Renewal Properties of Fade and Interfade Duration on Land Mobile Satellite Channel

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Abstract—This paper investigates the renewal properties of the fade and interfade duration processes on a Land Mobile Satellite (LMS) channel and proposes a suitable Markov chain model to reproduce the renewal properties of the original fading channel. The investigated radio link is an L-band terrestrial-satellite channel, measured on board of a moving vehicle on highway; therefore the channel was affected by multipath propagation impairments. In the paper we introduce a novel approach with transforming the analog received power process into a digital fading process in order to characterize the digital fade and interfade duration and calculate their high order statistics. The digital fade and interfade duration process allows determining their renewal properties.

To correctly reproduce the renewal property of the fade and interfade duration processes and model them with a Markov chain an appropriate discrete time and discrete state homogenous Markov chain has to be selected. We will show that the three investigated Markov chain models (Gilbert, Gilbert-Elliott and Fritchman) are having different reproducing capability in point of view of the renewal property. Finally, we will propose the most suitable model for the fade and interfade duration process.

Index Terms—multipath fading, fade and interfade duration, high order statistics, renewal property, Markov chain

I. INTRODUCTION

The high frequency radio connections operating between a satellite and a moving terrestrial receiver are often affected by the multipath propagation and shadowing effects which may cause remarkable attenuation [1]. The first and second order statistics of the attenuation are important characteristics of the radio connection and it has to be taken in consideration in the engineering phase of its design to select the transmitting power levels, applying the right coding scheme or fade mitigation techniques [2].

The fade and interfade duration is one of the most important second order statistics which can be applied in the above mentioned design phase. The ITU-R recommends a statistical model to describe the fade duration distribution as the function of different physical link parameters [3]. In [4] the authors of this paper are proposed a Markov chain based model to calculate the fade and interfade duration statistics.

Multipath fading is a stochastic process, but there are known distributions that can be applied to characterize it. In our contribution we analyze the fade and interfade duration on the investigated LMS radio link and catch the renewal property of these processes.

The renewal theory deals with the dependence of consecutive random variables [5]. In our case we consider the duration of fade and interfade events as random variables and investigate their dependence even at higher orders. The goal of this research is to select the right Markov chain model which is capable to reproduce the renewal properties of the original fading channel.

II. PARAMETERS OF THE MEASURED RADIO LINK

Our investigations are based on the measurement data of a land mobile satellite channel received on board of a moving vehicle with a speed of 60 km/h on highway. The observable channel impairments are producing even 30-40 dBm decrease of the received power.

The measurements are performed by DLR during 1984-87 [6].

The measured analog time function of the received power has been sampled with 300.5 Hz frequency and digitalized by an A/D converter. A normalization of the data was carried out to set the second moment of the fading amplitude process (power) equal to 1 (0 dB). An attenuation time series was calculated by subtracting the received power from the median value.

The parameters of the measurement are detailed in Table 1.

<table>
<thead>
<tr>
<th>Table 1. DESCRIPTION OF THE MEASURED SATELLITE LINK</th>
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<tbody>
<tr>
<td>Satellite</td>
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<tr>
<td>Elevation</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Sampling rate</td>
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<tr>
<td>Vehicle speed</td>
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<td>Measurement duration</td>
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III. FADE AND INTERFADE DURATION AS A DIGITAL PROCESS

Fade duration is the time interval between two crossings at the same attenuation threshold, where the attenuation is higher than the threshold. Similarly, the interfade duration is defined as the time interval where the attenuation is below the same threshold [3]. According to the definitions, fade and interfade duration are calculated for different thresholds providing an important second order statistics of the investigated radio connection.

In [7] a method has been introduced to characterize the error intervals and cluster density in channel modeling. On the analogy of this approach this paper introduces the...
concept of the Digital Fading Process (DFP). The DFP consists of consecutive events, denoted by \( e_i \), that takes the value of 1 if the attenuation is higher than the given threshold and 0 if it is lower than the threshold. In Fig. 1, a segment of the attenuation process and the corresponding DFP is depicted for 10 dB threshold.

