N, q_1) = \phi(\xi_N + \varphi(\xi_N, q_1), q_1) = 0$, which implies $\xi_N = z_N - \varphi(\xi_N, q_1) = z_N$. Since $\xi_1 + \varphi(\xi_1, q_1)$ strictly increases in $\xi_1$, for any $\xi_1 > \xi_N$, $\xi_1 + \varphi(\xi_1, q_1) > z_N$ which implies $\varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1) = 0$; for any $\xi_1 < \xi_N$, $\xi_1 + \varphi(\xi_1, q_1) < z_N$ which implies $\varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1) > 0$.

In case $z_N$ does not exist, $\phi(z, q_1) > 0$ for all $z \in (0, q_1)$ and thus $\varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1) > 0$ for all $\xi_1 \in (0, q_1 - \phi(q_1, q_1)]$. By the same approach, we can show the converse is also true.

[Step 2(iv)]: Given $\varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)$, we can take the derivative:

$$
\frac{\partial \varphi(\xi_1, q_1)}{\partial \xi_1} = \frac{\partial \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial z} \frac{\partial (\xi_1 + \varphi(\xi_1, q_1))}{\partial \xi_1}.
$$

EC.15

Rearranging the terms, we obtain

$$
\frac{\partial \varphi(\xi_1, q_1)}{d\xi_1} = \frac{\partial \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{d\xi_1} \left(1 - \frac{\partial \varphi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial z}\right).
$$

EC.16

When $\varphi(\xi_1, q_1) > 0$, $\phi(\xi_1 + \varphi(\xi_1, q_1), q_1) > 0$ and follows Equation (EC.9) by Lemma EC.1. Hence,

$$
\frac{\partial \varphi(\xi_1, q_1)}{d\xi_1} = \frac{\beta \partial \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial \xi_1} \frac{\partial \varphi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial q_1} - \frac{(1 - \beta) \gamma(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1))}{\partial \xi_1} + \frac{\beta \gamma(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1))}{\partial q_1} - \frac{(1 - \beta) \gamma(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1))}{\partial \xi_1} + \frac{\beta \gamma(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1))}{\partial q_1}
$$

EC.17

The last equality holds because $\varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)$.

If Equation (EC.14) does not hold at some $\xi_1$ then we must have $\varphi(\xi_1, q_1) = 0$ and $\frac{\partial \varphi(\xi_1, q_1)}{\partial \xi_1} = 0$, which corresponds to the properties of the function $\phi(z, q_1)$ (Lemma EC.1(ii)); moreover, $\xi_N$ must exist and $\xi_1 \geq \xi_N$. ■
Overview: The intuition of the characterization of the equilibrium has been explained in the paper (see subsection 5.1). Here we prove the theorem. We proceed with two main steps, Steps 1 and 2, which prove the results corresponding to the semi-separating and semi-pooling parts of the equilibrium, respectively. In particular, Step 1 proves the first branches of the equilibrium market value and channel stuffing functions in Equations (14) and (16); Step 2 proves the second branches in Equations (14) and (16).

[Step 1, Semi-separating] consists of two sub-steps, Steps 1.1 and 1.2, in which we temporarily extend the domain of the market valuation function, $P(z,q_1) = v_1 (\phi(z,q_1), z - \phi(z,q_1), q_1)$, to $z \in [0,q_1]$. Given $P(z,q_1) = v_1 (\phi(z,q_1), z - \phi(z,q_1), q_1)$ for any $z \in [0,q_1]$:  

- In [Step 1.1], we show that for any $\xi_1 \in [0,q_1 - \phi(q_1,q_1)]$, the manager’s optimal strategy is to pad the sales by an amount of $x^* = \varphi(\xi_1,q_1)$. We divide the proof into two cases for any $\xi_1 \in [0,q_1 - \phi(q_1,q_1)]$: In [Step 1.1, Case A], $\varphi(\xi_1,q_1) > 0$. We show that in such a scenario, padding sales by $x^* = \varphi(\xi_1,q_1)$ is the unique solution of the first-order condition of the manager’s channel stuffing decision and it maximizes the manager’s total utility. In [Step 1.1, Case B], $\varphi(\xi_1,q_1) = 0$. We show that in such a scenario, the first derivative of the manager’s utility function with respect to channel stuffing is non-positive at any level of padding; i.e., this is a boundary scenario and $x^* = \varphi(\xi_1,q_1) = 0$ is the manager’s optimal strategy. Therefore, when $\xi_1 \in [0,q_1 - \phi(q_1,q_1)]$, the manager’s optimal channel stuffing strategy can be fully characterized by the function $\varphi(\xi_1,q_1)$, and the investors’ belief, $\phi(z,q_1)$, of the padded sales is consistent with the manager’s strategy.

- In [Step 1.2], we discuss a boundary scenario. We show that for any $\xi_1 \in (q_1 - \phi(q_1,q_1), q_1)$, the first derivative of the manager’s utility function with respect to channel stuffing is always positive at any level of padding, and thus, to pad all the leftover inventory ($x^* = q_1 - \xi_1$) is optimal for the manager. This result reveals that a fully separating equilibrium does not exist in our model and a further correction of the market value $P(z,q_1)$ at $z = q_1$ is needed in order for an equilibrium to hold.

[Step 2, Semi-pooling] “corrects” the market valuation function $P(z,q_1)$ at $z = q_1$. We prove that given $q_1$ and $\phi(z,q_1)$, a unique threshold $\xi_1(q_1) \in [0,q_1 - \phi(q_1,q_1)]$ exists such that if $P(z,q_1) = v_1 (\phi(z,q_1), z - \phi(z,q_1), q_1)$ when $z \in [0,q_1]$ and $P(q_1,q_1) = \mathbb{E}_{q_1} \left[ v_1 ((q_1 - \eta_1)^+, \eta_1, q_1) | \eta_1 \geq \xi_1(q_1) \right]$ then the manager’s optimal strategy is to pad the sales by $\varphi(\xi_1,q_1)$ when $\xi_1 < \xi_1(q_1)$ and by $(q_1 - \xi_1)^+$ when $\xi_1 \geq \xi_1(q_1)$, which establishes the whole equilibrium.

[Step 1, Semi-separating]. Now we present the proof following the outline in the overview. In this step, we extend the domain of the market valuation function, $P(z,q_1) = v_1 (\phi(z,q_1), z - \phi(z,q_1), q_1)$, to $z \in [0,q_1]$. We will refine the market value for $z = q_1$ in Step 2.

[Step 1.1]. (When $\xi_1 \in [0,q_1 - \phi(q_1,q_1)]$) In this substep, we show that for any $\xi_1 \in [0,q_1 - \phi(q_1,q_1)]$, the manager’s optimal strategy is to pad the sales by $x^* = \varphi(\xi_1,q_1)$. We first derive the first derivative of the manager’s utility function with respect to the padded sales $x$. We then divide the discussion into two cases, Case A with $\varphi(\xi_1,q_1) > 0$ and Case B with $\varphi(\xi_1,q_1) = 0$, as described in the overview.

