Nonlinear filtering for phase image denoising

J.V. Lorenzo-Ginori, K.N. Plataniotis and A.N. Venetsanopoulos

Abstract: The problem of phase image denoising through nonlinear (NL) filtering is addressed. There are various imaging systems in which the phase information is utilized to generate useful imaging data. However, the presence of noise makes it difficult to obtain the appropriate phase image. The authors apply NL vector filtering techniques to denoise the complex data from which the phase image is extracted. A study was realized in which several NL filters were applied to a simulated complex image. The effects of filtering were determined through a Monte Carlo simulation in which the image was successively contaminated with six different noise models. The effectiveness of the filters was measured in terms of normalised mean square error, signal-to-noise ratio and the number of eliminated phase residues. Results indicate a significant noise reduction, especially when NL filters based on angular distances are applied to the noisy input.

1 Introduction

In many imaging systems, the phase information associated to a received signal is used to produce a useful image. Examples include among others magnetic resonance imaging (MRI), current density imaging (CDI) and interferometric synthetic aperture radar (IFSAR) [1, 2].

Different works have been devoted to the problem of noise reduction in the context of the above-mentioned techniques. MRI noise filtering has been achieved by means of wavelet and wavelet-packets shrinkage denoising [3-5]. However, these references address only the problem of magnitude image denoising. More specifically, phase image denoising was performed through application of wavelet shrinkage to the IFSAR complex image in [6], while an application of morphological and scalar nonlinear (NL) filters directly to the phase image can be found in [7].

In many situations, the complex data from which the phase image is extracted are available, as occurs in the imaging techniques already mentioned. In these cases, the application of NL vector filters can be a valuable alternative for noise filtering. NL vector filters have demonstrated excellent noise reduction properties in applications to colour image and multichannel signal processing [8-13].

This paper addresses the problem of phase image denoising through nonlinear filtering. In this work, a number of NL vector filters commonly used for multichannel signal processing were applied to the complex images. Their properties in reducing phase image noise and eliminating phase residues are studied. The latter problem is important, as phase residues introduced by noise can deteriorate seriously the process of phase unwrapping.

Results indicate a significant noise reduction in terms of signal-to-noise ratio (SNR) and suppression of phase residues, especially for the angular-distance based NL filters.

2 Phase unwrapping

The introduction of phase residues in the wrapped phase images is the most negative effect of noise on the bidimensional (2-D) phase unwrapping problem. Several examples of imaging applications such as IFSAR, CDI and MRI that require 2-D phase unwrapping are presented in [1]. In these cases, filtering is needed to reduce the number of phase residues introduced by noise, and its use can facilitate considerably the process of 2-D phase unwrapping.

When the phase is extracted from an actual complex image, only the values that lie in the interval \((-\pi, \pi]\) are obtained. This is the so-called wrapped phase. Phase unwrapping is the process employed to obtain the appropriate phase values from the wrapped phase. There are different methods for 2-D phase unwrapping [1, 14-16].

Solving the problem of 2-D phase unwrapping requires the evaluation of the line integral

\[
\phi(r) = \oint_C \nabla \phi(r) \cdot dr + \phi(r_0)
\]  

where \(\phi(r)\) is the phase, \(C\) is any path in a domain \(D\) connecting the points \(r_0\) and \(r\), and \(\nabla \phi\) is the phase gradient given by

\[
\nabla \phi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j}
\]

To obtain a reasonable phase unwrapping, the line integral (1) should be path-independent, for which the condition is

\[
\oint_C \nabla \phi(r) \cdot dr = 0
\]
In general, this condition is difficult to be satisfied by the unwrapped phase images. In these images, the residue theorem for phase unwrapping dictates that:

$$\phi(r) = \oint_C \phi(r) \cdot dr = 2\pi K$$

Here $K$ is an integer that accounts for the number of phase residues enclosed by $C$. Consistent phase unwrapping is possible if, and only if, all integration path do not include unbalanced residues.

Phase residues can occur in areas where the complex function from which they are calculated is well behaved. In fact, in many practical situations, zeros are residue generators in the extracted phase. Dealing with phase residues is a central issue on phase unwrapping, and it determines the nature of different algorithms.