From the DFP we can derive the digital fade and interfade duration processes. A Digital Fade Duration (DFD) event \( F_i \) is always starting with a one following a zero and ending with the first occurrence of zero, therefore it characterizes the fade bursts in the process. The Digital Interfade Duration (DIFD) event \( I_i \) is always starting with a zero following a one and ending with the first occurrence of one, characterizing the interfade bursts in the fading process. This approach embeds the interfade periods in the DFD process as fade events with the length of one, and the fading periods in the DIFD process as interfade events with the length of one. In this manner the duality between the two processes can be expressed.

In Fig. 2, a segment of the DFP is depicted and the embedded \( F_i \) and \( I_i \) processes are denoted.

![DFP and embedded processes](image)

The elements \( e_{i+1} \cdots e_{j-1} \) of the event are identically ones, while \( e_{j-1} = 0 \). The length of the event is \( L_F = j - i \). The PMF (Probability Mass Function) of the fade duration process gives the probability that the length of the \( i \)th fade duration event is \( \ell \):

\[
f_F(\ell) = \Pr(F_i = \ell) = \Pr(\bigcap_{m=i+1}^{j-\ell} e_m = 0, e_{j-\ell} = 1)
\]

(2)

The CDF (Cumulative Distribution Function) can be calculated from the PMF according to the next equation:

\[
F_F(n) = \sum_{i=1}^{n} f_F(i) = \sum_{i=1}^{n} \Pr(\bigcap_{m=i+1}^{j-\ell} e_m = 0, e_{j-\ell} = 1)
\]

(3)

In the same way a single DIFD event is:

\[
I_i = \{e_{i+1}, e_{i+2}, \ldots, e_{j-1}, e_{j-\ell}\}
\]

(4)

The elements \( e_{i+1} \cdots e_{j-\ell} \) of the event are identically zeros, while \( e_{j-\ell} = 1 \). The length of the event is \( L_I = \ell \). The PMF of the interfade duration process gives the probability that the length of the \( i \)th interfade duration event is \( \ell \):

\[
f_I(\ell) = \Pr(I_i = \ell) = \Pr\left(\bigcap_{m=i+1}^{j-\ell} e_m = 0, e_{j-\ell} = 1 | e_j = 1\right)
\]

(5)

The CDF can be expressed with the next equation:

\[
F_I(n) = \sum_{i=1}^{n} f_I(i) = \sum_{i=1}^{n} \Pr(\bigcap_{m=i+1}^{j-\ell} e_m = 0, e_{j-\ell} = 1 | e_j = 1)
\]

(6)

V. INVESTIGATION OF THE RENEWAL PROPERTY

In this section the higher order statistics of the DFD and DIFD processes will be investigated. This will lead up to analytical expressions to characterize the renewal properties of these processes.

We define the multiple DFD and DIFD event, \( F_i \) and \( I_i \) as \( r \) consecutive single event, respectively:

\[
F^r = \{F_{i_1}, F_{i_2}, \ldots, F_{i_r}\}, I^r = \{I_{i_1}, I_{i_2}, \ldots, I_{i_r}\}
\]

(7, 8)

The PMF of the multiple fade and interfade duration process is:

\[
f_F^r(\ell) = \Pr(L_F^r = \ell) = \Pr\left(\sum_{m=i+1}^{j+1} F_m = \ell\right)
\]

(9)

\[
f_I^r(\ell) = \Pr(L_I^r = \ell) = \Pr\left(\sum_{j=i+1}^{r} I_j = \ell\right)
\]

(10)

To indicate the renewal property of these processes the variation-coefficients will be applied [7, 8]. The variation-coefficient is the normalized second order moment of the multiple fade and interfade duration process. The normalization is made with the second order moment of the multiple fade and interfade duration process of a DMC.
On a DMC the appearance of the events – in our case the digital fading events – is statistically independent. The PMF of the multiple fade duration process can be expressed as a negative binomial distribution [9]:

\[
f_{FDMC}^{(r)}(\ell) = \Pr(\ell | F_{DMC}^{(r)} = \ell) = \binom{\ell - 1}{r - 1} p^r q^{\ell - r}
\]

where the average fading probability is denoted with \( p \) and the non-fading probability is denoted with \( q = 1 - p \).

