Nonlinear traveling-wave field effect transistors for amplification of short electrical pulses

Koichi Narahara\textsuperscript{a)} and Shun Nakagawa

Graduate School of Science and Engineering, Yamagata University, 4–3–16 Jonan, Yonezawa, Yamagata 992–8510, Japan
\textsuperscript{a)} narahara@yz.yamagata-u.ac.jp

Abstract: We investigated the properties of pulse propagation on nonlinear traveling-wave field effect transistors (TW-FET) to develop a method for amplifying short electrical pulses. TW-FETs are a special type of FET whose electrodes are employed not only as electrical contacts but also as transmission lines. Due to the presence of electromagnetic couplings between the gate and drain lines, two different propagation modes called the c mode and $\pi$ mode are developed on a TW-FET. Moreover, the Schottky contact beneath the gate electrode creates an ideal source of nonlinearity for soliton-like propagation. We can design the TW-FET to amplify only soliton-like pulses carried by one of the two modes and attenuate the ones carried by the other mode. This paper discusses the fundamental properties of a nonlinear TW-FET, including the width and velocity of a soliton-like pulse carried by c and $\pi$ modes, and gives design criteria of amplification of soliton-like pulses.

Keywords: solitons, nonlinear transmission lines (NLTLs), traveling-wave FETs, pulse amplification

Classification: Microwave and millimeter wave devices, circuits, and systems

References

1 Introduction

A nonlinear transmission line (NLTL) is defined as a lumped transmission line containing a series inductor and a shunt Schottky varactor in each section. NLTLs are used for the development of solitons [1]. Moreover, the operation bandwidth of carefully designed Schottky varactors goes beyond 100 GHz; therefore, they are employed in ultrafast electronic circuits including the subpicosecond electrical shock generator [2]. However, the line resistance generally attenuates the pulse amplitude, making pulses very small for an NLTL to exhibit nonlinearity. To bring out the potential of NLTLs, we considered a traveling-wave field effect transistor (TW-FET) to compensate for the wave attenuation by utilizing the transistors’ gain. Because of the Schottky contact beneath the gate, the gate line of a TW-FET simulates an NLTL. In contrast, the drain line is modeled by a linear transmission line coupled with the gate line via the gate-to-drain capacitance, mutual inductance and transconductance. Two different propagation modes called the c mode and π mode develop on a coupled line. Each mode has its own velocity, characteristic impedance and voltage fraction between the gate and drain lines. It is found that the nonlinearity introduced by the gate-to-source Schottky capacitors succeeds in compensating for the distortions caused by dispersion for pulses carried by either c or π modes; although, the nonlinear pulses are greatly attenuated due to the presence of finite electrode resistances irrespective of the carrying mode. For gate voltages higher than the threshold, it is expected that the transconductance relaxes pulse attenuation. Possibly, only a pulse carried by one of the modes gains amplitude, while a pulse carried by another mode decays [3]. Elaborating the design method enables the amplification of pulses that are not affected by both dispersion and attenuation.

In this paper, we first describe the fundamental properties of nonlinear pulses observed in a subthreshold TW-FET. Then, we discuss the design criteria for amplifying the nonlinear pulses in a TW-FET, together with several results of numerical evaluations that validate this method.

2 Nonlinear TW-FETs

Figure 1 shows the diagram of a TW-FET. The gate and drain lines are modeled by an NLTL and a linear line, respectively. These two are coupled via the gate-to-drain capacitance $C_m$, the mutual inductance $L_m$, and the drain-to-source current $I_{ds}$. The per-unit-cell series inductance and shunt capacitance of the gate (drain) line are denoted by $L_g$ ($L_d$) and $C_g$ ($C_d$), respectively. Note that $C_g$ shows the Schottky varactor whose capacitance-
voltage relationship is generally given by

\[ C_g(V_X) = \frac{C_0}{(1 - \frac{V_X}{V_J})^m}, \tag{1} \]

where \( V_X \) is the voltage between the terminals. \( C_0, V_J, \) and \( m \) are the optimizing parameter. Note that \( V_X < 0 \) for reverse bias.

