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Topology Optimization of Continuum Structures Based on a New Bionics Method

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Wolff’s law in biomechanics states that bone microstructure and local stiffness tend to align with the stress principal directions to adapt to their mechanical environments. In this paper, a new method for topology optimization of continuum structure based on Wolff’s law is presented. The major idea of the present approach is to consider the structure to be optimized as a piece of bone that obeys Wolff’s law and the process of finding the optimum topology of a structure is equivalent to the bone remodeling process. A second rank positive and definite fabric tensor, which is viewed as the design variable in design domain, is introduced to express the porosity and anisotropy properties of material points. The update rule of the design variables is established as: during the iteration process of the optimization of a structure, at any material point, the eigenvectors of the stress tensor in the present step are those of the fabric tensor in the next step based on Wolff’s law; the increments of the eigenvalues of the fabric tensor depend on the principal strains and an interval of reference strain, which is corresponding to the dead zone in bone mechanics. The process is called anisotropic growth if all the fabric tensors are proportional to the second rank identity tensor in the simulation. Numerical examples show that the present method is appropriate to solve large-scale problems, such as 3D structures and the geometrical nonlinear structures. As an application in biomechanics, it is extended to predict the mass distribution of the proximal femur and the results are the same as those obtained by using the other models.

Keywords Topology Optimization, Bionics, Wolff’s Law, Fabric Tensor, Geometrical Nonlinearity, Proximal Femur

1. INTRODUCTION

Topology optimization is often regarded as layout optimization or generalized shape optimization and works as a powerful tool assisting designers in the selection of suitable initial structural topology from an optimal point of view. Owing to its complexity, topology optimization has become an intellectually challenging field and attracted much attention in the past years. Over the last two decades, substantial efforts of fundamental research have been devoted to the development of efficient and reliable numerical models and algorithms for solution of such problems. Two different kinds of topology design processes, i.e. the Material or Microstructure Technique and the Geometrical or Macrostructure Technique, are generally focused on by the researchers.

The importance of topology optimization of continuum structures optimization lies in the fact that the choice of the appropriate topology of a continuum structure in the conceptual phase is generally the most decisive factor for the efficiency of a novel product. Moreover, topology or layout optimization will also be very valuable as preprocessing tools for sizing and shape optimization, because the usual sizing and shape optimization cannot change the structural topology during the solution process and a solution obtained by one of these methods will have the same topology as that of the initial design. Much more attention has been focused on the development of topology optimization methods for continuum structures and several methods have been presented by researchers. Bendsoe and Kikuchi [1] and Bendsoe [2] studied the homogenization design method (HDM) of continuum structures. Mattheck and Burhardt [3] gave a biological growth method that was further improved by Baumgartner et al. [4]. In the work by Rozvany et al. [5], the solid isotropic microstructure with penalization (SIMP) method was introduced. Xie and Steven [6] proposed the evolutionary structural optimization (ESO) method and the BESO (Bidirectional ESO) method was established by Querin et al. [7, 8]. Eschenauer et al. [9] introduced the so-called bubble method. A systematical review about the early work on the
structural topology optimization has been given in [10]. Some recent work can be found in [11–14].

The microstructural method has also been developed maturely [2, 5, 15, 16] for structural topology optimization in the past years. The microstructural method is called free material design approach to find the topology of the structure as well as the material properties at each point in the structure. In the free material approach, there are no limitations placed on the predicted material properties. So the number of the design variables, i.e., the components of the elastic tensor are so large, e.g., there are 21 independent design variables at a full anisotropic material point or there are 9 elastic constants plus 2 Euler angles at an orthotropic material point. On the one hand, in order to achieve a reasonable optimal result, the free material approach is the choice, while its computational efficiency is a problem. On the other hand, in order to improve the computational efficiency, the SIMP method is the choice, but it may lose some optimal topology. In order to overcome the dilemma, a new approach to express the relationship between the geometry of the microstructure and the elastic properties of a material point is presented in the current research.

