The Higgs Mass in the MSSM at two-loop order beyond minimal flavour violation

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Soft supersymmetry-breaking terms provide a wealth of new potential sources of flavour violation, which lead to very tight constraints from precision experiments. This has posed a challenge to construct flavour models to both explain the structure of the Standard Model Yukawa couplings and how their consequent predictions for patterns in the soft supersymmetry-breaking terms do not violate these constraints. While such models have been studied in great detail, the impact of flavour violating soft terms on the Higgs mass at the two-loop level has been assumed to be small or negligible. In this letter, we show that large flavour violation in the up-squark sector can give a positive or negative shift to the SM-like Higgs of several GeV, without being in conflict with any other observation. We investigate in which regions of the parameter space these effects can be expected.

I. INTRODUCTION

The discovery of the Higgs boson [1, 2] has already celebrated its third anniversary, and in the meantime its properties are measured with an impressive precision. In particular the mass is known up to an uncertainty of 0.25 GeV. This measurement is much better than the theoretical prediction of the mass in any model beyond the standard model (SM). The most studied extension of the SM is the minimal supersymmetric standard model (MSSM), in which the uncertainty is estimated to be of order of a few GeV [3]. It is often assumed that the origin of this uncertainty are the unknown electroweak corrections at two-loop as well as higher order corrections. While contributions from squarks and quarks at one-loop order are known exactly, the widely used two-loop corrections make the approximation of including only third generation states.

The impact of first and second generation (s)quarks can safely be neglected under the assumption that the only source of flavour violation is the CKM matrix of the SM. In this ansatz, known as ‘minimal flavour violation’ [4–6], the three generations of fermions are aligned with the corresponding sfermions and the soft-breaking terms do not introduce any additional flavour violation. However, there is no fundamental reason why this alignment should be present. In particular in models where SUSY breaking is transmitted via gravity, this is often a very strong and hard to motivate assumption. It can be motivated in models with pure gauge mediation, but these models have significant difficulties in explaining the Higgs mass – hence recent interest in non-minimal gauge mediation models with direct couplings between the messenger and visible sectors, which may as a consequence lead to flavour violation [7–12].

For these reasons, non-minimal flavour violation in the MSSM has been studied for several years already: the focus has been mainly on the collider phenomenology [13–18], and the impact on flavour precision observables, see for instance Ref. [19] and references therein. Interestingly, a large mixing between stops and charginos could explain some recent flavour anomalies [20]. It is also known that large flavour mixing involving stops can have an important consequence for the Higgs mass calculation at one-loop [21–23]. However, it was not yet studied how significant these effects can be at two loops. We close this gap here. We shall show that the usually neglected two-loop corrections can shift the Higgs mass by several GeV.

This letter is organised as follows: in sec. II we introduce our conventions to parametrise flavour violation in the MSSM, before we show the numerical results in sec. III. We discuss the results in sec. IV.
II. THE MSSM WITH GENERAL FLAVOUR VIOLATION

We shortly introduce our conventions for the discussion in the following. We stick closely to the SLHA 2 conventions for the definition of our basis [24], and the superpotential reads

\[ W = Y_u^{ij} \tilde{L}_i \tilde{E}_j \tilde{H}_d + Y_d^{ij} \tilde{Q}_i \tilde{D}_j \tilde{H}_d + Y_u^{ij} \tilde{Q}_i \tilde{U}_j \mu + \mu \tilde{H}_u \tilde{H}_d \]  

(1)

where the sums over colour and isospin indices are implicit. In general, the Yukawa couplings \( Y_X (X = e, d, u) \) are 3 \times 3 complex matrices. However, since there is no source of lepton flavour violation in the MSSM, \( Y_e \) has to be diagonal: \( Y_e = \text{diag}(y_e, y_e, y_e) \). Moreover, one can always perform a rotation into the super-CKM basis where quark Yukawa couplings become diagonal as well:

\[ Y_d = \text{diag}(y_d, y_d, y_b), \quad Y_u = \text{diag}(y_u, y_d, y_u) \]  

(2)

The entire information of flavour violation is then included in the CKM matrix \( V \) which is defined as

\[ V = (U_L^d)^\dagger U_L^d \]  

(3)

where \((U_L^d)\) and \((U_R^u)\) rotate the left-handed down- and up-quarks which are assumed to be aligned with the corresponding superfields.

