Survivability Quantification of Communication Services

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Abstract

Our society is heavily dependent on a wide variety of communication services. These services must be available even when undesirable events like sabotage, natural disasters, or network failures happen. The network survivability as defined by the ANSI T1A1.2 committee [1] is the transient performance from the instant an undesirable event occurs until steady state with an acceptable performance level is attained.

In this paper we assess the survivability of a network with virtual connections exposed to link or node failures. We have developed both simulation and analytic models to cross validate our assumptions. In order to avoid state space explosion while addressing large networks we decompose our models first in space by studying the nodes independently and then in time by decoupling our analytic performance and recovery models which gives us a closed form solution. The modeling approaches are applied to two network examples. The results show very good correspondence between the transient loss and delay performance in our simulations and in the analytic approximations.

1 Introduction

Our society is heavily dependent on a wide variety of communication services to support our demands for everything from pure entertainment to commerce, banking and life critical services. The current trend is to integrate all these services on the same communication platform. Such an integrated multi-service network must provide virtual connections with highly differing QoS requirements with differentiated resilience requirements [6]. Management of such virtual connections is a challenging task since the set of operational virtual connections need to be continuously updated as the traffic load changes, new virtual connections need to be almost immediately available when established connections are affected by failures, and new or repaired network elements must be put into operation without unnecessary delays with priority to highly critical services. A large variety of management techniques exist and are under development. They apply to different network layers, use pre-planned or reactive techniques, and utilize various setup methods with different resource utilization on local or global operational domain and scope of repair. For an excellent classification of recovery techniques and current state of art the readers are referred to [6].

A model for the evaluation of the virtual connection management needs to consider both the behavioral as well as the structural aspects of the system. This means that the model must capture how the performance of the virtual connection is affected by routing and rerouting, by failures, by traffic load variations, by changes in network capacities, and by different service requirements. Structural dependability models typically focus on the probabilities of terminal connectivity, while behavioral models, e.g., as proposed in [13], take the network dynamics into account and provide steady state service availability. Combining structural and behavior aspects is typically done using Markov dependability models or queuing network models for performance analysis. Further, combined study of performance and dependability is carried out by Markov reward type models [23, 14].

In this paper our main concern is the survivability, or the transient performance of the virtual connections after an undesired event as defined by the ANSI T1A1.2 committee [1]. Survivability models are close cousins of performability models [23, 14] with the focus on transient performance immediately after an undesired event. This makes sense because, ideally, the QoS and resilience requirements should still be met even when the network suffers from overload or network element failures, irrespectively of whether this is caused by physical or logical bugs, sabotage or a natural disaster. The awareness of survivability has gained much attention due to acknowledgement of the society’s dependence on communication services clearly demonstrated by some unfortunate events. Well-
known examples are the unforeseen failure of the Galaxy IV communication satellite in 1998, which affected millions of pager users and several national radio stations lost their services, and the events of September 11 that exposed the vulnerability of telecommunications services when the need for the communication service was outmost.

The management of virtual connections must be designed to plan for the expected and react to the unexpected. Evaluation of different schemes is important. However, it is not obvious how to evaluate the management schemes, not only because the size and complexity of the problem is a huge modeling challenge, but also because the definition, metrics, and quantification methods of survivability are anything but clear. Different frameworks have been proposed and applied under different scenarios [12, 19, 20, 29]. The survivability definition we have chosen is from ANSI T1A1 [1]:

Suppose a measure of interest $M$ has the value $m_0$ just before a failure occurs. The survivability behavior can be depicted by the following attributes: $m_a$ is the value of $M$ just after the failure occurs; $m_u$ is the maximum difference between the value of $M$ and $m_a$ after the failure; $m_s$ is the restored value of $M$ after some time $t_s$; and $t_r$ is the relaxation time for the system to restore the value of $M$.

The attributes are illustrated in Figure 1. The measure of interest $M$ will in this paper be performance metrics like the loss probability and mean delay of non-lost packets. Specifically, the transient system behavior immediately after the occurrence of a failure can be analyzed under our proposed approach. Early related work can be found in [5, 28] and more recent work in [9, 22, 21].

