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Optimal Targets for the Bank Shot in Men's Basketball

Larry M. Silverberg, Chau M. Tran, and Taylor M. Adams

Abstract

The purpose of this study was to gain an understanding of the bank shot and ultimately determine the optimal target points on the backboard for the bank shot in men's basketball. The study used over one million three-dimensional simulations of basketball trajectories. Four launch variables were studied: launch height, launch speed, launch angle, and aim angle. The shooter's statistical characteristics were prescribed to yield a 70 percent free throw when launching the ball seven feet above the ground with 3 Hz of back spin. We found that the shooter can select a bank shot over a direct shot with as much as a 20 percent advantage. The distribution over the court of preferences of the bank shot over the direct shot was determined. It was also shown that there is an aim line on the backboard independent of the shooter's location on the court. We also found that at 3.326 inches behind the backboard, there exists a vertical axis that aids in finding the optimal target point on the backboard. The optimal target point is the crossing of the vertical axis and the aim line that is in the shooter's line of sight.

KEYWORDS: basketball, bank shot, optimal, backboard

Introduction

For the spectator, the bank shot is distinctive and even a bit mystical. It demands shooting a basketball farther than a direct shot and aiming the ball to the side. Yet, most lay ups are bank shots and there are locations on the court where the probability of a successful bank shot is considerably higher than the probability of a successful direct shot.

Shooters perfect their bank shot technique by performing shooting drills. Initially, however, the shooter can benefit from understanding the best launch conditions. At what aim angle should the ball be launched? Where should the ball make contact with the backboard? How does the contact point on the backboard change with launch distance, launch angle, and launch height? These are difficult questions to answer and, if left unanswered, prevent the shooter from perfecting a most effective bank shot.

The optimal launch conditions for the bank shot are not obvious because of the large number of factors. In practice, a prohibitively large number of bank shots must be studied to gain a complete understanding of the optimal launch conditions. An alternate approach is to perform computer simulations, where millions of shots can be investigated in a relatively short amount of time.

Previous simulation studies of the basketball shot considered trajectories launched from general locations on the court, as well as from the free throw line, while apparently no detailed studies of the bank shot have been conducted. The main contributors are Shibakuwa (1975), Brancazio (1981), Tan and Miller (1981), Hamilton and Reinschmidt (1997), Huston and Grau (2003), and Tran and Silverberg (2008). This paper studies the bank shot in detail and develops targets on the backboard for the perfection of the bank shot technique.

Methods

Silverberg, Tran, and Adcock (2003) developed a general-purpose numerical procedure for simulating basketball trajectories. Their model extended earlier work as follows:

(1) The ball is assumed to be a thin lightly-damped elastic body that undergoes rolling and/or sliding contact with the backboard and the rim.
(2) The ball undergoes any combination of consecutive bounces off the backboard and the rim.
(3) The statistical characteristics of the skill level of the shooter are incorporated in the procedure, making it possible to predict the probability of a successful shot.

Their model neglects three secondary effects. In order of decreasing importance, the neglected effects are: vibration of the backboard and ring; aerodynamic drag and Magnus force on the ball; and the bridge surface between the backboard and ring. Their model has been tested extensively, producing reliable results with errors in basketball simulations of less than 1%, and is used throughout this paper. The dimensions of the court, backboard, and ring that influence the bank shot are the same for international competition (International Basketball Federation, 2006), US collegiate competition (National Collegiate Athletic Association, 2001), and US professional competition (National Basketball Association, 2006). However, the conclusions reached in the present study apply only to men's basketball because in woman's basketball the ball is smaller and lighter (Fig. 1).