To avoid the effects of phase residues, 2-D unwrapping algorithms usually place 'branch cuts' between residues of opposite sign. If these branch cuts are not crossed, any closed path will enclose an equal number of positive and negative residues, or no residues at all. The phase can be appropriately unwrapped along any path that does not cross the branch cuts.

In this work the 2-D unwrapped phase was extracted from the noise-contaminated phase using a bidimensional version of the Itoh’s algorithm [1]. This is a simple one-dimensional algorithm in which the unwrapped phase is obtained as the integral (sum) of the wrapped differences of adjacent pixels, provided that the phase increments between adjacent pixels is always $|\Delta \phi| < \pi$. Starting from an initial point with phase $\phi(0)$, the unwrapped phase at a point $m$ is given by

$$\phi(m) = \phi(0) + \sum_{n=0}^{m-1} W|\Delta|W|\phi(n))|$$

where $W$ is the wrapping operator which maps the unwrapped phase values in the interval $(-\pi, \pi]$, and $\Delta|W|\phi(n) = \phi(n + 1) - \phi(n)$ is the difference operator.

The 2-D Itoh’s algorithm starts by unwrapping the phase along the first row in the phase image. Then it unwraps each column using the respective upper-most unwrapped phase as the initial value for that column. It is known that this algorithm cannot work properly in the presence of phase residues. However, this does not preclude here a clear comparison between the original and the filtered unwrapped phases, as the suppression of phase residues is one of the performance factors to be evaluated.

### 3 Nonlinear vector filters for complex images

NL vector filters have been used in various examples of multichannel image processing, as is the case for colour TV. A review of the different techniques can be found in [8, 9]. In this work, vector median filters [10], vector directional filters [11, 12], directional distance filters [13] and four examples of adaptive fuzzy filters [9] were used to process simulated complex data from which the phase images are derived. The noise suppression properties of these filters were determined for this application through extensive simulation experiments.

The filters mentioned involve the minimisation of an error criterion and use similarity measures that are usually defined in terms of $n$-dimensional vectors. The similarity measures can be expressed in terms of phasors, as bidimensional vectors, when the image to be processed is complex.

Let $W$ be the processing window of size $n$ and $\{x_i\}$, $i = 1, 2, \ldots, n$ be the complex numbers (phasors) in $W$, where

$$\chi_i = \rho_i e^{i\phi_i} \quad (7)$$

In the following analysis, the Cartesian and the polar representation of the complex numbers are completely equivalent. Let $\{x_i\}$ be the input set and let $D_i$ be a cumulative distance corresponding to $\chi_i$ defined as

$$D(i) = \sum_{j=1}^{n} \|x_i - x_j\|, \quad i = 1, 2, \ldots, n \quad (8)$$

where $\|\|$ is an appropriate vector norm. The phasor $\chi_i$ for which

$$D(i) \leq D(j) \quad \forall \quad j = 1, 2, \ldots, n \quad (9)$$

is the output of the vector median filter (VMF). For this filter, the most commonly used distances are the $L_1$ (city block) and $L_2$ (Euclidean) norms. These can be expressed in terms of phasors as

$$D_1(i) = \sum_{j=1}^{n} |\text{Re}(x_i - x_j)| + |\text{Im}(x_i - x_j)| \quad (10)$$

and

$$D_2(i) = \sum_{j=1}^{n} |x_i - x_j| \quad (11)$$

respectively.

This concept of vector distance serves to establish an ordering of the phasors $\{\chi_i\}$. If the distances are ordered as

$$D_1(i) \leq D_2(i) \leq \ldots \leq D_2(n) \quad (12)$$

this implies the same ordering for the phasors,

$$\{\chi^{(1)} \}, \{\chi^{(2)} \}, \ldots, \{\chi^{(n)} \} \quad (13)$$

While $\chi^{(1)}$ is the output of the VMF, the first $r$ phasors in the set $\{\chi^{(r)} \}$ can be selected through an appropriate membership function to obtain the filter output by means of some mathematical operations [9]. This is the case of the fuzzy vector median filter (FVMF) designed and tested in this work. The characteristics of the membership function will be discussed later.