In this paper the network survivability is quantified by the transient performance of virtual connections that are exposed to link or node failures. The performance depends on the resources available in the network for the virtual connections. In Section 2 the phased recovery model and the performance metrics are introduced. The recovery model captures the changes in resource availability as the rerouting and restoration mechanisms take effect. The remainder of the paper shows different network survivability models where this phased recovery model approach is applied. Section 3 describes two survivability models; a discrete event simulation model to cross-validate against a Stochastic Reward Net (SRN) model. In order to obtain the performance metrics numerically from the SRN model, underlying Continuous Time Markov Chain (CTMC) is generated and solved. This will cause state space explosion when the number of nodes and resources in the network increases. In Section 4 we reduce this problem by space decomposition assuming independence between the nodes in the network. To further improve the scalability, in Section 5 we also do time decomposition exploiting the fact that the steady-state performance in each recovery phase is reached quickly compared to the duration of the phase. Apart from the avoidance of largeness, a key advantage of the decomposition is that a closed form solution of the transient behavior of the performance metrics is found. The model scalability is discussed in Section 6 and the model assumptions in Section 7. Errors incurred by the space and time decomposition approximations are numerically studied by comparing against exact SRN model solutions and discrete event simulations in Section 8. The results are summarized and directions for further work are given in Section 9.

2 Phase dependent performance

In this paper we consider the performance of virtual connections between different peering nodes in a network with stochastic routing. The network is exposed to undesired events that cause links and nodes to fail. Such an undesired event is typically followed by a sudden change in the number of available resources for the virtual connections and a corresponding performance degradation. Network resources here include bandwidth of the transmission links, and finite queueing positions (memory) and processor capacity in the nodes. Gradually the resources are restored through rerouting around the problem and by restoration of the failed links and nodes, which results in improved performance. In this section a phased recovery model of the rerouting and restoration is introduced. In addition, the corresponding phase dependent performance metrics are described that will be applied in the survivability quantification models and examples later in this paper.

2.1 Phased recovery model

The phased recovery model describes the “cycle” from an undesired event that causes one or multiple links or nodes to fail, and until the system is back to the state just before
Phase IV can include any combination of multiple simultaneous node and link failures. The phased recovery model can be modified to refine the rerouting phases to also model the gradual changes in the routing probabilities or multiple steps in the virtual connection management scheme, or to model other failure modes like intermittent link failures.

2.2 Performance metrics

As mentioned in the introduction the survivability is quantified by the transient performance of the system after an undesired event [1]. In this paper the performance metric \( M \) includes the transient loss rate \( L(t) \) at time \( t \), the loss probability \( l(t) \) at time \( t \), the number of packets in the system \( N(t) \) at time \( t \), and the mean end-to-end delay of packets that are not lost in the virtual connection, \( D(t) \). In the survivability models in the following sections the composite CTMC state \((y,\bar{x})\) denotes the phase \( y \) and \( \bar{x} = (x_1, \ldots, x_n) \), where \( x_i \) is the number of packets in node \( i \). The reward rates are assigned for node \( i \) as follows:

1. For computation of the loss rate:
   \[
   f_{L_i}(t, y, x_i) = \begin{cases} 
   \Gamma_i(y) & \text{if } (x_i = n_i) \text{ at time } t \\
   0 & \text{otherwise}
   \end{cases}
   \]

2. For computation of the mean number of packets:
   \[
   f_{N_i}(t, y, x_i) = x_i
   \]

where \( n_i \) is the maximum number of packets and \( \Gamma_i(y) \) is the arrival rate to node \( i \) in phase \( y \). The transient probabilities, \( p(t, y, \bar{x}) \), are obtained from the composite CTMC models and the performance metrics become

**Expected total loss rate at time \( t \):**

\[
E[L(t)] = \sum_{y=1}^{IV} \sum_{i=1}^{n} \sum_{x_i=0}^{n_i} f_{L_i}(t, y, x_i) p(t, y, \bar{x})
\] (1)

**Expected total loss probability at time \( t \):**

\[
E[l(t)] = E[L(t)]/\gamma
\] (2)

