Broadcasting Efficient Opportunistic Network Coding for Multisource-Multidestination Scenarios

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Abstract—Network coding across multicast transmission can significantly reduce the number of retransmissions for wireless broadcasting. However, most of the existing network coding schemes for these scenarios base on the assumption that each source has priori knowledge of the others, which restricts the application of network coding seriously. In this paper, we propose a multi-relay opportunistic network coding (MR-OPP-NC) scheme for multisource-multidestination (MSMD) broadcasting scenarios without the above mentioned assumption. In specific, a joint network coding and opportunistic selected scheme is designed in decode-and-forward strategy. Simulation results show that the MR-OPP-NC scheme proposed outperforms the pure opportunistic relaying scheme in delay performance.

I. INTRODUCTION

Network coding [1] is a significant breakthrough in information theory, which allows intermediate nodes to perform processing operations on the incoming packets, in addition to just forwarding them. Network coding has emerged as a promising technique to reduce the system delay [2], [3], improve the throughput performance [4], [5]. Based on the broadcast nature of wireless channels, network coding has been applied to wireless communications smartly and effectively.

MSMD is a fundamental network structure of much interest to researchers, which is applied to some practical wireless communication system, especially wireless multimedia transmission system based on mobile terminals. Different base stations broadcast information for different applications, such as GPRS, DAB/DVB, WLAN and so on, and mobile terminals receive different base station messages by traditional scheduling methods. With the increase of time efficiency in spreading information, it is a potential truth delay performance may be critical to the satisfaction of the users. The objective of this paper is to develop a technique to reduce delays in MSMD scenarios and quantify the gains contrast to the traditional scheduling methods.

In MSMD scenarios, [6], [7] study the capacity region with network coding and propose a new explicit bound for the capacity region for arbitrary acyclic networks. [8] is one of the first tries to apply network coding to MSMD scenarios, in which the key idea is to encourage source nodes collaborating with each other. Analysis shows the scheme can obtain maximum stable arrival rate. However, such a scheme requires the strong assumption that each source needs the priori information of the others. [9] proposes the application of network coding to MSMD scenarios with amplify-and-forward (AaF) strategy and provide formal analysis for the improvement of system delay. Simulations show that the scheme can reduce averaged number of required retransmissions and system delay obviously.

In this paper, we study the impact of network coding on system delay for MSMD scenarios with decode-and-forward (DaF) strategy, and propose MR-OPP-NC scheme to reduce system delay. In specific, the two-hop transmission strategy is focused and the use of relays is introduced into MSMD scenarios. In multi-relay network, how to select appropriate relay is an unavoidable problem. MR-OPP-NC scheme uses opportunistic-relaying selected strategy [10] for reference. In MR-OPP-NC scheme proposed, many sources broadcast messages in turn, but each source should transmit continuously to make one relay, at least, can receive it’s messages successfully. Every relay mixes packets decoded rightly with random linear network coding (RLNC) [11], and the relay with the best channel quality is selected to broadcast its mixed packet. If only a destination can receive mixed packet sufficiently, it can decode all source messages. Contrasting with multi-relay opportunistic relaying (MR-OPP) mode, We give numerical results and theoretic simulation for gains from proposed scheme. Monte-Carlo simulation results also provide demonstration of the performance.

The rest of this paper is organized as follows. Section II describes the system model. Opportunistic relaying mode and Opportunistic Network Coding mode are described respectively in section III and section IV. Numerical results and Simulation are presented in Section V. The conclusions are given at the end.

II. SYSTEM MODEL

We consider a MSMD broadcast scenario as Fig. 1. Sources broadcast messages of fixed length to all destinations by relays without straight links, in other words, every destination should communicate with every source to receive different kinds of data. The set of sources is denoted by \( S \), \( M \) is the number of sources, \( S_m \) is the \( m \)th source (\( m \in [1, M] \)). Similarly, the set of \( J \) relays is denoted by \( R \); the set of \( J \) destinations is denoted by \( D \). The relays can overhear the pilot signals from destinations, from which they assess how appropriate each of them is for information relaying. Assuming broadcast channel is orthogonalized in time, we are interested in the average number required transmissions to make source
messages received by all destinations, where the transmissions have to be done over a time varying channel with Rayleigh fading.