[First derivative of manager’s utility function]: We use the notation $u(x,\xi_1,q_1)$ in this proof to denote the manager’s utility function with padded sales $x$, demand realization $\xi_1$ and...
initial inventory $q_1$. Given $P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)$ for $z \in [0, q_1]$, the manager solves:

$$
\max_{x \in [0, q_1 - \xi_1]} u(x, \xi_1, q_1) = \beta P(\xi_1 + x, q_1) + (1 - \beta) v_1(x, \xi_1, q_1).
$$

(EC.18)

Taking the first derivative of the manager’s utility function with respect to $x$ yields:

$$
\frac{\partial u(x, \xi_1, q_1)}{\partial x} = \beta \frac{\partial P(\xi_1 + x, q_1)}{\partial z} \frac{d (\xi_1 + x)}{dx} + (1 - \beta) \frac{\partial v_1(x, \xi_1, q_1)}{\partial x}
= \beta \frac{\partial P(\xi_1 + x, q_1)}{\partial z} - (1 - \beta) \gamma'(x).
$$

(EC.19)

Given $P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)$, we derive

$$
\frac{\partial P(z, q_1)}{\partial z} = \frac{\partial v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)}{\partial \xi_1} - \left( \frac{\partial v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)}{\partial \xi_1} + \gamma'(\phi(z, q_1)) \right) \frac{\partial \phi(z, q_1)}{\partial z}
$$

(EC.20)

Substituting Equation (EC.20) into Equation (EC.19) yields

$$
\frac{\partial u(x, \xi_1, q_1)}{\partial x} = \beta \frac{\partial v_1(\phi(\xi_1 + x, q_1), \xi_1 + x - \phi(\xi_1 + x, q_1), q_1)}{\partial \xi_1}
- \beta \left( \frac{\partial v_1(\phi(\xi_1 + x, q_1), \xi_1 + x - \phi(\xi_1 + x, q_1), q_1)}{\partial \xi_1} + \gamma'(\phi(\xi_1 + x, q_1)) \right) \frac{\partial \phi(\xi_1 + x, q_1)}{\partial z}
- (1 - \beta) \gamma'(x).
$$

(EC.21)

**Step 1.1, Case A with $\varphi(\xi_1, q_1) > 0$:** Given any $\xi_1 \in [0, q_1 - \phi(q_1, q_1))$, if $\varphi(\xi_1, q_1) > 0$ then $\varphi(\xi_1, q_1)$ follows the ODE in Equation (EC.14) by Lemma EC.2; accordingly, $\varphi(\xi_1 + \varphi(\xi_1, q_1), q_1) = \varphi(\xi_1, q_1) > 0$ and thus follows the ODE in Equation (EC.9) by Lemma EC.1. We show below that $x = \varphi(\xi_1, q_1)$ is the unique solution of the first-order condition of the manager’s channel stuffing decision and it maximizes his utility.

- First, we show $x = \varphi(\xi_1, q_1)$ is the solution of the first-order condition. We substitute $x = \varphi(\xi_1, q_1)$ into Equation (EC.21), which yields

$$
\frac{\partial u(\varphi(\xi_1, q_1), \xi_1, q_1)}{\partial x} = \beta \frac{\partial v_1(\varphi(\xi_1 + \varphi(\xi_1, q_1), q_1), \xi_1 + \varphi(\xi_1, q_1) - \phi(\xi_1 + \varphi(\xi_1, q_1), q_1), q_1)}{\partial \xi_1}
- \beta \left( \frac{\partial v_1(\varphi(\xi_1 + \varphi(\xi_1, q_1), q_1), \xi_1 + \varphi(\xi_1, q_1) - \phi(\xi_1 + \varphi(\xi_1, q_1), q_1), q_1)}{\partial \xi_1} + \gamma'(\varphi(\xi_1 + \varphi(\xi_1, q_1), q_1)) \right) \frac{\partial \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial z}
- (1 - \beta) \gamma'(\varphi(\xi_1, q_1)).
$$

(EC.22)
Since \( \phi(\xi_1 + \varphi(\xi_1, q_1), q_1) \) follows Equation (EC.9), we have

\[
\frac{\partial \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial z} = \frac{\beta \frac{\partial \varphi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial \xi_1} \cdot (\xi_1 + \varphi(\xi_1, q_1)) - (1 - \beta) \gamma'(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1))}{\beta \frac{\partial \varphi(\xi_1 + \varphi(\xi_1, q_1), q_1)}{\partial \xi_1} + \beta \gamma'(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1))}.
\] (EC.23)

Substituting Equation (EC.23) into Equation (EC.22), we obtain

\[
\frac{\partial u(\varphi(\xi_1, q_1), \xi_1, q_1)}{\partial x} = (1 - \beta) \gamma'(\phi(\xi_1 + \varphi(\xi_1, q_1), q_1)) - (1 - \beta) \gamma'(\varphi(\xi_1, q_1)).
\] (EC.24)

Given \( \varphi(\xi_1, q_1) = \phi(\xi_1 + \varphi(\xi_1, q_1), q_1), \frac{\partial u(\varphi(\xi_1, q_1), \xi_1, q_1)}{\partial x} = 0 \). Therefore, the first-order condition of the manager's channel stuffing decision problem is satisfied at \( x = \varphi(\xi_1, q_1) \).

- Second, we show the solution to the first-order condition is unique. We set the first derivative of the manager's utility (i.e., Equation (EC.21)) to zero, rearrange the terms, and obtain the first-order condition as

\[
\frac{\partial \phi(\xi_1 + x, q_1)}{\partial z} = \frac{\beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} \cdot (\xi_1 + x - \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)) - (1 - \beta) \gamma'(x)}{\beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} + \beta \gamma'(\phi(\xi_1 + x, q_1))}.
\] (EC.25)

As long as \( \phi(\xi_1 + x, q_1) \) follows the ODE in Equation (EC.9), we have

\[
\frac{\partial \phi(\xi_1 + x, q_1)}{\partial z} = \frac{\beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} \cdot (\xi_1 + x - \phi(\xi_1 + \varphi(\xi_1, q_1), q_1)) - (1 - \beta) \gamma'(\phi(\xi_1 + x, q_1))}{\beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} + \beta \gamma'(\phi(\xi_1 + x, q_1))}.
\] (EC.26)

Comparing Equations (EC.25) and (EC.26) reveals \( x = \phi(\xi_1 + x, q_1) \). From Lemma EC.2, there is a unique solution of \( x = \phi(\xi_1 + x, q_1) \); that is, \( \varphi(\xi_1, q_1) \).

Now, we show by contradiction that when \( \varphi(\xi_1, q_1) > 0 \), the first-order condition, Equation (EC.25), cannot hold at any \( x \) where \( \phi(\xi_1 + x, q_1) \) does not follow the ODE in Equation (EC.9). Suppose it holds in such a scenario. According to Lemma EC.1, when \( \phi(\xi_1 + x, q_1) \) does not follow Equation (EC.9), \( \phi(\xi_1 + x, q_1) = 0 \) and \( \frac{\partial \phi(\xi_1 + x, q_1)}{\partial \xi_1} = 0 \); moreover, \( z_N(= \xi_N) \) exists and \( \xi_1 + x \geq z_N \).

Thus the first-order condition (i.e., Equation (EC.25)) can be rewritten as

\[
0 = \frac{\beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(x)}{\beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} + \beta \gamma'(0)}.
\] (EC.27)

Equation (EC.27) would imply \( \beta \frac{\partial \varphi(\xi_1 + x, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(x) = 0 \), which however cannot hold for the following reason: Since \( \varphi(\xi_1, q_1) > 0 \), we must have \( \xi_1 < z_N \). Thus, given \( \xi_1 + x \geq z_N \), we
have $x > 0$. By the definition of $z_N$, we know $\beta \frac{\partial v(0,z_N,q_1)}{\partial q_1} - (1 - \beta) \gamma'(0) \leq 0$. Since $\frac{\partial v(0,z_1,q_1)}{\partial q_1}$ is a function that decreases in $\xi_1$ and $\gamma'(x)$ strictly increases in $x$, we have $\beta \frac{\partial v(0,z_1+x,q_1)}{\partial q_1} - (1 - \beta) \gamma'(x) < \beta \frac{\partial v(0,z_N,q_1)}{\partial q_1} - (1 - \beta) \gamma'(0) \leq 0$ given $x > 0$, which shows the contradiction.

Hence the solution to the first-order condition, if it exists, is unique; that is, $\varphi(\xi_1, q_1)$.