**Expected total number of packets at time \( t \):**

\[
E[N(t)] = \sum_{y=1}^{IV} \sum_{i=1}^{n} \sum_{x_i=0}^{n_i} f_{N_i}(t, y, x_i) p(t, y, \bar{x})
\] (3)

**Expected total delay of non-lost packets at time \( t \):**

\[
E[D(t)] = E[N(t)]/(\gamma(1 - E[l(t)]))
\] (4)

Here \( \gamma \) is the external arrival rate.
3 Exact network survivability models

This section presents two different modeling approaches to determine the exact transient probabilities, \( p(t, y, \vec{x}) \), and the above four performance metrics. The modeling assumptions in the simulation and Stochastic Reward Net (SRN) models are identical with exponentially distributed inter-event times and time-independent but phase-dependent routing probabilities. This allows cross-validation and comparison of their solution efficiency. The reason for the use of SRN is to simplify the tedious and error-prone task of CTMC construction.

3.1 Stochastic Reward Net model

Stochastic Reward Net (SRN) is a powerful paradigm for modeling and evaluation of the performance of networks. Figure 3(a) shows the SRN model of the 4-node network of Figure 4 that is used as a case study in Section 8.2. The packets are tokens that are generated by the timed transition “arrival” into the place “InQ1”. If there are less than \( n_1 \) tokens in place “Node1” the immediate transition “Q1” is enabled. If not, the “loss1” transition is enabled; upon its firing, the token is removed and a packet loss is counted. The same structure is replicated for each node. The routing is determined by probabilities on the immediate transitions in the simulation and Stochastic Reward Net (SRN) models are identical with exponentially distributed inter-event times and time-independent but phase-dependent routing probabilities. This allows cross-validation and comparison of their solution efficiency. The reason for the use of SRN is to simplify the tedious and error-prone task of CTMC construction.

3.2 Simulation model

The process-oriented discrete event simulation model is shown in Figure 3(b). The source process is generating packets at the ingress router. The handling of a packet in a node is modeled as a node process that describes the packet life cycle which may be interrupted by a failure process at instances of undesired events. In each node, when the “InBuffer” contains at least one packet (token) the node proceeds to the next step and checks if the number of tokens exceeds the maximum buffer size and if the node is currently working. Then the packet is served and sent to the next node by a random selection among the currently available buffers. If no buffers are available due to failure and all routing probabilities are 0, then the packet is counted as lost. The failure is modeled to the right in the figure. At the instant of a failure (phase I) the “working” attribute of the “Node j” process is set to “false”. After the rerouting time (indicated as double lined rectangle) all routing probabilities into the failed node are set to 0 (phase II). After repair and rerouting (phase III), the routing probabilities are restored back to their initial values and the “working” attribute id is set to “true” again (phase IV).

The performance metrics in the simulator are obtained by a measurement process that reads counters at regular time intervals \( \Delta t \). Let \( c_{r,i,t} \) be the value of the counter in node \( i \) at time \( t \) in simulation replication \( r \). The average at time \( t \) over \( R \) transient simulation replications is estimated by

\[
E(c_{i,t}) = \frac{1}{R} \sum_{r=1}^{R} c_{r,i,t}
\]
The average packet loss rate $L_i$ and number of packets $N_i$ are obtained by (5) where the counter $c_{r,i,t}$ is the number of losses and the number of packets in node $i$, respectively. The packet loss probability is estimated by $\hat{L}_i = L_i / \gamma$ and the average delay of served packets by $\hat{D}_i = N_i / \gamma(1 - \hat{L}_i)$. The latter is biased because it is the ratio of two estimators.

4 Space decomposed model

It is challenging to obtain the transient probabilities, $p(t, y, x)$ because the state space becomes huge as the network size increases. We decompose the problem and approximate the global probabilities by the product $p(t, y, x) = p_1(t, y, x_1) \cdots p_n(t, y, x_n)$ where $p_i(t, y, x_i)$ is the transient probability of $x_i$ packets in node $i$ at time $t$ in phase $y$. This is akin to a product-form solution of Jackson [16] or BCMP networks [2]. The space decomposition applied in Section 4.2 splits the survivability model into independent node models and obtains the arrival rate to each node $\Gamma_i$ by solving a set of traffic equations.

