Cooperative Algorithms for MIMO Amplify-and-Forward Relay Networks

Kien T. Truong*, Student Member, IEEE, Philippe Sartori, and Robert W. Heath, Jr., Fellow, IEEE

Abstract

Interference alignment is a signaling technique that provides high multiplexing gains in interference channels. It can be extended to multi-hop interference channels, where relays aid transmission between sources and destinations. In addition to coverage extension and capacity enhancement, relays enhance the multiplexing gain in the interference channel. In this paper, three cooperative algorithms are proposed for a multiple-antenna amplify-and-forward two-hop interference channel. The algorithms use the spatial dimensions at the transmitters and the relays to forward interference to the receivers so that interference can be aligned and canceled. The first algorithm minimizes the sum power of enhanced noise from the relays and interference at the receivers. The second and third algorithms rely on a connection between mean square error and mutual information to solve the end-to-end sum-rate maximization problems with either equality or inequality power constraints via matrix-weighted sum mean square error minimization. The resulting iterative algorithms converge to stationary points of the corresponding optimization problems. Our simulations show that amplify-and-forward relays achieve higher end-to-end sum-rates and multiplexing gains than decode-and-forward relays and direct transmission.

Index Terms

Interference alignment, relay-aided interference alignment, two-hop interference channel, relay interference channel, relay beamforming, joint source-relay design.

The authors are with the Department of Electrical and Computer Engineering, 1 University Station, C0806, The University of Texas at Austin, Austin, TX, 78712-0240 (email: kientruong@utexas.edu and rheath@ece.utexas.edu, phone: (512) 686 8225, fax: (512) 471 6512).

P. Sartori is with Huawei Technologies, Inc. (email: philippe.sartori@huawei.com)

This work was supported by a gift from Huawei Technologies, Inc.
I. INTRODUCTION

Two-hop interference channels model a network where a stage of intermediate nodes, called relays, help multiple transmitters communicate with their receivers using shared radio resources [1]–[4]. Relay communication is considered a cost-effective solution for coverage extension and capacity enhancement [5], [6]. Recent results on interference alignment show that the single-hop interference channel (without relays) is not interference limited [7]–[11]. Although these single-hop results can be applied separately for the transmitter-relay hop and for the relay-receiver hop, even higher sum-rates can be achieved if the relays are configured jointly [12], [13]. Obtaining the most from two-hop interference channels requires advanced interference management strategies that jointly configure the transmitters, relays, and receivers.

Multiplexing gain is an important performance metric of interference networks. The multiplexing gain, also known as the total number of degrees of freedom, of a network is a first-order approximation of its sum capacity at high signal-to-noise ratio (SNR) [14]. Interference alignment is a multiplexing gain maximizing signaling technique for the single-hop interference channel [11]. The idea is to arrange the transmitted signals such that interference is constrained within only a portion of the signal space observed by each receiver, leaving the remaining portion for interference-free detection of the desired signal [7]. The maximum multiplexing gain achievable through interference alignment, however, depends on the characteristics of interference channels. For a symmetric multiple-input multiple-output (MIMO) interference channel with constant channel coefficients, the maximum multiplexing gain is upper-bounded by the sum of the number of antennas at the transmitter and the receiver of a pair regardless the number of pairs [9], [10]. Note that the bound is tight in certain cases and corresponds to the total available spatial dimensions of a pair. Increasing the number of spatial dimensions in the network, using for example relays, is one way to improve the maximum achievable multiplexing gain.

Relays can be classified based on their signal processing operation, among which the most popular are decode-and-forward (DF - the relays decode the received signals then re-encode before retransmitting) and amplify-and-forward (AF - the relays apply linear signal processing to the signal before forwarding). Without decoding the received signals, AF relays need no knowledge of the codebooks used by the transmitters and require less computational power. Transparent to the modulation and coding of the signals, AF relays are more suitable for applications in heterogeneous networks comprising many nodes of different complexity or even standards [15]. Thus, AF relays may be attractive in practice thanks to lower complexity, higher flexibility, and faster signal processing. In this paper, we focus on a MIMO
two-hop interference channel where the transmitters must ask for the help of a stage of half-duplex AF relays to send data to the receivers. Since half-duplex relays cannot transmit and receive at the same time, thus they are more practical than full-duplex relays.

Several interference management strategies designed specifically for the one-way AF two-hop interference channel have been proposed in [13], [16]–[27]. Although relays cannot improve the multiplexing gains of the single-antenna fully-connected interference channel with time-varying or frequency-selective channel coefficients [16], they are beneficial for reducing the number of independent channel extensions needed to align interference at the receivers [17]. Prior work often considers networks operating in special circumstances. It is assumed in [13], [18]–[22] that there are enough antennas at the relays to cancel all interference on the reception and then to nullify all interference on the retransmission, allowing multiplexing gains to scale linearly with the number of users. Other prior work considers only small networks with up to three pairs to derive some kind of closed-form strategy [23]–[25]. Prior work in [26] considers design problems with different objective functions including sum power minimization and minimum SINR maximization. In this paper, we consider a general system model in the sense that we assume no special constraints on the number of relays or the number of antennas at a relay. The closest AF relay model to ours is considered in [27], which is only for single-antenna transmitters and receivers. Further, the authors of [27] assume no crosslinks from relays to receivers, resulting in a significantly simplified design problem.

We develop three cooperative interference management algorithms for the MIMO AF two-hop interference channel with constant channel coefficients. We assume that global channel state information (CSI) is available at a central processing unit for designing jointly the transmitters, relays and receivers. The first algorithm is an extension of the total leakage minimization interference alignment algorithm for the single-hop interference channel in [17], [28]–[30]. Note that the leakage signals in AF relay interference networks consist of interference signals and the enhanced noise from the relays. The first algorithm is useful for gaining insights into the interference alignment feasibility of the MIMO AF two-hop interference channel. The second algorithm aims at finding directly a stationary point of the end-to-end sum-rate maximization problem with equality power constraints; while the third algorithm deals specifically with inequality power constraints. Similar to the single-hop results in [31], [32], our second and third algorithms are inspired by a connection between mutual information and mean square error (MSE). The formulated optimization problems are non-convex and NP-hard; finding their globally
optimal solutions is challenging. We propose to use alternating minimization where in each iteration we fix all but one variable and focus on determining the remaining variable. We are able to prove that the proposed algorithms always converge to stationary points of the associated optimization problems. Note that the power constraints at the relays depend on both the transmit precoders at the transmitters and the processing matrices at the relays, adding more constraints to the design problems. Thus, it is not straightforward to extend the methods used for the single-hop design problems to solve the two-hop design problems. Our initial results in this paper were reported in [33]. Compared with [33], our paper presents three different algorithms, has more discussion of convergence and provides simulations that emphasize the achievable end-to-end sum-rates and multiplexing gains.

We use Monte Carlo simulation to evaluate the average end-to-end sum-rates and multiplexing gains achievable through the proposed algorithms. First, the numerical results confirm the convergence of the proposed algorithms as we expect since the proposed algorithms are able to find the global optimum of the corresponding single-variable optimization problems in each iteration. Second, we observe that over the iterations of the total leakage minimization algorithm, the true interference is dominant at the beginning, but it is canceled quickly; after that, the enhanced noise from the relays becomes dominant. This means that relay-aided interference alignment should take into account the enhanced noise from the relays. Third, the total leakage minimization algorithm achieves lower average end-to-end sum-rates than the others at low-to-medium SNR values because it ignores the desired signal power and noise power at the receivers. Nevertheless, the MSE-based algorithms result in unfairness, i.e., some users have much smaller rates than the others. Thus, the MSE-based algorithms achieve lower average end-to-end sum-rates and multiplexing gains than the total leakage minimization algorithm at high SNR. One reason for this is that the MSE-based algorithms are not guaranteed to find the global optima of the end-to-end sum-rate maximization problems. Fourth, we show that for fixed numbers of antennas at the transmitters and at the receivers, even with half-duplex loss, AF relays can provide larger end-to-end multiplexing gains than DF relays or direct transmission. Finally, the results show that AF relays provide larger average achievable end-to-end sum-rates than DF relays. The proposed algorithms also provide higher achievable end-to-end sum-rates than the baseline AF relaying strategies that do not align interference at the receivers.

The organization of the remainder of this paper is as follows. Section II describes the system model. The proposed algorithms are presented in detail in Section III and Section IV. Section V discusses several properties of the proposed algorithms. The end-to-end sum-rates and multiplexing gains achievable
through the proposed algorithms are evaluated numerically in Section VI. Section VII concludes this paper and suggests future research.