- Third, we show $x = \varphi(\xi_1, q_1)$ maximizes the manager’s utility. Given the manager’s utility function is continuous and a unique solution of the first-order condition exists in the region $[0, q_1 - \xi_1]$, it suffices to show $\frac{\partial u(\xi_1, q_1)}{\partial x} > 0$ at $x = 0$. Thus we obtain from Equation (EC.21):

$$
\frac{\partial u(0,\xi_1, q_1)}{\partial x} = \beta \frac{\partial v_1(\varphi(\xi_1, q_1), \xi_1 - \varphi(\xi_1, q_1), q_1)}{\partial \xi_1} - \left( \beta \frac{\partial v_1(\varphi(\xi_1, q_1), \xi_1 - \varphi(\xi_1, q_1), q_1)}{\partial \xi_1} + \beta \gamma'(\varphi(\xi_1, q_1)) \right) \frac{\partial \varphi(\xi_1, q_1)}{\partial z} - (1 - \beta) \gamma'(0).
$$

(EC.28)

Given $\varphi(\xi_1, q_1) > 0$, $\xi_1 < \xi_N = z_N$ and thus $\varphi(\xi_1, q_1)$ follows the ODE in Equation (EC.9); that is,

$$
\frac{\partial \varphi(\xi_1, q_1)}{\partial z} = \beta \frac{\beta \frac{\partial v_1(\varphi(\xi_1, q_1), \xi_1 - \varphi(\xi_1, q_1), q_1)}{\partial \xi_1}}{\beta \frac{\partial v_1(\varphi(\xi_1, q_1), \xi_1 - \varphi(\xi_1, q_1), q_1)}{\partial \xi_1} + \beta \gamma'(\varphi(\xi_1, q_1))} - (1 - \beta) \gamma'(\varphi(\xi_1, q_1)).
$$

(EC.29)

Substituting Equation (EC.29) into Equation (EC.28), we obtain

$$
\frac{\partial u(0,\xi_1, q_1)}{\partial x} = (1 - \beta) \gamma'(\varphi(\xi_1, q_1)) - (1 - \beta) \gamma'(0) > 0,
$$

(EC.30)

given $\gamma'(x)$ strictly increases in $x$ and $\varphi(\xi_1, q_1)$ is positive. This completes the proof for Case A.

**Step 1.1, Case B with $\varphi(\xi_1, q_1) = 0$:** We show below that if $\varphi(\xi_1, q_1) = 0$ at the given $\xi_1$ then the first derivative of the manager’s utility function, $\frac{\partial u(\xi_1, q_1)}{\partial x}$, is non-positive for any $x \in [0, q_1 - \xi_1]$ and thus the manager’s optimal channel stuffing decision is $x^* = \varphi(\xi_1, q_1) = 0$.

For any $x \in [0, q_1 - \xi_1]$,

- As long as $\varphi(\xi_1 + x, q_1)$ follows the ODE in Equation (EC.9), we can write the first derivative of the manager’s utility function as (substituting the ODE of $\varphi(\xi_1 + x, q_1)$ into Equation (EC.21)):

$$
\frac{\partial u(x,\xi_1, q_1)}{\partial x} = (1 - \beta) \gamma'(\varphi(\xi_1 + x, q_1)) - (1 - \beta) \gamma'(x).
$$

(EC.31)

Given $\varphi(\xi_1, q_1) = 0$, according to Lemma EC.2, we can conclude that either (a) $\xi_1 = 0$ or (b) $\xi_N$ exists and $\xi_1 \geq \xi_N$. For (a), we have $\frac{\partial u(x,\xi_1, q_1)}{\partial x} = (1 - \beta) \gamma'(\varphi(x, q_1)) - (1 - \beta) \gamma'(x)$. From Lemma
EC.1, we know $\phi(z, q_1) \leq z$, which implies $x \geq \phi(x, q_1)$. Thus $\frac{\partial u(x, 0, q_1)}{\partial x} < 0$ for $x > 0$. For (b), given $\xi_1 \geq \xi_N$, we have $\xi_1 + x \geq \xi_N$ and $\phi(\xi_1 + x, q_1) = 0$, which implies $\frac{\partial u(x, \xi_1, q_1)}{\partial x} = (1 - \beta)\gamma'(0) - (1 - \beta)\gamma'(x) < 0$ for $x > 0$. Therefore it is not optimal to set $x > 0$.

- In case $\phi(\xi_1 + x, q_1)$ does not follow the ODE in Equation (EC.9), we have $\phi(\xi_1 + x, q_1) = 0$, $\frac{\partial \phi(\xi_1 + x, q_1)}{\partial x} = 0$, and $\xi_1 + x \geq z_N$. Then we can write the first derivative of the manager’s utility function (i.e., Equation (EC.21)) to:

$$\frac{\partial u(x, \xi_1, q_1)}{\partial x} = \beta \frac{\partial v_1(0, \xi_1 + x, q_1)}{\partial \xi_1} - (1 - \beta)\gamma'(x).$$

(EC.32)

By the definition of $z_N$, we know $\beta \frac{\partial v_1(0, z_N, q_1)}{\partial \xi_1} - (1 - \beta)\gamma'(0) \leq 0$, which implies $\beta \frac{\partial v_1(0, \xi_1 + x, q_1)}{\partial \xi_1} - (1 - \beta)\gamma'(x) < 0$ for any $x > 0$. Therefore it is not optimal to set $x > 0$.

Hence, for both Cases A and B, we have shown that under the market value function $P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)$ for $z \in [0, q_1]$, the manager’s optimal channel stuffing strategy for any demand realization $\xi_1 \in [0, q_1 - \phi(q_1, q_1)]$ is to pad the sales by an amount $x^* = \varphi(\xi_1, q_1)$.

**[Step 1.2]** (When $\xi_1 \in (q_1 - \phi(q_1, q_1), q_1]$) As discussed in the overview, in this substep, we analyze another boundary scenario. We show below that when $\xi_1 \in (q_1 - \phi(q_1, q_1), q_1)$, the derivative of the manager’s utility function is always positive for any $x \in [0, q_1 - \xi_1]$ and thus the manager’s optimal strategy is to pad all the leftover inventory (i.e., $x^* = q_1 - \xi_1$).

Notice if $z_N$ exists for the function $\phi(z, q_1)$ then $\phi(z, q_1) = 0$ for any $z \in [z_N, q_1]$. Consequently, we will have $\phi(q_1, q_1) = 0$, which implies $q_1 - \phi(q_1, q_1) = q_1$ and the set $(q_1 - \phi(q_1, q_1), q_1)$ is empty.

Therefore, in this substep, we only need to focus on the scenario where $z_N$ as well as $\xi_N$ does not exist and hence $\phi(z, q_1)$ and $\varphi(\xi_1, q_1)$ always follow the ODEs in Equations (EC.9) and (EC.14).

We first discuss a special case where $\xi_1 = q_1 - \phi(q_1, q_1)$ in Case A of Step 1.1. According to Lemma EC.2, when $\xi_1 = q_1 - \phi(q_1, q_1)$, $\varphi(q_1 - \phi(q_1, q_1), q_1) = \phi(q_1, q_1)$ and thus $\xi_1 + \varphi(\xi_1, q_1) = q_1$.