Node A transmits signal $x_A$ to node B, the received signal is given by
\[
y_B = h_{AB}x_A + n_B
\]
where $h_{AB} \sim CN(0, \Omega_{AB})$ is the channel gain between the link A $\rightarrow$ B, and $n_B \sim CN(0, N_0)$ is the additive white Gaussian noise (AWGN) at node B. For each link, let $\gamma_{AB} \triangleq |h_{AB}|^2$ be the instantaneous squared channel strength, which obeys an exponential distribution with hazard rate $1/\Omega_{AB}$, denoted by $\gamma_{AB} \sim \Upsilon(1/\Omega_{AB})$. Also, $\mathbb{R}$ denotes the end-to-end spectral efficiency in bps/Hz and SNR denotes the end-to-end transmit signal-to-noise ratio, namely SNR is the ratio of transmission energy $P_A$ to the white noise energy $N_0$ added at node B. The received signal-to-noise ratio $snr_B$ is
\[
.snrr_B = \frac{P_B}{N_0} = \frac{P_A |h_{AB}|^2}{N_0} = SNR \cdot \gamma_{AB}
\]
Outage probability is another denotation of the channel capability. There is an outage event when channel capability can’t sustain the rate required by user. If spectral efficiency is required to satisfy $\mathbb{R}$, the outage probability at node B can be given by
\[
Pr\{out\} = P\{\mathbb{R} > C (snr_B) = \log_2 (1 + snr_B)\}
= P\{snr_B < 2^R - 1\}
= P\{\gamma_{AB} < \frac{2^R - 1}{SNR} \}
= \int_0^{2^R - 1} P(\gamma_{AB}) \cdot d_{\gamma_{AB}}
\]
Considering $\gamma_{AB} \sim \Upsilon(1/\Omega_{AB})$, we have
\[
Pr\{out\} = \int_0^{2^R - 1} \frac{1}{\Omega_{AB}} e^{-\frac{\gamma_{AB}}{\Omega_{AB}}} d_{\gamma_{AB}} = 1 - e^{-\frac{\kappa}{SNR}}
\]
where $\kappa = \frac{2^{2^R} - 1}{SNR}$. Since outage probability analysis is performed based on DaF strategy, we think node B can decode signal $x_A$ successfully when no outage event happens in the link A $\rightarrow$ B. In our paper, we assume all nodes’ transmission power is equal and white Gaussian noise with unity energy is added at every node, so SNR is identical in our paper. For a convenient expression, we also assume all channel gains obey Gaussian distribution with the same parameter $\Omega_{SR}$ in all sources and all relays; all channel gains obey Gaussian distribution with the same parameter $\Omega_{RD}$ in all relays and all destinations.

Opportunistic relay is proposed in [10], which develops and analyzes a distributed method to select the best relay based on relays’ local measurements of the instantaneous channel conditions. Analysis shows that the method can achieve the same diversity-multiplexing tradeoff as achieved by distributed space-time coding for relay nodes, but it is more simple and feasible to perform. Opportunistic relay selection can be performed by using the method of distributed timers, where each relay estimates its own instantaneous channel paths towards destinations. In DaF strategy, when one relay receives the pilot signals from destinations, it starts a timer with duration proportional to a function that depends only on it’s the worst channel gains towards destinations. The timer of the best relay expires first and a broadcast message notifies the rest of the network about its availability. The selected single relay is then used for information relaying.

