General Classical Electrodynamics, part two

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Abstract General Classical Electrodynamics part two. The energy density of electrodynamic sources are evaluated in the context of Maxwell's theory and in the context of General Classical Electrodynamics.

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1 Introduction

Generalized energy interaction terms are derived by means of *General Classical Electrodynamics* (GCED). Firstly, the following is defined.

$$\delta(\mathbf{x}) = \frac{-1}{4\pi}\Delta\left(\frac{1}{|\mathbf{x}|}\right) \tag{1.1}$$

$$\mathbf{F}(\mathbf{x}) = \int_{V'} \mathbf{F}(\mathbf{x}')\,\delta(\mathbf{x}-\mathbf{x}')\,\mathrm{d}^3x' \tag{1.2}$$

The fundamental theorem of vector algebra is as follows: a vector function $\mathbf{F}(\mathbf{x})$ can be decomposed into two unique vector functions $\mathbf{F}_l(\mathbf{x})$ and $\mathbf{F}_t(\mathbf{x})$, such that

$$\mathbf{F}(\mathbf{x}) = \mathbf{F}_l(\mathbf{x}) + \mathbf{F}_t(\mathbf{x}) \tag{1.3}$$

$$\mathbf{F}_l(\mathbf{x}) = -\frac{1}{4\pi}\nabla\int_{V'} \frac{\nabla'\cdot\mathbf{F}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x' \tag{1.4}$$

$$\mathbf{F}_t(\mathbf{x}) = \frac{1}{4\pi}\nabla\times\int_{V'} \frac{\nabla'\times\mathbf{F}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,\mathrm{d}^3x' \tag{1.5}$$

The lowercase subindexes 'l' and 't' will have the meaning of *longitudinal* and *transverse* in this paper. The longitudinal vector function $\mathbf{F}_l$ is curl free ($\nabla\times\mathbf{F}_l = \mathbf{0}$), and the transverse vector function $\mathbf{F}_t$ is divergence free ($\nabla\cdot\mathbf{F}_t = 0$). We assume that $\mathbf{F}$ is well behaved ($\mathbf{F}$ is zero if $|\mathbf{x}|$ is infinite). Let us further introduce the following notations and definitions.

ρ	Net electric charge density, in C/m^3
$\mathbf{J} = \mathbf{J}_l + \mathbf{J}_t$	Net electric current density, in A/m^2
Φ	Net electric charge (scalar) potential, in V
$\mathbf{A} = \mathbf{A}_l + \mathbf{A}_t$	Net electric current (vector) potential, in V·s/m
$\mathbf{E}_\Phi = -\nabla\Phi$	Electric field, in V/m
$\mathbf{E}_L = -\partial_t \mathbf{A}_l$	Field induced divergent electric field
$\mathbf{E}_T = -\partial_t \mathbf{A}_t$	Field induced rotational electric field
$\mathbf{E} = \mathbf{E}_\Phi + \mathbf{E}_L + \mathbf{E}_T$	
$= -\nabla\Phi - \partial_t \mathbf{A}$	Superimposed electric field
$B_\Phi = -\partial_t \Phi$	Field induced scalar field, in V/s
$B_L = -\nabla\cdot\mathbf{A}_l$	Scalar magnetic field, in T = V·s/m^2
$\mathbf{B}_T = \nabla\times\mathbf{A}_t$	Vector magnetic field, in T = V·s/m^2
$B = -\frac{1}{c^2}\partial_t\Phi - \nabla\cdot\mathbf{A}$	Superimposed scalar magnetic field
$\phi_0 \ll \epsilon_0^2\mu_0$	Polarizability of vacuum, in F·s^2/m^3
μ_0	Permeability of vacuum: $4\pi 10^{-7}$ H/m
ϵ_0	Permittivity of vacuum: 8.854^{-12} F/m