Notation: We use normal letters (e.g., \( a \)) for scalars, lowercase and uppercase boldface letters (e.g., \( h \) and \( H \)) for column vectors and matrices. \( I_N \) and \( 0_N \) are the identity matrix and all-zero matrices of size \( N \times N \). \( \nu_{\text{min}}^n (A) \) gives the eigenvectors corresponding to the \( n \) smallest eigenvalues of \( A \). For a matrix \( A \), \( A^T \) is the transpose matrix, \( \|A\|_F^2 \) the Frobenious norm, \( A^* \) the conjugate transpose, and \( \text{tr}(A) \) the trace. \( \text{vec}(A) \) denotes the vec operator to transform \( A \) into a while \( \text{vec}^{-1}(a) \) denotes the inverse operator. \( \otimes \) is the Kronecker product. \( \mathbb{E}[\cdot] \) is the statistical expectation operator. \( (\cdot)^{(n)} \) denotes iteration index. \( (\cdot)_T \) is used for transmitters’ parameters, \( (\cdot)_R \) for receivers’, and \( (\cdot)_X \) for relays’.

II. System Model

Consider a two-hop interference channel where \( M \) half-duplex AF relays aid the one-way communication between \( K \) pairs of transmitters and receivers, as illustrated in Fig. 1. Each transmitter has data for only one receiver and vice versa. Each pair is assigned a unique index \( k \in \mathcal{K} \triangleq \{1, \cdots, K\} \). Transmitter \( k \) has \( N_{T,k} \) antennas while receiver \( k \) has \( N_{R,k} \) antennas for \( k \in \mathcal{K} \). Similarly, each relay is assigned a unique index \( m \in \mathcal{M} \triangleq \{1, \cdots, M\} \). Relay \( m \) has \( N_{X,m} \) antennas for \( m \in \mathcal{M} \). The half-duplex relays cannot transmit and receive at the same time, thus the transmission procedure consists of two stages. In the first stage, the transmitters send data to the relays. In the second stage, the relays apply linear processing to the received signals and forward to the receivers. We assume the direct channels between the transmitters and the receivers are ignored by the second-stage receivers.

We denote \( H_{m,k} \in \mathbb{C}^{N_{X,m} \times N_{T,k}} \) as the matrix channel from transmitter \( k \) to relay \( m \) and \( G_{k,m} \in \mathbb{C}^{N_{R,k} \times N_{X,m}} \) as the matrix channel from relay \( m \) to receiver \( k \) for \( k \in \mathcal{K} \) and \( m \in \mathcal{M} \). We assume
that knowledge of $H_{m,k}$ and $G_{k,m}$ for $k \in \mathcal{K}$ and $m \in \mathcal{M}$ is available at a central processing unit. Let $s_k \in \mathbb{C}^{d_k \times 1}$ be the transmit symbol vector at transmitter $k$, where $d_k \leq \min\{N_{T,k}, N_{R,k}\}$ is the number of data streams from transmitter $k$ to receiver $k$ for $k \in \mathcal{K}$. The transmit symbols are independent identically distributed (i.i.d.) such that $\mathbb{E}(s_k s_k^*) = I_{d_k}$. Transmitter $k$ uses a linear transmit precoder $F_k \in \mathbb{C}^{N_{T,k} \times d_k}$ to map $s_k$ to its transmit antennas. The transmit power at transmitter $k$ is $P_{T,k} = \text{tr}(F_k^* F_k)$. Let $P_{T,k}^{\text{max}}$ be the maximum transmit power.

Let $n_{X,m}$ be spatially white, additive Gaussian noise at relay $m$ with covariance $\mathbb{E}(n_{X,m} n_{X,m}^*) = \sigma^2_{X,m} I_{N_{X,m}}$ for $m \in \mathcal{M}$. With perfect synchronization, relay $m$ observes the following signal

$$y_{X,m} = \sum_{k=1}^{K} H_{m,k} F_k s_k + n_{X,m}. \quad (1)$$

Let $U_m \in \mathbb{C}^{N_{X,m} \times N_{X,m}}$ be the processing matrix at relay $m$. The transmit signal at relay $m$ is given by

$$x_{X,m} = U_m y_{X,m} = \sum_{k=1}^{K} U_m H_{m,k} s_k + U_m n_{X,m}. \quad (2)$$

Thus, the transmit power at relay $m$ is computed as follows

$$P_{X,m} = \sum_{k=1}^{K} \text{tr}(U_m H_{m,k} H_{m,k}^* U_m^*) + \sigma^2_{X,m} \text{tr}(U_m U_m^*). \quad (3)$$

There are two possible types of power constraints at the relays: i) a set of individual power constraints at the relays and ii) a sum power constraint at all the relays. Per-relay power constraints are often considered in work on cellular systems [34], [35]. In work on ad hoc networks, a sum power constraint may be considered to extend the lifetime of battery-powered relays [36]–[38]. We denote the maximum transmit power at relay $m$ as $P_{X,m}^{\text{max}}$ and the maximum sum transmit power at all the relays at $P_X^{\text{max}}$. When power control is considered, the per relay power constraints are

$$P_{X,m} \leq P_{X,m}^{\text{max}}, \forall m \in \mathcal{M}, \quad (4)$$

whereas the sum relay power constraint is

$$\sum_{m=1}^{M} P_{X,m} \leq P_X^{\text{max}}. \quad (5)$$

Without power control, the inequalities in (4) and (5) are replaced by equalities. In the following sections, we focus on the sum relay power constraint. We discuss the reason for this assumption and the applicability of the per-relay power constraints in Section V.

We denote $n_{R,k}$ as spatially white, additive Gaussian noise at receiver $k$ with covariance $\mathbb{E}(n_{R,k} n_{R,k}^*) = \sigma^2_{R,k} I_{N_{R,k}}$. The transmit symbol vector at receiver $k$, where $d_k \leq \min\{N_{T,k}, N_{R,k}\}$ is the number of data streams from transmitter $k$ to receiver $k$ for $k \in \mathcal{K}$. The transmit symbols are independent identically distributed (i.i.d.) such that $\mathbb{E}(s_k s_k^*) = I_{d_k}$. Receiver $k$ uses a linear receive filter $Q_k \in \mathbb{C}^{N_{R,k} \times d_k}$ to map $s_k$ to its receive antennas. The receive power at receiver $k$ is $P_{R,k} = \text{tr}(Q_k^* Q_k)$. Let $P_{R,k}^{\text{max}}$ be the maximum receive power.
\[ \sigma^2_{R,k} I_{N_{R,k}}. \]

We denote \( G_{k,m} = G_{k,m} U_m \). Receiver \( k \) observes the following signal

\[ y_k = \sum_{m=1}^{M} G_{k,m} x_{m,k} + n_{R,k} \]

\[ = \sum_{q=1}^{K} \sum_{m=1}^{M} G_{k,m} H_{m,q} s_q + \sum_{m=1}^{M} G_{k,m} n_{X,m} + n_{R,k}, \]

where \( T_{k,q} \) is the effective end-to-end channel from transmitter \( q \) to receiver \( k \) for \( k,q \in K \). Applying a linear receive filter \( W_k \in \mathbb{C}^{N_{R,k} \times d_k} \) to \( y_k \), receiver \( k \) obtains

\[ \tilde{y}_k = W_k^* T_{k,k} s_k + \sum_{q=1}^{K} \sum_{q \neq k}^{K} W_k^* T_{k,q} s_q + \sum_{m=1}^{M} W_k^* G_{k,m} n_{X,m} + W_k^* n_{R,k}. \]

(6)

The interference-plus-noise covariance matrix at receiver \( k \) is

\[ R_k = \sum_{q=1}^{K} T_{k,q} T_{k,q}^* + \sum_{m=1}^{K} \sigma^2_{X,m} G_{k,m} G_{k,m}^* + \sigma^2_{R,k} I_{d_k}. \]

For notational convenience, we denote \( \{ F \} \triangleq \{ F_k \}_{k=1}^{K} \), \( \{ U \} \triangleq \{ U_m \}_{m=1}^{M} \) and \( \{ W \} \triangleq \{ W_k \}_{k=1}^{K} \), which are the main designed variables. We denote \( F_{-k} \triangleq \{ F_1, \cdots, F_{k-1}, F_{k+1}, \cdots, F_K \} \) for \( k \in K \).

We use \( \{ F \} \) and \( (F_k; F_{-k}) \) interchangeably. Similarly, we define \( U_{-m} \) for \( m \in M \) and \( W_{-k} \) for \( k \in K \).

We also denote \( U_{m,k} \triangleq U_m H_{m,k} \) and \( W_{k,m} \triangleq W_k^* G_{k,m} \) for \( k \in K \) and \( m \in M \).

### III. Total Leakage Minimization Algorithm

This section presents an interference alignment algorithm for the multiple-antenna amplify-and-forward relay interference channel, which is inspired by those for the single-hop interference channel in [17], [28]–[30]. The underlying observation for this approach is that when interference alignment is feasible, the sum power of the interference at all the receivers, also known as the leakage, is zero.