The soft-SUSY breaking sector of the model is parameterized by

\[ -\mathcal{L} = (T_u^{ij}\tilde{E}_i \tilde{H}_d + T_d^{ij}\tilde{Q}_i \tilde{D}_j \tilde{H}_d + T_u^{ij}\tilde{Q}_i \tilde{U}_j \mu + B_u \tilde{H}_d \tilde{H}_u + \text{h.c.}) 
+ (M_1 \lambda_B \lambda_B + M_2 \lambda_W \lambda_W + M_3 \lambda_G \lambda_G + \text{h.c.}) 
+ m_{\phi,ij}^2 \tilde{\phi}^* \tilde{\phi} + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 \]  

(4)

with \( \phi = u, d, q, e, l \). In the limit of minimal flavour violation, \( T_i = A_i Y_i \) \((i = d, u, e)\) would hold, but we want to study explicit deviations from this. We concentrate in the following on the up-squark sector. In general, the mass matrix squared for the six up-squarks, \( M_U^2 \), in the basis \((\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)\) is given by

\[ M_U^2 = \begin{pmatrix} V m_d^2 V^\dagger + \frac{1}{2} v_u^2 Y_u^2 + D_{LL} & X^\dagger \\ X & m_u^2 + \frac{1}{2} v_u^2 Y_u^2 + D_{RR} \end{pmatrix} \]  

(5)

with the 3 \times 3 matrix

\[ X = -\frac{v_d}{\sqrt{2}} \mu^* Y_u + \frac{v_u}{\sqrt{2}} T_u \]  

(6)

and the \( D \)-term contributions are expressed in diagonal matrices \( D_{LL} \) and \( D_{RR} \). We assume further that the only sources of additional flavour violation are the \((2,3)\) and \((3,2)\) entries of \( T_u \) and that \( T_u \) as well as \( T_{u,11} \) are vanishing. In this case, we can parametrize the squark sector by:

\[ m_{u,33}, m_{u,22}, m_{q,33}, m_{q,22} \]

\[ \tilde{m} \equiv m_{q,11} = m_{u,11} = m_{d,ii} \]

\[ T_{u,33}, T_{u,32}, T_{u,23} \]

\[ \mu, \tan \beta \]

For simplicity, we assumed a universal mass \( \tilde{m} \) for all squarks not mixing with the stops, and take this value also for all slepton soft masses. The other remaining parameter is the gluino mass \( M_3 \), which will be important in the following.

III. NUMERICAL RESULTS

For the numerical analysis we make use of the combination of the public codes SPheno [25, 26] and SARAH [27, 32]. SARAH generates all necessary routines to calculate the Higgs mass in a given model at the two-loop order [33, 34]. The only approximations made are that (i) the gaugeless limit is used, i.e. electroweak contributions are neglected, (ii) the momentum dependence is not included. However, all generations of (S)fermions are automatically taken into account. We will refer to this calculation as \( m_h^{\text{approx}} \) in the following, keeping in mind that these provisos exist. It has been shown that the obtained results for the MSSM are in perfect agreement with widely used results of Refs. [33, 39], if first and second generation (S)quarks are neglected in the generated routines (this will be called \( m_h^{\text{approx}} \) in the following). If they are taken into account in the limit of minimal flavour violation, the differences are still very small. We shall study what happens if we are far away from minimal flavour violation.

A. Exploring the MSSM with large stop flavour violation

We fix in the following the parameters which have a less important impact on the two-loop Higgs mass corrections as follows:

\[ M_1 = 100 \text{ GeV}, M_2 = 200 \text{ GeV}, \tilde{m} = 1500 \text{ GeV} \]

\[ \mu = 500 \text{ GeV}, M_3^2 = (1000 \text{ GeV})^2, \tan \beta = 10 \]

For the other parameters, we scan over the following ranges:

\[ M_3 \in [1, 3] \text{ TeV} \]

\[ m_{u/q,33} \in [0.2, 2] \text{ TeV}, m_{u/q,22} \in [1.2, 2.5] \text{ TeV} \]

\[ T_{u,ij} \in [-4, 4] \text{ TeV} (i, j = 2, 3) \]

and keep only points where the exact calculation including all generation of squarks at two-loop \((m_h^{\text{exact}})\) is larger than 120 GeV.
FIG. 1. $\delta m_h$ of the point with the maximal $|\delta m_h|$ per bin is shown, as function of different ratios of important soft-breaking parameters.