In biomechanics, Wolff’s law [17] states that the bone microstructure and the local stiffness tend to align with the stress principal directions according to the mechanical environments. Wolff’s law has been accepted widely and validated by several investigations [18–23]. Recently, the development of computational mechanics and high performance computers has led to the implementation of various mathematical continuum formulations of adaptive bone remodeling, which are used to make specific predictions of bony morphology. In this field, there are two hot topics. The first one considers bone adaptation as an optimization process [24–29]. Another one considers the rule of cancellous bone adaptation as a basic tool for the optimal design of continuum structures [30]. Different from the work mentioned above, a new bionics method based on Wolff’s law for obtaining optimal topology of continuum structure is developed in the current research.

The major idea of the present approach is to regard a structure as a piece of bone that obeys Wolff’s law and the topology optimization process is equivalent to the bone remodeling process. The optimal structural topology is obtained when the bone is in a state of remodeling equilibrium.

In the initial design, the material is assumed to be isotropic and distributed uniformly. With the redistribution of the material during the computation, each material point may become porous and anisotropic. In order to express these physical and geometric properties accurately, the fabric tensor methods are employed and detailed definitions and formulations are given in Section 2, where a second rank fabric tensor is introduced as the design variable at a material point in the design domain and adopted to express the geometry and elasticity of the material point. Some aspects, such as the basic equations of the elasticity theory, the formulations of the topology optimization problem, the update rule of the design variables and the optimization procedure, are described in Section 3. In Section 4, several numerical examples are given to show the validity and capability of the model and algorithm developed. Finally, some important characteristics of the developed method are summarized and discussed.

2. MATERIAL PROPERTIES

2.1. Notations

Tr and Det denote the trace and the determinant of a square matrix or an even-rank tensor, respectively. \( \nabla (\cdot) \) denotes the gradient operator. Italic bold letters denote vectors. Capital letters are used for the higher-rank tensors. The scalar product of two vectors or tensors is designed by symbols \(
\cdot
\) and \(\cdot\). The ten- sorial products are designed by symbols \(\otimes\) and \(\boxtimes\). For any given triplet of arbitrary second-rank tensors \(A, B\) and \(C\), the following equations are satisfied

1. \(A \cdot B = Tr (A \cdot B)\)
2. \((A \otimes C): B = (C; B) A\)
3. \((A \boxtimes C): B = \frac{1}{2} (A \cdot B \cdot C^{T} + A \cdot B^{T} \cdot C^{T})\)

where the symbol \((\cdot)^{T}\) denotes transpose of matrix.

2.2. Definitions of Fabric Tensor and Elastic Constitutive Tensor

The mechanical properties of multiphase or damaged materials are related closely to the distribution of the underlying microstructure or crack [31]. It is clear that the porous ratio alone is not sufficient to characterize the geometry of the local solid microstructure of a porous material. In the past decades, much work has been done [18–20, 23, 32] to develop the theoretical foundation for the establishment of accurate relationships between the macroscopic anisotropic properties of materials and their microstructures. It has been found that microstructural properties can be described in the invariant form by a set of even rank fabric tensors and assessed accurately by using the method given in [33]. One of the most important conclusions is that the principal directions of the fabric tensor coincide with those of the orthotropic elastic tensor.

The linear elastic properties of the anisotropic porous materials characterized by a fourth rank tensor are dependent on both the solid volume fraction of the material and the geometry of the material microstructure. In most applications, the microstructural anisotropy seems to be sufficiently well described by a scalar and a symmetric second-rank fabric tensor, which restricts the material symmetry to orthotropy. In the work by Zysset and Curnier [33], a general approach was introduced to express the anisotropic elasticity with the fabric tensors. Following the model given in [33], a second rank positive and definite fabric tensor with two material parameters to express the elasticity tensor is suggested in the current work.

