An age replacement maintenance policy is considered here, in which a system is restored whenever it fails, or ages without failure up to a preventive maintenance epoch (whichever comes first). The duration of the restoration activity is random, and depends on whether it was precipitated by a failure or by a preventive maintenance action. The case where the preventive maintenance epoch is deterministic has been studied previously, and shown to be optimal in a certain sense. Here, we consider the case where the preventive maintenance epoch is randomized, which is more realistic for many systems. The system availability is the long run proportion of time that the system is operational (i.e., not undergoing repair or preventive maintenance). The optimal rate of preventive maintenance to maximize availability is considered, along with sufficient conditions for such an optimum to exist. The results obtained herein are useful to systems engineers in making critical design decisions.

Keywords: Aging; equilibrium distribution; preventive maintenance; reliability; renewal theory.

1. Introduction

Consider a system that is maintained according to an age replacement maintenance policy. Such a policy dictates that the system is restored whenever it fails or ages without failure to a random preventive maintenance epoch (whichever comes first).
Specifically, the evolution of the system in continuous time is assumed as follows. The system starts at time 0, fully operational, with lifetime $X$, having cumulative distribution function (cdf) $F_1$ satisfying $F_1(x) = 0$ for $x < 0$. We assume $0 < E(X) < \infty$. Simultaneously with time 0, a preventive maintenance clock is started, by drawing a random time $Y$, having cdf $F_2$ satisfying $F_2(y) = 0$ for $y < 0$. If $X > Y$, then the system is taken out of operation at time $Y$, and full preventive maintenance is performed (i.e., the system is restored to “new” condition), which takes a random amount of time $U > 0$, having a distribution with finite mean $E(U) = 1/m > 0$ for some constant $m > 0$. After the preventive maintenance is completed, the system is set into operation again. The value $Y$ is sometimes referred to as the preventive maintenance “trigger”. If $X \leq Y$, the system is completely repaired starting at time $X$ (i.e., the system is also restored to new condition), which takes a random amount of time $V > 0$, having a distribution with finite mean $E(V) = 1/\mu > 0$ for constant $\mu > 0$. Once the repair is completed, the system is set into operation again. Thus, a complete cycle length, starting at time 0 and ending at the time the system has been restored to new condition, is given by

$$Z = \min\{X, Y\} + UI(X > Y) + VI(X \leq Y).$$

(1)

Here, $I(A)$ is the indicator function for the event $A$: $I(A) = 1$ if $A$ occurs, and 0 otherwise. If we assume that the succession of cycles are formed by independent and identically distributed copies of the random vector $(X, Y, U, V)$, and apply the renewal reward theorem (see for example, Theorem 3.6.1 of Ref. 1), we see that the limit, as $t \to \infty$, of the proportion of time in $[0, t]$ that the system is operational is given by

$$A = \frac{E(\min\{X, Y\})}{E(\min\{X, Y\}) + E(UI(X > Y)) + E(VI(X \leq Y))}.$$  

(2)

In this context, (2) is sometimes referred to as asymptotic average availability. If the distribution of $Z$ is nonlattice, then it follows by the alternating renewal process theorem (see for example, Theorem 3.4.4 of Ref. 1) that (2) is the limiting probability (as $t$ approaches infinity) of finding this system operational at time $t$, and is referred to in systems reliability analysis simply as “availability”. Under either interpretation, in what follows we will simply refer to (2) as availability. Note that result (2) is fairly general, and no assumptions have been made thus far about the joint distribution of $(X, Y, U, V)$. In fact, if $(U, V)$ is independent of $(X, Y)$, then (2) depends on the distribution of $(U, V)$ only through $E(U)$ and $E(V)$, and $A$ reduces to

$$A = \frac{E(\min\{X, Y\})}{E(\min\{X, Y\}) + (1/m)P\{X > Y\} + (1/\mu)P\{X \leq Y\}}.$$  

(3)

Finally, if we assume that $X, Y, U,$ and $V$ are mutually independent, then (3) becomes

$$A = \left[1 + \frac{(1/m)P\{X > Y\} + (1/\mu)P\{X \leq Y\}}{\int_0^\infty (1 - F_1(t))(1 - F_2(t))dt}\right]^{-1}.$$  

(4)
This expression was also derived in Ref. 2 using the semi-Markov structure of the process. Our objective is to determine the preventive maintenance schedule (i.e., determine $F_2$) in a way that maximizes (4).