The latter implies the manager pads all the remaining inventory to the sales. In Case A of Step 1.1, we have shown that as long as $\varphi(\xi_1, q_1)$ follows the ODE in Equation (EC.14), $x = \varphi(\xi_1, q_1)$ satisfies the first-order condition of the manager’s problem and it maximizes the manager’s utility. Given $x = \varphi(q_1 - \phi(q_1, q_1), q_1) = \phi(q_1, q_1)$ maximizes the manager’s utility, the first derivative of the
manager’s utility function must satisfy \( \frac{\partial u(x, q_1 - \phi(q_1, q_1), q_1)}{\partial x} > 0 \) for any \( x < \phi(q_1, q_1) \), or equivalently, for any \( 0 < \varepsilon < \phi(q_1, q_1) \),

\[
\frac{\partial u(\phi(q_1, q_1) - \varepsilon, q_1 - \phi(q_1, q_1), q_1)}{\partial x} = \beta \frac{\partial P(q_1 - \varepsilon, q_1)}{\partial z} - (1 - \beta) \gamma'(\phi(q_1, q_1) - \varepsilon) > 0.
\] (EC.33)

Now consider \( \xi_1 = q_1 - \phi(q_1, q_1) + \hat{\varepsilon} \) with some small \( \hat{\varepsilon} > 0 \). Suppose the manager pads the sales fewer than \( q_1 - \xi_1 \), that is, to pad \( x = \phi(q_1, q_1) - \hat{\varepsilon} - \varepsilon \) with any \( 0 < \varepsilon < \phi(q_1, q_1) - \hat{\varepsilon} \) and report \( z = \xi_1 + x = q_1 - \varepsilon \). Then the first derivative of the manager’s utility function satisfies:

\[
\frac{\partial u(\phi(q_1, q_1) - \hat{\varepsilon} - \varepsilon, q_1 - \phi(q_1, q_1) + \hat{\varepsilon}, q_1)}{\partial x} = \beta \frac{\partial P(q_1 - \varepsilon, q_1)}{\partial z} - (1 - \beta) \gamma'(\phi(q_1, q_1) - \hat{\varepsilon} - \varepsilon) > \frac{\partial u(\phi(q_1, q_1) - \varepsilon, q_1 - \phi(q_1, q_1), q_1)}{\partial x} > 0,
\]
given \( \gamma'(x) \) is a strictly increasing function. Hence, when \( \xi_1 = q_1 - \phi(q_1, q_1) + \hat{\varepsilon} \), the manager’s optimal strategy is to pad all the leftover inventory; i.e., \( x^* = q_1 - \xi_1 \). By iteration, we can show the same result for any \( \xi_1 \in (q_1 - \phi(q_1, q_1), q_1) \). This completes the proof for Step 1.2.

**[Step 2, Semi-pooling]**. Step 1.2 has revealed that when the market valuation function follows \( P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1) \) for all \( 0 \leq z \leq q_1 \), the manager pads all the leftover inventory to the sales once the demand realization \( \xi_1 \) exceeds the level of \( q_1 - \phi(q_1, q_1) \) and then pooling arises. Hence the market value at \( z = q_1 \) needs to be corrected. In this step, we will prove that given \( q_1 \) and \( \phi(z, q_1) \), a unique threshold \( \hat{\xi}(q_1) \in [0, q_1 - \phi(q_1, q_1)) \) exists such that if the market valuation function follows

\[
P(z, q_1) = \begin{cases} v_1(\phi(z, q_1), z - \phi(z, q_1), q_1), & 0 \leq z < q_1 \\ E_{\eta_1} \left[ v_1((q_1 - \eta_1)^+, \eta_1, q_1) \right] | \eta_1 \geq \hat{\xi}(q_1) & z = q_1, \end{cases}
\] (EC.34)
to pad the sales by \( x = \phi(\xi_1, q_1) \) and report \( z = \xi_1 + \phi(\xi_1, q_1) < q_1 \) is the manager’s optimal strategy when \( \xi_1 < \hat{\xi}(q_1) \), and to pad the sales by \( x = (q_1 - \xi_1)^+ \) and report \( z = q_1 \) is the manager’s optimal strategy when \( \xi_1 \geq \hat{\xi}(q_1) \). This result will complete the proof for the theorem.

Notice that if the manager follows the strategy \( x = \phi(\xi_1, q_1) \) to pad the sales then the market value, \( P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1) \), matches the real value of the firm; that is,

\[
P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1)
\] (EC.35)
the equilibrium, as we will discuss afterwards. Results will be sufficient to show the existence of the desired threshold, \( \hat{\xi} \), and \( \xi = LHS(\xi, q_1), q_1) \).

Thus, when padding \( x = \varphi(\xi_1, q_1) \), the manager’s utility follows:

\[
    u(\varphi(\xi_1, q_1), \xi_1, q_1) = \beta v_1(\varphi(z, q_1), z - \varphi(z, q_1), q_1) + (1 - \beta) v_1(\varphi(\xi_1, q_1), \xi_1, q_1) \tag{EC.36}
\]

\[
    = v_1(\varphi(\xi_1, q_1), \xi_1, q_1).
\]

Now suppose the investors value the firm by \( P(q_1, q_1) = E_{\eta_1} \left[v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi} \right] \) when \( z = q_1 \). If the manager pads the sales by \( q_1 - \xi_1 \) and reports \( z = q_1 \), his utility will be

\[
    u(q_1 - \xi_1, \xi_1, q_1) = \beta E_{\eta_1} \left[v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi} \right] + (1 - \beta) v_1(q_1 - \xi_1, \xi_1, q_1). \tag{EC.37}
\]

Hence the manager makes the decision by

\[
    \max \left\{ u(\varphi(\xi_1, q_1), \xi_1, q_1) = v_1(\varphi(\xi_1, q_1), \xi_1, q_1) \right\}
\]

\[
    u(q_1 - \xi_1, \xi_1, q_1) = \beta E_{\eta_1} \left[v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi} \right] + (1 - \beta) v_1(q_1 - \xi_1, \xi_1, q_1). \tag{EC.38}
\]

To show the existence of the desired threshold \( \hat{\xi}(q_1) \in [0, q_1 - \varphi(q_1, q_1)) \), we rearrange the terms in Equation (EC.38) and define

\[
    LHS(\xi) \equiv v_1(\varphi(\xi, q_1), \xi, q_1) - (1 - \beta) v_1(q_1 - \xi, \xi, q_1) \tag{EC.39};
\]

\[
    RHS(\xi) \equiv \beta E_{\eta_1} \left[v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi} \right].
\]

In the following, we will show that when \( \xi \in [0, q_1 - \varphi(q_1, q_1)) \), \( LHS(\xi) \) is strictly decreasing in \( \xi \) and \( RHS(\xi) \) is strictly increasing in \( \xi \), and when \( \xi = q_1 - \varphi(q_1, q_1) \), \( RHS(\xi) > LHS(\xi) \). These results will be sufficient to show the existence of the desired threshold, \( \hat{\xi}(q_1) \), and the structure of the equilibrium, as we will discuss afterwards.

- [First, \( LHS(\xi) \) is strictly decreasing in \( \xi \) when \( \xi \in [0, q_1 - \varphi(q_1, q_1)) \)): Expanding the terms in \( LHS(\xi) \), we obtain

\[
    LHS(\xi) = v_1(\varphi(\xi, q_1), \xi, q_1) - (1 - \beta) v_1(q_1 - \xi, \xi, q_1) \tag{EC.40}
\]
\[ = p \min(q_1, \xi) - cq_1 - h(q_1 - \xi)^+ - \gamma (\varphi(\xi, q_1)) + \Pi_2(\xi, q_1) \]

\[ - (1 - \beta) \left( p \min(q_1, \xi) - cq_1 - h(q_1 - \xi)^+ - \gamma (q_1 - \xi) + \Pi_2(\xi, q_1) \right) \]

\[ = \beta v_1(0, \xi, q_1) + (1 - \beta) \gamma (q_1 - \xi) - \gamma (\varphi(\xi, q_1)). \]

Taking the derivative,

\[
\frac{d \text{LHS}(\xi)}{d \xi} = \beta \frac{\partial v_1(0, \xi, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(q_1 - \xi) - \gamma'(\varphi(\xi, q_1)) \frac{\partial \varphi(\xi, q_1)}{\partial \xi_1} \tag{EC.41}
\]

\[ < \beta \frac{\partial v_1(0, \xi, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(\varphi(\xi, q_1)) - \gamma'(\varphi(\xi, q_1)) \frac{\partial \varphi(\xi, q_1)}{\partial \xi_1}. \]

The inequality holds because \( \varphi(\xi, q_1) < q_1 - \xi \) when \( \xi \in [0, q_1 - \phi(q_1, q_1)) \) and \( \gamma'(x) \) is a strictly increasing function.