2.2.1. Fabric Tensor

The fabric tensor used in the current study is defined as

\[ B = \sum_{i=1}^{3} b_{i} q_{i} \otimes q_{i} \]  (1)
2.2.2. Stiffness Tensor

From an experimental point of view, the anisotropic elasticity tensor \( D_0 \) of a specified material can be identified by using two independent material constants \( \lambda_0 \) and \( \mu_0 \), a second rank fabric tensor \( B \) and an exponent \( \alpha \) [33], i.e.

\[
D_0 = \lambda_0 B^\alpha \otimes B^\alpha + 2\mu_0 B^\alpha \otimes B^\alpha
\]

In this paper, a particular elasticity model with \( \lambda = \lambda_0, \mu = \mu_0 \) and \( \alpha = 1.0 \) in Eq. (2) is adopted, i.e.

\[
D = \lambda B \otimes B + 2\mu B \otimes B
\]

The compliance tensor is expressed as follows

\[
D^{-1} = \frac{-\lambda}{2\mu(3\lambda + 2\mu)} B^{-1} \otimes B^{-1} + \frac{1}{2\mu} B^{-1} \otimes B^{-1}
\]

where \( \lambda \) and \( \mu \) are Lamé constants of the base material which is the solid phase in porous medium.

It is clear that the material symmetries of orthotropy, transverse isotropy and isotropy correspond to the cases of three, two and one distinct eigenvalues of the fabric tensor, respectively. There are 3 eigenvalues and 2 eigenvectors of fabric tensor with two Lamé constants (considering orthotropy of those vectors) to express the elastic tensor.

2.3. Effective Volume Fraction of a Material Point

Homogeneity hypothesis: The anisotropy of the elastic constitutive law is independent of the size or physical units of the microstructure of the material.

Definition 1: The process of separating a piece of porous medium into a simply connected solid and a void is called as condensation, in which the porous medium satisfies the homogeneity hypothesis.

In definition 1, the shape of the simply connected solid may be random. In order to obtain the volume fraction of the solid, a tool called the stiffness-equivalence-based condensation (SEC) is adopted [34]. The orthogonal-solid-strut unit cell shown in Figure 1(b) is obtained by the operation of three dimensional (3D) SEC on the unit cell shown in Figure 1(a). Thus, this volume fraction of the unit cell shown in Figure 1(b) is called the effective volume fraction (EVF) of the porous unit cell in Figure 1(a). The distribution of the EVFs is used to display the final topology of the optimal structure in the follows. It is noticed that the EVF may not right equal to the real volume fraction (relative density) of the porous unit cell.

3. OPTIMIZATION MODEL

3.1. Basic Equations of Linear Elasticity Theory

In the present work, the loading process is quasi-static and the deformation process is an isothermal course simultaneously. The basic governing equations and boundary conditions are given as follows

\[
\begin{cases}
\sigma = D : \varepsilon \\
\varepsilon = \frac{1}{2} (\nabla u + (\nabla u)^T) + \nabla u \cdot (\nabla u)^T \\
\nabla (F : \sigma) + f = 0 \\
\n\Gamma_0 : (F \cdot \sigma) \cdot n = F^* \\
\Gamma_0 : u = u^*
\end{cases}
\]

where \( \sigma \) is the 2nd Piola-Kirchhoff stress tensor, \( \varepsilon \) is the Green strain tensor, \( D \) is the stiffness tensor; \( F \) is the deformation gradient, \( u \) is the displacement vector, \( f \) is the body force vector,
\( F^* \) is the boundary force acted on \( \Gamma_\sigma \) with the normal direction \( \mathbf{n} \), \( u^* \) is the assigned displacement on the boundary \( \Gamma_u \).

### 3.2. Formulations of Topology Optimization Problem

Formulations of topology optimization problem can be defined as

\[
\begin{align*}
\text{Find :} & \quad \{ B_m(b_{i,m}, q_{i,m}) \} \\
\text{to satisfy :} & \quad |\varepsilon_{i,k,m}| \in \left[ \varepsilon_{i,\inf}^{\ref}, \varepsilon_{i,\sup}^{\ref} \right] \\
\text{s.t.} & \quad b_{i,m} \in [\beta, 1.0], \quad (i = 1, 2, 3)
\end{align*}
\]