### III. OPPORTUNISTIC RELAYING

In MR-OPP mode, sources take turns to broadcast through the best relay selected to all destinations. One source transmits messages continuously to make sure that one relay, at least, can receive its messages rightly during the first phase. In one broadcast, the probability of that all relays can’t receive the messages is
\[
P(out) = \prod_{i=1}^I Pr\{\gamma_{S_m R_i} < \kappa^*\} = (1 - e^{-\frac{\kappa^*}{SNR}})^I
\]
where $\kappa^* = \frac{2^{2^R} - 1}{SNR}$. Since communication happens in half-duplex hops, the required spectral efficiency per hop is equal to $2\mathbb{R}$ so that the end-to-end spectral efficiency is $\mathbb{R}$. The average number of required transmissions in $S_m \rightarrow R$ can be given
\[
N_{S_m R}^{opp} = \sum_{k=1}^\infty k Pr(N_{S_m R} = k)
= \sum_{k=1}^\infty k P(out)^{k-1} [1 - P(out)]
\]
In DaF strategy, the transmissions, during the second phase, are performed only by a subset $U$, defined by
\[
U \triangleq \{R_i | \log_2 (1 + SNR \cdot \gamma_{S_i R_i}) \geq 2\mathbb{R} , i \in [1, I] \}
\]
where the decoding at relay $R_i$ is assumed to be successful if $\log_2 (1 + SNR \cdot \gamma_{S_i R_i}) \geq 2\mathbb{R}$, i.e., no outage event happens during the first phase. Let $\Gamma_k$ defines the event that the $k^{th}$ transmission make messages to be received by relays for the first time, similarly, $U_{\gamma}$ defines the event that the number of relays in the subset $U$ is $\gamma$. The joint probability of that in
The $k^{th}$ transmission the messages happened to be received by $\zeta$ relays can be found as
\[
\Pr(U_{\zeta}, \Gamma_k) = \prod_{i \in \zeta} \Pr\{\gamma_{S_m, R_i} \geq \kappa^*\} \prod_{i \notin \zeta} \Pr\{\gamma_{S_m, R_i} < \kappa^*\} P_{\text{out}}^{k-1}
\]
\[
= \prod_{i \in \zeta} e^{-\frac{\gamma_{S_m, R_i}}{\rho_{SR}}} \prod_{i \notin \zeta} \left(1 - e^{-\frac{\gamma_{S_m, R_i}}{\rho_{SR}}}\right) (1 - e^{-\frac{\gamma_{S_m}}{\rho_{SR}}})^{I(k-1)}
\]
\[
= e^{-\frac{\gamma_{S_m}}{\rho_{SR}}} \prod_{i \notin \zeta} \left(1 - e^{-\frac{\gamma_{S_m, R_i}}{\rho_{SR}}}\right)^I
\]
(8)

Then, the average number of required transmissions in $S_m \rightarrow D$ can be given:
\[
\mathcal{N}_{\text{opp}}^{S_m D} = \sum_{k=1}^{\infty} \left\{k + \{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\}\right\} \Pr(\Gamma_k)
\]
\[
= \sum_{k=1}^{\infty} k \Pr(\Gamma_k) + \sum_{k=1}^{\infty} \left\{\sum_{i=1}^{I} \sum_{\zeta} \{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\} \Pr(U_{\zeta}, \Gamma_k)\right\} \Pr(\Gamma_k)
\]
(9)
\[
= \sum_{k=1}^{\infty} k P_{\text{out}}^{k-1} [1 - P_{\text{out}}] + \sum_{k=1}^{\infty} \sum_{\zeta} \sum_{i=1}^{I} \left[1 + \sum_{t=1}^{\infty} \left(1 - \prod_{j=1}^{t} \left\{\sum_{\zeta} \{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\} \Pr(U_{\zeta}, \Gamma_k)\right\}\right) e^{-\frac{\gamma_{S_m}}{\rho_{SR}}} \left(1 - e^{-\frac{\gamma_{S_m}}{\rho_{SR}}}\right)^{I-1} k-\zeta\right]
\]
(15)

