Wavelet Based Adaptive Filtering Algorithms for Acoustic Noise Cancellation

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Abstract – This paper prefers Acoustic Noise cancellation (ANC) system using Wavelet based adaptive filtering algorithms. The Acoustic Noise canceller is implemented using adaptive algorithms like LMS (Least Mean Square), NLMS (Normalized Least Mean Square), RLS (Recursive Least Square), and FRLS (Fast Recursive Least Square). The inclusion of wavelet based transformation in ANC reduces the number of samples to be processed and increase the efficiency of the system by minimizing the processing time. The simulation results shows that the wavelet transform based adaptive algorithms produce improvement in SNR (Signal to Noise Ratio) with less execution time compared to conventional adaptive algorithms. Copyright © 2014 Praise Worthy Prize S.r.l. - All rights reserved.

Keywords: Acoustic Noise Cancellation, Least Mean Square, Normalized Least Mean Square, Recursive Least Square, Fast Recursive Least Square, Signal to Noise Ratio

Nomenclature

\begin{tabular}{ll}
\(\mu\) & Step size parameter \\
\(d(n)\) & Noise corrupted speech signal \\
\(x(n)\) & Back ground noise signal \\
\(\hat{v}(n)\) & Tap weight coefficient value of the filter \\
\(L\) & Length of the adaptive filter \\
\(a(n)\) & Clean speech signal \\
\(e(n)\) & Error signal \\
\(y(n)\) & Adaptive filter output \\
\(\lambda\) & Small positive constant \\
\(k(n)\) & Gain in RLS algorithm \\
\(\pi(n)\) & Intermediate quantity in the gain \\
\(\varphi(n)\) & Priori forward prediction error \\
\(\varphi(t)\) & Mother wavelet \\
\(W(a,b)\) & Continuous wavelet transform \\
\(a\) & Scaling parameter \\
\(b\) & Location parameter \\
\(W(l,m)\) & Discrete wavelet transform \\
\(l\) & Discrete translation \\
\(m\) & Discrete dilation
\end{tabular}

I. Introduction

Noise cancellation is performed in order to enhance the quality of noise corrupted speech. Acoustic noise cancellation (ANC) is required to cancel the background noise as in the case of mobile phone users using hands free equipment.
The NLMS uses variable step size algorithm to improve the convergence speed, stability and performance of LMS algorithm [19].

In order for the system to function efficiently, the value of the signal to noise ratio (SNR) has to be sufficient. Wavelet transform [20] can be used to raise the SNR value. Wavelet transform is a fairly recent method which is being used for signal analysis, especially for the investigation of speech, image and sonar signals. The advantage of wavelet transform [21] is that at high frequency regions and at low frequency regions, better time and frequency resolutions are respectively obtained. This helps in tracking the constantly varying transients of the signal. It is also computationally simpler than Fourier transforms. The disadvantages of adaptive filtering algorithms for ANC are their low SNR values. This paper prefers the analysis of Discrete Wavelet transform (DWT) in adaptive filtering algorithm in form as DWT-LMS, DWT-NLMS, DWT-RLS and DWT-FRLS to increase the SNR values for ANC application. This paper is structured as follows: Adaptive algorithms for Acoustic Noise Cancellation is briefed in section II, Proposed wavelet based adaptive algorithm for Acoustic Noise Cancellation is detailed in section III, Simulation and Results are discussed in section IV and Conclusion is achieved in section V.

II. Adaptive Algorithm for Acoustic Noise Cancellation

II.1. Acoustic Noise Cancellation

In order to decrease the overall noise of the system, a noise polluted speech signal and noisy reference signal, consisting of just noise are utilized which is shown in Fig. 1. A noise corrupted speech signal \( d(n) \) and a background noise signal \( x(n) \) as inputs to the filter.

