Symmetric Interaction in Channel Allocation for Bi-Directional In-Band Full-Duplex Network

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Abstract—An adaptive channel allocation scheme for bi-directional in-band full-duplex networks is proposed by modifying a channel allocation scheme for half-duplex networks on the basis of the property of bilateral symmetric interaction. To adapt schemes for half-duplex networks to full-duplex networks, we modify them so that channel update process successfully converges to a Nash equilibrium, i.e., it does not cycle. Specifically, the sum of products of transmission power and received interference power (instead of the sum of received interference power) at a communication pair is used as the novel payoff function. Then, the proposed channel allocation scheme is proved to be a potential game, and thus, the game is guaranteed to converge in a finite number of steps. Simulation results reveal that the proposed scheme is able to effectively allocate channels in bi-directional in-band full-duplex networks.

I. INTRODUCTION

Bi-directional in-band full-duplex (FD) operation [1]–[3] has recently attracted much attention as a technology that enables radio nodes in wireless communication networks to simultaneously transmit and receive signals over the same frequency band. Enabling FD operations of radio nodes has a potential to double the spectral efficiency [3]. However, in FD networks, multi-user interference and self-interference are performance limiting factors, compared with half-duplex (HD) networks. To resolve this, several papers [4]–[7] have discussed resource allocation in FD networks.

Adaptive channel allocation schemes are one of resource allocation technologies for efficiently managing a limited number of radio channels. Decentralized channel allocation algorithms configure radio channels autonomously and intelligently on the basis of received interference power from adjacent nodes. In these algorithms, nodes can be considered as multiple decision-making entities that act for their own communication performance and interact with each other. An effective framework for analyzing interaction among decision-making entities is game theory [8], [9].

Main contributions of this study are

• Exemplifying that an unilateral improvement dynamics can cycle when we simply adapt the channel allocation scheme for HD networks to FD networks.
• Modification of the payoff function so that it has the property of symmetric interaction for FD networks.
• Proof that the proposed channel allocation game is a potential game.

This paper is organized as follows. Section II introduces the system model. Section III formulates the problem as a channel allocation game and expresses important properties of strategic games and potential games. Section IV introduces the proposed channel allocation scheme. Section V presents simulation results and discusses the impact of the number of communication pairs on channel allocation and potential value. The concluding remarks are shown in Section VI.

II. SYSTEM MODEL

We focus on a distributed FD networks [1]. Fig. 1 shows the system model. There are \( N \) stationary transmitter-receiver communication pairs which operate in FD mode: they can be a transmitter and a receiver simultaneously. Pair \( i \in \mathcal{N} := \{1, \ldots, N\} \) selects one channel \( a_i \in \mathcal{A}_i \), where \( \mathcal{A}_i \) represents the set of channels available to pair \( i \). Each pair consists of two nodes that communicate with each other. Let two nodes of pair \( i \) be denoted by node \( x_i \) and node \( y_i \). In Fig. 1, \( G_{ij} \) represents the link gain between node \( x_i \) and node \( y_j \), where \( x, y \in \mathcal{Q} := \{(a), (b)\}. p_i^x \) represents the transmission power of node \( x_i \). \( \text{SI}_i^x(a_i) \) shows the self-interference power. \( \text{SI}_i^x(a_i) \) is assumed to have frequency selectivity. In this

The channel allocation scheme proposed in [17] is suitable for HD networks. However, the scheme is not directly adapted to networks where nodes operate in FD mode (FD networks). Specifically, the channel allocation scheme could cycle when we adapt the scheme to FD networks.

In the present paper, we propose a novel channel allocation scheme for FD networks. The channel allocation problem in FD networks is formulated as a game by utilizing the property of bilateral symmetric interaction [19], and thus is proved to be a potential game. To allocate channels effectively with high probability, we utilize spatial adaptive play (SAP) which is a kind of dynamics [17], [20]. In channel allocation games, we regard the set of communication pairs as the set of players that have their own payoff functions, that is, the sum of products of transmission power and received interference power.
At Nash equilibrium, the payoff does not increase if a player unilaterally changes its strategy. Therefore, players have no incentive to change their strategies once in a Nash equilibrium.