It is clear from (3) that a necessary condition for preventive maintenance to be effective in increasing availability is for $E(U) = 1/m < 1/\mu = E(V)$, i.e., on average, performing preventive maintenance takes less time than repairing a failed system. This is equivalent to the condition $0 < \mu/m < 1$. This constraint will be assumed in what follows.

Before continuing, it is useful to place our problem into the more general context of minimizing general maintenance costs. Following Barlow and Proschan, let $c_1 > c_2 > 0$ be the costs of performing a corrective maintenance action and a preventive maintenance action, respectively. Let $C(t)$ be the expected total accumulated maintenance cost up to time $t > 0$. Using the elementary renewal theorem, it is shown in Ref. 3 that

$$
\lim_{t \to \infty} \frac{C(t)}{t} = \frac{c_2 P\{X > Y\} + c_1 P\{X \leq Y\}}{\int_0^\infty (1 - F_1(t))(1 - F_2(t))dt}.
$$

Thus, the right-hand side of (5) is the long-run expected maintenance cost per unit time, and therefore represents an appropriate objective function to minimize as a function of the preventive maintenance schedule. Taking

$$
c_1 = 1/\mu, c_2 = 1/m
$$

in (5), this gives the ratio of expected down-time to expected up-time, and is seen to correspond with part of the denominator in (4). Since minimizing this quantity is the same as maximizing availability, it follows through the correspondence (6) that in principle, taking the objective function to availability is no different from the more general maintenance cost objective function in (5).

Barlow and Proschan had solved the case of a deterministic preventive maintenance trigger for age replacement; i.e., the case $P\{Y = T\} = 1$ for a constant $T > 0$. In this situation, (4) reduces to

$$
A(T) = \left[1 + \frac{F_1(T)}{\mu \int_0^T (1 - F_1(t))dt} + \frac{1 - F_1(T)}{m \int_0^T (1 - F_1(t))dt}\right]^{-1}.
$$

Using (6) and $F = F_1$ in Eq. (2.2) of Chap. 4 in Ref. 3, it is shown there that when $F_1$ has a density, a necessary condition for a finite $T$ to maximize (7) is a finite solution in $T$ to the equation

$$
r_1(T) \int_0^T (1 - F_1(t))dt - F_1(T) = \frac{1/m}{1/\mu - 1/m},
$$

where $r_1(t) = -d \ln(1 - F_1(t))/dt$ is the hazard rate function for $F_1$. It is further pointed out in Ref. 3 that a finite solution to (8) which maximizes (7) will exist if $r_1$ is increasing and

$$
\frac{\sqrt{\text{var}(X)}}{E(X)} < 1 - \frac{\mu}{m}.
$$
Otherwise, the solution \( T = \infty \) (i.e., never perform preventive maintenance) gives the maximum availability. Note that it is impossible for (9) to be satisfied if \( F_1 \) is the exponential cumulative distribution function, since in that case \( \sqrt{\text{var}(X)} = E(X) \).

Of course, the fact that preventive maintenance does not improve reliability for systems with a constant failure rate is well-known. However, due to the mathematical definition of increasing and decreasing functions, the exponential distribution is simultaneously IHR (increasing hazard rate) and DHR (decreasing hazard rate). Thus, (9) provides a meaningful criterion only for nonexponential distributions that are IHR.

Barlow and Proschan\(^3\) also show that no form of randomized preventive maintenance (i.e., a nondegenerate \( F_2 \)) in this setting can yield an optimum availability that exceeds the optimum availability achieved by the deterministic case outlined above. However, in practice, enforcing a deterministic preventive maintenance trigger can be impossible, impractical, or dangerous. Examples include: changing the oil in an automobile at exactly 5000 mile intervals; having your annual physical exam at exactly one-year intervals; taking a defensive system such as an artillery-locating radar offline for maintenance during an unexpected military engagement. In circumstances like these, it is more realistic to assume a preventive maintenance policy that has a nondegenerate distribution for \( F_2 \).