In a similar way, vector directional filters (VDF) can be defined by incorporating an angular distance measure. Here the cumulative distance is

$$\alpha(i) = \sum_{j=1}^{n} A(x_i, x_j) \quad i = 1, 2, \ldots, n \quad (14)$$

and the distance measure is defined as the scalar value

$$A(x_i - x_j) = |\arg(x_i \cdot \bar{x}_j)|, \quad 0 \leq A(x_i, x_j) \leq \pi \quad (15)$$

where $\bar{x}_j$ is the conjugate of $x_j$. An ordering of the $\alpha(i)$ such that

$$\alpha^{(1)} \leq \alpha^{(2)} \ldots \leq \alpha^{(n)} \quad (16)$$

implies the same order in the corresponding phasors. The basic vector directional filter (BVDF) is that whose output is $\chi^{(1)}$, and the generalised vector directional filter (GVDF) is that whose output is defined as the first $r$ phasors of the ordered set

$$\{\chi^{(r)} \} = \chi^{(1)}, \chi^{(2)}, \ldots, \chi^{(n)} \quad (17)$$
where

\[ x^{(t)} \in \{ x_j, \ j = 1, 2, \ldots, n \} \quad \forall t = 1, 2, \ldots, r \]  

(18)

The value of \( r \) can be determined according to certain membership rule, defining a fuzzy filter. In the fuzzy filter, the output is calculated from these \( r \) phasor values through some mathematical operation, as is explained in (21)-(25). A fuzzy vector directional filter based on magnitude distance, FVDMA, was designed and tested in this work. In this filter the trimmed mean of the selected \( r \) phasors included in the window is calculated as the filter output. Another filter was the fuzzy vector directional filter based on angular distance, FVDANG, in which only the angular information is taken into account to calculate the trimmed mean. In the latter case, the current phasors are substituted by unitary phasors with the same argument. The trimmed mean is a mean value calculation in which the outliers are discarded.

The phasor magnitude and angle distances can be combined to obtain the directional distance filter (DDF),

\[ \Omega_i = D_i^{(p)} \cdot \alpha_i^p \quad i = 1, 2, \ldots, n \]  

(19)

where \( p \in \{0, 1\} \) is a parameter used to weight the magnitude and angular distances. Therefore, magnitude and angular distances are particular cases of the directional distance. Again, an ordering

\[ \Omega^{(1)} \leq \Omega^{(2)} \leq \ldots \leq \Omega^{(n)} \]  

(20)

implies the ordering of the input phasors as in (13). The output of the DDF is \( \Omega^{(1)} \). A subset of (20) can be selected adaptively to calculate the filter output, as in the fuzzy directional distance filter (FDDF). After an experimental determination of \( p = 0.8 \), a DDF filter (DDF8), as well as the corresponding FDDF filter were designed and tested.

The general definition for the adaptive fuzzy filter [9] is

\[ \hat{y} = \sum_{k=1}^{r} w(k) x_k \]  

(21)

Here \( w(k) \) is the normalised \( k \)th membership function, given by

\[ w(k) = \frac{f(\mu_k)}{\sum_{l=1}^{r} f(\mu_l)} \]  

(22)

where \( f(\mu) = \mu^{\lambda} \) is a function adaptively determined on the basis of local context [9] with \( \mu_k \) being the membership function of input \( x_k \) and \( \lambda \in [0, \infty) \) a parameter.