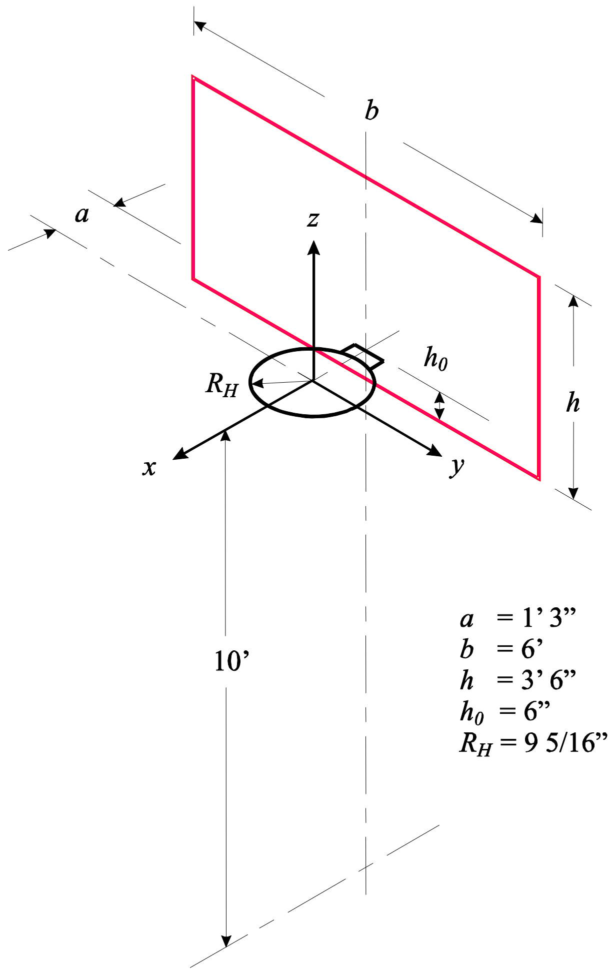

Figure 1. Dimensions

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In Figure 2, the ball is launched from a particular location that can be expressed in terms of the rectangular coordinates (x y z) or equivalently in terms of the cylindrical coordinates (r θ z) in which r denotes radial distance and θ denotes polar angle. The coordinates are located at the center of the ball. The ball is launched with a launch speed v, a launch angle α and an aim angle β. Notice that β is the angle between the plane of the trajectory and a horizontal line parallel to the x axis. When $\beta = \theta$, the player is shooting a direct shot. The shooter also imparts to the ball a back spin ω about an axis that is perpendicular to the vertical plane of the ball's approach to the basket. Out-of-plane components of back spin can be imparted too, but these effects are neglected because of their typically small magnitude.

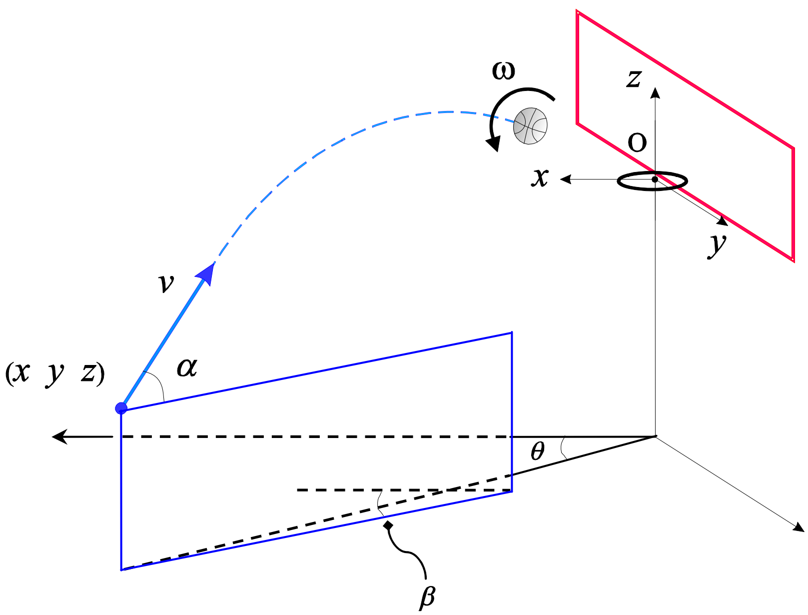

Figure 2. Launch conditions

The shooter's ultimate success depends on two factors. The first is his understanding of the desired shot. Of course, the desired shot is not precisely the optimal one. The second factor is the shooter's consistency. The actual shot will deviate from the desired shot because of the inevitable variability in shooting movements. The selection of the desired shot and the standard deviations in the launch conditions completely determine the chances of a shot being successful.

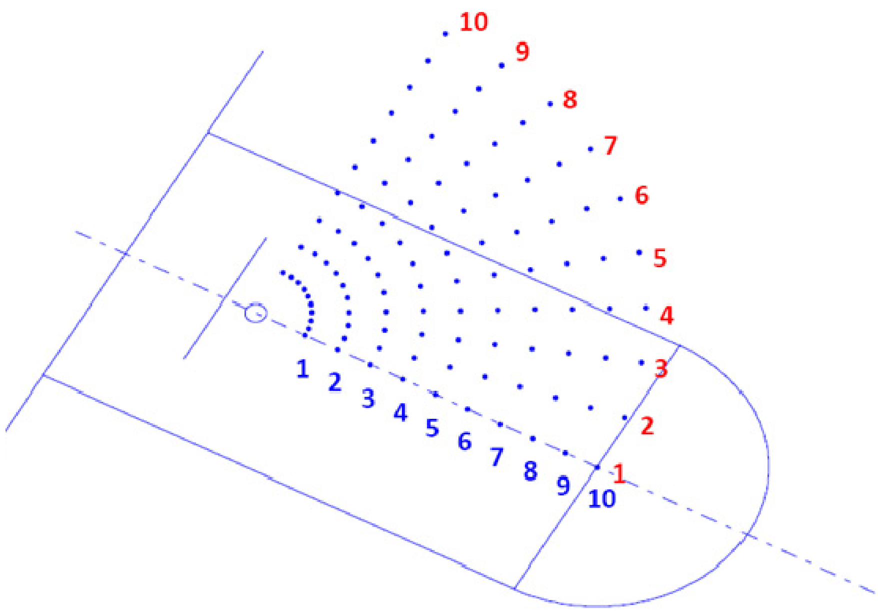

Figure 3. Grid of 100 court locations

radial distance (ft): 1.969 3.281 4.593 5.905 7.218
8.530 9.842 11.15 12.47 13.75
polar angle (^{o}): 0 10 20 30 40 50 60 70 80 90

In this study, bank shots are launched from the 100 court locations shown in Fig. 3. From each location, a set of direct shots and a set of bank shots are launched. The vertical planes of the trajectories are centered, that is, the vertical planes pass through the center point of the ring. The aim angles of the centered bank shots are determined from the formula shown in Fig. 4 (See Appendix). The ball is launched 6 ft, 7 ft, and 8 ft above the ground. It was shown in the case of the foul shot that imparting about 3 Hz (revolutions per second) of back spin is optimal, so we let ω = 3 Hz here, too (Tran and Silverberg, 2008).