#### A. Total Interference Plus Enhanced Noise Power Minimization Problem Formulation

From (6), there are three groups of undesired signals at each receiver: i) interference, ii) enhanced noise from the relays, and iii) local noise. The interference depends on \( \{ F \}, \{ U \}, \{ W \} \), and the channels on two hops while the enhanced noise from the relays depends on only \( \{ U \}, \{ W \} \) and the channels on the second hop. Note that the enhanced noise from the relays can be treated as the sum of interference signals at the receivers of a virtual single-hop interference channel with \( (M + 1) \) users. We denote \( \mathcal{I}(\{ F \}, \{ U \}, \{ W \}) \) as the total interference power of the two-hop AF relay interference channel. By
evaluating the expectation and exploiting the independence of transmit signals $s_k$ for $k \in K$ and using the equality $\|A\|_F^2 = \text{tr}(AA^*)$, we obtain

$$I(\{F\}, \{U\}, \{W\}) = \sum_{k=1}^{K} \sum_{q=1}^{K} \mathbb{E} \|W_k^* T_{k,q} S_q\|_F^2$$

$$= \sum_{k=1}^{K} \sum_{q=1}^{K} \text{tr}(W_k^* T_{k,q}^* T_{k,q} W_k).$$

We denote $N(\{U\}, \{W\})$ as the sum power of enhanced noise from the relays. By evaluating the expectations and exploiting the independence of the noise vectors at the relays, we obtain

$$N(\{U\}, \{W\}) = \sum_{k=1}^{K} \sum_{m=1}^{M} \mathbb{E} \|W_k^* G_{k,m} n_{X,m}\|_F^2$$

$$= \sum_{k=1}^{K} \sum_{m=1}^{M} \sigma_{X,m}^2 \text{tr}(W_k^* G_{k,m}^* G_{k,m} W_k).$$

In our opinion, the high SNR regime of the relay interference channel corresponds to high transmit power at both the transmitters and the relays. As a result, in addition to eliminating completely interference, we also need to eliminate the enhanced relay noise; otherwise, the enhanced relay noise power scales with the desired signal power, preventing the system from achieving high multiplexing gains. In addition, note that scaling down the transmit power at either the transmitters or the relays decreases the total leakage power at the receivers. For example, if $\{(1/a)F\}, \{U\}, \{W\}$ is used instead of $\{F\}, \{U\}, \{W\}$ where $\{(1/a)F\} = \{(1/a)F_1, \cdots, (1/a)F_K\}$ and $a > 1$, then both the actual transmit power at the transmitters and the total leakage power decrease $a^2 > 1$ times. Therefore, we require equality power constraints at both the transmitters and at the relays to make it a meaningful design problem. Assuming the sum relay power constraint without power control, we propose to formulate the following total leakage minimization problem, denoted as $(\mathcal{TL})$, for the AF relay interference channel as follows

$$\min_{\{F\}, \{U\}, \{W\}} I(\{F\}, \{U\}, \{W\}) + N(\{U\}, \{W\})$$

subject to

$$\text{tr}(F_k^* F_k) = p_{T,k}^\text{max}, \ k \in K,$$

$$\sum_{m=1}^{M} \sum_{k=1}^{K} \text{tr}(F_k^* H_{m,k}^* U_m^* U_m H_{m,k} F_k) + \sum_{m=1}^{M} \sigma_{X,m}^2 \text{tr}(U_m U_m^*) = p_{X}^\text{max}.$$

Remark 1: $(\mathcal{TL})$ is nonconvex. In general, finding the globally optimal solution to $(\mathcal{TL})$ is NP-hard, i.e., it is impossible to find it with reasonable computational complexity.
Remark 2: \((\mathcal{T}\mathcal{L})\) is always feasible. Indeed, \((\mathcal{T}\mathcal{L})\) at least has the following feasible solution

\[
F_{0,k} = \sqrt{\frac{p_{\text{max}}^{T,k}}{d_k}} I_{N_{T,k} \times d_k}, k \in \mathcal{K},
\]

\[
W_{0,k} = \sqrt{\frac{1}{d_k}} I_{N_{U,k} \times d_k}, k \in \mathcal{K},
\]

\[
U_{0,m} = \sqrt{\alpha p_{\text{max}}^{X,m}} I_{N_{X,m} \times N_{X,m}}, m \in \mathcal{M},
\]

where \(\alpha = \left( \sum_{k=1}^{K} \frac{p_{\text{max}}^{T,k}}{d_k} \sum_{m=1}^{M} \text{tr}(H_{m,k}^{*} H_{m,k}) + \sum_{m=1}^{M} N_{X,m} \sigma_{X,m}^2 \right)^{-1}.\) Any feasible solution to \((\mathcal{T}\mathcal{L})\) can be used as initial solutions for our algorithm. Note that we claim the feasibility of \((\mathcal{T}\mathcal{L})\), but not the feasibility of interference alignment.

Remark 3: The total leakage minimization problem formulated in [27] for an AF relay network is a simplified version of \((\mathcal{T}\mathcal{L})\). It is assumed in [27] that the transmitters and receivers are equipped with a single antenna. Each pair is aided by a dedicated multiple-antenna AF relay. The formulation in [27] does not consider power constraints at the relays. In addition, it is assumed that there are no cross-links for the transmissions from relays to receivers, i.e. \(G_{k,q} = 0\) for all \(k, q \in \mathcal{K}\) and \(k \neq q\). As a result, for fixed \(\{F\}\) and \(\{W\}\), the algorithm in [27] can determine each \(U_m\) independently.

Remark 4: As in Section III.B in [30], we can always take into account the local noise in the design problem. Due to space limitations, however, we omit this discussion and claim that the extension is straightforward based on the results in this paper and those in [30].

B. An Alternating Minimization based Relay Interference Alignment Algorithm

We adopt an alternating minimization approach to develop an iterative algorithm to find a stationary point of \((\mathcal{T}\mathcal{L})\), which we refer to as Algorithm 1. In each iteration, we alternatively fix \((2K + M - 1)\) variables and determine the remaining variable by solving a single-variable optimization problem. The optimization problem in each iteration is always feasible since it has the outcome of the previous iteration as a feasible solution. After initialization with a feasible solution to \((\mathcal{T}\mathcal{L})\), the algorithm is repeated until a convergent point is reached. There are three classes of design subproblems in Algorithm 1: i) receiver filter design, ii) relay processing matrix design and iii) transmit precoder design. The following subsections present in detail how to solve these subproblems.

1) Receive Filter Design for \((\mathcal{T}\mathcal{L})\): We can rewrite the cost function as follows

\[
\mathcal{I}(\{F\}, \{U\}, \{W\}) + \mathcal{N}(\{U\}, \{W\}) = \sum_{k=1}^{K} \text{tr}(W_{k}^{*} Z_{k} W_{k}),
\]
where $Z_k = \sum_{q=1}^{K} T_{k,q} T_{k,q}^* + \sum_{m=1}^{M} \sigma_{X,k,m}^2 G_{k,m} G_{k,m}^*$. Since $W_k$ for $k \in K$ are decoupled in (7), when $\{F\}$ and $\{U\}$ are fixed, we can determine simultaneously each $W_k$ for $k \in K$ by solving

$$(T \mathcal{L} - W_k) : W_k = \arg\min_{X \in \mathbb{C}^{N_k \times d_k}} \text{tr}(X^*Z_k X).$$

It follows from [39] that a global optimum of $(T \mathcal{L} - W_k)$ is as follows

$$W_k = \nu_{\text{min}}^{d_k}(Z_k). \quad (7)$$

2) Relay Processing Matrix Design for $(T \mathcal{L})$: For some $m \in M$, fixing $\{F\}$, $\{W\}$ and $U_{-m}$, we focus on determining $U_m$ that minimizes $I(\{F\}, (U_{-m}, U_m), \{W\}) + N((U_{-m}, U_m), \{W\})$. After some manipulation, we obtain $(T \mathcal{L} - U_m)$, the single-variable optimization problem for designing $U_m$ from $(T \mathcal{L})$, as follows

$$\min_{X \in \mathbb{C}^{N_m \times 1}} \sum_{k=1}^{K} \sum_{q=1}^{K} \text{tr} \left( X \mathcal{H}_{m,q} \mathcal{H}_{m,q}^* X^* \mathcal{W}_{k,m}^* \mathcal{W}_{k,m} \right) + \sigma_{X,m}^2 \sum_{k=1}^{K} \text{tr} (X^* \mathcal{W}_{k,m}^* \mathcal{W}_{k,m} X)$$

$$+ 2 \Re \left\{ \sum_{k=1}^{K} \sum_{q=1}^{K} \sum_{n=1}^{M} \sum_{q \neq k}^{M} \text{tr} \left( X \mathcal{H}_{m,q} \mathcal{H}_{n,q}^* U_n^* \mathcal{W}_{k,n}^* \mathcal{W}_{k,n} \right) \right\}$$

subject to $\text{tr} \left( X \left( \sum_{k=1}^{K} \mathcal{H}_{m,k} \mathcal{H}_{m,k}^* + \sigma_{X,m}^2 I_{N_k,m} \right) X^* \right) = \eta_{U,m}$,

where

$$\eta_{U,m} = p_{X,m}^{\max} - \sum_{n=1}^{N_m} \sum_{n \neq m}^{M} \text{tr} (U_n \mathcal{H}_{n,k} \mathcal{H}_{n,k}^* U_n^*) - \sum_{n=1}^{M} \sum_{n \neq m}^{N_m} \sigma_{X,n}^2 \text{tr} (U_n U_n^*).$$

Because of the special form of the first term in the cost function of $(T \mathcal{L} - U_m)$, it is not straightforward to use the methods for the single-hop interference channel like those in [30] to solve $(T \mathcal{L} - U_m)$.