In this investigation, for the case of the fuzzy filters FVMF, FVDANG, FVDMA and FDDF designed and applied to the complex image, a linear membership function was defined as

\[ \mu_k = \frac{d_{\max} - d_k}{d_{\max} - d_{\min}} \]  

(23)

where \( d \) stands for any of the distances previously defined, ‘\( \text{max} \)’ and ‘\( \text{min} \)’ denote the maximum and minimum value of \( d \), and \( \lambda = 1 \) defines a ‘centre of gravity’ strategy. A threshold \( s = 0.2 \) was established experimentally to determine the phasor membership whenever \( \mu_k \geq 1 - s \), or equivalently when

\[ d_k \leq d_{\min} + s(d_{\max} - d_{\min}) \]  

(24)

It should be noted that VMF and BVDF are particular cases of the fuzzy filter in which the maximum defuzzifier strategy was used. This strategy consists in

\[ w(k) = \begin{cases} 1, & \text{if } \mu_k = \mu_{\text{max}} \\ 0, & \text{if } \mu_k \neq \mu_{\text{max}} \end{cases} \]  

(25)

### 4 Methods

To evaluate the properties of the previously described filters in denoising phase images, a Monte Carlo experiment was performed. This experiment consisted of successive noise contamination (using 20 trials) of a simulated complex image and then the application of the filters to reduce the noise level (and the number of phase residues) in the phase image. The performance of the filters was then measured quantitatively both in terms of improvements in signal to noise ratio, and of reduction of the number of phase residues due to noise.

The simulated image was constructed according to an accepted practice for this type of study [4, 17]. The different numerical parameters, however, were defined here to meet appropriately the objectives of this work. A magnitude image consisting of a 64 \( \times \) 64 pixel square of amplitude 210 was centered in a 128 \( \times \) 128 pixel square of amplitude 90. The original unwrapped phase image was a bidimensional Gaussian function given by

\[ \varphi_{\text{wr}} = A \exp \left( \frac{(u - 64)^2 + (v - 64)^2}{\sigma_1^2 + \sigma_2^2} \right) \]  

(26)

where \( u \) and \( v \) represent the coordinate variables. Values \( A = 7\pi, \sigma_1^2 = 3500 \) and \( \sigma_2^2 = 1000 \) were used.

A complex image was formed from these magnitude and phase. The wrapped phase image was obtained as a natural consequence of the inverse process. This is because when the phase is calculated again from the complex data, only values that lie in the interval \((-\pi, \pi]\) can be obtained. Then the complex image was contaminated with any of the six noise models described in Table 1. These models combine in some cases Gaussian and impulsive noise to pose a demanding situation that could facilitate a clear discrimination between the behavior of the different filters. In applications such as MRI, the Gaussian model is widely accepted for this type of study [4, 17]. The different numerical parameters, however, were defined here to meet appropriately the objectives of this work. A magnitude image consisting of a 64 \( \times \) 64 pixel square of amplitude 210 was centered in a 128 \( \times \) 128 pixel square of amplitude 90. The original unwrapped phase image was a bidimensional Gaussian function given by

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Noise power was divided evenly between real and imaginary parts for the Gaussian noise model. For impulsive noise, the probability of an impulse for the real or imaginary part was calculated according to

\[ p = 1 - \sqrt{1 - F_i} \]  

(27)

<table>
<thead>
<tr>
<th>Table 1: Noise models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise model</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

The measures used to quantify the effects of noise were the normalised mean square error (NMSE), the SNR and the number of phase residues. The NMSE was defined as

\[ NMSE = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \| \phi(i,j) - \hat{\phi}(i,j) \|^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} \| \phi(i,j) \|^2} \]  

(28)

where \( \phi \) denotes the original unwrapped phase, \( \hat{\phi} \) the recovered unwrapped phase after filtering, and \( (i,j) \) the pixel indexes along the directions \( u \) and \( v \).

The SNR was calculated as

\[ SNR = 10 \log_{10} \left( \frac{1}{NMSE} \right) \]  

(29)

The number of phase residues, both, in the noisy and in the recovered phase was calculated by systematic application of (4) to the phase image.

The filters used to process the complex image previous to phase image calculation were VMF, FVMF, BVDF, FVDMAG, FVDFANG, DDF8 and FDDF. The city block distance was employed for the magnitude in all cases. Table 2 summarises the different filters employed and the corresponding parameters where appropriate.

For the NL vector filters used here, the number of distances that are to be calculated in an \( n \times n \) elements window is \( N = n!/(2(n-2)! \). This means that the computational burden increases rapidly with the window size and can be high, especially in the case of angular distances. Taking this into account, only \( 3 \times 3 \) windows were considered here.