![Fig. 1. Adaptive filter for Acoustic Noise Cancellation](image)

The input noise signal, \( x(n) \) is an Mx1 vector given as:

\[
x(n) = [x(n), x(n-1), ...., x(n-M+1)]^T \tag{1}
\]

The length of the filter is denoted by \( L \) and \( \hat{w}(n) \) is tap weight coefficient value of the filter. The corrupted speech signal is denoted as:

\[
d(n) = a(n) + x(n) \tag{2}
\]

where \( a(n) \) is the clean speech signal, the Enhanced error signal \( e(n) \) which is the difference between the noisy speech and the filter output \( y(n) \) is given as:

\[
e(n) = d(n) - y(n) \tag{3}
\]

The LMS, NLMS, RLS and FRLS algorithms have been utilized for this purpose.

II.2. Adaptive LMS and MLMS Algorithms

The function of the adaptive filter is to be able to estimate the noise in the speech signal. The correction of the speech and the output of the filter, as well as the background noise input are used to manipulate the filter coefficients, which are continuously restructured via the adaptive filtering algorithms.

The LMS algorithm is one of the most commonly used and the simplest of adaptive algorithms [3]. The fact that it is less complex and more stable than other algorithms is its major advantages and the algorithm appears to be fairly robust against implementation errors. The LMS algorithm is the first adaptive filtering algorithm that is implemented in this paper.

As previously discussed, the adaptive filtering algorithms undergo the filtering and the adaptive procedure. During the filtering part, two values are estimated. First, the value of the filter output is generated:

\[
y(n) = \sum_{i=0}^{m-1} w_i x(n-i) = w^T(n)x(n) \tag{4}
\]

The LMS algorithm aims at curtailing the mean square error by amending the tap weight vectors. The tap weight adjustment at time \( n+1 \) is:

\[
\hat{w}(n+1) = \hat{w}(n) + \mu x(n)e^*(n) \tag{5}
\]

where \( \mu \) is the step size parameter, the value of the error is found by subtracting the filter output from the wanted reaction. In the adaptive process, the filter regulates its factors in agreement to the wanted reaction:

\[
e(n) = d(n) - y(n) = d(n) - w^T(n) \tag{6}
\]

The LMS algorithm suffers from a slow convergence rate and when the input vector is too large, there arises a complication called gradient noise amplification problem. The NLMS algorithm is the second type of adaptive filtering algorithm which has been implemented [5]. It is analogous to the LMS algorithm, but with a few changes made.
The NLMS algorithm is improvised in terms of a better accuracy rate and also a higher rate of convergence than the LMS algorithm. Since the method by which the step size parameter is different for NLMS, it shows better solidity as well. The tap weight adjustment at time \( n+1 \) is given by:

\[
\hat{w}(n+1) = \hat{w}(n) + \frac{\mu}{|x(n)|^2} x(n) e^*(n) \tag{7} \]

where \( \mu \) variable step size parameter. This way, a greater value of convergence is obtained for the NLMS algorithm.

But we constantly try to increase this convergence rate in order to come up with adaptive filtering algorithms of higher caliber. This leads us to examine the Recursive Least Squares (RLS) algorithm.

### II.3. Adaptive RLS and FRLS Algorithms

The Recursive Least Squares (RLS) algorithm, a deterministic process differs from the previous two algorithms, which happen to be stochastic processes. All the algorithms differ in the methodology used for the calculation of gain. In the RLS algorithm, [7] the element of randomness is eliminated. This is achieved by introducing a term called the “forgetting factor” which ensures only select data, and not all of it, is sent for processing. This leads to a higher rate of convergence.

The fact that matrix inversion is not required is another advantage of the RLS algorithm. The value of gain is computed as:

\[
k(n) = \frac{\pi(n)}{\lambda + \pi^H(n)\pi(n)} \tag{8} \]

where \( \lambda \) is a small positive constant and \( \pi(n) \) is an intermediate quantity in the calculation of the gain.

The tap weight vector is estimated as:

\[
\hat{w}(n) = \hat{w}(n-1) + k(n)e^*(n) \tag{9} \]

The Fast Recursive Least Squares algorithm was introduced in order to surmount the complexity of the RLS algorithm. The main aim of this algorithm is to combine the merits of the LMS and the RLS algorithm, thereby resulting in a superior algorithm that is fast and also simple to execute. Thus this becomes a highly efficient noise cancelling method. This algorithm redefined the methodology in which the gain was calculated. The FRLS algorithm [8] is classified into a filtering element and a prediction element. The filtering element receives a gain value from the prediction element in order to categorize the unidentified system.