where the fabric tensor \( B_m \) is the design variable of the \( m \)-th material point. \( \varepsilon_{i,k,m} \) is the \( i \)-th principal strain of the \( m \)-th material point in the \( k \)-th step of iteration. \( \left[ \varepsilon_{i,\inf}^{\ref}, \varepsilon_{i,\sup}^{\ref} \right] \) is the interval of reference strain (\( \varepsilon_{i,\inf}^{\ref} \geq 0 \) and \( \varepsilon_{i,\sup} \) are the infimum and the supremum of the interval, respectively), which is corresponding to the dead zone of bone in biomechanics. It is noted that the objective function in Eq. (10) is not the weight or the strain energy of the structure; instead, the optimization problem is described as if \( |\varepsilon_{i,k,m}| \in \left[ \varepsilon_{i,\inf}^{\ref}, \varepsilon_{i,\sup}^{\ref} \right] \) occurs at each material point, and then the optimal distribution of material and the optimal topology are achieved.

### 3.3. Update Rule of Design Variables

The rule of updating the design variables in the present work is the approach to renew the fabric tensors. It contains the updating of the eigenpairs of the tensors. According to Wolff’s law, the update rule of the eigenvectors of a fabric tensor can be expressed as follows: During the optimization process, the eigenvectors of a fabric tensor at the \((k+1)\)-th step should be identical to those of the stress tensor at the \( k \)-th step for the same material point.

The update rule of eigenvalues of a fabric tensor can be defined briefly as: The increments of the eigenvalues of the fabric tensor at a material point are called the growth strain and are determined by the absolute values of the principal strain along the corresponding directions. If any of the absolute values is out of the given interval of reference strain, then the growth speed is non-zero. The opposite case is that, if all of the absolute values of its principal strains locate in the interval, the growth speed is equal to zero, i.e., the material point reaches the equilibrium state. If all the material points are in the state of equilibrium, the optimal topology of the structure is obtained. Mathematically, the update rule of the eigenvalues of the fabric tensor at the \( m \)-th material point at the \( k \)-th iteration step can be expressed as

\[
b_{i,k+1,m} = \begin{cases} 
1.0 & \text{if } b_{i,k+1,m} \geq 1.0 \\
\beta & \text{if } b_{i,k+1,m} < 1.0
\end{cases}
\]

where

\[
\Delta b_{i,k,m} = \begin{cases} 
g_1 & \text{if } |\varepsilon_{i,k,m}| > \varepsilon_{\inf}^{\ref} \\
0 & \text{if } \varepsilon_{\inf}^{\ref} \leq |\varepsilon_{i,k,m}| \leq \varepsilon_{\sup}^{\ref} \\
g_2 & \text{if } |\varepsilon_{i,k,m}| < \varepsilon_{\inf}^{\ref}
\end{cases}
\]

\( g_1 \) is the deposition speed and \( g_2 \) is the dissipation speed. Generally, the magnitudes of those two speeds are kept within the interval \([0.001, 0.2]\).

The new fabric tensor is given as

\[
B_{k+1,m} = Q_{k,m} \begin{bmatrix} b_{1,k+1,m} & 0 & 0 \\ 0 & b_{2,k+1,m} & 0 \\ 0 & 0 & b_{3,k+1,m} \end{bmatrix} Q_{k,m}^T
\]

where \( Q_{k,m} = [q_{1,k,m} \ q_{2,k,m} \ q_{3,k,m}] \) is the transfer tensor from the global Cartesian coordinate system to the local one. At the same time, the new elastic tensor can be expressed as

\[
D_{k+1,m} = \lambda^* B_{k+1,m} \otimes B_{k+1,m} + 2\mu B_{k+1,m} \otimes \overline{B}_{k+1,m}
\]

where

\[
\lambda^* = \begin{cases} 
\lambda & \text{Plane strain or 3D} \\
\frac{2\lambda \mu}{\lambda + 2\mu} & \text{Plane stress}
\end{cases}
\]

In an optimization process when both of the eigenvectors and the eigenvalues of the fabric tensors are changed, the process is called an anisotropic growth (AG). For a special case, if the fabric tensors of the material points are kept to be proportional to the second rank identity tensor, the process of optimization is called as an isotropic growth (IG). For an isotropic growth, the update rule of a fabric tensor is given below

\[
B_{k+1,m}^{iso} = \frac{1}{N} Tr(B_{k+1,m}) I_N
\]

where \( N \) is the spatial dimension of the structure to be analyzed (\( N=2 \) or \( 3 \)), \( I_N \) is the \( N \)-dimensional second rank identity tensor, \( B_{k+1,m}^{iso} \) has been given in Eq. (13).