### Results

The main results are shown in Tables 3–6. Table 3 summarises results in terms NMSE and SNR, obtained for phase image denoising using different filters, while Table 4 indicates performance in terms of phase residues. The standard deviations of NMSE and the number of residues are also shown. In the simulations that correspond to these two tables, the noise models 1–3 of Table 1 were employed. A high degree of noise contamination was used to better illustrate the performance of the various methods under consideration.

Results indicate that filters based on angular distances, BVDF, FVDFANG and FVDFMAG perform the best in terms of SNR improvement and in reducing the number of phase residues. The ability of these filters to eliminate the phase residues introduced by noise is of particular importance. This was above 95% for the worst case of noise model 3.

Tables 5 and 6 summarise the results obtained for noise models 4–6. These are consistent with the previous values. The relatively low effectiveness of the VMF can be seen in all cases. This performance improves when FVMF is considered, as expected, but it is not as good as that obtained by the angular distance filters BVDF, FVDFANG and FVDFMAG, which showed the best performance.

DDF8 and FDDF results are closer to the angular-distance filters than to the magnitude-distance filters. This was expected, given that \( p = 0.8 \), which means that the directional distance is heavily biased towards the angular distance. The first column of Table 6 shows no phase residues, while in column 3, the phase residues are completely cleared out by the filters. Column 5 again reveals the advantage shown by the filters based on angular distances, which produced the lowest mean values of remaining residues.

The graphical results for noise reduction in phase images are shown in Fig. 1. Here the wrapped and
Table 4: Results of filtering in terms of phase residues, noise models 1-3

<table>
<thead>
<tr>
<th>Filter</th>
<th>Noise model 1 Nres</th>
<th>stdv</th>
<th>Noise model 2 Nres</th>
<th>stdv</th>
<th>Noise model 3 Nres</th>
<th>stdv</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>883.70</td>
<td>38.25</td>
<td>1391.85</td>
<td>42.33</td>
<td>2155.85</td>
<td>47.52</td>
</tr>
<tr>
<td>VMF</td>
<td>29.35</td>
<td>8.27</td>
<td>86.65</td>
<td>9.87</td>
<td>264.60</td>
<td>28.44</td>
</tr>
<tr>
<td>BVDF</td>
<td>3.40</td>
<td>2.76</td>
<td>18.05</td>
<td>5.31</td>
<td>101.45</td>
<td>16.76</td>
</tr>
<tr>
<td>DDF8</td>
<td>5.30</td>
<td>3.84</td>
<td>23.60</td>
<td>7.26</td>
<td>122.40</td>
<td>21.41</td>
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<tr>
<td>FVMF</td>
<td>16.40</td>
<td>6.95</td>
<td>49.35</td>
<td>13.27</td>
<td>186.85</td>
<td>24.09</td>
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<tr>
<td>FVDMAG</td>
<td>1.80</td>
<td>2.17</td>
<td>17.00</td>
<td>6.78</td>
<td>99.10</td>
<td>11.10</td>
</tr>
<tr>
<td>FVDANG</td>
<td>2.30</td>
<td>2.36</td>
<td>16.70</td>
<td>7.34</td>
<td>101.10</td>
<td>17.04</td>
</tr>
<tr>
<td>FDDF</td>
<td>4.50</td>
<td>2.14</td>
<td>17.55</td>
<td>7.72</td>
<td>101.80</td>
<td>16.03</td>
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</table>