One way to choose \( F_2 \) is to view the random age replacement policy as a small random perturbation of the deterministic case, choosing \( F_2 \) to be a distribution that is highly concentrated near the optimal \( T \) for the deterministic problem. However, this approach is not interesting, as the solution is predictably just an asymptotic expansion of the deterministic solution in a small neighborhood of \( T \), and the achievable availability will be correspondingly close to the optimally achievable value. Also, this type of model would be appropriate only when the operating and maintenance environment is predictable. On the other hand, if the environment in which the system operates can be highly entropic (as in the aircraft example, or in a battlefield scenario) then there will be many instances where the deviations from the optimal \( T \) are much larger than small perturbations. For such systems, where the randomness of the environment makes it difficult to characterize the distribution of the preventive maintenance trigger, it would make sense to choose \( F_2 \) according to the principle of maximum entropy. In the statistical context, entropy is a measure of randomness, or in the case of a distribution, how "diffuse" or spread-out the distribution is. Thus, we choose \( F_2 \) to be the distribution of a positive random variable that has the maximum entropy subject to a given mean. This, of course, is known to be the exponential distribution. To a certain extent, optimizing availability using this choice for \( F_2 \) leads to a model that captures what is optimally achievable in an environment that is highly unpredictable in terms of when preventive maintenance can be undertaken.

Finally, we point out that the conditions sufficient for an optimal deterministic maintenance trigger (i.e., that the distribution of \( X \) have a strictly increasing continuous failure rate function increasing to infinity) in the work by Barlow and
Proschan are relatively restrictive and difficult to verify. The analogous conditions for an optimal random preventive maintenance policy (assuming an exponential distribution for $F_2$) are much less restrictive.

In this paper, we take $F_2$ to be the exponential cdf with finite mean $\delta^{-1}$, $\delta > 0$, and consider the problem of maximizing (4) as a function of $\delta$. In so doing, many parallels will be seen with the deterministic case. For example, a condition similar to (9), along with taking $F_1$ to have a certain aging property (e.g., IHR, or decreasing mean residual life) will be sufficient for an optimal solution to exist.

2. The Optimal Randomized Age Replacement Policy

Using the exponential distribution with mean $1/\delta$ for $F_2$ in (4), and denoting, for $\delta \geq 0$, the Laplace transform of $1-F_1$ by

$$\psi(\delta) = \int_0^\infty \exp(-\delta t)(1-F_1(t))dt,$$

(10)

it follows that the availability (4), as a function of $\delta$, is given by

$$A(\delta) = \left[1 + \frac{1}{\mu\psi(\delta)} + \delta(1/m - 1/\mu)\right]^{-1}.$$  

(11)

For positive $\delta$, we have $\psi(\delta) = E(\min\{X,Y\}) \leq E(Y) = 1/\delta$, and consequently $1 + \frac{1}{\mu\psi(\delta)} + \delta(1/m - 1/\mu) \geq 1 + \delta/m$. As a result, it is seen that (as expected)

$$\lim_{\delta \to \infty} A(\delta) = 0, \quad A(0) = \frac{E(X)}{E(X) + 1/\mu}.$$  

(12)

Suppose that $X$ has a constant failure rate $\lambda > 0$; i.e., $F_1$ is the cdf of the exponential distribution with mean $1/\lambda$. In this case, the expression inside the brackets in (11) reduces to $1 + \delta/m + \lambda/\mu$, and availability is maximized by taking $\delta = 0$. Thus, in addition to assuming that $0 < \mu/m < 1$, we must also make some assumption regarding “aging” with respect to the distribution of $X$. This will be made clear in the analysis that follows.

Maximizing $A(\delta)$ in (11) is tantamount to minimizing $1/A(\delta)$, which is the expression inside the brackets in (11). Taking a derivative of this, it follows that necessarily, the optimizing value of $\delta$, if it exists, is either $\delta = 0$ or a solution to the equation

$$g(\delta) \equiv 1 + \frac{\psi'(\delta)}{(\psi(\delta))^2} = \frac{\mu}{m}.$$  

(13)

From the general formula $E(X^n) = \int_0^\infty nt^{n-1}P\{X > t\}dt$, (10) implies $\psi(0) = E(X)$, $\psi'(0) = -E(X^2)/2$, $\delta\psi(\delta) = E(\min\{\delta X, E_1\})$, and $\delta^2\psi'(\delta) = -(1/2)E[\min\{\delta X, E_1\}]^2$ where $E_1$ denotes an exponentially distributed random variable with mean 1, independent of $X$. Since $\lim_{\delta \to \infty} \min\{\delta X, E_1\} = E_1I(X > 0)$ with probability 1, $\min\{\delta X, E_1\} \leq E_1$, and $E_1$ has finite moments of all non-negative orders, the dominated convergence theorem implies that $(\delta\psi(\delta))^2 \to 1 - F_1(0)$ and
\[ \delta^2 \psi'(\delta) \to -(1 - F_1(0)) \] as \( \delta \to \infty. \) Since \( E(X) > 0 \) implies \( 1 - F_1(0) > 0, \) we thus have
\[
\lim_{\delta \to 0} g(\delta) = 1 - \frac{E(X^2)}{2(E(X))^2}, \quad \lim_{\delta \to \infty} g(\delta) = 0, \tag{14}
\]
where the first of these may be \(-\infty\) in general.