The augmented gain vector \( k_{L+1}(n) \) is first calculated as the intermediate step to calculating the gain vector.

This requires the calculation of the forward prediction coefficient \( A(n) \). Next, the backward prediction coefficient \( G(n) \) is calculated. Using all the intermediate steps, the required gain value \( k(n) \) is arrived at and calculated as:

\[
k(n) = (1 - \varphi(n) r_e(n))^{-1}(k_{L+1} + r_e(n)G(n-1)) \tag{10} \]

where \( \varphi(n) \) is the priori forward prediction error.

While implementing the above time domain adaptive algorithms for Acoustic Noise cancellation, produces low value of SNR with high execution time.

Which implies the noise cancellation performance is less in case of conventional adaptive algorithm and having poor convergence rate. To improve noise cancellation performance with less execution time wavelet based adaptive algorithms are proposed.

### III. Proposed Wavelet based Adaptive Algorithm for Acoustic Noise Cancellation

#### III.1. Wavelet Transform

Wavelet Transform mapped \( L^2(R) \) into \( L^2(R^2) \), but having good time-frequency localization. There are two types of wavelet transform are continuous wavelet transform (CWT) and discrete wavelet transforms (DWT) [22]. The Continuous Wavelet Transform (CWT) is defined in terms of dilations and translations of a mother wavelet \( \varphi(t) \) and it is given by:

\[
\varphi_{ab}(t) = \frac{1}{\sqrt{a}} \varphi\left(\frac{t-b}{a}\right) \tag{11} \]

The CWT of a given signal \( f(t) \) is given by:

\[
W(a,b) = \int_{-\infty}^{\infty} \varphi_{ab}(t) f(t) dt \tag{12} \]

where ‘a’ is scaling parameter and ‘b’ is the Location parameter. Wavelet is translated for a given scaling parameter a by varying the parameter b. CWT of a signal \( f(t) \) will undergo many dilation and translation of mother wavelet, which results in redundant information.

This is the main cause for Discrete Wavelet transform. Discrete Wavelet transform (DWT) is the discretization of the CWT through sampling particular wavelet coefficients. Sampling of CWT is achieved by letting \( a=2^l \) and \( b=m2^l \), in \( W(a,b) \). Where ‘l’ is the discrete translation and ‘m’ is the discrete dilations. DWT is given signal \( f(t) \) is given by:
\[ W(l,m) = \int_{-\infty}^{\infty} f(t) 2^{l/2} \varphi(2^l t - m) f(t) dt \] (13)

DWT is easier to implement and also reduce the computation time. The signal at diverse frequencies with diverse resolutions can be analyzed by using Multi Resolution Analysis (MRA).

The given signal is decomposed into approximation component and detail component, where approximation consists of low frequency information and detail consists of high frequency information. On consecutive decomposition on approximation gives multiple levels, which is shown in Fig. 2. The decomposition level [23] can be extended until the standard deviation of the approximation component is less than the standard deviation of the original signal. This is given by an equation:

\[ \frac{\sigma_{dk}}{\sigma_s} < 0.1 \] (14)

\( \sigma_{dk} \) is the standard deviation of the approximation coefficients for the \( k^{th} \) level, and \( \sigma_s \) is the standard deviation of the original input signal.

\[ d(m) = \sum_{k} c_{j,k} \varphi_{j,k} (m) + \sum_{j \geq j_0} \sum_{k} f_{j,k} \psi_{j,k} (m) z \] (15)

The better correlation between the wavelet and signal gives higher value of the transform. In Discrete Wavelet Transform, high pass filters and low pass filters are employed to scrutinize the signal. The high frequency regions of the signal are transmitted through high pass filters and the low frequency regions are transmitted through low pass filters for analysis. In this paper, discrete wavelet transform is implemented as a preprocessing step to the adaptive filtering algorithms, which is shown in Fig. 3. The first stage that is implemented is called decomposition. The signal length has to be adjusted and extended. Next, the convolved input noisy speech signal undergoes a process called down sampling, which involves one half of the data being sent through a low pass filter and the other half, through a high pass filter. The content that passes through the low pass filter is normally the more important part.