4.1 Network performance model

The network is a graph $\mathcal{G} = (v, e)$ where $v$ is the set of nodes and $e$ is the set of links. The single- or multi-path routing of a virtual connection between source node $s$ and destination $d$ reduces the $\mathcal{G}$ to a directed graph $\mathcal{G}_{[s,d]}$. In the following only one virtual connection is considered to simplify the notation.

The network model is Markovian with Poisson external arrivals and exponential service time distribution with an FCFS service discipline at each node. The routing between node $i$ and $j$ is stochastic with time-independent probability $q_{ij}$. Each packet occupies one buffer position and the state $x_i$ in the CTMC is the number of packets in node $i$. The node capacity $n_i$ is the number of buffer positions. With finite $n_i$, the arrival rate $\Gamma_i$ is obtained by solving the linear system of traffic equations

$$\Gamma_i = \gamma_i + \sum_{j \neq i} q_{ij} \Gamma_j (1 - \pi_j(n_i))$$

where $\pi_j(n_i)$ is the steady state probability that node $j$ will reject an incoming packet. In the case of infinite $n_j$ the $\pi_j(n_j) = 0$; $\forall j$, and the model is an open BCMP-type queuing network [2]. A single path virtual connection is modeled as a special case where each hop has a single link $j$ with $q_{ij} = 1$ and 0 for all other links. The total external traffic $\gamma_i$ to node $i$ is $\gamma_i = 0$ for all $i \neq s$ and $\gamma_s = \gamma$. In this paper we assume finite buffers which makes the product-form solution an approximation even for steady-state computations, let alone for the transient behavior that we need for the survivability quantification. As illustrated in Section 8 the approximation is found to be very good though.

Initial value of the measure of interest, $M$ from Section 2.2, is obtained as reward measures from a continuous time Markov chain (CTMC) model of the resource utilization. Assume that each node is an $M/M/1/n_i$ queue. With $\Gamma_i$ and $\mu_i$ independent of the state $x_i$ and let $\rho_i = \Gamma_i / \mu_i$, then the steady state probability $\pi_i(x_i)$ of the state $x_i$ in node $i$ has the following closed form solution [27]

$$\pi_i(x_i) = \frac{1 - \rho_i}{1 - \rho_i^x + x_i \rho_i^x}$$

4.2 Survivability model

As per our space decomposition approximation we model the transient behavior in each node separately. The node dynamics depends on whether a link connected to this node, or the node itself, has failed. To give an example consider the four node case in Figure 4 where node $j = 2$ has failed. For this the following two CTMC models describe the failed node ($j = 2$) and the non-failed nodes ($i = 1, 3, 4$);

1. CTMC model for the node that has failed (see Figure 5). Immediately after the undesired event all the packets that are sent to node $j$ are lost, hence all transitions lead to state $(I, n_j)$ where all resources are unavailable and no packets will be served.

2. CTMC model for the non-failed nodes (see Figure 6). Immediately after the undesired event the network state is changed from $(IV, x_i)$ to $(I, x_i)$. This means that no packets are lost but the arrival rates $\Gamma_i(IV)$ are changed to $\Gamma_i(I)$. For some nodes the arrival rates

![Figure 4. Network example with 4 nodes](image-url)
are unchanged, but for the nodes that used to receive packets from node \( j \) the arrival rate is reduced. In phase \( II \) the rerouting is completed and the arrival rate is changed to \( \Gamma_i(II) \) depending on the position of \( i \) relative to \( j \) and on the routing probabilities given by Eq. (6).