At this point, we consider assumptions about the aging properties of \( F_1. \) A relatively nonrestrictive, and easy-to-verify aging property is “New Better than Used in Expectation”, or NBUE. We say that a life distribution’s cdf \( F, \) or its survival function \( 1 - F \equiv F, \) is NBUE if it has a finite mean, \( F(0) < 1, \) and if for all \( t \) for which \( F(t) > 0, \) we have
\[
\int_0^\infty F(x)dx \geq \int_0^\infty \frac{F(t + x)}{F(t)}dx. \tag{15}
\]
The following moment inequality for NBUE distributions will be a key component in what follows.

**Lemma 1.** Let \( F \) be an NBUE life distribution, and denote \( \lambda_r = \int_0^\infty x^r dF(x)/\Gamma(r + 1) \) for \( r > 0, \) where \( \Gamma(\cdot) \) is the usual gamma function. Then \( \lambda_{r+1} \leq \lambda_1 \lambda_r. \) In particular, if \( W \) has NBUE distribution \( F, \) then \( E(W^2) \leq 2(E(W))^2 \) with strict inequality unless \( F \) is the exponential distribution.

The proof of the basic inequality in Lemma 1 can be found in Ref. 4, p. 197. The strict inequality statement follows from the result of Brown and Ge\(^5\) that if the random variable \( W \) has an NBUE life distribution with cdf \( F, \) then
\[
\sup_{x \geq 0} |F(x) - \exp(-x/E(W))| \leq \frac{4\sqrt{6}}{\pi} \sqrt{\frac{E(W^2)}{2(E(W))^2} - 1}. \tag{16}
\]
Therefore, the inequality must be strict unless \( F \) is the cdf of an exponential distribution. This fact will be important later, to exclude the exponential distribution from our class of aging distributions.

We can now establish a result on the existence of solutions to (13). This result should be compared to (9).

**Theorem 1.** If \( F_1 \) is NBUE, then provided
\[
0 < \frac{\mu}{m} < 1 - \frac{E(X^2)}{2(E(X))^2} \tag{17}
\]
there will be at least one solution to (13).

To prove Theorem 1, simply note that if (17) is true, then by (14) and continuity of the function \( g, \) it must cross the level \( \mu/m \) at least once.

Theorem 1 guarantees that there will be at least one critical point for the availability function when (17) is true. Examining the derivative of the function \( g, \) we see that \( g \) will be decreasing in \( \delta \) and non-negative if \( E(X^2) \leq 2(E(X))^2 \) and
\[
\psi(\delta)\psi''(\delta) \leq 2(\psi'(\delta))^2 \tag{18}
\]
for all $\delta \geq 0$. Indeed, (18) is the condition for $g'(\delta) \leq 0$ for all $\delta \geq 0$. If we now denote $W_\delta = \min\{X, Y\}$, i.e., a random variable that has survival function
\[ \overline{H}(t) = \exp(-\delta t)(1 - F_1(t)) \] (19)
we see that (18) is equivalent to
\[ E(W_\delta)E(W_\delta^2) \leq (3/2)(E(W_\delta^3))^2. \] (20)
Let us denote the equilibrium survival function corresponding to $W_\delta$ (i.e., the survival function corresponding to the equilibrium distribution for a renewal process with inter-arrival times distributed like $W_\delta$) by
\[ \overline{H}_e(t) = \frac{1}{E(W_\delta)} \int_t^\infty \overline{H}(x)dx. \] (21)
Then if $\overline{H}_e$ is NBUE, applying Lemma 1 to the moments of $\overline{H}_e$ with $r = 1$ we arrive at
\[ \int_0^\infty 2t\overline{H}_e(t)dt \leq 2 \left( \int_0^\infty \overline{H}_e(t)dt \right)^2. \] (22)
The right-hand side of (22) is
\[ 2(E(W_\delta^2)/(2E(W_\delta)))^2 = (1/2)(E(W_\delta^2))/(E(W_\delta))^2. \] (23)
The left-hand side of (22) is
\[ \frac{1}{E(W_\delta)} \int_0^\infty 2t \int_t^\infty \overline{H}(x)dxdt = \frac{1}{E(W_\delta)} \int_0^\infty \overline{H}(x) \int_0^x 2tdtdx \]
\[ = \frac{1}{E(W_\delta)} \int_0^\infty x^2\overline{H}(x)dx = \frac{1}{3} \frac{E(W_\delta^3)}{E(W_\delta)}. \] (24)
Putting (23) and (24) back into (22) then gives (20). If both $F_1$ and the equilibrium distribution $H_e$ are NBUE for all $\delta \geq 0$, and condition (17) is met, then $F_1$ cannot be the cdf of an exponential distribution, and therefore neither $\overline{H}$ nor $\overline{H}_e$ can correspond to exponential distributions, so it follows by Lemma 1 that the inequality in (20) is strict (so $g'(\delta) < 0$ for all $\delta \geq 0$), $g$ is strictly decreasing, and (13) has a unique solution in $\delta$. Note that from (11) and (13), we have $d(1/A(\delta))/d\delta = 1/m - g(\delta)/\mu$, and since $g' < 0$, it follows that $d^2(1/A(\delta))/d\delta^2$ is positive, so that the unique solution to (13) yields the maximum value of (11). We summarize this result as:

**Theorem 2.** There is a unique solution $\delta > 0$ to (13) that maximizes $A(\delta)$ given by (11) if the following three conditions are met:

1. The distribution of $X$ is NBUE.
2. $0 < \frac{m}{\mu} < 1 - \frac{E(X^2)}{2[E(X)]^2}$.
For all $\delta > 0$ and $Y$ having an exponential distribution with mean $1/\delta$, the equilibrium distribution of a renewal process with inter-renewal intervals that are independent and identically distributed as $\min\{X, Y\}$ is NBUE.

There are two immediate corollaries to Theorem 2 that are worth mentioning.

**Corollary 1.** If $F_1$ is IHR, then condition (17) is sufficient for there to exist a unique solution $\delta > 0$ to (13) that maximizes $A(\delta)$ given by (11).

To prove Corollary 1, note that if $F_1$ is IHR, then $H$, which is the survival function of $\min\{X, Y\}$, is also IHR, and by Proposition 4.C.6 of Ref. 4, the equilibrium distribution with survival function $H_e$ given by (21) is also IHR. Since IHR implies NBUE, the result follows. Corollary 1 is comparable to the result of Barlow and Proschan$^3$ for the deterministic age replacement policy discussed in the previous section. However, note that here, the IHR property, which is the property that for all $x \geq 0$ and all $t$ for which $1 - F_1(t) > 0$, the function

$$F_1(t + x) - F_1(t) \quad \frac{1 - F_1(t)}{1 - F_1(t)}$$

is increasing in $t$ and $F_1(0+) = 0$, does not require that the hazard rate function $r_1(t) = -\frac{d\ln(1 - F_1(t))}{dt}$ exist (i.e., that a density exists for $F_1$) or, in the case where a density does exist, that the function $r_1$ is continuous and strictly increasing. Thus, while this corollary is analogous to the result of Barlow and Proschan$^3$ for the deterministic age replacement policy, its conditions are much less restrictive.

**Corollary 2.** Let $F_1$ be NBUE, and suppose that $H$ in (19) has a decreasing mean residual life (DMRL) for each $\delta \geq 0$; that is,

$$m(t) \equiv \int_0^\infty \frac{H(t + x)}{H(t)} \, dx \quad (25)$$

is a decreasing function of $t$ for $t \geq 0$. Then condition (17) is sufficient for there to exist a unique solution $\delta > 0$ to (13) that maximizes $A(\delta)$ given by (11).

To prove Corollary 2, note that the hazard rate function corresponding to the equilibrium distribution $H_e$ given by (21) exists and is equal to $1/m(t)$, which is increasing, which implies that $H_e$ is NBUE.

### 3. Final Remarks

When a solution to (13) exists, it can be found by the secant method. Specifically, given positive starting values $\delta_0 \neq \delta_1$ close to the solution, the sequence

$$\delta_{n+1} = \delta_n - g(\delta_n) \left( \frac{\delta_n - \delta_{n-1}}{g(\delta_n) - g(\delta_{n-1})} \right), \quad n = 1, 2, \ldots \quad (26)$$