Next, we discuss strategy updates at each time. We assume that only one pair can change its strategy at each time $k$. Let the set of strategies at time $k$ be denoted by $(a^k_i, a^k_{-i}) \in \mathcal{A}$. At time $k$, assuming that pair $i$ has a chance to change its strategy, if there exists a strategy $a^k_{i} \neq a^k_i$ that satisfies

$$a^k_{i} \in \arg \max_{a_i \in \mathcal{A}_i} u_i(a_i, a^k_{-i})$$

then, the strategy of pair $i$ is changed from $a^k_i$ to $a^k_{i} \neq a^k_i$ to optimize its payoff with respect to the other player’s strategy profile. The above dynamics is the most standard form of dynamics, which is called best response dynamics [21], [22].

**C. Spatial Adaptive Play**

In this section, we describe the channel allocation dynamics, which is known as spatial adaptive play [20], [23]. The possibility to escape after encountering a local maximizer (strategy profile) of a potential function is retained in this dynamics. At each channel update time $k$, a stochastic update is conducted with the following probability

$$\pi_i(a^k_i, a^k_{-i}) = \frac{e^{\beta u_i(a^k_i, a^k_{-i})}}{\sum_{a_i \in \mathcal{A}_i} e^{\beta u_i(a_i', a^k_{-i})}}$$

for some exploration parameter $\beta \geq 0$. A player $i$ is randomly chosen to update its strategy with probability $\pi_i$.

The constant $\beta$ determines how likely player $i$ is to select a sub-optimal strategy. If $\beta = 0$, player $i$ selects a strategy among all strategies $a_i \in \mathcal{A}_i$ with equal probability. On the other hand, if $\beta$ has a large value, player $i$ will select the best response with high probability.

**D. Potential Game**

A potential game forms a class of strategic form games. In potential games, unilateral improvement dynamics of players automatically lead to improving not only their own payoff but also the potential of the whole network.

A game $(\mathcal{N}, \mathcal{A}, (u_i)_{i \in \mathcal{N}})$ is referred to as an exact potential game (EPG) if there is a function $f : \mathcal{A} \to \mathbb{R}$ that satisfies

$$u_i(a_i, a_{-i}) - u_i(a'_i, a_{-i}) = f(a_i, a_{-i}) - f(a'_i, a_{-i}),$$

$$a_i, a'_i \in \mathcal{A}_i, a_{-i} \in \mathcal{A}_{-i}, \forall i \in \mathcal{N},$$

where the function $f$ is referred to as a potential function. In addition, a game $(\mathcal{N}, \mathcal{A}, (u_i)_{i \in \mathcal{N}})$ is called to a weighted potential game (WPG) if there is a function $f : \mathcal{A} \to \mathbb{R}$ and a set of positive numbers $\{\alpha_i\}_{i \in \mathcal{N}}$ such that

$$u_i(a_i, a_{-i}) - u_i(a'_i, a_{-i}) = \alpha_i(f(a_i, a_{-i}) - f(a'_i, a_{-i})),$$

$$a_i, a'_i \in \mathcal{A}_i, a_{-i} \in \mathcal{A}_{-i}, \forall i \in \mathcal{N},$$

In a strategic-form game, there is not necessarily a pure-strategy Nash equilibrium, i.e., best response dynamics is not always guaranteed to converge. On the other hand, the
potential function monotonically increases when using unilateral improvement dynamics, and because a potential game with a finite set of strategies always has a Nash equilibrium, the unilateral improvement dynamics converge to a Nash equilibrium in a finite number of steps [10].

E. Bilateral Symmetric Interaction Games

A game is called a bilateral symmetric interaction (BSI) game [19], if there exist functions \( w_{ij} : A_i \times A_j \rightarrow \mathbb{R} \) and \( s_i : A_i \rightarrow \mathbb{R} \) such that
\[
u_i(a) = \sum_{j \in \mathcal{N} \setminus \{i\}} w_{ij}(a_i, a_j) - s_i(a_i),
\]
where \( w_{ij}(a_i, a_j) \) holds \( w_{ij}(a_i, a_j) = w_{ji}(a_j, a_i) \) for every \((a_i, a_j) \in A_i \times A_j\). A BSI game is an EPG. In addition, according to [19], the potential function \( V \) of every BSI game has the following form:
\[
V(a) = \frac{1}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} w_{ij}(a_i, a_j) - \sum_{i \in \mathcal{N}} s_i(a_i).
\]
When conditions (6) and (7) are satisfied, a Nash equilibrium exists and best response dynamics are guaranteed to converge to a Nash equilibrium in a finite number of steps, as stated in Section III-D [19].