### 3.4. Optimization Procedure

Step 1: Model the continuum structure with finite elements, initiate parameters, such as the growth speeds \( g_1, g_2 \); the interval of reference strain \([\varepsilon_{\inf}^{\ref}, \varepsilon_{\sup}^{\ref}]\) and to give the initial design, i.e., let initial eigenvalues of fabric tensors equal \( b_0 \), let \( k = 1 \);

Step 2: Analyze the structure to obtain the displacement, strain and stress fields by using Eqs. (8) and (9);

Step 3: Update the fabric tensor at each material point (whether Gaussian point or element): 1) take orthogonal transform of the stress tensor of the element to obtain the eigenvectors of the fabric tensor based on Wolff’s law;
2) orthogonal transform the strain tensor of the fabric tensor to get the eigenvalues and update the eigenvalues of fabric tensor by using Eqs. (11) and (12), calculate the relative density of each element by using Eq. (5) or (6) and calculate \( R_{g}^{*k} \) by using Eq. (7).

**Step 4:** Iteration determination: If \( |R_{g}^{*k} - R_{g}^{*k-1}|/R_{g}^{*k-1} \leq \zeta \) or \( k \) is equal to the given maximum number of iteration, Stop, else let \( k = k + 1 \) and go to Step 2, where \( \zeta \) is the tolerance parameter.

Obviously, all the initial values of design variables \( (b_0) \) and their increments \( (g_1 \) and \( g_2) \) effect the efficiency of the algorithm. To improve the efficiency, one can set \( b_0 \) to be a constant within [\( \beta \ 1.0 \)] and set the growth speeds \( g_1 \) and \( g_2 \) to be little large values in the first several steps and then decreased gradually in the later steps.

### 4. NUMERICAL EXAMPLES

It is assumed that the volume of the material in solid form is less than those which cover the entire admissible domain for the continuum structure. Hence for the initial design it is normally to distribute the material uniformly in the porous media with microstructures by initializing all the eigenvalues of the fabric tensors to be a positive scalar (\( b_0 \)) less than unity, over the admissible design domain. A fixed finite element mesh is used to describe the geometry and mechanical response within the entire design domain. The design variables are assumed to attain constant values within each finite element.

The analysis procedure for 3D cases and geometrical non-linear cases are compiled in APDL of the commercial software ANSYS.

Four numerical cases are given in the section. They are: 1) the case of 2D isotropic growth/anisotropic growth; 2) the case of 3D isotropic growth; 3) the case of 2D geometrical non-linear isotropic growth; 4) prediction of mass distribution of 2D proximal femur as an application in biomechanics.

#### 4.1. The Case of 2D Isotropic Growth/Anisotropic Growth

**4.1.1. The Second Principal Direction Angle of the Fabric Tensor**

For a 2D problem solved by the anisotropic growth simulation, each element (e.g. the \( m \)-th) has three independent design variables, i.e., two eigenvalues \( (b_{1,m}, b_{2,m}) \) and one angle \( (\theta_m) \). The definition of the angle is given below and shown in Figure 2.

**Definition 3:** The second principal direction angle \( (SPDA) \) is obtained by rotating anti-clockwise from X-axis to the second principal direction \( (q_2) \) of the fabric tensor of the material point \( A \) in the global coordinate system \( XOY \) (see Figure 2). Generally, \( SPDA \in [0^\circ \ 180^\circ] \).