In the CTMC model of the failed node \( j \) the \( p_j(t, y, x_j) \) is obtained with the initial condition \( p_j(0, I, n_j) = 1 \) while for the non-failed nodes \( (i \neq j) \) the \( p_i(t, y, x_i) \) is obtained with the initial condition \( p_i(0, I, x_i) = \pi_i(x_i) \). The \( \pi_i(x_i) \) are the steady state probabilities in (7) from Section 4.1. Finally, the global state probabilities are obtained by product form approximation

\[
p(t, y, \bar{x}) \approx \prod_{i=1}^{n} p_i(t, y, x_i)
\]  

(8)

There is no easy way to obtain closed form solutions of \( p_i(t, y, x_i) \) from the models in Figures 5 and 6. But, numerical solutions can be obtained by means of tools like SHARPE [26, 15] and SPNP [7] for rather large systems. However, as the size of the network node model increases, caused either by a more complicated recovery model or increasing the number of buffers \( n_i \), the solution becomes very resource demanding and slow.

5 Time-space decomposed model

The space decomposition of the exact model into node independent models improves the scalability significantly. However, even the decomposed CTMC model of a single node might be too complex to obtain symbolic closed form solution, and too large for numerical solution. This section proposes time decomposition [23, 14] where we assume steady state performance in each state and need only obtain transient solution of the phased recovery model described in Section 2.1.

5.1 Phased recovery model

The phased recovery model in Figure 2 describes rerouting and restoration. The system starts in phase \( I \) and we study the system until its return to phase \( IV \). The transient probabilities \( p(t,y) \) of the four phases \( y = I, \cdots, IV \), in the model can be obtained by the convolution integration approach [27]

\[
\begin{align*}
p(t, I) &= e^{-t\alpha_d} \\
p(t, II) &= \frac{\alpha_d}{\alpha_d - \tau} \left( e^{-t\tau} - e^{-t\alpha_d} \right) \\
p(t, III) &= \frac{\alpha_d}{\alpha_d - \tau} \left( \frac{e^{-t\alpha_d} - e^{-t\alpha_u}}{\alpha_d - \alpha_u} - e^{-t\alpha_u} e^{-t\tau} \right) \\
p(t, IV) &= 1 - p(t, I) - p(t, II) - p(t, III)
\end{align*}
\]  

(9)

under the assumption \( \alpha_d \neq \tau \neq \alpha_u \). For the case \( \alpha_d = \alpha_u \) as applied in the examples in Section 8 the solution is even simpler.

5.2 Survivability model

The time decomposition is a decoupling of the performance and recovery models. This means that the steady-state probabilities in the performance models and the transient solution of the phased recovery model are obtained separately and independently of each other, and
\[ p_i(t, y, x_i) \approx p(t, y) \cdot \pi_i(x_i, y) \]. The transient probabilities \( p(t, y) \) are from (9) while the steady-state probabilities \( \pi_i(x_i, y) \) are the \( \pi_i(x_i) \) from (7) for different phase dependent arrival intensities \( \Gamma_i(y) \). Observe that we assume steady state performance in each phase. The approximation is good when upon a phase change, the steady-state performance in the new phase is reached quickly compared to the duration of the phase which is the case in our network models when \( (\Gamma_i, \mu_i) \gg (\alpha_d, \tau, \alpha_u) \). The quality of the time decomposed Markov model approximation depends on the degree of coupling between the “performance block” and “phase block” in our Markov matrix \([10, 23, 4]\). So far we have justified the approximation by discussion of typical system parameters in Section 7 and validated by numerical results in Section 8.

Then, the global state probabilities are obtained by approximation using both space decomposition from (8) and the time decomposition

\[
p(t, y, x) \approx \prod_{i=1}^{n} p_i(t, y, x_i) \approx p(t, y) \cdot \prod_{i=1}^{n} \pi_i(x_i, y) \tag{10}
\]

When the phased recovery model is as simple as the example in Figure 2 we have a closed form solution. This enables efficient evaluation of very large networks with large, and even infinite, server and buffer capacities, \( n_i \). With a much more complex phased recovery model where a closed form transient solution is hard or impossible, this time decomposition approach is still advantageous since the numerical solution is significantly faster compared to the node model in Section 4 because the number of states is reduced.

6 Model scalability

The main purpose of the approximations proposed in this paper is to reduce the computational effort of obtaining transient solutions in large network models without an undue loss in the accuracy. The assumptions made are discussed in Section 7. The accuracy is demonstrated by two small network models in Section 8.