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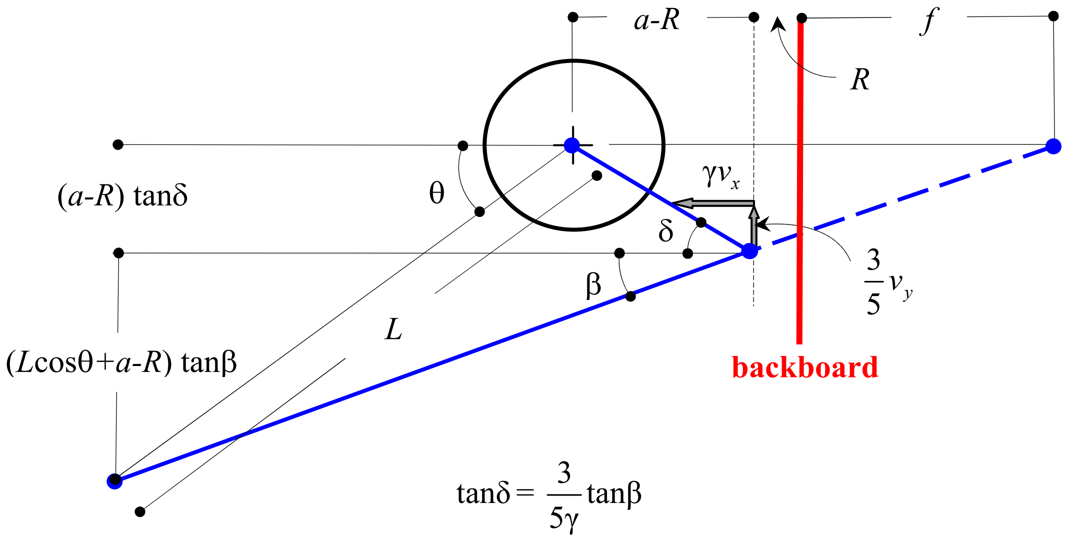

$$\tan\delta = \frac{3}{5\gamma}\tan\beta$$

$$\tan\beta = \frac{\sin\theta}{\cos\theta + \left(1+\frac{3}{5\gamma}\right)\left(\frac{a-R}{L}\right)}$$

Figure 4. The aim angle β of the bank shot

Figure 5 shows the launch speed v versus the launch angle α for successful bank shots from a set of 40,000 bank shots launched at r = 9.842 ft, θ = 60º and 7 ft above the ground. As shown, the region of successful shots has the shape of a horseshoe. To gain a greater appreciation of the different types of bank shots encountered, shots 1 through 11 are depicted around the figure. The trajectories shown are the center-lines of the basketball. Shots 6, 5, 4, 1, 7, 8, and 9 are along the outer (left) edge of the horseshoe, shots 10, 3 and 11 are along the inner (right) edge, and shot 2 is in the middle of the horseshoe.

The shots along the outer edge have the smallest launch velocities for a given launch angle and they first bank and then bounce off of the back of the ring. The shots along the inner edge have the largest launch velocities for a given launch angle and they bank and then bounce off of the front of the ring. Shot 2, located in the middle of the v-α region, is optimal (Tran and Silverberg, 2008). It strikes the backboard and then swishes through the ring.