During the second phase, through opportunistic relaying, the best relay $b^*$ is chosen among $\zeta$ relays in the decoding subset $U_{\zeta}$ in a distributed fashion that requires each relay to know its own instantaneous signal strength between the links $R_i \rightarrow D$. The choose of the best relay $b^*$ is to maximize the minimum of the weighted channel strengths between the links $R_i \rightarrow D$ for all $R_i \in U_{\zeta}$:
\[
b^* = \arg \max_{R_i \in U_{\zeta}} W_i
\]
(10)

where $W_i = \min\{\gamma_{R_i, D_1}, \ldots, \gamma_{R_i, D_j}, \ldots, \gamma_{R_i, D_I}\}$. Recall that
\[
W_i \sim \frac{1}{\Omega_{R_i, D_j}} \Rightarrow \sum_{j=1}^{I} \frac{1}{\Omega_{R_i, D_j}} = \frac{J}{\Omega_{R, D_j}}
\]
(11)

which follows from the fact that the minimum of $J$ independent exponential random variables (r.v’s) is again an exponential r.v with a hazard rate equal to the sum of the $J$ hazard rates. Hence, we can obtain the outage probability for $R \rightarrow D$ as follows:
\[
P_{\text{out1}}^{RD} = P \left\{ W_{b^*} < \frac{2^{2R} - 1}{SNR} \right\} = P \left\{ \max_{R_i \in U_{\zeta}} W_{R_i} < \frac{2^{2R} - 1}{SNR} \right\}
\]
\[
= \prod_{i=1}^{\zeta} P \left\{ W_{R_i} < \frac{2^{2R} - 1}{SNR} \right\} = \left(1 - e^{-\kappa^* \frac{1}{\rho_{RD}}}\right) \zeta
\]
(12)

where $P_{\text{out1}}^{RD}$ defines the joint probability of that some destinations can’t receive the messages of $b^*$ successfully. Now, Let $P_{\text{out1}}^{RD}$ defines the probability of that the destination $D_j$ can’t receive successfully. Considering $P_{\text{out1}}^{RD} = 1 - \left(1 - P_{\text{out1}}^{RD}\right)^{\zeta}, we have
\[
P_{\text{out1}}^{RD} = 1 - \left[1 - e^{-\kappa^* \frac{1}{\rho_{RD}}}\right]^\zeta
\]
(13)

Since we have to assure all destinations can receive the messages, the average number of required transmissions in $R \rightarrow D$ can be given as follows [3]:
\[
\left\{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\right\} = 1 + \sum_{t=1}^{\infty} \left[1 - \prod_{j=1}^{t} \left\{\sum_{\zeta} \{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\} \Pr(U_{\zeta}, \Gamma_k)\right\}\right] e^{-\frac{\gamma_{S_m}}{\rho_{SR}}} \left(1 - e^{-\frac{\gamma_{S_m}}{\rho_{SR}}}\right)^{I-1} k-\zeta
\]
(14)

Therefore, in MR-OPP mode, substituting (6), (8) and (14) to (9), we can have the average number of required transmissions in $S_m \rightarrow D$ as
\[
\mathcal{N}_{\text{opp}}^{S_m D} = \mathcal{N}_{\text{opp}}^{S_m R} + \sum_{k=1}^{\infty} \sum_{\zeta} \sum_{i=1}^{I} \left\{\sum_{\zeta} \{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\} \Pr(U_{\zeta}, \Gamma_k)\right\}
\]
\[
= \sum_{k=1}^{\infty} \sum_{i=1}^{I} \left[1 + \sum_{t=1}^{\infty} \left(1 - \prod_{j=1}^{t} \left\{\sum_{\zeta} \{\mathcal{N}_{\text{opp}}^{RD} | U_{\zeta}, \Gamma_k\} \Pr(U_{\zeta}, \Gamma_k)\right\}\right) e^{-\frac{\gamma_{S_m}}{\rho_{SR}}} \left(1 - e^{-\frac{\gamma_{S_m}}{\rho_{SR}}}\right)^{I-1} k-\zeta\right]
\]