**4.1.2. Numerical Results**

The design domain shown in Figure 3(a) is a unit square \((1.0 \ m \times 1.0 \ m)\) plate with thickness of 10 mm at the initial state. It is subjected to a vertical concentrated force \( P = 1000N \) at the middle point of the upper side and the bottom is fixed. The properties of the base material are Young’s modulus \( E = 200.0GPa \) and Poisson’s ratio \( \nu = 0.25 \). Let all the initial eigenvalues of fabric tensors are equal to \( b_0 = 0.3 \). The infimum and supremum of the interval of reference strain are assigned as \( \varepsilon_{ref}^{inf} = \varepsilon_{ref}^{sup} = 0.2e \ - \ 5 \); during the isotropic growth process, the growth speeds are \( g_1 = 0.05 \) and \( g_2 = -0.03 \), respectively; and \( g_1 = 0.005 \) and \( g_2 = -0.003 \) are used in anisotropic growth process.

Figures 3(b) and (c) are the optimal topologies of the structure obtained by using the IG and the AG models, respectively. The final shape looks like a trunk of a tree [4]. It can be found that the two topologies are in close agreement with each other except size difference. The rod in Figures 3(b) is thinner than that in Figure 3(c). Figures 3(d) and (e) are the gray level images of \{ \( b_1 \) \} and \{ \( b_2 \) \}, respectively. Figure 3(f) shows the distribution of \( SPDAs \). By comparing the bars shown in Figures 3(d) and (e), it can be found that the second principal value of the fabric tensor is greater than the first one at the same point. Figure 3(f) implies that the \( SPDAs \) are nearly 90\(^\circ\), which implies that the stiffness of the rod along the longitudinal direction is greater than that along the level direction at the same point (see Figures 3(d) and (e)).

Figure 4 shows the iteration histories of the relative weights of the structure by using the IG and AG models and the relative weights converge at 3.2\% and 2.4\%, respectively. That is to say, the final optimal structure obtained by using the AG model is much lighter than that obtained by using the IG model. The reason is that the anisotropic material can supply the same stiffness with less material than the isotropic one when their base materials are the same with each other.

#### 4.2. The Case of 3D Isotropic Growth

The unit cube \((l_1 = 1.0 \ m)\) is divided into \(40 \times 40 \times 40\) 8-node brick elements (see Figure 5). The properties of the base material are Young’s modulus \( E = 68.0 \ GPa \) and Poisson’s ratio \( \nu = 0.333 \). All of the initial eigenvalues of the fabric tensors of each finite element are set to be \( b_0 = 0.5 \). During the growth process, from the 1st to the 15th iteration step, the growth speeds are...
FIG. 3. Numerical results: (a) design domain with load and support conditions; (b) optimal topology of structure obtained by using the IG model; (c) optimal topology of structure obtained by using the AG model; distributions of eigenvalues of fabric tensors, (d) $b_1$, (e) $b_2$, (f) $\theta$ (SPDA).

FIG. 4. Iteration histories of the relative weights of the structure by using the IG and AG models.

FIG. 5. Design domains: (a) under loading condition A; (b) under loading condition B.
$g_1 = 0.12$ and $g_2 = -0.09$. From the 16th to the 60th step, $g_1 = 0.04$ and $g_2 = -0.03$ are used in the computation. Two loading conditions are considered:

**Loading condition A:** Two intersecting loads $P = 10.0$ kN/m are symmetrically applied on the upper surface ($l_2 = 0.5$ m) (see Figure 5(a)). Four lower corner points of the cube are fixed. Let $\varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{inf}} = 0.5e^{-4}$.

**Loading condition B:** A concentrated force $P = 1.0$ kN is applied at the center of the upper surface. Four lower corner points of the cube can slide on the horizontal plane (Figure 5(b)). Let $\varepsilon_{\text{inf}}^{\text{ref}} = \varepsilon_{\text{sup}}^{\text{ref}} = 0.5e^{-5}$.

Figures 6(a) and (b) give the optimal topologies of the structure under different loading conditions. They are identical to those reported by Eschenauer and Olhoff [10].

Figure 7 shows the iteration histories of the relative weights of the structure under different loading conditions. They are identical to those reported by Eschenauer and Olhoff [10].