The underlying CTMC model of the exact Stochastic Reward Net has a state space that is proportional to \( \prod_{i=1}^{n} n_i \times n_p \) where \( n_p \) is the number of phases. The space decomposed CTMC model will reduce the state space of the transient solution to \( \sum_{i=1}^{n} n_i \times n_p \) while for the time-space decomposed model a transient model with only \( n_p \) states needs to be solved.

The simulation model is considered to be attractive when the analytical model fails due to too restrictive modeling assumptions or intractable or inefficient solutions. The strong side of the simulation approach is that an arbitrary level of detail applies that suits the study of interest. In many cases it is easy to change the modeling assumptions and to make the model as close to reality as required. However, it is important to point out that the efficiency of the simulation approach depends on the network size and the stiffness of the model, i.e., the ratio between the packet arrival and departure rates, and the repair and rerouting rates \([25]\). In addition, simulations become inefficient when the events of importance to the metrics of interest are infrequent, e.g. the packet losses are very rare. Then, numerical solutions might become less computer intensive compared to simulation unless some rare event simulation technique like importance sampling, RESTART or importance splitting \([17]\) can be applied.

7 Modeling assumptions

In the network models in this paper we have assumed that external packet arrivals to the source of a virtual connection is a Poisson process. If a bursty arrival process is required we may use an MMPP or MAP type process \([27]\) which will produce larger CTMC models. The packet service time distribution is assumed to be exponential. The service time distribution is influenced by a combination of the packet size distribution, the aggregation level, and the header processing time. Non-exponential empirical distributions are observed, for instance in \([24]\) where the empirical distribution of single router service time was fitted to a Weibull distribution. A phase-type fit for Weibull distribution is possible and will once again yield larger CTMC models.

In our space decomposition model we assumed independence between the network nodes. Independence is not a fully realistic assumption but a good approximation in networks with low loss probability and with high aggregation, i.e., with multiplexing of a large number of connections.

We have decoupled the performance and recovery models. We assumed that in each phase we have steady-state performance. This approximation is good when the steady-state performance in a phase is reached quickly after the change of phase compared to the expected duration of the phase. As a rule of the thumb, the approximation is good if there is at least two order of magnitude difference between the time granularity of the events in the performance model and in the recovery model. E.g. in medium loaded (30-50%) high capacity networks (100Mbit/s -10 Gbit/s) you will observe 3-300 packets/ms, while the routing, rerouting and repair (at IP level) is in the order of 100s of ms. This means a few hundred to several thousand packets are expected in each phase.

The phase time distribution in the recovery model is for simplicity assumed to be exponential but any general distribution can be accommodated using semi-Markov models \([27]\).
The simulation model assumptions are the same as the assumptions in SRN model to cross validate and to validate the following model decomposition approximations. The assumptions in the simulation model are easily relaxed, e.g., to change to general arrival process or general service time distributions. To model the network protocols in more detail, alternatives to DEMOS/SIMULA like ns-2 [11] will be considered provided that the network size is not too large and the packet rate is not too high compared to the expected duration of the transient period after the undesired event.

8 Network examples

In this section two small network examples are studied by simulation and the analytic approaches described in the paper.

8.1 Evaluation tools

The simulations in this paper are done by a process-oriented simulator implemented using the programming language SIMULA [18] with the DEMOS (Discrete Event Simulation on SIMULA [3]) class library. The analytic evaluations of the Stochastic Reward Net are conducted both in SHARPE [15, 26] and SPNP [7], while the CTMC models are numerically solved in SHARPE and the symbolic closed form solution is checked in Mathematica.

8.2 A 4 node example

The first example is a network with \( n = 4 \) nodes as depicted in Figure 4. The performance of the virtual connection between \( s = 1 \) and \( d = 4 \) is evaluated after the failure of node 2 at time \( t = 500 \). Each node \( i \) is an \( M/M/1/n_i \) system with the parameters given in Table 1. The parameters in the phased recovery model are \( \alpha_d = \alpha_u = 0.01 \) and \( \tau = 0.001 \).