We propose a method for transforming $(T \mathcal{L} - U_m)$ into an equivalent optimization problem that is more readily solvable. We define a new variable $u_m = \text{vec}(U_m) \in \mathbb{C}^{N_k \times 1}$. We can obtain $U_m$ from $u_m$ by the $\text{vec}^{-1}$ operator. We define the following matrices that are independent of $u_m$

$$A_{1,m} = \sum_{k=1}^{K} \left( \sum_{q=1}^{K} \mathcal{H}_{m,q} \mathcal{H}_{m,q}^* + \sigma_{X,m}^2 I_{N_k,m} \right)^T \otimes (\mathcal{W}_{k,m}^* \mathcal{W}_{k,m})$$

$$A_{2,m} = \text{vec} \left( \sum_{k=1}^{K} \sum_{q=1}^{K} \sum_{n=1}^{M} \mathcal{W}_{k,m}^* \mathcal{W}_{k,n} U_n \mathcal{H}_{n,q} \mathcal{H}_{m,q}^* \right),$$

$$A_{3,m} = \left( \sum_{k=1}^{K} \mathcal{H}_{m,k} \mathcal{H}_{m,k}^* + \sigma_{X,m}^2 I_{N_k,m} \right)^T \otimes I_{N_k,m}. \quad (8)$$
Note that with probability one, \( A_{3,m} \) is Hermitian and positive definite while \( A_{1,m} \) is Hermitian and positive semidefinite. Then, we use the following equalities, \( \text{tr}(ABA^*C) = (\text{vec}(A))^*(B^T \otimes C) \text{vec}(A) \), \( \text{tr}(A^*BA) = \text{tr}(AIA^*B) = (\text{vec}(A))^*(I \otimes B) \text{vec}(A) \) and \( \text{tr}(AB^*) = (\text{vec}(B))^* \text{vec}(A) \) \cite{40}, to transform both the cost function and the constraint of \((T\mathcal{L} - u_m)\) into quadratic expressions of \( u_m \). We obtain \((T\mathcal{L} - u_m)\), a quadratically constrained quadratic program (QCQP) of \( u_m \), as follows

\[
 u_m = \arg \min_{x \in \mathbb{C}^{n^2 \times 1}} \quad x^* A_{1,m} x + a_{2,m}^* x + x^* a_{2,m} - \theta \eta u_m.
\]

subject to \( x^* A_{3,m} x = \eta u_m. \) (9)

To guarantee convergence, we need to find a global optimum of \((T\mathcal{L} - u_m)\). The existence of this global optimum is stated in Proposition [1].

**Proposition 1:** The problem \((T\mathcal{L} - u_m)\) has a unique globally optimal solution with probability one.

**Proof:** Let \( \theta \) be the Lagrange multiplier associated with the constraint (9). The corresponding Lagrangian function is defined as

\[
 \mathcal{L}_1(x, \theta) = x^*(A_{1,m} + \theta A_{3,m}) x + a_{2,m}^* x + x^* a_{2,m} - \theta \eta u_m.
\]

Any optimal solution to \((T\mathcal{L} - u_m)\) must satisfy the following KKT conditions

\[
 (A_{1,m} + \theta A_{3,m}) x + a_{2,m} = 0, \quad \text{(10)}
\]

\[
 x^* A_{3,m} x - \eta u_m = 0. \quad \text{(11)}
\]

Since \( A_{1,m} + \theta A_{3,m} \neq 0 \) with probability one, then it follows from (10) that

\[
 x = (-1)^* (A_{1,m} + \theta A_{3,m})^{-1} a_{2,m}. \quad \text{(12)}
\]

By substituting (12) into (11), we obtain

\[
 a_{2,m}^* (A_{1,m} + \theta A_{3,m})^{-1} A_{3,m} (A_{1,m} + \theta A_{3,m})^{-1} a_{2,m} = \eta u_m. \quad \text{(13)}
\]

We denote \( \mathcal{Z} = \{ z \in \mathbb{R} : A + zB \text{ is a positive definite matrix} \} \). The left-hand side of (13) has the form of \( g(z) = a^* (A + zB)^{-1} B (A + zB)^{-1} a \), where \( z \in \mathcal{Z} \), \( A, B \in \mathbb{C}^{n \times n} \), \( A \) is a positive semidefinite matrix, \( B \) is a positive semidefinite matrix, and \( a \in \mathbb{C}^{n \times 1} \) is a column vector. Note that there exists a nonsingular matrix \( S \) such that \( A = S C S^* \) and \( B = S D S^* \), where \( C = \text{diag}(c_1, \cdots, c_n) \) and \( D = \text{diag}(d_1, \cdots, d_n) \) are the diagonal matrices with \( c_i, d_i > 0 \) for \( i = 1, \cdots, n \) \cite{41}. If we denote \( S a = (y_1, \cdots, y_n) \), then we can rewrite

\[
 g(z) = \sum_{i=1}^{n} \frac{d_i}{c_i + zd_i} y_i^2. \quad \text{(14)}
\]
Thus, $g(z)$ is a monotonically decreasing function of $z \in \mathbb{Z}$, i.e., the left-hand side of (13) monotonically decreases in $\theta$. In addition, Remark 2 ensures that (13) always has a solution. Therefore, (13) has a unique solution of $\theta$, and equivalently, $(\mathcal{TL}-\mathbf{u}_m)$ has a unique global optimum.

Based on the proof of Proposition 1 we propose a method for finding $\mathbf{U}_m$ in three steps. First, we perform a simple 1-D search to find the unique solution $\theta^*$ of (13). Then we substitute $\theta^*$ into (12) to obtain the global optimum of $(\mathcal{TL}-\mathbf{u}_m)$. Finally, we use $\text{vec}^{-1}()$ to obtain $\mathbf{U}_m$ from $\mathbf{u}_m$.

3) Transmit Precoder Design for $(\mathcal{TL})$: We now focus on designing $\mathbf{F}_k$ for some $k \in K$ assuming that $\mathbf{F}_{-k}$, $\{\mathbf{U}\}$, and $\{\mathbf{W}\}$ are fixed. Conditioning on the other variables, we obtain $(\mathcal{TL}-\mathbf{F}_k)$, the optimization problem from $(\mathcal{TL})$ to determine $\mathbf{F}_k$, as follows

$$
\min_{\mathbf{X} \in \mathbb{C}^{N_t,k \times N_s,k}} \quad \text{tr} \left( \mathbf{X}^* \left( \sum_{q=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{M} \mathbf{U}_{m,k}^* \mathbf{W}_{q,m}^* \mathbf{W}_{q,n} \mathbf{U}_{n,k} \right) \mathbf{X} \right)
$$

subject to

$$
\text{tr}(\mathbf{X}^* \mathbf{X}) = p_{T,k}
$$

$$
\text{tr} \left( \mathbf{X}^* \left( \sum_{m=1}^{M} \mathbf{U}_{m,k}^* \mathbf{U}_{m,k} \right) \right) = \eta_{F,k},
$$

where

$$
\eta_{F,k} = \frac{p_{T,k}^{\max}}{\sum_{q=1}^{K} \sum_{m=1}^{M} \text{tr} \left( \mathbf{F}_{q}^* \mathbf{U}_{m,q}^* \mathbf{U}_{m,q} \mathbf{F}_{q} \right) - \sum_{m=1}^{M} \sigma_{X,m}^2 \text{tr} \left( \mathbf{U}_m \mathbf{U}_m^* \right)}. \quad (15)
$$

In general, $(\mathcal{TL}-\mathbf{F}_k)$ is non-convex and NP-hard. Recall that the counterpart transmit precoder design problem in [30] has a single equality constraint, making it possible to find its globally optimal solution by using the Lagrange multiplier method with simple 1-D search. Nevertheless, $(\mathcal{TL}-\mathbf{F}_k)$ has two equality constraints, the use of the Lagrange multiplier method requires a more complicated 2-D search. Thus, $(\mathcal{TL}-\mathbf{F}_k)$ needs to be solved by another method.