As long as \( \varphi(\xi, q_1) \) follows the ODE in Equation (EC.14), i.e., \( \frac{\partial \varphi(\xi, q_1)}{\partial \xi_1} = \frac{\beta (0, \xi, q_1) - (1 - \beta) \gamma'(\varphi(\xi, q_1))}{\gamma'(\varphi(\xi, q_1))} \), we can obtain

\[
\frac{d \text{LHS}(\xi)}{d \xi} < \beta \frac{\partial v_1(0, \xi, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(\varphi(\xi, q_1)) - (1 - \beta) \gamma'(\varphi(\xi, q_1)) \frac{\partial \varphi(\xi, q_1)}{\partial \xi_1} = 0. \tag{EC.42}
\]

The last equality holds since \( \frac{\partial v_1(0, \xi, q_1)}{\partial \xi_1} \) is independent of \( x \). Thus \( \text{LHS}(\xi) \) strictly decreases in \( \xi \).

In case \( \varphi(\xi, q_1) \) does not follow the ODE in Equation (EC.14), \( \varphi(\xi, q_1) = 0, \frac{\partial \varphi(\xi, q_1)}{\partial \xi_1} = 0 \), and \( \xi \geq \xi_N (= z_N) \). Substituting \( \varphi(\xi, q_1) = 0 \) and \( \frac{\partial \varphi(\xi, q_1)}{\partial \xi_1} = 0 \) into \( \frac{d \text{LHS}(\xi)}{d \xi} \) yields

\[
\frac{d \text{LHS}(\xi)}{d \xi} < \beta \frac{\partial v_1(0, \xi, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0). \tag{EC.43}
\]

From the definition of \( z_N, \beta \frac{\partial v_1(0, z_N, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \), which implies

\[
\frac{d \text{LHS}(\xi)}{d \xi} < \beta \frac{\partial v_1(0, \xi, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq \beta \frac{\partial v_1(0, z_N, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0. \tag{EC.44}
\]

Hence \( \text{LHS}(\xi) \) is strictly decreasing in \( \xi \).

- [Second, \( \text{RHS}(\xi) \) is strictly increasing in \( \xi \) when \( \xi \in [0, q_1 - \phi(q_1, q_1)) \): We derive

\[
\frac{d \text{RHS}(\xi)}{d \xi} = \frac{d}{d \xi} \left[ \int_{\xi}^{\infty} v_1((q_1 - \eta_1)^+, \eta_1, q_1) \frac{f_1(\eta_1)}{F_1(\xi)} \, d\eta_1 \right] \tag{EC.45}
\]
which asserts \( \text{RHS}(\xi) \) is strictly increasing in \( \xi \).

- [Third, when \( \xi = q_1 - \phi(q_1, q_1) \), \( \text{RHS}(\xi) > \text{LHS}(\xi) \)]: Recall from Lemma EC.2 that \( \varphi(\xi, q_1) = q_1 - \xi \) when \( \xi = q_1 - \phi(q_1, q_1) \). Thus, when \( \xi = q_1 - \phi(q_1, q_1) \),

\[
\text{RHS}(\xi) - \text{LHS}(\xi) = \beta \mathbb{E}_{\eta_1} \left[ v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \xi \right] - \beta v_1(q_1 - \xi, \xi, q_1) \quad \text{(EC.46)}
\]

\[
= \beta \int_{\xi}^{\infty} \left( v_1((q_1 - \eta_1)^+, \eta_1, q_1) - v_1(q_1 - \xi, \xi, q_1) \right) \frac{f_1(\eta_1)}{F_1(\xi)} \, d\eta_1 > 0.
\]

From the above three results, we can conclude that (a) either a unique \( \hat{\xi}(q_1) \in [0, q_1 - \phi(q_1, q_1)) \) exists such that \( \text{RHS}(\xi) \geq \text{LHS}(\xi) \) when \( \xi \geq \hat{\xi}(q_1) \), or (b) \( \text{RHS}(0) > \text{LHS}(\xi) \) for any \( \xi \). In case (a), using \( \hat{\xi}(q_1) \) to characterize the market value for \( z = q_1 \), we will have \( \text{RHS}(\hat{\xi}(q_1)) \geq \text{LHS}(\xi) \), or equivalently, \( u(q - \xi, \xi, q_1) \geq u(\varphi(\xi, q_1), \xi, q_1) \) when \( \xi \in [0, q_1 - \phi(q_1, q_1)] \). In case (b), setting \( \hat{\xi}(q_1) = 0 \) and using it to characterize the market value for \( z = q_1 \), we will have \( \text{RHS}(\hat{\xi}(q_1)) > \text{LHS}(\xi) \), or equivalently, \( u(q_1 - \xi_1, q_1) > u(\varphi(\xi_1, q_1), \xi_1, q_1) \) for any \( \xi \in [0, q_1 - \phi(q_1, q_1)] \). Thus, if the market value follows \( P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1) \) when \( 0 \leq z < q_1 \) and \( P(z, q_1) = \mathbb{E}_{\eta_1} \left[ v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi}(q_1) \right] \) when \( z = q_1 \), to pad the sales by \( q_1 - \xi_1 \) \( \varphi(\xi_1, q_1) \) is the manager’s optimal strategy when \( \xi_1 \in [0, q_1 - \phi(q_1, q_1)] \).

Note that when \( \xi_1 = q_1 - \phi(q_1, q_1) \), \( \varphi(\xi_1, q_1) = \phi(q_1, q_1) = q_1 - \xi_1 \). As a result, given \( u(q_1 - \xi_1, \xi_1, q_1) > u(\varphi(\xi_1, q_1), \xi_1, q_1) \) when \( \xi_1 = q_1 - \phi(q_1, q_1) \), we can observe from Equation (EC.38) that \( P(q_1, q_1) = \mathbb{E}_{\eta_1} \left[ v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi}(q_1) \right] > v_1(\phi(q_1, q_1), q_1 - \phi(q_1, q_1), q_1) \).

This result indicates that \( P(q_1, q_1) = \mathbb{E}_{\eta_1} \left[ v_1((q_1 - \eta_1)^+, \eta_1, q_1) \mid \eta_1 \geq \hat{\xi}(q_1) \right] > P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1) \) as \( z \to q_1 \); that is, a jump in the market value exists at \( z = q_1 \).

For \( \xi_1 \in (q_1 - \phi(q_1, q_1), q_1) \), we have shown in Step 1.2 that if \( P(z, q_1) = v_1(\phi(z, q_1), z - \phi(z, q_1), q_1) \) for all \( 0 \leq z \leq q_1 \) then the manager’s optimal strategy is to pad the sales by \( q_1 - \xi_1 \). Therefore, as
long as \( P(q_1, q_1) = \mathbb{E}_{\eta_1} \left[ v_1 ((q_1 - \eta_1)^+, \eta_1, q_1) | \eta_1 \geq \hat{\xi}(q_1) \right] > P(z, q_1) = v_1 (\phi(z, q_1), z - \phi(z, q_1), q_1) \) as \( z \to q_1 \), the manager still prefers to pad the sales by \( q_1 - \xi_1 \) for any \( \xi_1 \in (q_1 - \phi(q_1, q_1), q_1) \).