IV. OPPORTUNISTIC NETWORK CODING

In MR-OPP-NC mode, all source take turns to broadcast messages continuously to make one relay, at least, can receive its messages rightly during the first phase. Be the same with the MR-OPP mode, the average number of required transmissions in $S_m \rightarrow R$ can be given
\[
\mathcal{N}_{\text{opp}}^{S_m R} = \sum_{k=1}^{\infty} k \Pr(\mathcal{N}_{\text{opp}}^{S_m R} = k)
\]
\[
= \sum_{k=1}^{\infty} k P_{\text{out}}^{k-1} [1 - P_{\text{out}}]
\]
(16)

where $P_{\text{out}} = (1 - e^{-\frac{\gamma_{S_m}}{\rho_{SR}}})^I$ and $\kappa^* = \frac{2^{2R} - 1}{SNR}$.

During the second phase, RLNC is performed at all relays, and any linear combination of packets decoded successfully can be transmitted. Assuming $\eta$ packets are decoded at one relay, RLNC is applied to a finite set of $\eta$ packets. Each packet can be viewed as a dimensional vector over a finite field, $F_q$ of size $q$. Typically, $F_{2^8}$ (i.e. $F_{256}$) is used [12]. The relay generates a random linear combination of the currently stored packets, say $P_1, \ldots, P_\eta$, by selecting uniformly at random the coefficients $\beta_1, \ldots, \beta_\eta$ over the field $F_q$, and generates: $P_{\text{new}} = \sum_{r=1}^\eta \beta_r P_r,$ i.e. generational packets. This new combination, $P_{\text{new}}$, along with its coefficients in terms of the original packets, is forwarded to destinations. When a
destination receives $\eta$ independent linear combinations, i.e., the destination reaches rank $\eta$, it can decode all original packets through matrix inversion.

Considering the outage probability is equal for all links $S_m \rightarrow R_i$, $m \in [1, M]$ $i \in [1, J]$, we can think all generational packets by RLNC at different relays have the same information measure, especially, when the number of sources is larger than the number of relays. Different from the best relay $b^*$ is chosen among the decoding subset $U_\mathcal{C}$ in MR-OPP mode, we can choose $b^*$ from all relays to broadcast. Moreover, if a generational packet transmitted by $b^*$ doesn’t contain the messages from one, even more, source in current slot, combining it with all generational packets received in latter slots, we can decode all original source messages very likely. Thereby, when the finite field $F_q$ is enough large, we say if only one destination received $M$ generational packets, the destination can decode all $M$ source messages.

Using equation (12) for reference, we have

$$P_{\text{out}2}^{RD} = P\left\{ W_{b^*} < \frac{2^{2R} - 1}{SNR} \right\} = P\left\{ \max_{R_i \in R} W_k < \frac{2^{2R} - 1}{SNR} \right\}$$

$$= \prod_{i=1}^{J} P\left\{ W_k < \frac{2^{2R} - 1}{SNR} \right\} = \left( 1 - \frac{2^{2R} - 1}{SNR} \right)^J$$

(17)

where $P_{\text{out}2}^{RD}$ defines the joint probability of that some destinations can’t receive the messages of $b^*$ successfully in MR-OPP-NC mode. Now, Let $P_{\text{out}2}^{RD_j}$ defines the probability of that the destination $D_j$ can’t receive successfully. Considering $P_{\text{out}2}^{RD} = 1 - \left( 1 - P_{\text{out}2}^{RD_j} \right)^J$, we have

$$P_{\text{out}2}^{RD_j} = 1 - \left[ 1 - \left( 1 - e^{-\kappa \Omega_{\mathcal{R}D}} \right)^J \right]$$