Noise is generally of high frequency. This process of sending the signal through a low pass filter and a high pass filter is known as approximations and detail coefficients respectively. In this proposed technique, HAAR and Daubechies wavelets are implemented.

![Fig. 2. Multilevel decomposition](image-url)

where \( S \) is the original signal, \( SA \) and \( SD \) represents approximate and detail components respectively:

\( S = SA_3 + SD_3 + SD_2 + SD_1 \)

III.2. Wavelet Transform based Adaptive Filter

In wavelet analysis, the signal that is under scrutiny is altered to a form that is more constructive to us. A wavelet undergoes two main processes, known as translation and scaling. During translation, the wavelet undergoes a shift in position and during scaling, the scale gets shifted.

Before estimating the noise signal from the noise corrupted speech by using adaptive filter, Noise corrupted speech is decomposed into approximation and detail coefficients by using Daubechies 2 (dB2) wavelet and Haar wavelet.

III.3. The Daubechies Wavelet Based Approach

The Daubechies Wavelet Transform was named after the mathematician Ingrid Daubechies. We have implemented the Daubechies 2 (db2) Wavelet transform. This wavelet transform also consists of a scaling function and a wavelet function. These have a wide range of applications. The wavelet coefficients used are derived from the scaling function coefficients. The wavelet expansion of a noise corrupted speech signal \( d(m) \) has the following:

\[ d(m) = \sum_{k} c_{j,k} \varphi_{j,k} (m) + \sum_{j \geq j_0} \sum_{k} f_{j,k} \psi_{j,k} (m) z \] (15)
There are two terms in wavelet expansion; the first term is the approximation is defined by:

\[ c_{jk} = \int d(m) \varphi_{jk}^*(m) \, dm \]  

(16)

and \( \varphi_{jk} \) are called scaling function:

\[ \varphi_{jk}(m) = \frac{1}{\sqrt{2}} \varphi \left( \frac{m-k2^j}{2^j} \right) \]  

(17)

The detail coefficients are:

\[ f_{jk} = \int d(m) \psi_{jk}^*(m) \, dm \]  

(18)

and \( \psi_{jk} \) are called wavelet function:

\[ \psi_{jk}(m) = \frac{1}{\sqrt{2}} \psi \left( \frac{m-k2^j}{2^j} \right) \]  

(19)

The wavelet expansion of a Noise reference signal \( x(m) \) has the following:

\[ x(m) = \sum_k a_{jk} \varphi_{jk}^* (m) + \sum_{j=0}^J \sum_k b_{jk} \psi_{jk} (m) \]  

(20)

The first term is the approximation of microphone signal is defined by:

\[ a_{jk} = \int x(m) \varphi_{jk}^* (m) \, dm \]  

(21)

and \( \varphi_{jk} \) are called scaling function:

\[ \varphi_{jk}(m) = \frac{1}{\sqrt{2}} \varphi \left( \frac{m-k2^j}{2^j} \right) \]  

(22)

The detail coefficients are:

\[ b_{jk} = \int x(m) \psi_{jk}^* (m) \, dm \]  

(23)

and \( \psi_{jk} \) are called wavelet function:

\[ \psi_{jk}(m) = \frac{1}{\sqrt{2}} \psi \left( \frac{m-k2^j}{2^j} \right) \]  

(24)

The approximated coefficients of noise corrupted speech signal consist of clean speech signal with some part of noise signal, whereas the detail coefficients of noise corrupted speech signal consist of majority of noise signal. The approximated coefficients are sufficient to separate the clean speech signal, so it is passed to the adaptive filter for cancellation of remaining noise signal.

The output of adaptive filtering algorithm gets up sampled by adding new samples to the signal. The wavelet based adaptive algorithm separates the clean speech signal from the noise signal with increase efficiency and taken less computation time compared to conventional adaptive filter. The proposed method is simulated using Matlab.