Hence, if the market value follows \( P(z, q_1) = v_1 (\phi(z, q_1), z - \phi(z, q_1), q_1) \) when \( 0 \leq z < q_1 \) and \( P(z, q_1) = \mathbb{E}_{\eta_1} \left[ v_1 ((q_1 - \eta_1)^+, \eta_1, q_1) | \eta_1 \geq \hat{\xi}(q_1) \right] \) when \( z = q_1 \), then to pad the sales by \((q_1 - \xi_1)^+\) (\( \phi(\xi_1, q_1) \)) is the manager’s optimal strategy for any \( \xi_1 \geq (\leq) \hat{\xi}(q_1) \), and thus the existence of a unique threshold \( \hat{\xi}(q_1) \) is confirmed which divides the semi-separating and semi-pooling of the equilibrium as presented in Theorem 1.

Note that the manager may be indifferent to pad the sales by \( \phi(\xi_1, q_1) \) or by \( q_1 - \xi_1 \) at the threshold \( \xi_1 = \hat{\xi}(q_1) \), we combine this scenario with the semi-pooling part of the equilibrium. Furthermore, given the above equilibrium structure, the reported sales \( z \), in equilibrium, will not appear in the region \( [\hat{\xi}(q_1) + \phi(\xi(q_1), q_1), q_1] \). The off-equilibrium path belief is specified as: the investors would believe the padded sales equal \( \phi(z, q_1) \) if a report with \( z \in [\hat{\xi}(q_1) + \phi(\xi(q_1), q_1), q_1] \) was released. This specification obviously supports the equilibrium based on the above proof.

Finally, the properties of the channel stuffing and market valuation strategies in the semi-separating part of the equilibrium (i.e., the properties of \( \phi(\xi, q_1) \) and \( \phi(z, q_1) \)) have been shown in Lemmas EC.1 and EC.2. This completes the proof for Theorem 1. ■

**Proof of Proposition 1:** This proposition is straightforward. If \( \beta = 0 \), the manager’s utility strictly decreases in the amount of padded sales, and thus no channel stuffing will arise. ■

**Proof of Proposition 2:**

**Overview:** In this proof, we show in part (i) that if \( \theta(q_1) \leq 0 \) then channel stuffing does not arise in the semi-separating part of the equilibrium; we show in part (ii) that if \( \theta(q_1) \leq 0 \) and at the same time \( a \) goes to zero then channel stuffing does not arise in the whole-equilibrium.

(i) According to Lemmas EC.1 and EC.2, if \( \theta(q_1) = \beta \frac{\partial v_1(0, 0, q_1)}{\partial q_1} - (1 - \beta) \gamma'(0) \leq 0 \) then \( \xi_N = z_N = 0 \) and hence both \( \phi(\xi_1, q_1) \) and \( \phi(z, q_1) \) remain at zero for any \( \xi_1 \) and \( z \).

(ii) If \( a \) goes to zero then \( \mathbb{E}_{\eta_1} [v_1 (0, \eta_1, q_1) | \eta_1 \geq q_1] = v_1 (0, q_1, q_1) \) (see the proof of Lemma 2). At the same time, if \( \theta(q_1) = \beta \frac{\partial v_1(0, 0, q_1)}{\partial q_1} - (1 - \beta) \gamma'(0) \leq 0 \) then \( \phi(z, q_1) \) is always equal to zero and thus the market value follows \( P(z, q_1) = v_1 (0, z, q_1) \) for all \( 0 \leq z \leq q_1 \). Consequently, given \( \theta(q_1) \leq 0 \),
for any \( \xi_1 \geq 0 \) and \( x \geq 0 \), the manager’s utility function satisfies: 
\[
\frac{\partial u(x, \xi_1, q_1)}{\partial x} = \frac{\beta \partial v_1(0, \xi_1 + x, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(x) \leq \frac{\beta \partial v_1(0, 0, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(0) \leq 0 \text{ (recall from Lemma 1 that the derivative } \frac{\partial v_1(0, \xi_1, q_1)}{\partial \xi_1} \text{ decreases in } \xi_1, \text{ and at the same time, } \gamma'(x) \text{ increases } x). \]

\[\text{Hence no channel stuffing will arise.} \]

**Proof of Proposition 3:**

**Overview:** In this proof, we show in part (i) that \( \phi(z, q_1) \) and \( \varphi(\xi_1, q_1) \) increases if \( \beta, h, \) or \( p \) increases or \( c \) decreases; we show in part (ii) that the threshold \( \xi(q_1) \) decreases if \( \beta, h, \) or \( p \) increases or \( c \) decreases.

(i). We first examine the two ODEs that characterize \( \varphi(\xi_1, q_1) \) and \( \phi(z, q_1) \):

\[
\begin{align*}
\frac{\partial \varphi(\xi_1, q_1)}{\partial \xi_1} &= \frac{\beta \partial v_1(\varphi(\xi_1, q_1), \xi_1, q_1)}{\partial \xi_1} - (1 - \beta) \gamma'(\varphi(\xi_1, q_1)) ; \\
\frac{\partial \phi(z, q_1)}{\partial z} &= 1 - \frac{\gamma'(\phi(z, q_1))}{\gamma'(\varphi(\xi_1, q_1))}.
\end{align*}
\]

Recall from Lemma 1 that

\[
\frac{\partial v_1(x, \xi_1, q_1)}{\partial \xi_1} = \begin{cases} 
(1 + a)pF_2(q_1 - (1 + a)\xi_1) + h, & 0 \leq \xi_1 \leq \bar{\xi}(q_1) \\
(1 + a)(p - c) + h, & \bar{\xi}(q_1) < \xi_1 < q_1. 
\end{cases}
\]

Therefore, we can rewrite Equations (EC.47) and (EC.48) to

\[
\begin{align*}
\frac{\partial \varphi(\xi_1, q_1)}{\partial \xi_1} &= \begin{cases} 
\frac{\beta (1 + a)pF_2(q_1 - (1 + a)\xi_1) + h}{\gamma'(\varphi(\xi_1, q_1))} - (1 - \beta), & 0 \leq \xi_1 \leq \min(\bar{\xi}(q_1), q_1 - \phi(q_1, q_1)) \\
\frac{\beta (1 + a)(p - c) + h}{\gamma'(\varphi(\xi_1, q_1))} - (1 - \beta), & \min(\bar{\xi}(q_1), q_1 - \phi(q_1, q_1)) < \xi_1 < q_1 - \phi(q_1, q_1) 
\end{cases} ; \\
\frac{\partial \phi(z, q_1)}{\partial z} &= \begin{cases} 
1 - \frac{\gamma'(\phi(z, q_1))}{\gamma'(\phi(\xi_1, q_1))} & 0 \leq \xi_1 \leq \bar{\xi}(q_1) \\
1 - \frac{1}{\beta ((1 + a)(p - c) + h) + \gamma'(\phi(z, q_1))} & \bar{\xi}(q_1) < \xi_1 < q_1 
\end{cases}.
\end{align*}
\]

Notice from Equation (EC.50) that for any given \( \xi_1 \) and \( \varphi(\xi_1, q_1) \), the derivative \( \frac{\partial \phi(\xi_1, q_1)}{\partial \xi_1} \) will be larger if \( \beta, p, h, \) or \( q_1 \) is larger or \( c \) is smaller. Similarly, for any given \( z \) and \( \phi(z, q_1) \), the derivative \( \frac{\partial \phi(z, q_1)}{\partial z} \) will be larger if \( \beta, p, h, \) or \( q_1 \) is larger or \( c \) is smaller.