$(\mathbf{x}, t) = (x, y, z, t)$	Place and time coordinates
$\partial_t = \frac{\partial}{\partial t}$	Partial time differential
$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$	Del operator
$\Delta = \nabla\cdot\nabla$	Laplace operator

$$\Delta\Phi = \nabla\cdot\nabla\Phi, \qquad \Delta\mathbf{A} = \nabla\nabla\cdot\mathbf{A} - \nabla\times\nabla\times\mathbf{A}$$

2 The energy density of electrodynamic sources

The electric power density of *sources* in the power theorem of GCED in the Whittaker premise are expressed as follows:

$$\begin{aligned} P &= -\mathbf{J}\cdot\mathbf{E} - c^2\rho B \\ &= \mathbf{J}\cdot\nabla\Phi + \rho\partial_t\Phi + \mathbf{J}\cdot\partial_t\mathbf{A} + c^2\rho\nabla\cdot\mathbf{A} \end{aligned} \tag{2.1}$$

Integration by parts of the first and the fourth term of this expression, over the entire space, results into:

$$P = -\Phi\nabla\cdot\mathbf{J} + \rho\partial_t\Phi + \mathbf{J}\cdot\partial_t\mathbf{A} - c^2\mathbf{A}\cdot\nabla\rho \tag{2.2}$$

The first term can be combined with the second term, by means of the equation of continuity of charge:

$$P = \partial_t(\rho\Phi) + \mathbf{J}\cdot\partial_t\mathbf{A} - c^2\mathbf{A}\cdot\nabla\rho \tag{2.3}$$

In case the specific (non general) condition $\frac{1}{c^2}\partial_t\mathbf{J} + \nabla\rho = \mathbf{0}$ is satisfied as well, then we can combine all terms into a single term:

$$P = \partial_t(\rho\Phi + \mathbf{J}\cdot\mathbf{A}) \tag{2.4}$$

The energy density of the electrodynamic sources is simply $\rho\Phi + \mathbf{J}\cdot\mathbf{A}$ for this particular case. Compare this result with the 'interaction terms' in the Lagrange theory (the Lagrangian) of the incorrect Maxwell-Lorentz electromagnetism.

3 The special case of net electric current, as the product of net charge and its velocity

In general one cannot attribute a particular velocity, $\mathbf{v}$ to the net charge density, ρ, that is present in macroscopic material (plasma, gas, fluid, solid), such that $\mathbf{J} = \mathbf{v}\rho$. For instance, the net charge, ρ, on a metallic wire that carries a current with density $\mathbf{J}$, mainly depends on the electric potential from the electric power source that is attached to the wire, so the net charge on the wire does not relate directly to the current by means of $\mathbf{J} = \mathbf{v}\rho$. Just the conservation of charge is generally true, which also relates the net charge density and net current density, and in most cases $\mathbf{J} \neq \mathbf{v}\rho$.

In case of a dynamic object that has a measurable velocity, $\mathbf{v}$, and that is electrically charged, one can say the object carries a current such that $\mathbf{J} = \mathbf{v}\rho$. If we consider the electrodynamic force on that object in the context of GCED, we can calculate the 'work density', W, for instance of time 'dt':

$$\begin{aligned} \partial_t W &= \mathbf{v} \cdot [\rho\mathbf{E} + \rho\mathbf{v} \times \mathbf{B}_T + \rho\mathbf{v}B_L] \\ &= \mathbf{J} \cdot \mathbf{E} + \rho v^2 B_L \end{aligned} \tag{3.1}$$

This expression must be equal to the power density expression of GCED in the Whittaker premise:

$$\mathbf{J} \cdot \mathbf{E} + \rho v^2 B_L = \mathbf{J} \cdot \mathbf{E} + \rho c^2 \mathbf{B} \tag{3.2}$$

such that:

$$\begin{aligned} B &= \frac{v^2}{c^2} B_L \\ v^2 B_L &= B_\Phi + c^2 \mathbf{B}_L \\ B_\Phi &= (v^2 - c^2) B_L \end{aligned} \tag{3.3}$$