4.3. The Case of 2D Geometrical Nonlinear Isotropic Growth

4.3.1. Study of Mesh-Dependent Problem in Topology Optimization

The design domain shown in Figure 8 is a $1.0m \times 1.0m$ rectangular plate with a thickness of 5 mm. The displacement along $Z$-direction of the center of the plate is specified to be equal to $u_Z = -0.05$ m. The displacements along $Z$-direction of those four edges are specified to be zero. The base material of the beam has Young’s modulus $E = 210.0$ GPa and Poisson’s ratio $\nu = 0.333$. The infimum and supremum of the interval of reference strain are assigned as $\varepsilon_{\text{inf}}^{\text{ref}} = \varepsilon_{\text{sup}}^{\text{ref}} = 0.5e^{-5}$. All of the initial eigenvalues of the fabric tensors are set to be $b_0 = 0.5$. During the growth process, from the 1st to the 15th iteration step, the growth speeds are $g_1 = 0.15$ and $g_2 = -0.09$; while from the 16th to the 25th step and from the 26th step, $g_1 = 0.05$ and $g_2 = -0.03$ and $g_1 = 0.017$ and $g_2 = -0.01$ are used, respectively. To test the mesh-dependent problem in topology optimization, the structure is discretized into the meshes of (a) $40 \times 40$; (b) $60 \times 60$ and (c) $80 \times 80$ by 4-node shell elements.
Figures 9 (a), (b) and (c) give the optimal topologies of the structure with different mesh schemes, i.e., (a) 40 × 40; (b) 60 × 60 and (c) 80 × 80 shell elements, respectively. The optimal topologies are the same as a cross. Only the material distributions around the center are slightly different. Figure 10 shows the iteration histories of the relative weights of the structure with different meshes. The convergence behaviors are the same. The relative weights of the optimal topologies converge at 14.7% for mesh scheme (a), 13.7% for mesh scheme (b), and 13.3% for mesh scheme (c) after 58 times of iteration. From these results, it can be concluded that the optimal topology is independent on the mesh scheme.

4.3.2. Study of the Effects of Interval of Reference Strain on Optimal Topology

The design domain shown in Figure 11 is a 2.0 m × 2.0 m rectangular plate with thickness of 0.001 m. Four bottom corners and four centers of the edges are fixed. The displacements of the four points (i.e., \( P_1, P_2, P_3 \) and \( P_4 \) in Figure 11) in the plane along the Z-direction are specified to be \( u_Z = -0.03 \) m. The base material of the beam has Young’s modulus \( E = 210 \) GPa and Poisson’s ratio \( \nu = 0.333 \). All of the initial eigenvalues of the fabric tensors are set to be \( b_0 = 5 \). The design domain is divided into 40 × 40 shell elements. During the growth process, from the 1st to the 15th iteration step, the growth speeds are \( g_1 = 0.15 \) and \( g_2 = -0.09 \); from the 16th to the 25th step, \( g_1 = 0.05 \) and \( g_2 = -0.03 \); from the 26th step, \( g_1 = 0.017 \) and \( g_2 = -0.01 \) are used. To study the dependent problem of optimal topology of structure on the interval of reference strain, the infimum and supremum of the interval are assigned as (a) \( \varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{inf}} = 0.1 \) e − 3; (b) \( \varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{sup}} = 0.25 \) e − 3; (c) \( \varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{sup}} = 0.5 \) e − 3, respectively.

Figure 12 shows the optimal topologies of the structure under the same loading condition but with different intervals of reference strain. The final topologies vary with the reference strain intervals greatly. Figure 13 shows that the relative weights converge at 83.2% with \( \varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{sup}} = 0.1 \) e − 3, at 48.3% with \( \varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{sup}} = 0.25 \) e − 3 and at 18.4% with \( \varepsilon_{\text{ref}}^{\text{inf}} = \varepsilon_{\text{sup}}^{\text{sup}} = 0.5 \) e − 3. Thus, it can be concluded that under the same loading condition the optimal topology of a structure will depend on the interval of reference strain used in the computational model.