In the following, we argue that \( \varphi(\xi_1, q_1) \) increases for any given \( \xi_1 \) if \( p \) becomes larger. Suppose there are two prices, \( p_a < p_b \). Let \( \varphi_a(\xi_1, q_1) \) and \( \varphi_b(\xi_1, q_1) \) denote the two channel stuffing functions associated with these two prices. Both \( \varphi_a(\xi_1, q_1) \) and \( \varphi_b(\xi_1, q_1) \) are equal to zero when \( \xi_1 = 0 \), but \( \frac{\partial \varphi_a(0, q_1)}{\partial \xi_1} < \frac{\partial \varphi_b(0, q_1)}{\partial \xi_1} \). As a result, \( \varphi_b(\xi_1, q_1) \) increases immediately above \( \varphi_a(\xi_1, q_1) \) as \( \xi_1 \) increases. Since both \( \varphi_a(\xi_1, q_1) \) and \( \varphi_b(\xi_1, q_1) \) are continuous, we can show by contradiction that no \( \xi_1 \) exists where \( \varphi_a(\xi_1, q_1) \) can cross \( \varphi_b(\xi_1, q_1) \) from the below to the above. Suppose such a demand realization
First, we prove when $\beta$ increases, $\xi_1(q_1)$ decreases. Notice $\mathbb{E}_{\eta_1}[v_1(0, \eta_1, q_1) | \eta_1 \geq \xi] > v_1(0, \xi, q_1)$ and $\gamma(q_1 - \xi) > \mathbb{E}_{\eta_1}[\gamma((q_1 - \eta_1)^+) | \eta_1 \geq \xi]$ since $v_1(0, \xi, q_1)$ is an increasing function of $\xi_1$ and $\gamma(x)$ is an increasing function of $x$. Furthermore, based on the proof in (i), if $\beta$ increases then $\varphi(\xi_1, q_1)$ increases. Hence $RHS(\xi) - LHS(\xi)$ increases in $\beta$. On the other hand, as shown in Step
2 in the Proof of Theorem 1, \(LHS(\xi)\) strictly decreases in \(\xi\), whereas \(RHS(\xi)\) strictly increases in \(\xi\), which implies \(RHS(\xi) - LHS(\xi)\) strictly increases in \(\xi\). Therefore, if \(RHS(\xi) = LHS(\xi)\) at \(\xi = \hat{\xi}(q_1)\), in order for this equality to hold, when \(\beta\) increases, the threshold \(\hat{\xi}(q_1)\) must decrease; if \(RHS(0) > LHS(\xi)\) for any \(\xi\), when \(\beta\) increases, \(RHS(0)\) becomes even larger than \(LHS(0)\) and thus \(RHS(0) > LHS(\xi)\) still holds for any \(\xi\) (given \(LHS(\xi)\) decreases in \(\xi\)), which shows \(\hat{\xi}(q_1)\) will remain at zero.

Second, we prove when \(p\) increases, \(\hat{\xi}(q_1)\) decreases. To do so, we take the derivative of \(RHS(\xi) - LHS(\xi)\) with respect to \(p\):

\[
\frac{d}{dp} (RHS(\xi) - LHS(\xi)) = \frac{d}{dp} (\mathbb{E}_{\eta_1} [v_1(0, \eta_1, q_1) | \eta_1 \geq \xi] - v_1(0, \xi, q_1)) + \frac{d\gamma(\varphi(\xi, q_1))}{dp}. \tag{EC.55}
\]

From Equation (EC.3), we derive:

\[
\frac{d}{dp} (\mathbb{E}_{\eta_1} [v_1(0, \eta_1, q_1) | \eta_1 \geq \xi] - v_1(0, \xi, q_1)) = \mathbb{E}_{\eta_1} \left[ \min(\eta_1, q_1) + \frac{d\Pi_2(\eta_1, q_1)}{dp} | \eta_1 \geq \xi \right] - \xi - \frac{d\Pi_2(\xi, q_1)}{dp}. \tag{EC.56}
\]

Given \(\Pi_2^*(\eta_1, q_1)\) in Equation (EC.2), we obtain by the envelope theorem:

\[
\frac{d}{d\eta_1} \left( \min(\eta_1, q_1) + \frac{d\Pi_2(\eta_1, q_1)}{dp} \right) = \begin{cases} 
(1 + a) F_2(q_1 - (1 + a) \eta_1), & 0 \leq \eta_1 \leq \bar{\xi}(q_1) \\
1 + a, & \bar{\xi}(q_1) < \eta_1 < q_1 \\
0, & \eta_1 \geq q_1 
\end{cases} \tag{EC.57}
\]

Therefore, \(\min(\eta_1, q_1) + \frac{d\Pi_2(\eta_1, q_1)}{dp}\) increases in \(\eta_1\), which asserts \(\frac{d(\mathbb{E}_{\eta_1} [v_1(0, \eta_1, q_1) | \eta_1 \geq \xi] - v_1(0, \xi, q_1))}{dp} > 0\). From part (i), \(\varphi(\xi_1, q_1)\) increases in \(p\) and thus \(\frac{d\gamma(\varphi(\xi, q_1))}{dp} > 0\) given \(\gamma(x)\) is convex. As a result, \(\frac{d(RHS(\xi) - LHS(\xi))}{dp} > 0\); that is, \(RHS(\xi) - LHS(\xi)\) strictly increases in \(p\). On the other hand, \(RHS(\xi) - LHS(\xi)\) strictly increases in \(\xi\), as shown in Step 2 in the Proof of Theorem 1. Hence, by the same argument which we used to prove the result for \(\beta\) in the above, we can conclude that \(\hat{\xi}(q_1)\) decreases if \(p\) increases.

It can be similarly shown that \(RHS(\xi) - LHS(\xi)\) increases in \(h\) and decreases in \(c\), which implies that the threshold \(\hat{\xi}(q_1)\) decreases in \(h\) and increases in \(c\). ■

**Proof of Proposition 4:** This proposition follows from the Proof of Proposition 3, part (i). ■
Figure EC.2 Comparative statics of the manager’s channel stuffing strategy, \( x(\xi_1, q_1) \) and the market valuation function \( \phi(z, q_1) \). In this experiment, we select the parameters in the base setting as \( q_1 = 6.8, \beta = 0.26, p = 10, c = 6, h = 1, a = 1.5, \gamma(x) = \gamma_1 x + \frac{1}{2} \gamma_2 x^2 \) where \( \gamma_1 = \gamma_2 = 4 \), and \( \eta_{1,2} \) follow a Gamma distribution with both mean and variance equal to 5. We illustrate the impact of \( a, \gamma_1 \) and \( \gamma_2 \), respectively, keeping the other parameters under the base setting. Note that \( x(\xi_1, q_1) \) consists of two parts: the function \( \varphi(\xi_1, q_1) \) when \( \xi_1 < \hat{\xi}(q_1) \) and the function \( (q_1 - \xi_1)^+ \) when \( \xi_1 \geq \hat{\xi}(q_1) \). The top panels show that the curve, \( \varphi(\xi_1, q_1) \), shifts up when \( a \) becomes larger and shifts down when \( \gamma_1 \) or \( \gamma_2 \) becomes larger; the threshold, \( \hat{\xi}(q_1) \), decreases in \( a \) and increases in \( \gamma_1 \) and \( \gamma_2 \). The bottom panels show that the curve, \( \phi(z, q_1) \), shifts up when \( a \) becomes larger and shifts down when \( \gamma_1 \) or \( \gamma_2 \) becomes larger.

Appendix B: Numerical Experiments
Appendix B.1: Comparative Statics

This subsection provides numerical illustrations for the comparative statics. In Figure EC.2, we illustrate the impacts of \( a \) and \( \gamma(x) \) on the equilibrium channel stuffing strategy \( x(\xi_1, q_1) \) and the market valuation function \( \phi(z, q_1) \). In Figure EC.3, we demonstrate how an increase of \( q_1 \) may influence the threshold \( \hat{\xi}(q_1) \) and the length of the interval, \( [\hat{\xi}(q_1), q_1] \) (i.e., the region of the semi-pooling part of the equilibrium).