Table 5: Results of filtering in terms of NMSE and SNR, noise models 4-6

<table>
<thead>
<tr>
<th>Filter</th>
<th>Noise model 4 NMSE</th>
<th>STDV</th>
<th>SNR</th>
<th>Noise model 5 NMSE</th>
<th>STDV</th>
<th>SNR</th>
<th>Noise model 6 NMSE</th>
<th>STDV</th>
<th>SNR</th>
</tr>
</thead>
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<tr>
<td>none</td>
<td>0.0004</td>
<td>4.96 × 10⁻⁶</td>
<td>34.29</td>
<td>0.0290</td>
<td>6.69 × 10⁻²</td>
<td>20.36</td>
<td>0.3989</td>
<td>3.59 × 10⁻¹</td>
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<tr>
<td>VMF</td>
<td>0.0001</td>
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<td>39.03</td>
<td>0.0002</td>
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</tr>
<tr>
<td>BVDF</td>
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<td>1.95 × 10⁻⁶</td>
<td>40.87</td>
<td>0.0002</td>
<td>3.20 × 10⁻⁶</td>
<td>37.92</td>
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<td>1.69 × 10⁻⁶</td>
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<td>1.77 × 10⁻⁶</td>
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<td>37.12</td>
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<td>FVDMAG</td>
<td>0.0001</td>
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<td>40.52</td>
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<td>4.30 × 10⁻⁶</td>
<td>37.72</td>
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<td>4.04 × 10⁻⁴</td>
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<tr>
<td>FDDF</td>
<td>0.0001</td>
<td>1.85 × 10⁻⁶</td>
<td>40.46</td>
<td>0.0002</td>
<td>3.70 × 10⁻⁶</td>
<td>37.66</td>
<td>0.0007</td>
<td>6.37 × 10⁻⁴</td>
<td>32.52</td>
</tr>
</tbody>
</table>

unwrapped phase images are shown for the noiseless, the noisy and the filtered cases. For the noisy case, the presence of phase residues in the wrapped phase image, manifests in the appearance of lines and strips in the unwrapped phase image, as is to be expected from the unwrapping algorithm used. On the other hand, the filtered image has two phase residues only, and a fairly good recovery of the unwrapped phase was possible.

Fig. 2 shows phase filtering of a real MRI phase image of a phantom with a FVDMAG filter. In this case, residues were reduced to 26.1% of the original amount of 6725. A mask was used to eliminate partially the noise outside the region of interest.

6 Discussion and conclusions

In this work a study of noise reduction in phase images was realized. Denoising was achieved through the application of NL vector filters. These NL vector filters were adapted to phasor processing, and used to filter the complex image from which the phase is obtained. The performance indicators utilised to evaluate filter behaviour were the NMSE, the SNR and the number of phase residues. The filters were tested for six different noise models, some of them including impulsive noise to pose a more demanding situation.

Results indicate that the NL vector filters can be effective in phase image denoising whenever the complex data associated
Fig. 1  Wrapped and unwrapped phase images for noiseless, noisy and filtered cases
Noise model is 6, phase filter is FVDANG, and a 3 × 3 window was used. The brightness is proportional to the magnitude
\( s = 2 \); noisy residues: 985; residues after filtering: 2
\( a \) Original; \( b \) Noisy; \( c \) Filtered

Fig. 2  Phase image nonlinear filtering of a real MRI phantom
The filter used was a FVDMAG, with a 3 × 3 window. Residues were reduced to 26.1% of the original 6725
\( a \) Original WP
\( b \) Filtered WP
\( c \) Mask from filtered image
\( d \) Filtered WP, masked
to the phase image are available. The results showed a positive effect in all cases, with clear advantage of the filters that use similarity measures based on angular distances, i.e., BVDF, FVDMAG, and FVDANG, in terms of all the performance indicators employed. Nevertheless, given that these filters are highly time consuming, the FVMF filter can be a good alternative when SNR is not too low. Based on these results, and balancing computational cost and preservation of image characteristics, the BVDF can be a good option for various applications when SNR is low.

For phase image filtering, only windows of size $3 \times 3$ were considered. This was due to the high computational load that larger windows imply, mostly in the case of filters based on angular distances.

Reduction in the number of phase residues due to noise showed values above 95% for the worst noise contamination tested. For the phase image, SNR improvements were above 13 dB for the best filters at the worst noise conditions.

The application of the NL filters studied in this work to phase image denoising is by no means restricted to a particular technique that uses phase images. Therefore, this can constitute a valuable tool whenever 2D phase unwrapping of noisy data is a necessity. These filters have the advantage of being capable of filtering impulsive noise that can occur in some situations like occurs in phase stepped interferometry.

7 References