Similar to Subsection III-B2, we propose a method for transforming $(\mathcal{TL}-\mathbf{F}_k)$ into an equivalent optimization problem and for solving for its global optimum. We start by defining a new variable $\mathbf{f}_k = \text{vec}(\mathbf{F}_k) \in \mathbb{C}^{N_{r,k}d_k \times 1}$. We also define the following matrices

$$
\mathbf{B}_{1,k} = \mathbf{I}_{d_k} \otimes \left( \sum_{q=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{M} \mathbf{U}_{m,k}^* \mathbf{W}_{q,m}^* \mathbf{W}_{q,n} \mathbf{U}_{n,k} \right),
$$

$$
\mathbf{B}_{2,k} = \mathbf{I}_{d_k} \otimes \left( \sum_{m=1}^{M} \mathbf{U}_{m,k}^* \mathbf{U}_{m,k} \right).
$$

Both $\mathbf{B}_{1,k}$ and $\mathbf{B}_{2,k}$ are Hermitian positive definite matrices. They are also independent of $\mathbf{f}_k$ and $\mathbf{f}_k^*$. Using $\text{tr}(\mathbf{A}^* \mathbf{B} \mathbf{A}) = (\text{vec}(\mathbf{A}))^* (\mathbf{I} \otimes \mathbf{B}) \text{vec}(\mathbf{A})$ [40], we transform $(\mathcal{TL}-\mathbf{F}_k)$ into the following equivalent
single-variable optimization problem

\[
(\mathcal{T} \mathcal{L}_k f_k) : f_k = \arg \min_{x \in \mathbb{C}^{N_T,k \times 1}} \quad x^* B_{1,k} x \\
\text{subject to} \quad x^* x = P_{T,k}^{\max}, \\
x^* B_{2,k} x = \eta_{F,k}.
\]

Note that \((\mathcal{T} \mathcal{L}_k f_k)\) is a complex-valued homogeneous QCQP with two equality quadratic constraints. Nevertheless, \((\mathcal{T} \mathcal{L}_k f_k)\) is still non-convex and NP-hard [42], [43].

In solving \((\mathcal{T} \mathcal{L}_k f_k)\), we introduce a new variable \(Y = xx^*\). It follows that \(Y\) is a rank-one Hermitian positive semidefinite matrix. In addition, since \(a^* B a = \text{tr}(Baa^*)\) for any matrix \(B\) and any vector \(a\) [41], we obtain an equivalent optimization problem of \((\mathcal{T} \mathcal{L}_k f_k)\) as follows

\[
(\mathcal{T} \mathcal{L}_k f_k^*) : \min_{Y \in \mathbb{C}^{N_T,k \times N_T,k}} \quad \text{tr}(B_{1,k} Y) \\
\text{subject to} \quad \text{tr}(Y) = P_{T,k}^{\max}, \\
\text{tr}(B_{2,k} Y) = \eta_{F,k}, \\
Y \succeq 0, \text{rank}(Y) = 1.
\]

While the cost function and all other constraints are convex, the rank constraint is nonconvex. This rank constraint is actually the main difficulty in solving \((\mathcal{T} \mathcal{L}_k f_k^*)\). Dropping this rank constraint, however, we obtain a relaxed version of \((\mathcal{T} \mathcal{L}_k f_k^*)\), which is a convex optimization problem and also known as a semidefinite relaxation (SDR) of \((\mathcal{T} \mathcal{L}_k f_k^*)\). Of particular interest, the relaxation is exact as stated in Proposition 2. This means that the SDR always has a rank-one global optimum.

**Proposition 2:** The SDR obtained from \((\mathcal{T} \mathcal{L}_k f_k^*)\) by relaxing the rank constraint is exact, i.e., it always has a rank-one global optimum.

**Proof:** The proof is immediate based on Theorem 3.2. in [43], which states that a complex-valued homogeneous QCQP with \(n\) constraints is guaranteed to have a global optimum with rank \(r \leq \sqrt{n}\). Therefore, having \(n = 2\) constraints, \((\mathcal{T} \mathcal{L}_k f_k^*)\) is guaranteed to have a rank-one global optimum.

The SDR of \((\mathcal{T} \mathcal{L}_k f_k^*)\) can be solved, to any arbitrary accuracy, in a numerically reliable and efficient manner by readily available (and currently free) software packages, e.g., the convex optimization toolbox CVX [44]. Because the SDR may have general-rank global optima besides its rank-one global optima, it is not guaranteed that solving the SDR by the available software packages provides a desired rank-one global optimum. Fortunately, we can always construct a rank-one global optimum of the SDR from any
of its general-rank global optimum, e.g., by using the rank reduction procedure in [43], which is an extension of the purification technique in [45]. This procedure also provides us with the decomposition of the rank-one global optimum, giving us $f_k$ from the resulting rank-one global optimum.

IV. MSE-BASED SUM-RATE MAXIMIZATION ALGORITHM

The algorithm in Section III aims at using the spatial dimensions for minimizing the total leakage power, but it does not take into account the desired signal power at the receivers. Therefore, it may not perform well in terms of end-to-end sum-rate maximization. In this section, we formulate end-to-end sum-rate maximization problems either with or without power control. We then develop algorithms for solving them based on a relationship between the end-to-end achievable rates and the MSE values at the receivers. Note that we are able to find stationary points of the sum-rate maximization problems, but not their global optima. Our simulation results show that the algorithms developed in the section outperform Algorithm 1 at low to medium SNR regimes at the expense of higher algorithmic complexity.

A. End-to-End Sum-Rate Maximization Problem Formulations

1) Mean Square Error Computation: From [6], the MSE of the estimate of $s_k$ based on $\tilde{y}_k$ is

$$MSE_k = \mathbb{E}(\|\tilde{y}_k - s_k\|^2) = \text{tr} \left( \left( W_k^* \mathcal{T}_{k,k} - I_{d_k} \right) \left( W_k - I_{d_k} \right) \right) + \sum_{q=1, q \neq k}^{K} \text{tr} \left( W_k^* \mathcal{T}_{k,q} \mathcal{T}_{k,q}^* W_k \right)$$

$$+ \sum_{m=1}^{M} \sigma_{X,m}^2 \text{tr} \left( W_k^* G_{k,m}^* G_{k,m} W_k \right) + \sigma_{R,k}^2 \text{tr} \left( W_k^* W_k \right)$$

$$= \text{tr} \left( W_k^* (\mathcal{T}_{k,k} \mathcal{T}_{k,k}^* + R_k) W_k \right) - \text{tr}(W_k^* \mathcal{T}_{k,k}) - \text{tr}(F_k^* \mathcal{T}_{k,k}^*) + d_k. \quad (18)$$

Note that $MSE_k$ depends on $\{F\}$, $\{U\}$, and $W_k$, but not on $W_{-k}$. We denote the MSE matrix for receiver $k$ as follows

$$E_k(\{F\}, \{U\}, W_k) = W_k^* (\mathcal{T}_{k,k} \mathcal{T}_{k,k}^* + R_k) W_k - W_k^* \mathcal{T}_{k,k} - \mathcal{T}_{k,k}^* W_k + I_{d_k}. \quad (19)$$

Thus, we have the following expression

$$MSE_k(\{F\}, \{U\}, W_k) = \text{tr} \left( E_k(\{F\}, \{U\}, W_k) \right). \quad (19)$$

It follows from (16) and (19) that $E_k$ is a Hermitian and positive semidefinite matrix for $k \in \mathcal{K}$. 

For a given set of \( \{F\} \) and \( \{U\} \), the receive filter \( W_k \) that minimizes \( \text{MSE}_k(\{F\}, \{U\}, W_k) \) can be determined based on the gradient of \( \text{MSE}_k \) with respect to \( W_k^* \), which is given by
\[
\frac{\partial \text{MSE}_k}{\partial W_k^*} = W_k (T_{k,k} T_{k,k}^* + R_k) - T_{k,k}.
\]
By solving \( \frac{\partial \text{MSE}_k}{\partial W_k^*} = 0 \), we actually obtain the linear MMSE receive filter
\[
W_k^{\text{MMSE}} = (T_{k,k} T_{k,k}^* + R_k)^{-1} T_{k,k}.
\]
Let \( E_k^{\text{MMSE}} \) be the MSE matrix corresponding to the use of \( W_k^{\text{MMSE}} \), which is a function of \( (\{F\}, \{U\}) \). By substituting (21) into (19), and applying the binomial inverse theorem [41], it follows that
\[
E_k^{\text{MMSE}} = I_d - T_{k,k}^* (T_{k,k} T_{k,k}^* + R_k)^{-1} T_{k,k}
\]
\[
= (I_d + T_{k,k} R_k^{-1} T_{k,k})^{-1}.
\]

2) Sum-Rate Maximization Problem Formulation: To make the analysis tractable, we assume that Gaussian signaling is used in the system. The maximum achievable rate for the transmission from transmitter \( k \) to receiver \( k \) when a linear receive filter is used at receiver \( k \) is given by
\[
R_k(\{F\}, \{U\}) = \log_2 \det (I_d + T_{k,k}^* T_{k,k} + R_k)^{-1}.
\]
The sum of the end-to-end achievable rates is defined as
\[
R_{\text{sum}}(\{F\}, \{U\}) = -\sum_{k=1}^{K} \log_2 \det (E_k^{\text{MMSE}}(\{F\}, \{U\})).
\]
For consistency with (TL), we consider first the end-to-end sum-rate maximization problem without power control, which is denoted as \( (\mathcal{SR}-\text{EQ}) \) and formulated as follows
\[
\min_{\{F\},\{U\}} -R_{\text{sum}}(\{F\}, \{U\})
\]
subject to
\[
\text{tr}(F_k F_k^*) = p_{T,k}^{\max}, k = 1, \cdots, K
\]
\[
\sum_{m=1}^{M} \sum_{k=1}^{K} \text{tr}(U_m H_{m,k} F_k F_k^* H_{m,k}^* U_m^*) + \sum_{m=1}^{M} \sigma_{X,m}^2 \text{tr}(U_m U_m^*) = p_{X}^{\max}.
\]
In general, \( (\mathcal{SR}-\text{EQ}) \) is nonconvex and finding a globally optimal solution to \( (\mathcal{SR}-\text{EQ}) \) is NP-hard. In this paper, we aim at developing an algorithm for finding at least a stationary point of \( (\mathcal{SR}-\text{EQ}) \) with reasonable computational complexity.