The overall results of the scan are summarised in Fig. 1. The specific settings in SPheno used for the two values of the Higgs mass are in the Flag 8 of Block SPhenoInput: the value is set to 3 for $m_h^{\text{exact}}$ (diagrammatic calculation) and to 9 for $m_h^{\text{approx}}$ (2-loop dominant, 3rd generation contributions; using routines based on Refs. [35–39]).
(i) A necessary condition for a large deficit (of several GeV) in $m_{h}^{\text{approx}}$ (i.e. $\delta m_{h} > 0$) is a large hierarchy between the third and second generation of the soft-masses $m_{q}$ or $m_{u}$. In particular, there is a preferred region visible around $r_{u} = 0.8$ or $r_{q} = 0.8$ in Fig. 2 (lower). On the other side, if this ratio is about 0.4 or larger, one finds that $m_{h}^{\text{approx}}$ is larger than $m_{h}^{\text{exact}}$ ($\delta m_{h} < 0$). This also becomes visible in Fig. 3, where we show $\delta m_{h}$ (for the maximal $|\delta m_{h}|$ per bin) as a function of $\min(r_{u}, r_{q})$, and in Fig. 2 (upper), where the bulk of points is within the area of $r_{u} \geq 0.4, r_{q} \geq 0.4$.

(ii) In the case that the gluino is lighter than the second generation of soft-masses, $m_{h}^{\text{exact}}$ is often larger than $m_{h}^{\text{approx}}$, while for a heavier gluino the additional corrections from flavour violation are negative.

(iii) The sign of the additional corrections depends strongly on the ratio of $T_{u,33}$ and the two off-diagonal couplings $T_{u,32}$ and $T_{u,23}$. If $|T_{u,32}|$ or $|T_{u,23}|$ are much bigger than $|T_{u,33}|$, the flavoured two-loop corrections are usually large and positive. Negative corrections appear in particular for the case that $\max(|T_{u,32}|, |T_{u,23}|) \simeq |T_{u,33}|$. This is shown in Fig. 1 (lower) and also in Fig. 4.

B. Examples

To further investigate the dependence on the different parameters, we pick two parameter points where the flavour effects at two-loop give either a positive or negative shift to the Higgs mass.

FIG. 3. $\delta m_{h}$ as function of $\min(m_{x,33}/m_{x,22})$ ($x = q, u$).

FIG. 4. $\delta m_{h}$ as function of $T_{u,33}/\max(T_{u,32}, T_{u,23})$, where $\max$ picks the entry whose absolute value is larger independent of the sign.

1. Positive contributions from flavour effects

The input parameters of the first example are

\begin{align*}
    m_{u,33} & = 300 \text{ GeV}, \quad m_{q,33} = 2000 \text{ GeV} \\
    m_{u,22} & = m_{q,22} = 2300 \text{ GeV} \\
    T_{u,33} & = T_{u,32} = -1800 \text{ GeV}, \quad T_{u,23} = 0, \\
    M_{3} & = 1550 \text{ GeV},
\end{align*}

Depending on the used two-loop calculation, we find the following values for the SM-like Higgs mass

\begin{align*}
    m_{h}^{\text{exact}} & = 123.1 \text{ GeV} \quad (9) \\
    m_{h}^{\text{approx}} & = 121.1 \text{ GeV} \quad (10)
\end{align*}

Thus, the approximation to consider only the third generation (s)quark effects at two-loop gives a result which is 2 GeV too small compared to the exact calculation. This is not one of the points which maximizes the difference between both calculations, but it can be used to see nicely the dependence on the different parameters as shown in Fig. 5, where the difference between both calculations quickly increases for smaller $M_{3}$ and $m_{u,33}$ as well as for increasing $|T_{u,32}|$. For decreasing $|T_{u,33}|$ the change in sign can also be observed.

There is one final comment at place: it is known that large trilinear couplings in the squark sector together with a sizeable splitting in the soft-masses can trigger charge and colour breaking \cite{40,44}. We checked the vacuum stability of this point with Vevacious \cite{15} with the possibility that the second and third generation of up-squarks can receive vacuum expectation values. We actually found that this happens at the global minimum of the scalar potential. However, the lifetime of this point calculated with CosmoTransitions \cite{46} turns out to be
FIG. 5. $m_h^{\text{exact}}$ (solid blue) and $m_h^{\text{approx}}$ (dashed red) as functions of $T_{u,32}$, $T_{u,33}$, $m_{u,33}$ and $M_3$. The other parameters are fixed to the values in eq. (8).