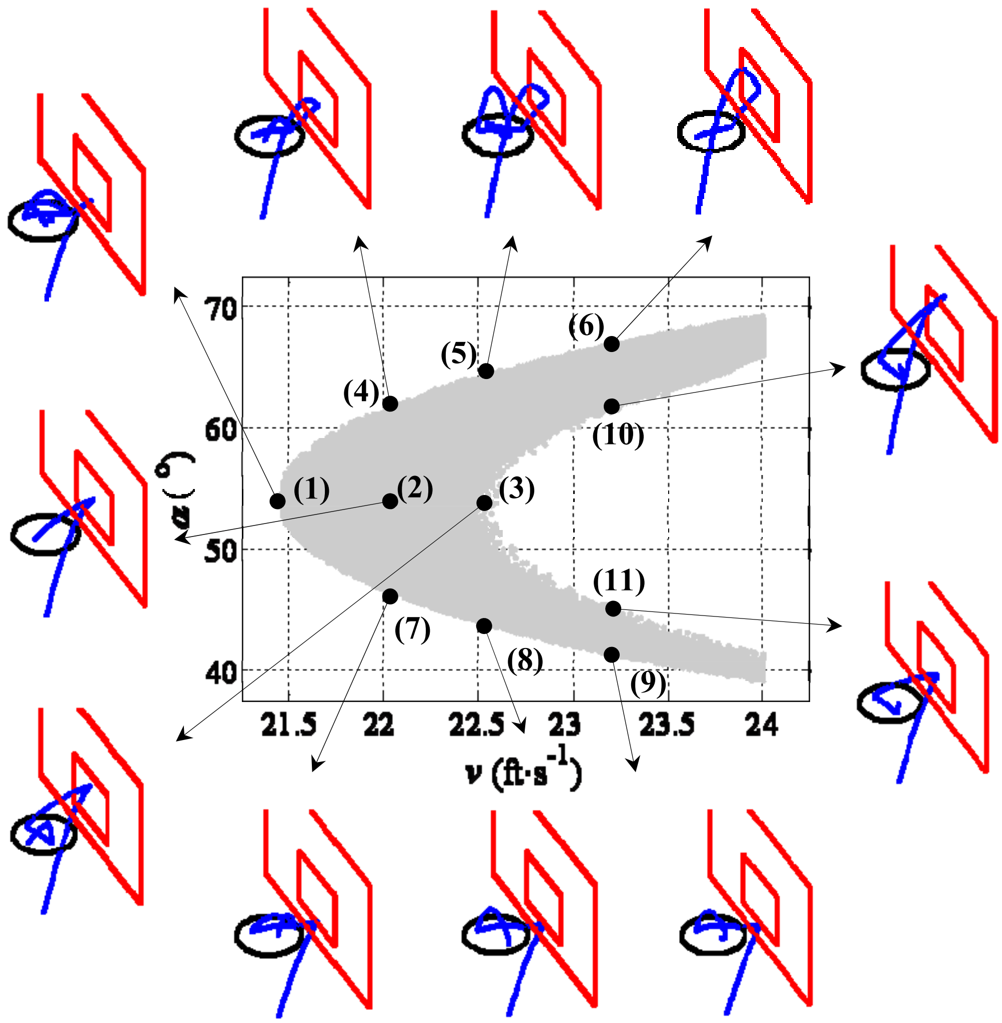

Figure 5. Launch speed versus launch angle for successful shots

Figure 6 shows the launch speed v versus the aim angle β for successful shots launched from another set of 40,000 bank shots launched again at r = 9.842 ft, θ = 60º and 7 ft above the ground. The launch angle for all of these shots is α = 54º, which is the optimal launch angle located in the center of the v-α curve in Fig. 5. Note that the lines of constant probability in Figs. 5 and 6 (not shown) are ellipses when v, α, and β are statistically independent and normally distributed and when the other launch variables are regarded as deterministic. The center of the largest probability ellipse fully contained in a region was taken as the desired shot. The desired shot is the optimal shot when the probability of that ellipse is sufficiently low (the probability of that ellipse is the calculation error). The desired shots considered throughout the paper were all optimal.

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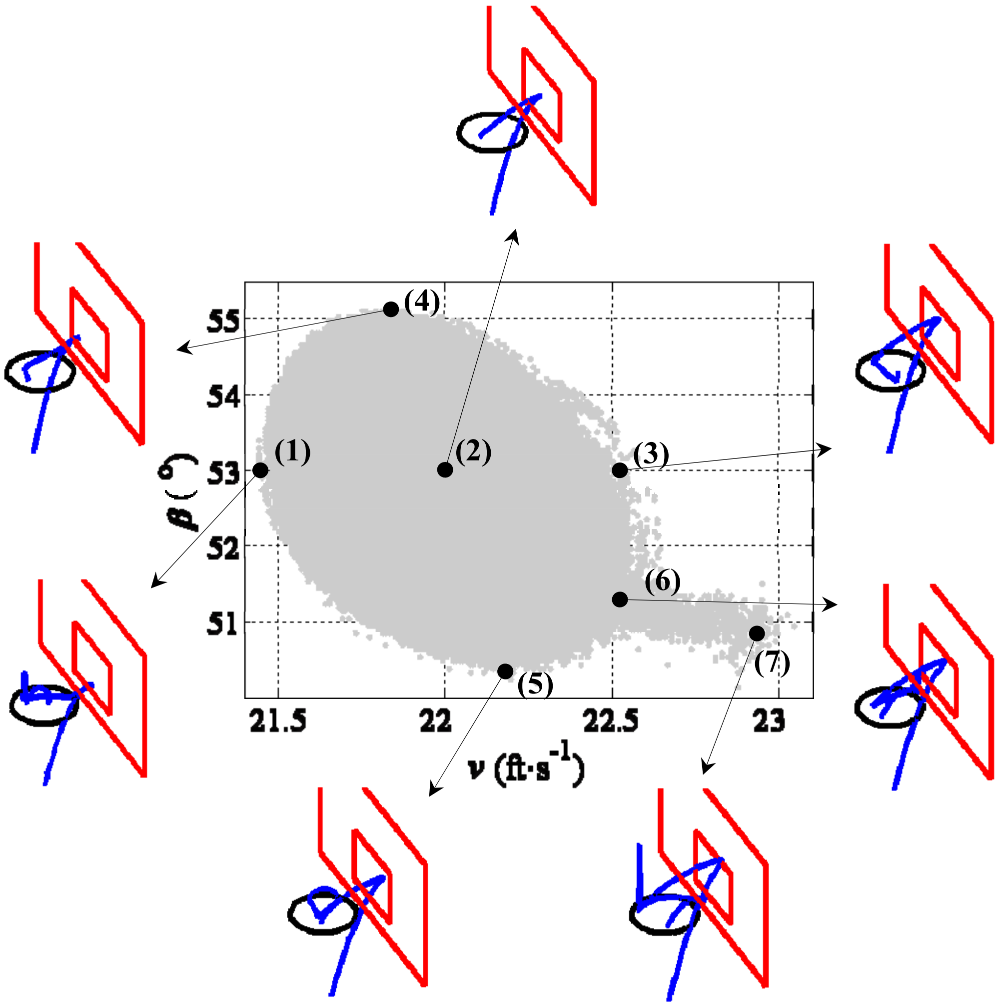

Figure 6. Launch speed versus aim angle for successful shots

Shot 1 has the lowest launch speed, shot 5 has the lowest aim angle, shot 3 has the highest aim angle, and shot 2, located in the middle of the v-β region, is optimal. Shots 1 and 4, which are launched at relatively low speeds, bounce low off of the backboard. Shots 3, 5, 6, and 7, which are launched at relatively high speeds, bounce high off of the backboard. Shot 2, the optimal shot that is launched at a moderate speed, strikes the middle of the backboard. Also, the optimal shot is centered; its aim angle produces a trajectory whose vertical plane passes through the center point of the ring.