**Remark 5:** If we replace the equality constraints of \( (\mathcal{SR}-\text{EQ}) \) by the inequality constraints, then we obtain the end-to-end sum-rate maximization with power control, which we refer to as \( (\mathcal{SR}-\text{NEQ}) \).
3) Linking Sum-rate Maximization and Weighted Sum-MSE Minimization Problems: We notice that $R_{\text{sum}}$ can be expressed as a function of the MSE matrices at all the receivers. Similar expressions of sum-rates as functions of MSE matrices are obtained for other systems such as MIMO broadcast channel [46], MIMO interference broadcast channel [31], and two-way relay channel [47], [48]. This relationship allows us to formulate a matrix-weighted sum-MSE minimization problem that has the same optimal solutions as does $(SR)$. We introduce auxiliary weight matrix variables $\{V\} \triangleq (V_1, \cdots, V_K)$ with $V_k \in \mathbb{C}^{d_k \times d_k}$ for $k \in K$. We define the matrix-weighted sum-MSEs as follows

$$WMSE_{\text{sum}}(\{F\}, \{U\}, \{W\}, \{V\}) = \sum_{k=1}^{K} \left( \text{tr}(V_k E_k(\{F\}, \{U\}, \{W\})) - \log_2 \det(V_k) \right).$$

We formulate the following matrix-weighted sum-MSE minimization problem, denoted as $(WMSE\text{-EQ})$:

$$\min_{\{F\}, \{U\}, \{W\}, \{V\}} WMSE_{\text{sum}}(\{F\}, \{U\}, \{W\}, \{V\})$$

subject to

$$\text{tr}(F_k F^*_k) = p_{T,k}^\text{max}, \ k \in K$$

$$\sum_{m=1}^{M} \sum_{k=1}^{K} \text{tr}(U_m H_{m,k} F_k F^*_k H^*_m U^*_m) + \sum_{m=1}^{M} \sigma^2_{X,m} \text{tr}(U_m U^*_m) = p_X^\text{max}.$$ 

Note that $\{V\}$ appears only in the objective function of $(WMSE\text{-EQ})$. The key observation for solving $(SR\text{-EQ})$ via solving $(WMSE\text{-EQ})$ is stated in Proposition 3.

**Proposition 3:** $(WMSE\text{-EQ})$ is equivalent to $(SR\text{-EQ})$ in the sense that they have the same stationary points.

**Proof:** In $(WMSE\text{-EQ})$, $W_k$ appears only in the term $\text{tr}(V_k E_k(\{F\}, \{U\}, \{W\}))$ in the objective function. When the other parameters are fixed, it follows that the optimal linear receive filter for $(WMSE\text{-EQ})$ is also $W_k^{\text{MMSE}}$, which is given in [21]. Since $(WMSE\text{-EQ})$ and $(SR\text{-EQ})$ have the same constraints, then to check their equivalence, we only need to check if the differentials of their objective functions are the same. These differentials are given by

$$d(-R_{\text{sum}}) = \sum_{k=1}^{K} \text{tr}(E_k^{-1} dE_k),$$

$$dWMSE_{\text{sum}} = \sum_{k=1}^{K} \text{tr}(V_k dE_k) + \sum_{k=1}^{K} \left( \text{tr}(E_k dV_k) - \text{tr}(V_k^{-1} dV_k) \right).$$
For any fixed \( \{F\}, \{U\}, \) and \( \{W\} \), then \( E_k(\{F\}, \{U\}, W_k) \) is independent of \( \{V\} \) for \( k \in \mathcal{K} \). The optimal matrix weights \( \{V^{\text{opt}}\}_{k=1}^K \) of \( (WMSE\text{-EQ}) \) are given by

\[
V^{\text{opt}}_k(\{F\}, \{U\}, W_k^{\text{MMSE}}) = E_k^{-1}(\{F\}, \{U\}, W_k^{\text{MMSE}}), \tag{23}
\]

\[
= I_{d_k} + \mathcal{T}^{-1}_{k,k} R^{-1}_k \mathcal{T}_{k,k}.
\]

Thus, if \( V_k = V^{\text{opt}}_k(\{F\}, \{U\}, W_k^{\text{MMSE}}) \) for \( k \in \mathcal{K} \), then \( \text{tr}(E_k d V_k) - \text{tr}(V_k^{-1} d V_k) = 0 \) for \( k \in \mathcal{K} \), leading to \( d(-R_{\text{sum}}) = dWMSE_{\text{sum}} \).

Essentially, Proposition 3 states that any stationary point of \( (WMSE\text{-EQ}) \) is also a stationary point of \( (SR\text{-EQ}) \) and vice versa. Therefore, we propose to find a stationary point of \( (SR) \) indirectly via solving \( (WMSE\text{-EQ}) \) rather than solving directly the NP-hard problem \( SR \).

Remark 6: The matrix-weighted sum-MSE value \( WMSE_{\text{sum}}(\{F\}, \{U\}, \{W\}, \{V\}) \) is convex with respect to \( F_k \) for \( k \in \mathcal{K} \) if we always choose \( V_k = V^{\text{opt}}_k(\{F\}, \{U\}, W_k^{\text{MMSE}}) \) according to (23). Indeed, from (17), we can check that \( MSE_k \) is convex with respect to \( F_q \) for all \( k, q \in \mathcal{K} \). By construction, \( V_k = V^{\text{opt}}_k(\{F\}, \{U\}, W_k^{\text{MMSE}}) \) is a Hermitian and positive semidefinite matrix for \( k \in \mathcal{K} \). Then, by definition \( WMSE_{\text{sum}}(\{F\}, \{U\}, \{W\}, \{V\}) \) is also convex with respect to \( F_k \) for \( k \in \mathcal{K} \).

B. An MSE-based Algorithm for End-to-End Sum-Rate Maximization without Power Control

In this section we propose an algorithm for solving \( (WMSE\text{-EQ}) \). Specifically, adopting an alternating minimization approach, we develop an algorithm for finding a stationary point of \( (WMSE\text{-EQ}) \) as well as \( (SR\text{-EQ}) \), which we refer to as Algorithm 2. The design subproblems in the iterations of Algorithm 2 belong to one of the following four categories.

1) Matrix Weight Design for \( (SR\text{-EQ}) \): Since the matrix weights \( V^{\text{opt}}_k \) for \( k \in \mathcal{K} \) are independent of each other, they can be updated simultaneously based on (19) and (23) for given \( \{F\}, \{U\}, \) and \( \{W\} \).

2) Receive Filter Design for \( (SR\text{-EQ}) \): As discussed in the proof of Proposition 3 the optimal solution of \( (WMSE\text{-EQ}) \) requires the receivers use the linear MMSE receive filters \( W_k^{\text{MMSE}} \) given in (21). Similar to Algorithm 1, the receive filters can be updated simultaneously for given \( \{F\}, \{U\}, \) and \( \{V\} \).