(18)

In respect that all destinations are promised to receive $M$ generational packets, the total number of required transmissions in $R \rightarrow D$ should be given as follows [3]:

$$N_{\text{oppp-nc}}^{RD} = M + \sum_{t=M}^{\infty} \left\{ 1 - \prod_{i=1}^{J} \left[ \sum_{\tau=t}^{M} \left( \frac{\tau - 1}{M - 1} \right) \right] \times \left( P_{\text{out}2}^{RD} \right)^{\tau - M} \left( 1 - P_{\text{out}2}^{RD_j} \right)^M \right\}$$

(19)

where $\binom{n}{m}$ gives the number of combinations of size $m$ of $n$ elements.

Therefore, in MR-OPP-NC mode, combining (16) and (19), we can have the average number of required transmissions in $S \rightarrow D$ as

$$N_{\text{oppp-nc}}^{SD} = N_{\text{oppp-nc}}^{SR} + N_{\text{oppp-nc}}^{RD}/M$$

$$= \sum_{k=1}^{\infty} k P(\text{out})^{k-1} \left[ 1 - P(\text{out}) \right] + 1$$

$$+ \frac{1}{M} \sum_{t=M}^{\infty} \left\{ 1 - \prod_{i=1}^{J} \left[ \sum_{\tau=t}^{M} \left( \frac{\tau - 1}{M - 1} \right) \right] \times \left( P_{\text{out}2}^{RD} \right)^{\tau - M} \left( 1 - P_{\text{out}2}^{RD_j} \right)^M \right\}$$

(20)

V. NUMERICAL RESULTS & SIMULATION

In this section, the performance of the proposed MR-OPP-NC scheme is evaluated with the comparison to the pure MR-OPP scheme in MSMD scenarios. 8 sources broadcast messages to 12 destinations by relays at the end-to-end spectral efficiency $R=2$ bps/Hz and $\Omega_{\mathcal{R}D}=1$, where the transmissions have to be done over a time varying channel with Rayleigh fading. Monte-Carlo simulation is also performed to demonstrate the delay performance.

Firstly, the outage probability is simulated for opportunistic relaying in $R \rightarrow D$. Fig. 2 shows the outage probability as a function of SNR for the DaF strategy with $\{1, 2, 4, 6, 8\}$ relays at single-hop spectral efficiency $R=2$ bps/Hz. As can be seen from Fig. 2, with the stepping up of the number of relays, the outage probability for $R \rightarrow D_j$ is decreasing gradually. Compare to MR-OPP scheme where the partial relays that received source’s messages successfully are applied to opp-
As Fig. 2, for an explicit expression, we use network coding instead of opportunistic relaying, because the low SNR (0 to 2, which is needed in two-way relay channels. The gains of SNR go to larger, the number of transmission approximates to MR-OPP scheme. As Fig. 3 shows in one slot can be used for all destinations; thus, more relay transmissions can be saved comparing to MR-OPP scheme. In MR-OPP-NC scheme, by right of network coding, the generational packet mission number required for the two schemes. In MR-OPP-NC scheme is smaller than the number in MR-OPP scheme.

Compare to MR-OPP scheme based the pure opportunistic relaying, MR-OPP-NC scheme proposed is proved to obtain significant system delay (or throughput) gains, through reducing the number of required transmission. Theoretical analysis and Monte-Carlo simulation have been provided to demonstrate the delay performance gains of the proposed network coded protocol.

VI. Conclusion

In this paper, we propose the application of network coding to MSMD broadcasting scenarios with DaF strategy and provided form analysis for the improvement of system delay. Compare to MR-OPP scheme based the pure opportunistic relaying, MR-OPP-NC scheme proposed is proved to obtain significant system delay (or throughput) gains, through reducing the number of required transmission. Theoretical analysis and Monte-Carlo simulation have been provided to demonstrate the delay performance gains of the proposed network coded protocol.

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