In the bank shot, the launch variable that the shooter finds particularly difficult to select is the aim angle. To assist with aiming, the shooter benefits from selecting a target point on the backboard toward which to aim. However, when the polar angle is large the contact point C of the ball on the backboard is not aligned with the plane of the trajectory, making it difficult for the shooter to aim toward (See Figure 7). To remedy this misalignment problem a shooter can more naturally aim toward point *A* produced by extending the trajectory to the backboard in the horizontal plane of the contact point. Point *A* is called the aim point. Later in the paper, collections of aim points on the backboard, called aim lines, will be studied.

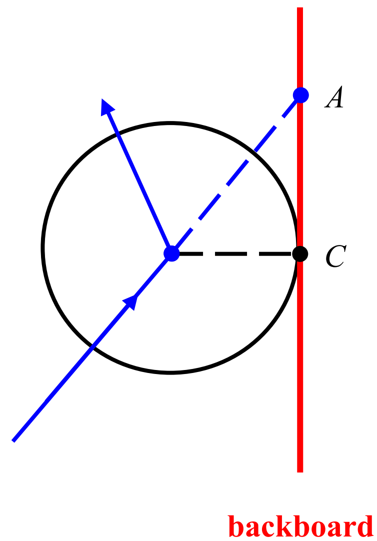

Figure 7. The contact point *C* and the aim point *A*

Results

The nominal parameters are the 100 court locations, centered aim angles, optimal launch angles, the launch height of 7 ft above the ground, and the back spin of 3 Hz. The results presented in this section use the nominal parameters and deviations from the nominal parameters.

The Court

Figures 5 and 6 showed the v-α curve and the v-β curve for shots launched from a single location on the court. The optimal (v α β) was located in the middles of the regions shown in these figures. We shall now assume that the optimal shot is the shooter's desired shot and that the shooter's consistency, quantified in terms of a standard deviation in launch speed (Tran and Silverberg, 2008), is a 70% direct shot from the free throw line, which is about the average free throw percentage in US collegiate competition as well as in the NBA. The same standard deviation will be assumed for shots launched anywhere on the court,

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although shooters actually tend to shoot more accurately from closer in. The 70% shooting percentage is representative but arbitrary in that the same trends shown below would be obtained for shooting percentages in the range of 65% to 85% with the values scaled up or down. With these assumptions, the probabilities of both the optimal bank shots and the optimal direct shots were calculated over the indicated 100 court locations. The results are symmetric; differences between the left and right sides of the court are being neglected. Note that the calculated percentages tend to under-estimate the shooter's performance since decreases in the shooter's standard deviations with distance to the ring are being neglected.

Figure 8a shows the probability of success of the bank shots. One observes probabilities that decrease with distance, a peak occurs in the neighborhood of polar angles of 75º where the backboard is fully utilized and a second increase in the neighborhood of 0º where the bank shot and the direct shot are aligned. As shown, the probability reaches over 90% close to the ring and drops to 60% at 12 ft distances with angles in the neighborhood of 45º. Next, referring to Fig. 8b showing the probability of success of the direct shots, one observes probabilities that decrease with distance, and a peak occurs, again, in the neighborhood of 0º. As shown, the probability of success at the free throw line is 70% (Recall, that is how the standard deviation in launch speed was set.), and increases to more than 90% as the shooter moves closer to the ring. Finally, Fig. 8c shows the difference between the probability of success of the bank shots and that of the direct shots. Therefore, a positive percentage indicates a level of preference of the bank shot over the direct shot and a negative percentage indicates the opposite. One observes that the bank shot is preferred in the red and pink regions and the direct shot is preferred in the other regions. Notice that bank shot preferences are on the order of 20% for polar angles of about 75º and in mid-range distances for polar angles of 0º. The bank shot is not preferred at very short distances to the ring where ball trajectories that bounce off of the backboard require very large launch angles, nor preferred close to the foul line, nor at very steep polar angles approaching 90º where the backboard is no longer effective.