IV. PAYOFF FUNCTIONS FOR CHANNEL ALLOCATION SCHEMES

In channel allocation games [17], [18], pairs of nodes are players, available channels are strategy spaces, and the received interference power is the payoff function. In these games, each pair is assumed to repeatedly select a channel to maximize its payoff function, and consequently, to communicate with a high SINR.

A. Payoff Function for HD Networks

We introduce two payoff functions for HD networks [17], [18]. These are suitable to use in a “canonical network model” [17]. A canonical network model shown in Fig. 2 consists of clusters that are spatially separated. Among these clusters, \( G_{ij} = G_{ji} \) holds. First, the payoff function \( u_1 \) introduced in [17] is expressed as follows:
\[
u_1(a_i, a_{-i}) = - \sum_{j \neq i} G_{ij} p_j \mathbb{1}_{\{a_i = a_j\}},
\]
where \( G_{ij} \) shows the link gain between nodes \( i \) and \( j \), and \( p_j \) shows the transmission power of node \( j \). \( \mathbb{1}_{\{condition\}} \) is the indicator function, which is one when \( condition \) is true and is zero otherwise. Game \((\mathcal{N}, \mathcal{A}, (u_1)_{i \in \mathcal{N}})\) has been proved to be a WPG [10], [17]. Second, the payoff function \( u_2 \) for HD networks proposed in [18] is expressed as follows:
\[
u_2(a_i, a_{-i}) = -p_i \sum_{j \neq i} G_{ij} p_j \mathbb{1}_{\{a_i = a_j\}}.
\]
Game \((\mathcal{N}, \mathcal{A}, (u_2)_{i \in \mathcal{N}})\) has been proved to be an EPG game [18]. On the basis of these payoff functions (8) and (9), we try to formulate payoff functions for FD networks.

B. Payoff Function for FD networks

We discuss a payoff function modified from (8) to FD networks and exemplify that the payoff function is not suitable for FD networks in terms of potential games. The example of the network is shown in Fig. 3.

We directly apply \( u_1 \) to FD networks as follows:
\[
u_3(a_i, a_{-i}) = - \sum_{s \in \mathcal{Q}} I^s_i(a_i, a_{-i}) - \sum_{s \in \mathcal{Q}} SI^s_i(a_i),
\]
\[
I^s_i(a_i, a_{-i}) = \sum_{j \in \mathcal{N} \setminus \{i\}} \sum_{t \in \mathcal{Q}} G^s_{ij} p^t_j \mathbb{1}_{\{a_i = a_j\}}.
\]
\( I^s_i(a_i, a_{-i}) \) represents the total multi-user interference power from nodes except in communication pair \( s \) to node \( i \) on the channel \( a_i \). In addition, \( SI^s_i(a_i) \) means the self-interference when node \( i \) communicates on channel \( a_i \).

In FD networks, payoff function \( u_3 \) is not suitable because the dynamics can cycle. We exemplify that the channel allocation process in the network shown in Fig. 3 cycles and
not converge to a Nash equilibrium. In this example, there are three communication pairs 1, 2, and 3. It is assumed that channels 1 and 2 are available for all communication pairs, i.e., \(A_i = \{1, 2\}, \forall i\). The channel update process is based on the best response dynamics. Now we assume \(p_i^{(a)} = P_S\), \(p_i^{(b)} = P_L, \forall i\), and \(P_S < P_L\).

First, pair 1 updates its channel \(a_1\) from 2 to 1 to maximize its payoff function \(u_3\) using the best response dynamics. Second, pair 2 updates its channel from 1 to 2 similarly. Third, pair 3 updates its channel from 2 to 1. After all of these steps have been conducted, the sequence of channel updates shown above repeats; the process never converges to a Nash equilibrium.

C. Proposed Payoff Function for FD networks

We discuss a payoff function modified from (9) for FD networks to avoid cycles in channel update process stated in Section IV-B. Proposed payoff function \(u_4\) for FD networks is defined as follows:

\[
u_4(a_i, a_{-i}) = -\sum_{s \in \mathcal{Q}} p^*_s I^b_s(a_i, a_{-i}) - \sum_{s \in \mathcal{Q}} p^*_s SI^a_s(a_i).
\]

(12)

Payoff function \(u_4\) depends on the amount of interference power.