Similarly, the PMF of the multiple DIFD process is:

\[
f_{IDMC}^{(r)}(\ell) = \Pr(\ell | I_{DMC}^{(r)} = \ell) = \binom{\ell - 1}{r - 1} p^r q^{\ell - r}
\]

The second moment in the case of DMC processes can be calculated as it follows:

\[
D^2(F_{DMC}^{(r)}) = \frac{r \cdot p}{q^2}, \quad D^2(I_{DMC}^{(r)}) = \frac{r \cdot q}{p^2}
\]

Now the variation coefficients of the real channel can be expressed from the investigated multiple DFD and DIFD processes as the normalized second order moment of the multiple fade and interfade duration process:

\[
K_{F}(r) = \frac{D^2(F_{DMC}^{(r)})}{D^2(F_{DMC}^{(r)})} = q^2 \frac{D^2(F_{DMC}^{(r)})}{r \cdot p}
\]

\[
K_{I}(r) = \frac{D^2(I_{DMC}^{(r)})}{D^2(I_{DMC}^{(r)})} = p^2 \frac{D^2(I_{DMC}^{(r)})}{r \cdot q}
\]

V. Renewing Properties of the LMS Radio Link

In this section the renewal properties of the DFD and DIFD processes will be investigated by calculating the variation coefficients of the LMS radio link (Fig. 1.) using Eq. (15, 16) in Fig. 3. The normalized variation coefficients \( K_{F}(r)/K_{F}(1) \) and \( K_{I}(r)/K_{I}(1) \) are depicted for orders \( r = 1 \) to 10.

The higher order values of \( K_{F}(r) \) and \( K_{I}(r) \) are indicating the renewal property of the process. If \( K(r) = K(1) \) for any \( r \), the process is renewal; contrarily, if the variation coefficient shows a change, it is not renewal. The further is the normalized \( K_{F}(r) \) and \( K_{I}(r) \) from 1, the less true is the assumption that the channel is memoryless. The correlation between the successive events shows the tendency of the event length variation. A positive correlation, i.e. the variation coefficient increases with the order, indicates that a short event will be followed by a short one and a long event with a long one. The negative correlation indicates that a short event will be followed by a long one and vice versa.

VI. Digital Models for Nonrenewal Channels

There are a number of different channel characteristics that can be appropriately modeled with stochastic methods.

In this section three different types of discrete time, discrete state homogenous first order Markov chain will be investigated in point of view of their capability of reproducing the renewal properties of the DFD and DIFD processes.

A. The Gilbert Model

The Gilbert model [10] is a two state Markov chain. This model has been developed originally to characterize the burst errors on digital channels. One of the states, denoted by G, represents the error free condition and the probability of the error in this state is zero, \( e_G = 0 \). The other state, denoted by B, represents the error condition and the probability of the error in this state is nonzero, \( e_B > 0 \). A 2*2 quadratic matrix gives the transition probabilities of the model:

\[
P = \begin{bmatrix}
P_{GG} & P_{GB} \\
P_{BG} & P_{BB}
\end{bmatrix}
\]

To investigate this model in the point of renewal property, a simulation has been performed with the model and the variation coefficients have been studied.

Before the simulation the parameters of the Gilbert model has to be approximated. The parameters can be extracted from the statistics of the measured process (see Fig. 1.) according to [10]. For the calculations the \( a, b, c \) parameters are defined as it follows:
\[ a = \Pr(e_i = 1) = \frac{p_{GB}}{p_{GB} + p_{BG}} \cdot e_B \] (18)

\[ b = \Pr(e_i = 1 | e_{i-1} = 1) = p_{BB} \cdot e_B \] (19)

\[ c = \Pr(e_i = 1 | e_{i-1} = 1, e_{i-1} = 1) = \frac{p_{BB} \cdot e_B}{p_{GB} \cdot p_{BG} + p_{BB}} \] (20)

From the measured DFP the probabilities in the equations (18-20) can be determined. Then by solving the equations the Gilbert model parameters \( p_{BB}, e_B \) and \( p_{GB} \) can be expressed, thus the model parameters are completely determined.