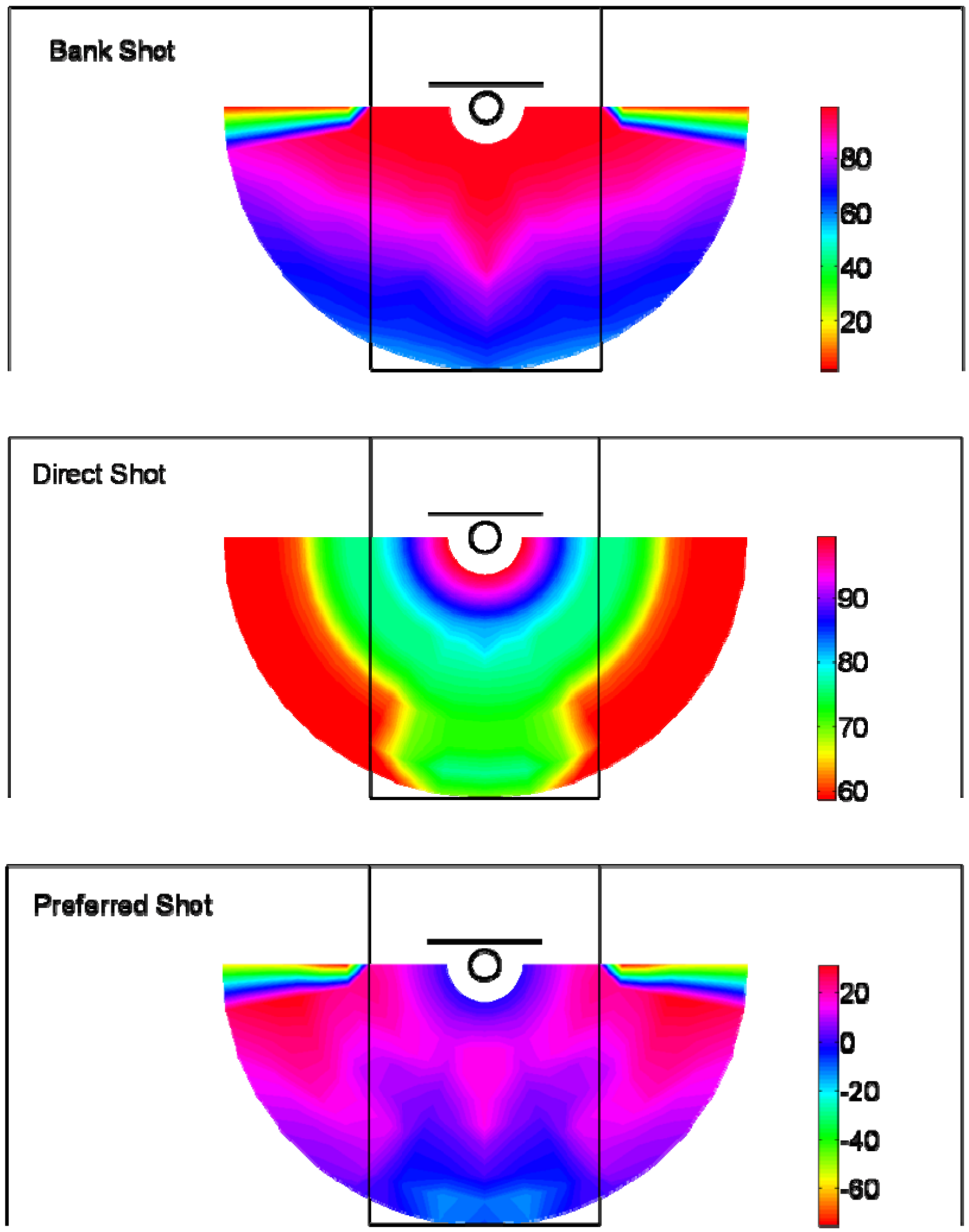

Figure 8: (a) bank shots, (b) direct shots, and (c) preferred shots

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Note that these results do not take into account particular court conditions. They mimic the conditions of a free throw shot during which play has been stopped. The comparison between the bank shot and the direct shot discounts such factors as the height and quickness of a defender, both of which influence the player's shot selection and hence change the presumed statistics.

Finally, keep in mind that the results presented above focus on consistency, which is the second factor mentioned in the beginning of the paper that determines whether or not a shot is successful. The first factor, the selection of the desired shot, was treated by assuming that the shooter selects the optimal shot as the desired shot. This is unattainable when the shooter does not have knowledge of the optimal shot so, toward finding the optimal shot, the next sub-section looks for the optimal targets on the backboard.

The Backboard

The locus of aim points on the backboard forms an aim line. The aim line is associated with optimal shots launched from a given radial distance and a given launch height from polar angles between 0^{o} and 90^{o}. Figure 9 shows aim points (black) and corresponding contact points (green) for radial distances of 5.905 ft, 9.842 ft, and 13.75 ft. Note that the horizontal distance between a point on an aim line and a point on a contact line increases with polar angle. At large polar angles, the large distance corresponds to a misalignment of the ball's trajectory, illustrating the necessity for the aim line.

Also, note that the rectangle on the backboard provides some guidance as to where the ball should make contact with the backboard. For an aim angle of 55^{o} it was found that the contact point is close to the upper corner of the rectangle. However, this does not imply that the shooter should aim toward the upper corner of the rectangle because of the large misalignment between aim point and contact point.

Furthermore Fig. 9 shows that the three aim lines corresponding to the three radial distances are very close to each other. The vertical distances between them (associated with one aim angle) is about ±2 inches. Although not shown, a range of launch heights from 6 ft to 8 ft were also considered. Again, it was found that these variations have little effect on the positions of the aim lines. Indeed, the aim lines are approximately independent of the shooter's location and launch height. Therefore, there is practically a unique (averaged) aim line, as shown in Fig. 10.