4.4. Prediction of Mass Distribution of the Proximal Femur

The solid phase of the bone is considered as isotropic material with Young’s modulus \( E = 14.3 \) GPa, Poisson’s ratio \( \nu = 0.43 \) and real density 2100 kg/m³. Figure 14 shows a specimen of proximal femur with length 136.5 mm, which is divided into 2205 3-node linear plane stress elements. Table 1 gives the loading conditions of the design domain. The dead zone or the interval of reference strain is \([6.999e-4 7.000e-4]\).

Figures 15 (a) and (b) are the numerical results obtained by using IG model and AG model, respectively. Figures 15 (c) and (d) are the numerical results obtained by using the isotropic Stanford model and anisotropic Stanford model [35], respectively. From Figure 15, it can be found that most of the femur has the same mass distribution at the same material point under different models. Only the masses in such regions as the head and
FIG. 12. Optimal topologies: (a) \( \varepsilon^{\text{ref}}_{\text{inf}} = \varepsilon^{\text{ref}}_{\text{sup}} = 0.1 \times 10^{-3} \); (b) \( \varepsilon^{\text{ref}}_{\text{inf}} = \varepsilon^{\text{ref}}_{\text{sup}} = 0.25 \times 10^{-3} \); (c) \( \varepsilon^{\text{ref}}_{\text{inf}} = \varepsilon^{\text{ref}}_{\text{sup}} = 0.5 \times 10^{-3} \).

FIG. 13. Iteration histories of the relative weights of the structure with different meshes.

FIG. 14. 2D proximal femur under different loading conditions

FIG. 15. Comparison between the present results and those obtained by using Stanford models [32] on the analysis of the 2D proximal femur under different loading conditions: (a) obtained by using the IG model; (b) obtained by using the AG model; (c) obtained by using Stanford isotropic model; (d) obtained by using Stanford anisotropic model.

The middle part of the neck are obviously different between the isotropic simulation results and the anisotropic results. The numerical results are very similar whether Stanford model or the present model is used. This demonstrates also the validity of the model developed in the current work. Of course, the results obtained by the two models are not compared numerically, because no explicit label scale can be obtained from the results given by Stanford model.

TABLE 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Ratio</th>
<th>Load value at the head (N)</th>
<th>Load direction ( (°) )</th>
<th>React value (N)</th>
<th>React direction at the abductor ( (°) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60%</td>
<td>2317</td>
<td>24</td>
<td>703</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>20%</td>
<td>1158</td>
<td>-15</td>
<td>351</td>
<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>20%</td>
<td>1548</td>
<td>56</td>
<td>468</td>
<td>35</td>
</tr>
</tbody>
</table>

Loading conditions (orientations are referred to the vertical direction)
5. CONCLUSIONS
Based on Wolff’s law in bone mechanics, a new bionics method is developed for topology optimization of continuum structures. In the current method, a second rank fabric tensor is adopted to express the geometry of the microstructure and the constitutive properties of a material point in a design domain. The update rule of design variables is not based on the mathematical programming methods, but on Wolff’s law in biomechanics. Numerical examples are computed to show the efficiency of the method developed. Several conclusions are summarized as follows:

1. Under the same loading condition, the relative weight of the optimal structure obtained by using the AG model is lighter than that obtained by using the IG model.
2. Because of using a rule method, not a mathematical programming method, to renew the design variables, the present method is more appropriate to solve the large scale problems such as 3D structures and geometrical nonlinear problems.
3. The results of the geometrical nonlinear cases show that the optimal topology is not mesh dependent, but dependent on the interval of reference strain adopted in the computational model. The later problem is addressed here and will be discussed in more detail in our future work.
4. By comparing the numerical results of the mass distribution of the proximal femur obtained by using the present model and the Stanford model, the present method is appropriate to predict the mass distribution in the bone remodeling process.

REFERENCES
7. O.M. Querin, G.P. Steven, Y.M. Xie, Improved Computational Efficiency using Bidirectional Evolutionary Structural Optimization (BESO), Abstract for 4th World Congress on Computational Mechanics, held in Argentina, 1997.