FIG. 6. $\delta m_h$ in the $(T_{u,32}, T_{u,33})$ and $(m_{u,33}, M_3)$ plane. The other parameters are fixed to the values in eq. (11).

2. Negative contributions from flavour effects

As second example we choose the point given by

\[ m_{u,33} = 720 \text{ GeV}, \quad m_{q,33} = 875 \text{ GeV} \]
\[ m_{u,22} = m_{q,22} = 2500 \text{ GeV} \]
\[ T_{u,33} = 1200 \text{ GeV}, \quad T_{u,32} = -1900 \text{ GeV}, \quad T_{u,23} = 0, \]
\[ M_3 = 2600 \text{ GeV}, \]  

(11)

The scalar potential is even stable for this point and charge/colour is unbroken at the global minimum. The approximate calculation turns out to predict a Higgs mass which is too large by about 3 GeV

\[ m_h^{\text{exact}} = 121.2 \text{ GeV} \]  
\[ m_h^{\text{approx}} = 124.0 \text{ GeV} \]  

(12) (13)

Thus, the estimate of 3 GeV for the theoretical uncertainty for the standard two-loop calculations is obviously
too small because the missing flavour effects are already of this size. We show the dependence on the trilinear squark couplings $T_{u,32}$ and $T_{u,33}$ as well as on $m_{u,33}$ and $M_3$ in Fig. 6.

Note, in the regions with large $|T_{u,32}|$ in the upper plot in Fig. 6 where $\delta m_h$ is very large, the electroweak potential becomes metastable and even short-lived. So, the constraints from charge and colour breaking minima are actually larger than the ones from flavour observables.

IV. DISCUSSION

We have analysed the effect of large flavour-mixing on the Higgs mass calculation, and compared it to the approximation that only the third generation contributes, finding that the discrepancy can be several GeV for parameter points that are consistent with all other observations. The size and the sign of the flavoured two-loop contributions depends mainly on the hierarchy in the soft-breaking squark masses, the size of the flavour violation trilinear soft-terms and the gluino mass. This raises several questions:

1. **Are the corrections proportional to the full Yukawa couplings?** To investigate this, we recalculated the corrections with only the top/bottom mass terms in the Yukawa couplings non-zero, and found very little difference.

2. **Are the differences mostly in $\alpha_t^2$ or $\alpha_t\alpha_s$ corrections?** By comparing results using specially modified versions of our code we have compared the “exact” and third-generation only results for the strong corrections only, and found that as usual the strong corrections are largest and thus exhibit the largest differences.

3. **Can the differences be explained by a one-loop shift in the stop masses?** A guess for the order of magnitude of two-loop corrections is to insert the one-loop corrected stop masses into the expression for the one-loop Higgs mass:

$$\delta^2m_h^2 \sim \delta^1m_h^2(m_{\tilde{t}_i}^{1L}) - \delta^1m_h^2(m_{\tilde{t}_i}^{DR}) - \delta^1m_h^2(m_{\tilde{t}_i}^{1L})_{|T_{23}=T_{32}=0}.$$  

Therefore, we should expect that there should be a relationship between the shifts in the stop masses with and without flavour-violating $A$-terms, and the two-loop $\delta m_h$:

$$\delta m_h^{(1L)} \sim \delta^1m_h^2(m_{\tilde{t}_i}^{1L}) - \delta^1m_h^2(m_{\tilde{t}_i}^{1L})_{|T_{23}=T_{32}=0}.$$  

However, as we show in Fig. 7 there is no correlation, and in fact the shift in the Higgs mass computed from this guess is notably smaller than that which we obtain in the exact expression (we even computed Fig. 7 using a top mass of 173 GeV).

A more precise way of phrasing the above question would be: **can $\delta m_h$ be explained by passing from the DR-scheme to on-shell scheme for stop masses?** Although the above results would hint that this is not the case, it would nevertheless be interesting to explore in future work.

From considering the above, we conclude that a sizeable contribution to $\delta m_h$ arises from the new diagrams involving the trilinear couplings $T_{23}, T_{32}$ mixing the generations, and that the effects can not be simply obtained from the existing approximate expressions. Hence, once these trilinear terms have magnitude comparable to the other soft terms we can no longer trust the...
third-generation-only approximation even for the two-loop mass calculation.

It would be interesting to consider consistent models realising such substantial flavour violation terms from a top-down perspective (along the lines of e.g. [9]). Moreover, since the result is largely independent of the charm mass, models mixing the stop and sup should yield very similar results.

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