### III.4. Haar Wavelet Based Approach

The HAAR wavelet, a square function, is the simplest of all wavelets. The HAAR wavelets are discrete-time orthonormal sequences \( \psi_{ij}(t) \), defined by:

\[ \psi_{ij}(t) = \psi_{i0}(t-2^j) \]  

(25)

and:

\[ \psi_{i0}(t) = \begin{cases} \frac{2^j}{2}, & 0 \leq t \leq 2^j - 1 \\ 0, & \text{elsewhere} \end{cases} \]  

(26)

The indices ‘i’ and ‘j’ correspond to the scale and translation respectively. Here ‘i’ is a natural number and ‘j’ is an integer.

Haar wavelet has two properties; the first property is that any function can be linearly combined with the scaling function, \( \varphi(x) \) and its shifted versions. The second property is that any function can be linearly combined with the Haar wavelet function, \( \psi(x) \) and its shifted versions. The LMS algorithm is used for the adaptation of filter coefficients \( \hat{w}_y \) i.e.:

\[ \hat{w}_y(t+1) = \hat{w}_y(t+1) + \mu \hat{r}_y(t) e(t) \]  

(27)

where \( \mu \) is the adaptation gain, \( e(t) \) the error between the desired signal and adaptive filter output:

\[ \hat{y}(t) = \sum_{(i,j) \in D} \hat{r}_y(i) \hat{w}_y(t) \]  

(28)

The index set \( D \) is the reduced order for modelling, \( \hat{r}_y(i) \) convolution of the input signal \( x(t) \) and wavelet \( \psi_y(t) \):

\[ \hat{r}_y(t) = \sum_i x(i) \psi_y(t) \]  

(29)

Procedure to carry out the Approximation and detailed coefficients use haar wavelet as follows. Consider a signal \( X = (y_1, y_2, y_3, \ldots, y_n) \). This signal can be decomposed into two coarser signals as follows:
\[ A = \left( \frac{y_1 + y_2}{\sqrt{2}}, \frac{y_2 + y_4}{\sqrt{2}}, \ldots, \frac{y_{N-1} + y_N}{\sqrt{2}} \right) \]
\[ D = \left( \frac{y_1 - y_2}{\sqrt{2}}, \frac{y_2 - y_4}{\sqrt{2}}, \ldots, \frac{y_{N-1} - y_N}{\sqrt{2}} \right) \]  

(30)

IV. Simulation and Results

In order to examine the performance of the proposed wavelet based adaptive algorithm, number of experiments like DWT-LMS, DWT-NLMS, DWT-RLS and DWT-FRLS have been performed in Matlab and results are compared with conventional time domain adaptive algorithms, namely the LMS, NLMS, RLS and FRLS algorithms.

For the simulation purpose, TIMIT database is used to generate noisy speech observations, with various acoustic environments, the room impulse response and additive noises. A noise corrupted speech signal and a noise reference signal are used, comprising of a sampling frequency of 8 KHz. A -21dB noisy speech signal that is around 15 seconds long, consisting of 1, 25,502 samples is used. Three different simulations were carried out using three different noise levels of 24dB, 27dB and 30dB to corrupt the clean speech signal. To evaluate the variation between these algorithms, the SNR value is calculated. The SNR values of the speech signal before and after the implementation of the adaptive filtering algorithms are noted:

\[
\text{SNR}_{in} = 10 \log_{10} \frac{\text{Variance (input speech)}}{\text{Variance (noise reference)}}
\]

(31)

\[
\text{SNR}_{out} = 10 \log_{10} \frac{\text{Variance (filtered speech)}}{\text{Variance (residual noise)}}
\]

(32)

Table I denotes the SNR values obtained for various values of input SNR in dB. Amongst the adaptive filtering algorithms implemented, FRLS displays the highest SNR values. The execution speed is also improved compared to RLS. Wavelet transform is introduced in order to reduce the complexity and to improve the SNR values.

The convergence rates show marked improvements. Table II and Table III illustrate the improved values of SNR as a result of the HAAR and DB2 wavelets respectively.

### References


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