3) Relay Processing Matrix Design for \( (SR\text{-EQ}) \): We need to determine \( U_m \) for some \( m \in \mathcal{M} \) assuming that \( \{F\}, \{W\}, \) \( \{V\} \), and \( U_{-m} \) are fixed. After some manipulation and with \( \eta_{U,m} \) defined in (8), we obtain \( (WMSE\text{-EQ-U}_m) \), the single-variable optimization problem for designing \( U_m \) from
(\text{WMSE-EQ}), as follows
\[
\min_{\mathbf{x} \in \mathbb{C}^{N \times m} \times N \times m} \sum_{k=1}^{K} \sum_{q=1}^{K} \text{tr} \left( \mathbf{X} \mathbf{H}_{m,q} \mathbf{H}^*_{m,q} \mathbf{X}^* \mathbf{W}_{k,m}^* \mathbf{V}_k \mathbf{W}_{k,m} \right) + \sigma^2_{\mathbf{x},m} \sum_{k=1}^{K} \text{tr} (\mathbf{X}^* \mathbf{W}_{k,m}^* \mathbf{V}_k \mathbf{W}_{k,m} \mathbf{X}) \\
-2\Re \left\{ \sum_{k=1}^{K} \sum_{q=1}^{K} \sum_{n=1}^{M} \sum_{q \neq k \neq n \neq m} \text{tr} \left( \mathbf{X} \mathbf{H}_{m,q} \mathbf{H}^*_{n,q} \mathbf{U}^*_n \mathbf{W}_{k,n}^* \mathbf{V}_k \mathbf{W}_{k,m} \right) \right\}
\]
subject to \[ \text{tr} \left( \mathbf{X}^* \left( \sum_{k=1}^{K} \mathbf{H}_{m,k} \mathbf{H}^*_{m,k} + \sigma^2_{\mathbf{x},m} \mathbf{I}_{N \times m} \right) \mathbf{X} \right) = \eta_{U,m}. \]

Note that (\text{WMSE-EQ-U}_m) differs from (\text{TL-U}_m) mainly due to the appearance of \( \mathbf{V}_k \) in the cost function. Thus, we follow the same steps in Subsection III-B2 to develop the method for solving for \( \mathbf{U}_m \).

In solving (\text{WMSE-EQ-U}_m), we define a new variable \( \mathbf{u}_m = \text{vec}(\mathbf{U}_m) \). We also define the following matrices that are independent of \( \mathbf{u}_m \) and \( \mathbf{u}^*_m \)
\[
\mathbf{C}_{1,m} = \sum_{k=1}^{K} \left( \sum_{q=1}^{K} \mathbf{H}_{m,q} \mathbf{H}^*_{m,q} + \sigma^2_{\mathbf{x},m} \mathbf{I}_{N \times m} \right)^T \otimes (\mathbf{W}_{k,m}^* \mathbf{V}_k \mathbf{W}_{k,m}),
\]
\[
\mathbf{c}_{2,m} = \text{vec} \left( -\sum_{k=1}^{K} \mathbf{W}_{k,m}^* \mathbf{V}_k \mathbf{H}^*_{m,k} + \sum_{n=1}^{M} \sum_{k=1}^{K} \mathbf{W}_{k,m}^* \mathbf{V}_k \mathbf{W}_{k,n} \mathbf{U}_n \mathbf{H}_{n,q} \mathbf{H}^*_{m,q} \right).
\]

Using the same manipulation as in Subsection III-B2 and denoting \( \mathbf{C}_{3,m} = \mathbf{A}_{3,m} \), which is defined in [8], we obtain an equivalent optimization problem of (\text{WMSE-EQ-U}_m) for determining \( \mathbf{u}_m \), which we refer to as (\text{WMSE-EQ-u}_m), as follows
\[
\mathbf{u}_m = \arg \min_{\mathbf{x} \in \mathbb{C}^{N \times m}} \mathbf{x}^* \mathbf{C}_{1,m} \mathbf{x} + \mathbf{c}_{2,m}^* \mathbf{x} + \mathbf{x}^* \mathbf{c}_{2,m}
\]
subject to \[ \mathbf{x}^* \mathbf{C}_{3,m} \mathbf{x} = \eta_{U,m}. \quad (24) \]

Note that (\text{WMSE-EQ-u}_m) has the same form as (\text{TL-u}_m), thus its global optimum can be found using the Lagrange multiplier method. First, we find the positive Lagrange multiplier \( \beta^* \) associated with the constraint (24) by solving the following equation
\[
\mathbf{c}_{2,m}^* (\mathbf{C}_{1,m} + \beta \mathbf{C}_{3,m})^{-1} \mathbf{C}_{3,m} (\mathbf{C}_{1,m} + \beta \mathbf{C}_{3,m})^{-1} \mathbf{c}_{2,m} = \eta_{U,m}.
\]
Second, we obtain the global optimum of (\text{WMSE-EQ-u}_m) as \( \mathbf{u}_m = -(\mathbf{C}_{1,m} + \beta \mathbf{C}_{3,m})^{-1} \mathbf{c}_{2,m} \). Finally, we use the \( \text{vec}^{-1} \) operator to get \( \mathbf{U}_m \) from \( \mathbf{u}_m \).

4) Transmit Precoder Design for (\text{SR-EQ}): Assuming \( \{\mathbf{W}\}, \{\mathbf{U}\}, \{\mathbf{V}\}, \) and \( \mathbf{F}_{-k} \) are fixed for some \( k \in \mathcal{K} \), we focus on determining \( \mathbf{F}_k \). For notational convenience in formulating the single-variable
optimization problem for transmit precoder design, we define the following matrices

\[
\begin{align*}
\mathbf{D}_{1,k} &= \sum_{q=1}^{K} \sum_{m=1}^{M} \sum_{n=1}^{M} \mathbf{U}_{m,k} \mathbf{W}_{q,m}^* \mathbf{V}_{q,n} \mathbf{U}_{n,k}, \\
\mathbf{D}_{2,k} &= \sum_{m=1}^{M} \mathbf{U}_{m,k} \mathbf{W}_{k,m}^* \mathbf{V}^* , \\
\mathbf{D}_{3,k} &= \sum_{m=1}^{M} \mathbf{U}_{m,k}^* \mathbf{U}_{m,k}, \\
\mathbf{D}_{4,k} &= \sum_{q=1}^{K} \sum_{q=1}^{K} \mathbf{V}_{q} \mathbf{W}_{q}^* \mathbf{T}_{q,q}^* \mathbf{T}_{q,q} \mathbf{W}_{q} - 2\Re \left( \sum_{q=1}^{K} \mathbf{V}_{q} \mathbf{W}_{q}^* \mathbf{T}_{q,q} \right) + \sum_{q=1}^{K} \sigma^2_q \mathbf{V}_{q} \mathbf{W}_{q} \mathbf{W}_{q}^* \\
&+ \sum_{q=1}^{K} \sum_{m=1}^{M} \sigma^2_{X,m} \mathbf{V}_{q} \mathbf{W}_{q}^* \mathbf{G}_{q,m} \mathbf{G}_{q,m}^* \mathbf{W}_{q} + \sum_{q=1}^{K} \sigma^2_{R,q} \mathbf{V}_{q} \mathbf{W}_{q} \mathbf{W}_{q}^*. 
\end{align*}
\]

After some manipulation of (WMSE-EQ) and with \( \eta_{T,k} \) defined in (15), we need to solve the following optimization problem (WMSE-EQ-F\(_k\)) for \( \mathbf{F}_k \)

\[
\mathbf{F}_k = \arg \min_{\mathbf{X} \in \mathbb{C}^{N_{T,k} \times d_k}} \text{tr}(\mathbf{X}^* \mathbf{D}_{1,k} \mathbf{X}) - \text{tr}(\mathbf{D}_{2,k}^* \mathbf{X}^*) - \text{tr}(\mathbf{D}_{2,k} \mathbf{X}^*) + \text{tr}(\mathbf{D}_{4,k}) \\
\text{subject to} \quad \text{tr}(\mathbf{X}^* \mathbf{X}) = p_T^\text{max}, \\
\text{tr}(\mathbf{X}^* \mathbf{D}_{3,k} \mathbf{X}) = \eta_{T,k}.
\]

Recall that we define \( \mathbf{f}_k = \text{vec}(\mathbf{F}_k) \). We introduce a new variable \( \mathbf{Y} = \begin{pmatrix} \mathbf{f}_k \\ 1 \end{pmatrix} \)

\[
= \begin{pmatrix} \mathbf{f}_k^* \\ 1 \end{pmatrix} \begin{pmatrix} \mathbf{f}_k^* & \mathbf{f}_k \\ \mathbf{f}_k & 1 \end{pmatrix}.
\]

It follows that \( \mathbf{Y} \) is a rank-one Hermitian positive semidefinite matrix with the bottom right entry equal to 1. Then, we transform (WMSE-EQ-F\(_k\)) equivalently into the following problem (WMSE-EQ-F\(_k\)\(_*\))

\[
(\text{WMSE-EQ-F}_k^*) : \min_{\mathbf{Y} \in \mathbb{C}^{(N_{T,k}^2 d_k+1) \times (N_{T,k}^2 d_k+1)}} \text{tr} \left( \begin{pmatrix} \mathbf{I}_{d_k} \otimes \mathbf{D}_{1,k} & -\text{vec}(\mathbf{D}_{2,k}) \\ -\text{vec}(\mathbf{D}_{2,k})^* & 1 \end{pmatrix} \mathbf{Y} \right) \\
\text{subject to} \quad \text{tr}(\mathbf{Y}) = p_T^\text{max} + 1, \\
\text{tr} \left( \begin{pmatrix} \mathbf{I}_{d_k} \otimes \mathbf{D}_{3,k} & 0_{N_{T,k}^2 d_k \times 1} \\ 0_{1 \times N_{T,k}^2 d_k} & 1 \end{pmatrix} \mathbf{Y} \right) = \eta_{T,k} + 1, \\
\begin{pmatrix} 0_{N_{T,k}^2 d_k \times N_{T,k}^2 d_k} & 0_{N_{T,k}^2 d_k \times 1} \\ 0_{1 \times N_{T,k}^2 d_k} & 1 \end{pmatrix} \mathbf{Y} = 1, \\
\mathbf{Y} \succeq 0, \text{rank}(\mathbf{Y}) = 1.
\]