is computed until $|\delta_{n+1} - \delta_n|$ is less than a prespecified error tolerance.
As an example of the application of our results, consider the case where \( F_1 \) is the hypoexponential distribution with two stages, having \( \lambda_1 = 0.002, \lambda_2 = 0.001, \) the preventive maintenance rate is \( m = 6, \) and the repair rate is \( \mu = 0.1, \) with all of these rates given on a per hour basis. The hypoexponential distribution, being the distribution of the convolution of independent exponentially distributed random variables, is IHR, and serves as a good model for a system that degrades in stages. Here, Corollary 1 applies, and the unique solution to (13) yielding the optimum availability, found by the iteration (26), is \( \delta = 0.007954, \) or a mean of about 125.72h. The optimal availability is 0.99686. It is interesting to consider how much better the optimal availability would be using the optimal deterministic age replacement policy, which (as discussed above) is shown to dominate random age replacement policies in Ref. 3. For this problem, using the scheme (26) to solve (8) yields the optimal deterministic preventive maintenance trigger \( T = 149.59h \) with optimal availability 0.99761. A plot of the availability for this system, for both policies, as a function of the mean preventive maintenance trigger is shown in Fig. 1. Availabilities are often compared using the measure \( -\log_{10}(1 - A) \) where \( A \) is the availability. For this example, using this measure to compare the two optimum availabilities, the optimal deterministic policy is only about 5% higher relative to that of the optimal random policy.

A related problem is to find the preventive maintenance rate that minimizes a more general maintenance cost function (5), subject to meeting (or exceeding) a fixed availability requirement \( A^* \) for (4). With \( m, \mu \) and \( F_1 \) fixed, and only the
preventive maintenance rate $\delta$ varying, it is easy to see that under the conditions of Theorem 2, this restricts the admissible values of $\delta$ in (5) to lie in a set of the form $[\delta_1, \delta_2]$ where $0 < \delta_1 \leq \delta_2$, and $A(\delta_1) = A(\delta_2) = A^*$, for the case where $\max_{\delta > 0} A(\delta) \geq A^*$. Obviously, no solution to the constrained optimization problem will exist if $\max_{\delta > 0} A(\delta) < A^*$. In the former restricted case, it is straightforward to find an optimal cost solution, which will either occur at one of the endpoints of $[\delta_1, \delta_2]$, or at an interior point of that set. If some of the other variables ($m$, $\mu$ and $F_1$) are also allowed to vary, this constrained optimization problem would be an interesting problem for further research.

References


About the Authors

John E. Angus is Professor of Mathematics in the School of Mathematical Sciences at Claremont Graduate University since 1990. He received his M.A. in Mathematics from UCLA in 1977, and his M.S. and Ph.D. in Statistics from the University of California at Riverside in 1981. After receiving his B.A. in Mathematics from the University of San Diego in 1975, he worked for Hughes Aircraft Company as a Systems Engineer until 1990, and has been an active consultant to Raytheon since 1996 in system engineering and algorithm development. His research interests include survival analysis, applied probability and statistics.

Meng-Lai Yin is Professor of Electrical and Computer Engineering at the California State Polytechnic University, Pomona. She received her M.S. and Ph.D. degree in Information and Computer Science from the University of California, Irvine, in 1989 and 1995, respectively. She also holds a Masters degree in Electrical and Computer Engineering from National Cheng-Kung University, Taiwan. She has many years of industrial experience at Raytheon as a system engineer. Her research interests include performance and reliability analysis, embedded systems and parallel processing.

Kishor Trivedi holds the Hudson Chair in the Department of Electrical and Computer Engineering at Duke University, Durham, NC. He has been on the Duke faculty since 1975. He is the author of a well-known text entitled “Probability and Statistics with Reliability, Queuing and Computer Science Applications”, published by Prentice-Hall; a thoroughly revised second edition (including its Indian
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edition) of this book has been published by John Wiley. He has also published two other books entitled “Performance and Reliability Analysis of Computer Systems”, published by Kluwer Academic Publishers and “Queueing Networks and Markov Chains”, John Wiley. He is a Fellow of the Institute of Electrical and Electronics Engineers. He is a Golden Core Member of IEEE Computer Society. He has published over 480 articles and has supervised 43 Ph.D. dissertations. He is on the editorial boards of Journal of Risk and Reliability, International Journal of Performability Engineering and International Journal of Quality and Safety Engineering. He is the recipient of IEEE Computer Society Technical Achievement Award for his research on Software Aging and Rejuvenation. His research interests are in reliability, availability, performance, performability and survivability modeling of computer and communication systems. He works closely with industry in carrying out reliability/availability analysis, providing short courses on reliability, availability, performability modeling and in the development and dissemination of software packages such as SHARPE and SPNP.