We denote the line voltages of the gate and drain lines at the \( n \)th cell as \( V_n \) and \( W_n \), respectively, and the line currents of the gate and drain lines at the \( n \)th cell by \( I_n \) and \( J_n \), respectively. Then, the transmission equations of a TW-FET are given by

\[
L_g \frac{dI_n}{dt} + L_m \frac{dJ_n}{dt} = V_{n-1} - V_n - R_g I_n, \tag{2}
\]

\[
L_d \frac{dJ_n}{dt} + L_m \frac{dI_n}{dt} = W_{n-1} - W_n - R_d J_n, \tag{3}
\]

\[
[C_m + C_g(V_n)] \frac{dV_n}{dt} - C_m \frac{dW_n}{dt} = I_n - I_{n+1}, \tag{4}
\]

\[
(C_m + C_d) \frac{dW_n}{dt} - C_m \frac{dV_n}{dt} = J_n - J_{n+1} - I_{ds}(V_n). \tag{5}
\]

When the pulse spreads over many cells, the discrete spatial coordinate \( n \) can be replaced by a continuous one \( x \), series-expanding \( V_{n\pm1} \) and \( W_{n\pm1} \) up to the fourth order of the cell length \( \delta \), we then obtain the evolution equation of the line voltage:

\[
(l_d c_m + l_d c_d - l_m c_m) \frac{\partial^2 W}{\partial t^2} + [l_m c_g(V) + l_n c_m - l_d c_m] \frac{\partial^2 V}{\partial t^2} + \frac{\partial W}{\partial x} + \frac{\partial^4 W}{12 \partial x^4}.
\]

\[
(l_m c_m + l_m c_d - l_g c_m) \frac{\partial^2 W}{\partial t^2} + [l_g c_g(V) + l_g c_m - l_n c_m] \frac{\partial^2 V}{\partial t^2} - r_g c_m \frac{\partial W}{\partial t} + \left[l_m \frac{dI_{ds}}{dt} + r_g c_m + r_g c_g(V) \right] \frac{\partial V}{\partial t} + l_g \frac{dV}{dt} \left(\frac{\partial V}{\partial t}\right)^2
\]

\[
= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^4 V}{12 \partial x^4}.
\tag{6}
\]

\[
(l_m c_m + l_m c_d - l_g c_m) \frac{\partial^2 W}{\partial t^2} + [l_g c_g(V) + l_g c_m - l_n c_m] \frac{\partial^2 V}{\partial t^2} - r_g c_m \frac{\partial W}{\partial t} + \left[l_m \frac{dI_{ds}}{dt} + r_g c_m + r_g c_g(V) \right] \frac{\partial V}{\partial t} + l_g \frac{dV}{dt} \left(\frac{\partial V}{\partial t}\right)^2
\]

\[
= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^4 V}{12 \partial x^4}.
\tag{7}
\]

Fig. 1. Unit cell of nonlinear TW-FETs.
where \( V = V(x,t) \) and \( W = W(x,t) \) are the continuous counterparts of \( V_n \) and \( W_n \). Moreover, \( l_{g,d} \) and \( c_{g,d,m} \) are the line inductance and capacitance per unit length defined as \( l = L/\delta \) and \( c = C/\delta \), respectively.

There are two different propagation modes, \( c \) mode and \( \pi \) mode, on a linear coupled line [4], and the same is true for a subthreshold TW-FET. Each mode has its own velocity and voltage fraction between the lines (= drain voltage/gate voltage). Hereafter, we denote the velocity of \( c \) mode, velocity of \( \pi \) mode, voltage fraction of \( c \) mode, and voltage fraction of \( \pi \) mode at long wavelengths as \( u_c, u_\pi, R_c, \) and \( R_\pi \), respectively. These are explicitly written as:

\[
\begin{align*}
  u_{c,\pi} &= \sqrt{\frac{x_1 + x_2 \pm \sqrt{(x_1 + x_2)^2 - 4x_3}}{2x_3}}, \\
  R_{c,\pi} &= \frac{x_1 - x_2 \pm \sqrt{(x_1 + x_2)^2 - 4x_3}}{2x_4},
\end{align*}
\]

where the upper (lower) signs are for \( c (\pi) \) mode. For concise notations, we define \( x_{1,2,3,4} \) as

\[
\begin{align*}
  x_1 &= c_g(V_0)l_g + c_m l_g - c_m l_m, \\
  x_2 &= c_d l_d + c_m l_d - c_m l_m, \\
  x_3 &= [c_g(V_0)c_d + c_g(V_0)c_m + c_d c_m] (l_d l_d - l_m^2), \\
  x_4 &= c_m l_g - c_d l_m - c_m l_m,
\end{align*}
\]