Theorem 1. A game \((\mathcal{N}, \mathcal{A}, (u_4)_i)\in \mathcal{N}\) is a BSI game with potential \(f(a_i, a_{-i}) = -\frac{1}{2} \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{Q}} p^*_s I^b_s(a_i, a_{-i}) - \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{Q}} p^*_s SI^a_s(a_i).
\)

(13)

Proof. Payoff function \(u_4\) satisfies (6) when we set \(w_{ij}(a_i, a_j) = -\sum_{s \in \mathcal{Q}} \sum_{i \in \mathcal{Q}} p^*_s G_{ij} p^a_s (a_{i}, a_{-i})\) and \(s_i(a_i) = \sum_{s \in \mathcal{Q}} p^*_s SI^a_s(a_i)\). That is why the game \((\mathcal{N}, \mathcal{A}, (u_4)_i)\in \mathcal{N}\) is a BSI game and the potential function is written as (13) by substituting them into (7).

As a result, the game \((\mathcal{N}, \mathcal{A}, (u_4)_i)\in \mathcal{N}\) is an EPG. Thus, when using payoff function \(u_4\), in FD networks, channel allocation dynamics successfully converge to a Nash equilibrium.

V. SIMULATION RESULTS

In this section, we confirm the feasibility of the proposed channel allocation scheme. In particular, we evaluate the channel allocation scheme using the payoff function \(u_4\).

We assume \(N = 12\) stationary FD communication pairs with omnidirectional antennas in a 100 m \(\times\) 100 m square area. One node in a communication pair \(i\) is placed randomly, and another node in pair \(i\) is randomly placed within a 10-m radius from the node. We simulate the proposed channel allocation scheme 20 times in a different position, and use average values as simulation results.

In this simulation, all transmitters are assumed to transmit at the same power level \(p_i = P, \forall i\). We set \(P = 13\) dBm. We use the free space path loss model. Note that they are operated in the 2.4 GHz band. We also assume that there are three available channels with equal bandwidth, i.e., \(A_i = \{1, 2\}, \forall i\). For the sake of simplicity, self-interference can be canceled perfectly, i.e., we assume \(SI^a_s(a_i) = 0\).

In this simulation, three settings for exploration parameter \(\beta\) are used, i.e., \(\beta = 3 \cdot 10^5 \times \log_{10}(1+k), 3 \cdot 10^4 \times \log_{10}(1+k), \) and \(10^6\) where these parameters are determined based on reference [23]. Here, we call these parameter settings “SAP w/ high \(\beta\), “SAP w/ low \(\beta\), and “nearBR", respectively.

First, we discuss the transition of channel allocation. Figs. 4, 5, and 6 describe transitions of channel allocation when parameter settings are SAP w/ high \(\beta\), SAP w/ low \(\beta\), and nearBR, respectively. As stated in Section III-C, when \(\beta\) is low, channels are allocated randomly and the convergence of channel allocation is not achieved. The results shown in Fig. 5 indicates this random allocation. In contrast, when \(\beta\) is high, allocation is performed successfully, as shown in Figs. 4 and 6. Hence, we need to use large \(\beta\) to allocate channel efficiently.

Second, we evaluate the impact of the number of communication pairs, \(N\), on the value of potential function which means the total interference power. Fig. 7 shows simulation results. As shown in Fig. 7, potentials of nearBR and SAP w/ high \(\beta\) are greater than that of SAP w/ low \(\beta\). The simulation results show that SAP w/ high \(\beta\) and nearBR are more appropriate compared to that of SAP w/ low \(\beta\) when the proposed payoff function is used for FD networks.

VI. CONCLUSION

We proposed a channel allocation scheme based on a potential game-theoretic framework for bi-directional in-band full-duplex networks. In addition, we showed that the proposed channel allocation game is an exact potential game. The sum
of the received power from other transmitters is not suitable for a payoff function for full-duplex networks. In the proposed channel allocation scheme, the sum of products of transmission power and received interference power is used as a payoff function and the payoff function has the property of bilateral symmetric interaction. Therefore, the payoff enabled channel allocation dynamics in full-duplex networks to successfully converge to a Nash equiribrium.

Fig. 5. Transition of channel allocation (SAP w/ low $\beta$).

Fig. 6. Transition of channel allocation (nearBR).

Fig. 7. Value of potential function vs. number of communication pairs $N$.

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