The network example is studied by simulations and all three analytical approaches. The estimated performance metrics from \( R = 90 \) simulation replica (Simulations) are compared against the analytical values of the Stochastic Reward Net model solved by SPNP (SRN model). The loss probability and the average number of packets in the system at different \( t \) are shown in Figures 7(a) and 7(b), respectively. The space and time-space decomposed models are indistinguishable so they are represented by only one curve (Decomposed CTMC model).

The main observations from the experiments are that the simulations and SRN models show perfect fit as expected since the modeling assumptions are identical, and that the decomposed CTMC models capture the transient performance very well. The time-space decomposed model with the closed form solution are a significant simplification that enables studies of large networks with many, and even infinite, buffers. The results show that when the transient performance is dominated by impairments in a single node this decomposed, product form approximation is a viable approach.

8.3 A 10 node example

The second example is a network with \( n = 10 \) nodes. The directed graph \( G_{10} \) for routing virtual connections

<table>
<thead>
<tr>
<th>( i )</th>
<th>( n_i )</th>
<th>( \mu_i )</th>
<th>( \Gamma_i(I) )</th>
<th>( \Gamma_i(II) )</th>
<th>( \Gamma_i(III) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>100.0</td>
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<td>80.0</td>
<td>80.0</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>100.0</td>
<td>46.9</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>100.0</td>
<td>31.2</td>
<td>31.2</td>
<td>78.1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>100.0</td>
<td>77.9</td>
<td>31.0</td>
<td>69.1</td>
</tr>
</tbody>
</table>


Figure 7. Performance in 4 node network
between $s = 1$ and $d = 10$ is depicted in Figure 8. The performance of the virtual connection is evaluated after the failure of node 4 at time $t = 500$. Again each node is an $M/M/1/n_{i}$ system with the parameters given in Table 2. The parameters in the phased recovery model are $\alpha_{d} = \alpha_{u} = 0.01$ and $r = 0.001$.

The network example is studied by simulations and one analytic approach. The estimated performance metrics from 90 simulation replica (Simulations) are compared with the analytical values of the time-space decomposed model (Decomposed CTMC model). The results are given in Figures 9(a) and 9(b), for the loss probability and average number of packets in the system at different $t$, respectively. The results show that the closed form solution from Section 5 captures the transient performance very well.

Figure 9(a) also includes a “rerouting model” which is $r(t) = q_{(1,4)}e^{-\alpha_{d}t}$, i.e. the probability that a packet is lost in the failed node at time $t$ after the instant of failure. The $q_{(1,4)} = q_{14} + q_{12}q_{23}q_{34}$ is the probability of visiting node 4 when starting from node 1 in the directed graph $\tilde{G}^{\gamma}_{[1,10]}$ in Figure 8. The results in Figure 9(a) show almost perfect overlap between $r(t)$ and $l(t)$ from (2) which means that with very low steady-state packet loss probability, the transient loss probability is dominated by $q_{(1,4)}$ with the decay rate equal to the reciprocal of the expected rerouting time $1/\alpha_{d}$. The same is not observed in the 4-node network because here steady-state loss probability is not negligible.

### 9 Closing remarks

The time-space decomposed model with the closed form solution is a significant simplification that can easily model huge networks with large, even infinite, buffers. In this paper we have cross-validated our analytical and simulation models, and checked the approximations of the decomposed analytical models. The results from the survivability studies show that when the transient performance impairment is dominated by a failure event the decomposed, product form approximation is a viable approach.

In cases where the space decomposition is inaccurate still time decomposition should be considered because it will help reduce the complexity of the transient solution significantly.

The assumptions made in our models will be relaxed in the future allowing for multiple failures, general distribu-
tions and multiple virtual connections. Currently the approach is applied to a larger network where transient delay distributions are obtained. We import routing probabilities from ns-2 simulations before and after a failure, and before and after a repair. This is ongoing research with promising preliminary results. The routing probabilities can also be obtained from operational networks. We plan to extend the phased recovery model to include more details regarding the virtual connection management at failure and repair, and possibly to include multiple failure modes.

References