for the case where the gate line is biased at \( V_0 \). Moreover, the short-wavelength waves travel slower than the long-wavelength waves due to dispersion, which results in the distortion of the baseband pulses having short temporal durations. In a nonlinear NLTL, this distortion can be compensated for by the nonlinearity introduced using Schottky varactors, regardless of the propagation mode. To quantify the compensation of dispersion by nonlinearity, we apply reductive perturbation [5] to the transmission equation of a coupled NLTL. We first series expand the voltage variables as

\[
\begin{align*}
  V(x,t) &= V_0 + \sum_{i=1}^{\infty} \epsilon^i V^{(i)}(x,t), \\
  W(x,t) &= W^{(0)}(x) + \sum_{i=1}^{\infty} \epsilon^i W^{(i)}(x,t),
\end{align*}
\]

for \( \epsilon << 1 \). To evaluate the influence of electrode resistances and transistor currents on the dynamics of soliton-like pulses, we restrict the values of \( r_{g,d} \) and \( I_{ds} \) to the order of \( \epsilon^{3/2} \). For convenience, we thus define \( r'_{g,d} \) as \( r_{g,d} = \epsilon^{3/2} r'_{g,d} \) and

\[
I_{ds}(V) = \epsilon^{3/2} \beta (V - V_{TO})^2.
\]

Moreover, the following transformations are applied: \( \xi = \epsilon^{1/2} (x - ut) \) and \( \tau = \epsilon^{3/2} t \), where \( u \) is a parameter determined as follows. By evaluating eqs. (6) and (7) for each order of \( \epsilon \), we can extract the equation that describes the developing soliton-like pulses. It has been shown that \( O(\epsilon) \) terms give
trivial identities, and $O(\epsilon^2)$ terms determine the allowed values of $u$, such that $u$ has to be equal to either $u_c$ or $u_\pi$. Moreover, $W^{(1)}$ becomes equal to $R_{c,\pi}V^{(1)}$ for $u = u_{c,\pi}$. Finally, we obtain the Korteweg–de Vries (KdV) equations for $W^{(1)}$ from $O(\epsilon^3)$ terms, which is given by

$$\frac{\partial W^{(1)}}{\partial \tau} + p\frac{\partial^3 W^{(1)}}{\partial \xi^3} + qW^{(1)}\frac{\partial W^{(1)}}{\partial \xi} + \nu W^{(1)} = 0. \quad (17)$$

The nonlinear and dispersive coefficients for $c$ and $\pi$ modes are given by

$$p_{c,\pi} = \frac{\delta^2}{24} u_{c,\pi}, \quad q_{c,\pi} = \pm c_g(V_0)m \frac{u_{c,\pi}^3}{2(V_0 - V_f)} \sqrt{(x_1 + x_2)^2 - 4x_3}$$

$$\times \left\{ l_g \left[ x_1 - x_2 \mp \sqrt{(x_1 + x_2)^2 - 4x_3} \right] - 2m x_4 \right\}. \quad (19)$$

where upper (lower) signs are for $c$ ($\pi$) mode. As a result, we obtain the following one-soliton solutions specified by $A_0$:

$$V = V_0 + A_0 \text{sech}^2 \left[ \frac{\sqrt{pA_0}}{12q} \left[ x - \left( u + \frac{pA_0}{3} \right) t \right] \right], \quad (20)$$

$$W = W^{(0)}(x) + RA_0 \text{sech}^2 \left[ \frac{\sqrt{pA_0}}{12q} \left[ x - \left( u + \frac{pA_0}{3} \right) t \right] \right], \quad (21)$$

where $W^{(0)}$ is the solution of the equation: $d^2W^{(0)}/dx^2 = r_d' I_{ds}(V_0)$ and $(p, q, u, R)$ is set to $(p_{c,\pi}, q_{c,\pi}, u_{c,\pi}, R_{c,\pi})$ for the $c$ ($\pi$)-mode soliton. Note that $A_0$ is set as positive (negative) for $p > (<)0$.