Appendix B2: Distortion of Initial Inventory Investment

This subsection illustrates how channel stuffing may influence the initial inventory decision. We first provide the derivation of Equation (19) in section 5.4; that is, the manager’s expected utility function with respect to the initial inventory decision \( q_1 \).
Figure EC.3  Impact of the initial inventory level $q_1$ on the semi-pooling part of the equilibrium. In this experiment, besides $\beta$, the other parameters are: $p = 10$, $c = 6$, $h = 1$, $a = 1.5$, $\gamma(x) = 4x + 2x^2$, and $\eta_{1,2}$ follow a Gamma distribution with both mean and variance equal to 5. The four top panels show that the threshold $\hat{\xi}(q_1)$ increases in $q_1$. The four bottom panels show that the length of the semi-pooling region, $[\hat{\xi}(q_1), q_1)$, that is, $q_1 - \hat{\xi}(q_1)$, first increases in $q_1$ when the threshold $\hat{\xi}(q_1)$ is a boundary solution (i.e., $\hat{\xi}(q_1) = 0$), and then decreases in $q_1$ when $q_1$ becomes relatively large and the threshold $\hat{\xi}(q_1)$ is an interior solution (i.e., $\hat{\xi}(q_1) > 0$); when $\beta$ and $q_1$ are both large, the length of the semi-pooling region, $q_1 - \hat{\xi}(q_1)$ may increase again in $q_1$ as shown in the right bottom panel.

**Derivation of Equation (19):** When the manager makes the initial inventory decision $q_1$, he takes into account the potential channel stuffing and the firm’s market valuation. In particular, for the semi-separating part of the equilibrium on channel stuffing and market valuation (i.e., $\xi_1 < \hat{\xi}(q_1)$), the investors can perfectly infer the real demand, and thus the market value of the firm equals the real value of the firm. For the semi-pooling part of the equilibrium (i.e., $\xi_1 > \hat{\xi}(q_1)$), the market value of the firm equals the expected real value of the firm over this region. Therefore, the manager’s expected utility with respect to $q_1$ follows:

$$
\mathbb{E}_{\eta_1} \left[ \beta P(\eta_1, q_1, q_1) + (1 - \beta) v_1(\eta_1, q_1), \eta_1, q_1) \right] = \mathbb{E}_{\eta_1 \in [0, \hat{\xi}(q_1)]} \left[ v_1(\eta_1, q_1), \eta_1, q_1) \right] + \beta \mathbb{E}_{\eta_1 \in [\hat{\xi}(q_1), \infty)} \left[ P(q_1, q_1) \right] + (1 - \beta) \mathbb{E}_{\eta_1 \in [\hat{\xi}(q_1), \infty)} \left[ v_1(\eta_1, q_1), \eta_1, q_1) \right] = \int_{\hat{\xi}(q_1)}^{\infty} \left[ -cq_1 + p \min(\eta_1, q_1) - h(q_1 - \eta_1)^+ + \Pi_2(\eta_1, q_1) - \gamma(x(q_1), q_1)) \right] f_1(\eta_1) d\eta_1.
$$

(EC.58)
\[ + \beta \int_{\xi(q_1)}^{\infty} \int_{\xi(q_1)}^{\infty} \left[ -cq_1 + \frac{p \min(q_1, q_1) - h(q_1 - \eta_1)}{\Phi_1(q_1)} \right] \frac{f_1(\eta_1)}{\Phi_1(q_1)} d\eta_1 d\xi \\
+ \int_{\xi(q_1)}^{\infty} \left[ -cq_1 + \frac{p \min(q_1, q_1) - h(q_1 - \eta_1)}{\Phi_1(q_1)} \right] f_1(\eta_1) d\eta_1 = -cq_1 + \mathbb{E}_{\eta_1} \left[ \frac{p \min(q_1, q_1) - h(q_1 - \eta_1)}{\Phi_1(q_1)} \right] - \mathbb{E}_{\eta_1} \left[ \frac{\gamma(x(q_1, q_1))}{\Phi_1(q_1)} \right], \]

which is Equation (19) in section 5.4. Notice that Equation (19) also represents the firm’s expected final real value as the firm incurs the channel stuffing cost.

**Numerical Example:** In the following, we illustrate by Figure EC.4 how the presence of channel stuffing incentives may distort the firm’s operations, where we compare the initial inventory level \((q^*_1)\) and the firm’s expected final real value \((V^*)\) in presence of channel stuffing with the first-best \((q^o_1\) and \(V^o\)) where channel stuffing is excluded (i.e., without the last term in Equation (19)).

We observe from Figure EC.4 that compared with the first-best investment, the manager first over-invests in inventory when \(\beta\) is small and then under-invests in inventory when \(\beta\) is large. To explain this finding, we express the expected channel stuffing cost as

\[ \mathbb{E}_{\eta_1} \left[ \frac{\gamma(x(q_1, q_1))}{\Phi_1(q_1)} \right] = \int_{\xi(q_1)}^{\xi(q_1)} \frac{\gamma(\varphi(q_1, q_1))}{\Phi_1(q_1)} f_1(\eta_1) d\eta_1 + \int_{\xi(q_1)}^{\xi(q_1)} \frac{\gamma(q_1 - \eta_1)}{\Phi_1(q_1)} f_1(\eta_1) d\eta_1. \]

Proposition 2 indicates that if \(\theta(q_1) \leq 0\) then \(\varphi(q_1, q_1)\) will remain at zero for \(q_1 \in [0, \xi(q_1))\). Such a situation arises when \(\beta\) is small. Consequently, the second term on the right-hand side in Equation (EC.59) exclusively determines the cost of channel stuffing. As shown in Appendix B1, an increase of \(q_1\) may shrink the interval \([\xi(q_1), \xi(q_1))\). Due to that effect, we observe in this experiment that the expected channel stuffing cost reduces if the manager installs relatively more initial inventory when \(\beta\) is small. As a result, the manager has incentive to over-invest in inventory. When \(\beta\) slightly increases, the manager may over-invest more to reduce the expected channel stuffing cost that is enhanced if \(\beta\) becomes larger. However, as \(\beta\) keeps increasing, \(\varphi(q_1, q_1)\) will become positive, and thus the first term on the right-hand side in Equation (EC.59) will play a role. Recall from Proposition 4 that the function \(\varphi(q_1, q_1)\) increases in \(q_1\). Moreover, when \(\varphi(q_1, q_1)\) is positive, if \(q_1\) increases, the total region where the manager pads the sales expands. Thus, we observe in this experiment that as \(\beta\) becomes relatively large, the total channel stuffing cost starts to increase in
Figure EC.4 The comparison between the firm’s initial inventory investments and expected final real values in the cases with and without channel stuffing, as $\beta$ increases. The parameters are $p = 10$, $c = 6$, $h = 1$, $a = 1.5$, $\gamma(x) = 4x + 2x^2$, and $\eta_1, \eta_2$ follow a Gamma distribution with both mean and variance equal to 5.

$q_1$. Therefore, under-investing in inventory will be a more beneficial strategy for the manager. The investment sharply reduces as $\beta$ becomes large. After $\beta$ exceeds some threshold, given a relatively low initial inventory level and a large $\beta$, in equilibrium, the manager always pads all remaining inventory. Once the structure of the equilibrium does not change any more, the initial inventory investment that the manager will make remains at some low level. We also observe from Figure EC.4 that the loss of the firm’s value due to channel stuffing always increases when $\beta$ increases. This observation is intuitive since the overall amount of channel stuffing increases in $\beta$ and thus the total channel stuffing cost increases.