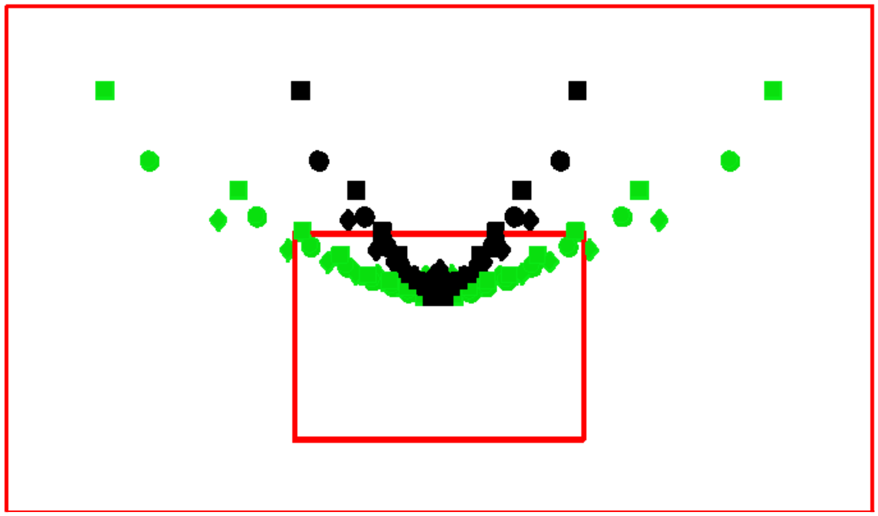

Figure 9: Aim points (black) and contact points (green)
r = 13.75 ft (square), r = 9.842 ft (circle), and r = 5.905 ft (diamond)

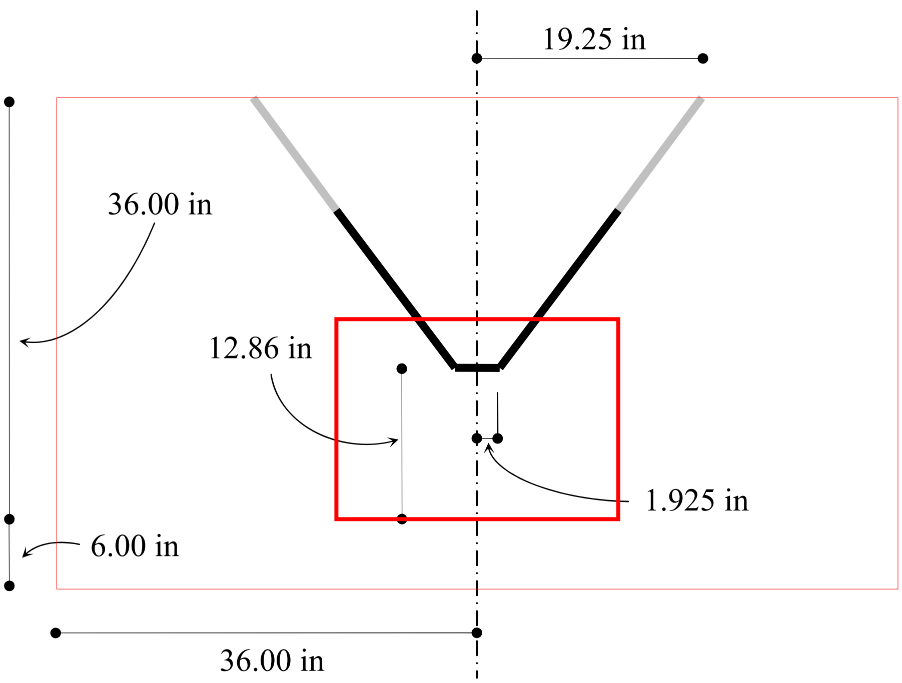

Figure 10: Aim line

As shown, the aim line is v-shaped with a flat bottom. At polar angles of 0º to 30º the shooter aims toward points on the flat bottom portion of the aim line. At polar angles of 40º to 80º the shooter aims toward points on the v-shaped portion of the aim line. This is an important result that helps the shooter recognize where to target the ball. However, it stills remains to know at what specific aim point on the aim line to direct the bank shot.

Training

Figure 10 demonstrated that there exists a single aim line on the backboard, although no guidance was offered above as to the target point on that line. It turns out, however, that the focal distance f in Fig. 4 is independent of the side angle β, from which we conclude that the vertical planes of the optimal bank shot trajectories all intersect at a single vertical axis f = 3.327 inches directly behind the center of the backboard. These results suggest an approach toward training players how to find the target point toward which to aim. The vertical axis could be a physical pole behind the backboard and the aim line could be drawn on the backboard, as shown in Fig. 11. The shooter could then look at the pole from any court location and it will cross the aim line along his line of sight at the optimal target point. Thus the shooter just aims toward the crossing.

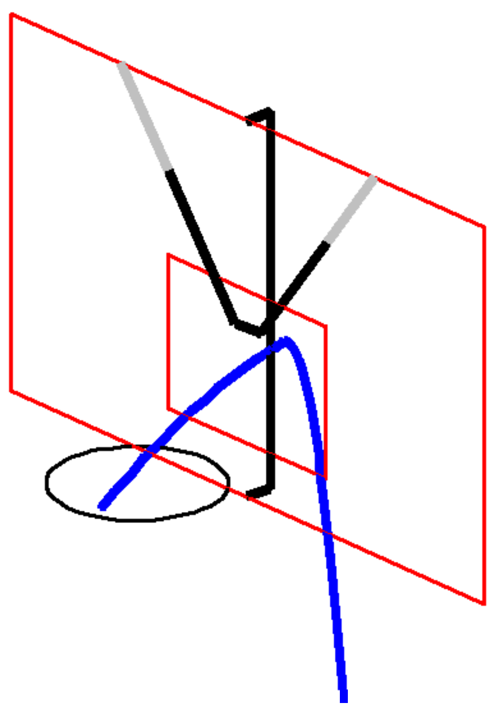

Figure 11: Finding the targets using the pole and aim line

Conclusions

In this paper, bank shots launched from 100 court locations were studied. About 40,000 bank shots and another 40,000 direct shots were launched from each location. These shots were launched from 7 ft above the ground, with 3 Hz of back spin, and assumed a standard deviation in launch speed that corresponds to a 70% direct shot from the free throw line. Shots were also launched at other launch heights to study the effect of launch height. In all, more than one million shots were launched. Our results permit us to draw the following conclusions about the bank shot in men's basketball:

(1) A typical 70% free throw shooter can select a bank shot over a direct shot and gain as much as a 20% advantage. This 20% advantage is significant in that a 70% shooter misses three times more than a 90% shooter. The court preferences of the bank shot over the direct shot were given.
(2) The corner of the rectangle on the backboard corresponds to the optimal contact point for an aim angle of 55º. The contact point is difficult to utilize since it is not aligned with the direction of aim and applies to just one aim angle.
(3) There exists a unique aim line on a backboard. The aim line is independent of the shooter's location on the court.
(4) The optimal target point can be pinpointed during a training session that employs the pole and aim line. It is the crossing of the pole and the aim line in the shooter's line of sight.

The results presented in this paper can form the basis for future studies aimed at establishing more effective ways of training players how to shoot the bank shot.

Appendix

The following shows that

$$\tan\beta = \frac{\sin\theta}{\cos\theta + \left(1 + \frac{3}{5\gamma}\right)\left(\frac{a-R}{L}\right)} \quad (1)$$

which appeared in Fig. 4. Figure 12 shows the free body diagram of the ball when it makes contact with the backboard. First refer to Fig. 12.

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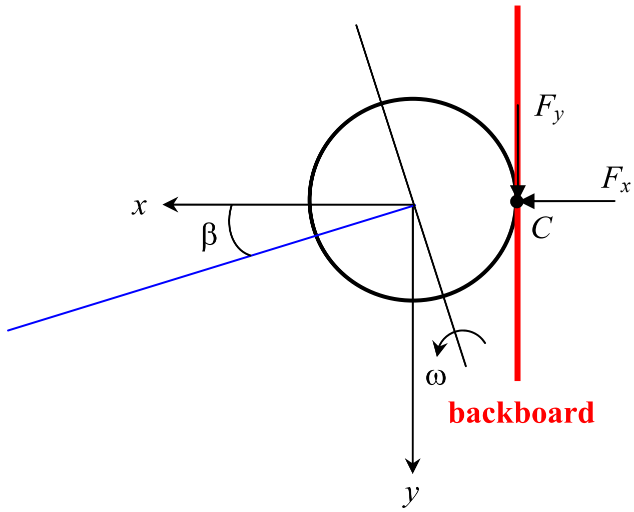

Figure 12: Free body diagram

The important equations are:

$$v_x' = \gamma v_x \tag{2a}$$

$$mv_y' - mv_y = \int F_y dt \tag{2b}$$

$$I\omega_z' = -R\int F_y dt \tag{2c}$$

$$\begin{gathered}\mathbf{0} = \mathbf{v}_C' = \mathbf{v}_{CG}' + \boldsymbol{\omega}' \times \mathbf{r}_{C/CG}, \quad \text{that is} \\ (0\ 0\ 0) = (v_x\ v_y\ v_z)' + (\omega_x\ \omega_y\ \omega_z)' \times (-R\ 0\ 0)\end{gathered} \tag{2d}$$

Equation (2a) follows from conservation of linear momentum in the x direction and assumes a linear visco-elastic collision in which v_x and v_x' denote the x components of velocity just before and after contact, and γ is the coefficient of restitution (Silverberg and Thrower, 2001). Equation (2b) follows from linear impulse-momentum in the y direction in which m denotes ball mass, F_y denotes the y component of force acting on the ball by the backboard, R is ball radius, and v_y and v_y' denote the y components of velocity just before and after contact. Equation (2c) follows from angular impulse-momentum about the z axis in which $I = \frac{2}{3}mR^2$ denotes the mass moment of inertia of a thin, spherical shell of radius R and ω_z' denotes the angular velocity of the ball about the z axis just after contact. Equation (2d) is a vector equation that expresses the kinematic constraints between the velocity vector of the contact point just after the collision

$\mathbf{v}_C'$, which is zero, and the velocity vector of the center of the ball $\mathbf{v}_{CG}'$. By expanding Eq. (2d), we get

$$v_y' = R\omega_z' \qquad v_z' = -R\omega_y' \tag{3a,b}$$

Substituting Eq. (2b) into Eq. (2c), and substituting the result into Eq. (3a), yields

$$v_y' = \frac{3}{5} v_y \tag{4}$$

Next, referring to Fig. 4, we know that

$$\tan\beta = \frac{v_y}{v_x} \text{ and } \tan\delta = \frac{3}{5\gamma}\tan\beta \tag{5}$$

Also, notice that

$$L\sin\theta = (L\cos\theta + a - R)\tan\beta + (a - R)\tan\delta \tag{6}$$

Substituting Eq. (5) into Eq. (6), yields Eq. (1).

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