3 Design criteria of pulse amplification

By the soliton perturbation theory, the amplitude $K$ of the one-soliton solution becomes time-dependent due to a perturbing term $\nu W^{(1)}$ as $dK/d\tau = -4\nu K/3$. Therefore, the soliton-like pulse gains amplitude for $\nu < 0$. For $l_m \neq 0$, the coefficient $\nu$ is given by

$$\nu_{c,\pi} = \pm \frac{g_m l_m}{2\sqrt{(x_1 + x_2)^2 - 4x_3}} \left( 1 - \frac{u_{c,\pi}^2}{u_s^2} \right)$$

$$\pm \frac{r_d' c_m}{\sqrt{(x_1 + x_2)^2 - 4x_3}} \left( 1 - \frac{u_{\pi}^2}{u_{\pi,c}^2} \right) \times \frac{(c_d + c_m) l_m - c_m l_g}{x_1 - x_2 \mp \sqrt{(x_1 + x_2)^2 - 4x_3}}$$

$$\pm \frac{r_g' [c_g(V_0) + c_m]}{2\sqrt{(x_1 + x_2)^2 - 4x_3}} \left[ \frac{c_m l_d}{(c_g(V_0) + c_m) l_m} - \frac{u_{\pi,s}^2}{u_{\pi,c}^2} - 1 \right], \quad (22)$$

where upper (lower) signs are for $c$ ($\pi$) mode. Moreover, $u_s$ is defined as

$$u_s = \sqrt{\frac{l_m}{c_m(l_d - l_m)}.} \quad (23)$$
By eq. (22) with $r'_{g,d} = 0$, it is found that the $c$-mode pulse is generically amplified when $u_s > u_c$, while the $\pi$-mode pulse is amplified when $u_s < u_\pi$ for nonzero $l_m$. Because $u_c$ is always greater than $u_\pi$, we can see that when the characteristic velocity $u_s$ is less than both $u_c$ and $u_\pi$, the slower mode is the unique amplified mode; in contrast, when $u_s$ is greater than both $u_c$ and $u_\pi$, the faster mode is the unique amplified mode. For $l_m = 0$, $\nu$ is given by

$$
\nu_{c,\pi} = \pm \frac{g_m c_m l_d l_d u_{c,\pi}^2}{2\sqrt{(x_1 + x_2)^2 - 4x_3}} + \frac{r'_{d}}{4l_d} + \frac{r'_{g}}{4l_g} + \frac{(r'_{d} - r'_{g})}{4l_g} \frac{x_1 - x_2}{\sqrt{(x_1 + x_2)^2 - 4x_3}}
$$

where upper (lower) signs are for $c$ ($\pi$) mode. As a result, when the electrode lines are only capacitively coupled, the $\pi$-mode pulse is always the unique mode to be amplified.

As far as a pulse travels on a unique mode, it is free from distortions caused by the dispersive difference between modes; therefore, a TW-FET succeeds in amplifying short electrical pulses.

We numerically solve eqs. (2)−(5) using a standard finite-difference time-domain method for TW-FETs. The total number of cells is 200. Throughout the calculations, we set $\beta$, $V_{TO}$, $C_0$, $V_J$, $m$, $V_0$, $L_g$, $R_g$, $R_d$, $C_d$, and $C_m$ to 1.5 mA/V$^2$, −1.8 V, 0.5 pF, 1.0 V, 0.5, −0.5 V, 2.0 nH, 1.0 nH, 0.15 Ω, 0.15 Ω, 0.5 pF, and 0.2 pF, respectively. Figure 2 (a) shows the calculated waveforms monitored at $n = 1$ (blue), 50 (red), 100 (green), and 150 (black) for $L_m = 0.5$ nH. For the present line parameters, the values of $\nu_\pi$ and $\nu_c$ are
calculated to be $7.62 \times 10^8 \text{s}^{-1}$ and $3.78 \times 10^8 \text{s}^{-1}$, respectively. Consistently, the pulses simply decay along the line in Fig. 2 (a). Next, we set $L_m$ to zero; therefore, the $\pi$-mode pulse is potentially amplified irrespective of the reactive design parameters. Actually, $\nu_\pi$ and $\nu_c$ are calculated to be $-7.16 \times 10^8 \text{s}^{-1}$ and $1.84 \times 10^9 \text{s}^{-1}$, respectively. Figure 2 (b) shows the calculated waveforms monitored at $n = 1$ (blue), 50 (red), 100 (green), and 150 (black). We can see that the $\pi$-mode pulse is successfully amplified as expected.