After the simulation of the channel with these parameters, the normalized variation coefficients \( K_F(r)/K_F(1) \) can be calculated. The simulation results are depicted in Fig. 4:

![Normalized variation-coefficients](image)

The parameters of the model are the following:

Transition matrix: 
\[ P = \begin{bmatrix} 0.9997 & 0.0003 \\ 0.03 & 0.97 \end{bmatrix} \]

Error probability in state B: \( e_B = 0.65 \)

Error probability in state G: \( e_G = 0.00048 \)

The result is depicted in Fig. 5:

![Normalized variation-coefficients](image)

The result of this simulation shows that the Gilbert-Elliott model reproduces the multiple DFD and DIFD process as nonrenewal, because the normalized variation coefficients are showing an increasing tendency even at high order. The variability expressed by this model is higher than the original channel’s variability, as it can be observed from the large values of the normalized variation coefficients.

### C. The Fritchman-Model

The Fritchman-model [13] is an N-state Markov chain which consists of two partitions of states. It was originally dedicated to characterize the error-free and error-cluster distributions of a binary channel. One partition contains the error states, the other the error-free states. The model doesn’t allow transitions between the states in the same partition. A simplified type of the Fritchman-model is when one of the two partitions is containing only one state. On the analogy of the binary channel errors, this model is applicable to simulate the digital fading events which are now the point of our interest. The multiple DFD and DIFD statistics can be derived from the simulated digital fading events. This model can be easily parameterized with the gradient method [14] based on the measured DFP. The number of required states depends on the measured data; 3-5 states are regular (Fig. 6.).

![Normalized variation-coefficients](image)

The result of this simulation shows that the Gilbert-Elliott model reproduces the multiple DFD and DIFD process as nonrenewal, because the normalized variation coefficients are showing an increasing tendency even at high order. The variability expressed by this model is higher than the original channel’s variability, as it can be observed from the large values of the normalized variation coefficients.
The variation coefficients of the DFD and DIFD processes are determined by the renewal/nonrenewal properties of these processes. The threshold of the DFD and DIFD can be calculated. The normalized variation coefficients of the Fritchman-model simulation are depicted in Fig. 7., applying the following state transition matrix (parameterized from the measurement data with the method described in [14]):

\[
P = \begin{bmatrix}
0.99998 & 0 & 0 & 0 & 0.00002 \\
0 & 0.99997 & 0 & 0 & 0.00003 \\
0 & 0 & 0.99994 & 0 & 0.00006 \\
0 & 0 & 0 & 0.98231 & 0.01769 \\
0.006210 & 0.13630 & 0.19640 & 0.47125 & 0.48927 \\
\end{bmatrix}
\]

The result of the simulation shows that similarly to the Gilbert-Elliott model, the Fritchman-model also reproduces the multiple DFD and DIFD processes as nonrenewal. The main difference comparing the Gilbert-Elliott model with the other models, that this model shows a nonrenewal property of a digital process. The results can be applied also for terrestrial links to characterize the fade and interfade duration caused by rain attenuation.

To continue this work further Markov chain models could be applied to select the most appropriate method to model the fade and interfade duration of radio links. The threshold dependency of the DFD and DIFD renewal properties are also the subject of further investigations.

**VIII. CONCLUSIONS**

In this paper the fade and interfade duration processes are investigated on a land mobile satellite radio link. A novel term, the digital fading process (DFP) has been introduced, which is a digital process originating from the measured attenuation time series of the radio signal and it depends on the selected attenuation threshold. The digital fade and interfade duration processes (DFD and DIFD) are embedded processes in the DFP and their high order statistics has been found as an appropriate tool to reveal the renewal/nonrenewal properties of these processes. The variation coefficients of the DFD and DIFD processes are pointing out the nonrenewal properties of the measured channel.

In the second part of this work three different types of discrete time, discrete state homogenous first order Markov chain are investigated in point of renewal/nonrenewal simulation capabilities. It has been shown that there are significant differences between the Gilbert, Gilbert-Elliott and Fritchman’s Markov model in their capability to simulate the nonrenewal property of a digital process.

According to the simulation results the Gilbert model is the most suitable to reproduce the renewal properties of the measured DFD and DIFD processes.

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**REFERENCES**