Similar to solving (TL-F\(_k\)_\(_*\)), we adopt the SDP method for solving (WMSE-EQ-F\(_k\)_\(_*\)). Specifically, dropping the non convex rank constraint, we obtain an SDR of (WMSE-EQ-F\(_k\)_\(_*\)), which is a convex
optimization problem. More importantly, since \((WMSE\text{-EQ-}f_k f_k^*)\) has \(n = 3\) constraints (excluding the rank-one constraint), the relaxation is exact. This means that the SDR of \((WMSE\text{-EQ-}f_k f_k^*)\) always has a rank-one global optimum. Thus, we can find a general-rank global optimum of the SDR of \((WMSE\text{-EQ-}f_k f_k^*)\) using readily available software packages, e.g., CVX toolbox. Then we can construct a rank-one global optimum of the SDR from the resulting general-rank global optimum using the rank-reduction procedure in [43]. We notice that the last entry of the column vector obtained by the decomposition of the rank-one global optimum may be a complex number with modulus of 1. By multiplying the resulting column vector with the conjugate of its last entry, we obtain a desired column vector in the form of \((f_k 1)^T\), which corresponds to another rank-one global optimum of \((WMSE\text{-EQ-}f_k f_k^*)\).

C. An MSE-based Algorithm for End-to-End Sum-Rate Maximization with Power Control

In this subsection, we develop an algorithm for solving \((SR\text{-NEQ})\), which is also based on the relationship between achievable end-to-end rates and MSE values. Similar to Subsection IV-A3, we can formulate the corresponding matrix-weighted sum-MSE minimization problem that has the same stationary points as does \((SR\text{-NEQ})\). We refer to it as \((WMSE\text{-NEQ})\). Using the same steps as in Subsection IV-B, we can develop an alternating minimization algorithm for solving for a stationary point of \((SR\text{-NEQ})\). We refer to this algorithm as Algorithm 3. Due to space limitation, we only compare and contrast the steps of Algorithm 3 and those of Algorithm 2 in this section. The details of Algorithm 3 are provided in [33]. First, the matrix weight and receive filter designs for \((SR\text{-NEQ})\) are exactly the same as those for \((SR\text{-EQ})\). Second, the relay processing matrix design for \((SR\text{-NEQ})\) can be solved by the Lagrangian multiplier method with the only difference is that the multiplier must be nonnegative. Finally, the optimization problem for the transmit precoder design for \((SR\text{-NEQ})\) is obtained by replacing the equality constraints in \((WMSE\text{-EQ-F}_k)\) by the corresponding inequality constraints. Fortunately, the resulting optimization problem is convex with respect to \(F_k\). In particular, it follows from Remark 6 that the objective function of the problem \((WMSE\text{-EQ-F}_k)\) is convex with respect to \(F_k\). In addition, since \(D_{3,k}\) is a Hermitian and positive semidefinite matrix, then we can easily check that the constraints of the resulting problem are also convex with respect to \(F_k\). Thus, any available software package could be used to solve for its unique global optimum \(F_k\).
V. DISCUSSION

In this section, we discuss the proposed algorithms in the following aspects: i) the convergence, ii) the quality of the solution, and iii) the assumption on power constraints at the relays.

In terms of convergence, all the proposed algorithms are guaranteed to converge to a stationary point of the corresponding multi-variable optimization problem. All the proposed algorithms are based on alternating minimization. In each iteration we actually aim to minimize the cost function of the original multi-variable optimization problem, which is either the total leakage power in ($\mathcal{T}_L$) or the matrix-weighted sum-MSEs in ($\mathcal{W}_M\mathcal{MSE}_{EQ}$) and ($\mathcal{W}_M\mathcal{MSE}_{NEQ}$). Since we are able to find a global optimum of the corresponding single-variable minimization problem in each iteration, the cost function of the original multi-variable optimization problem is non-increasing after each iteration.

All the proposed algorithms are not guaranteed to reach a global optimum of the corresponding multi-variable optimization problem. The quality of the solution found by the proposed algorithms depends significantly on the initial solution selected in the first iteration. One way to improve the performance of the proposed algorithms, either in terms of total leakage minimization for Algorithm 1 or in terms of end-to-end sum-rate maximization for Algorithm 2 and Algorithm 3, is to use multiple initializations, selecting the one with the best performance at the expenses of running time.

The assumption on power constraints at the relays is mainly due to the applicability of the method we use to solve the transmit precoder design problems in Algorithm 1 and Algorithm 2. Specifically, we propose to use the SDP method to find a global optimum of the transmit precoder design problems, which are nonconvex complex-valued homogeneous QCQPs with equality constraints. Prior work in [43], [45] shows that the relaxed SDR of the QCQPs is guaranteed to have a rank-one global optimum if there are at most for equality quadratic constraints. The sum relay power constraint in either ($\mathcal{T}_L$) or ($\mathcal{SR}_{EQ}$), and hence ($\mathcal{W}_M\mathcal{MSE}_{EQ}$), is crucial since it results in only two equality quadratic constraints. Note that with per-relay power constraints, the number of quadratic constraints of the resulting QCQP is ($M + 1$). Therefore, when power control is not considered, we can always formulate and solve similar total leakage minimization problem and end-to-end sum-rate maximization problem with per-relay power constraints using the same steps if there are at most three relays, i.e., $M \leq 3$. The sum relay power constraint, however, is not crucial to the formulation of ($\mathcal{SR}_{NEQ}$), and hence ($\mathcal{W}_M\mathcal{MSE}_{NEQ}$), and the development of Algorithm 3. For the end-to-end sum-rate maximization problem with power control, regardless of the type of power constraints at the relays, the single-variable optimization problems for
designing the relay processing matrices and the transmit precoders are always convex with respect to the corresponding variable \(33\). Thus, we are always able to solve for their global optimum in each iteration.

VI. SIMULATIONS

This section presents Monte Carlo simulation results to investigate the average end-to-end sum-rate performance and to gain insights into the achieved multiplexing gains of the proposed algorithms. We consider only symmetric relay systems, which are denoted as \((N_R \times N_T, d)^K + N_X^M\), where \(N_{R,k} = N_R\), \(N_{T,k} = N_T\), \(d_k = d\) and \(N_{X,m} = N_X\) for \(k \in K, m \in M\). The power values are normalized such that \(\sigma_{R,k} = \sigma_{X,m} = 1\), \(p_{T,k}^{\text{max}} = P_T\), and \(p_{X,m}^{\text{max}} = P_X\) for \(k \in K, m \in M\). The channel realizations are flat in time and frequency. The channel coefficients on the two hops are generated as i.i.d. zero-mean unit-variance complex Gaussian random variables. Note that there is no path loss assumed in the simulations, thus there average power of all cross-links on the same hop is the same. The plots are produced by averaging over 100 random channel realizations. In each channel realization, the initial transceivers are chosen randomly subject to the power constraints at the transmitters and at the relays. In each iteration, either one transmitter or one relay is allowed to update its transceiver, after that all the receivers are allowed to update their receive filters. We use the CVX toolbox \(44\) to solve convex optimization problems in each iteration.

A. Convergence

Fig. 2 and Fig. 3 illustrate the convergence behavior of the proposed algorithms. Fig. 2 provides the analysis of the sum power of post-processed leakage signals of Algorithm 1 for a random channel realization of the \((4 \times 4, 2)^3 + 4^3\) system. We observe that the sum power of leakage signals decreases monotonically over iterations. Interestingly, the interference and the enhanced relay noise change their roles during the process of Algorithm 1. The interference is the dominant component of the leakage signal at the beginning, however, it can be aligned and then cancelled quickly in a few iterations. After this point, the enhanced relay noise becomes dominant - its sum power is thousands time larger than the interference sum power. Unfortunately, given that many spatial dimensions are devoted to minimizing the sum interference power, it becomes challenging for Algorithm 1 to align and cancel the enhanced relay noise power. Intuitively, the enhanced relay noise can be thought of as a source of single-hop interference from “virtual uncoordinated relays” that impacts directly the receivers. Thus, we need to take into account both the interference and enhanced relay noise in the design of interference alignment strategies for AF.
two-hop interference channel. Fig. [3] provides the values of $WMSE_{sum}$ achieved by Algorithm 2 and by Algorithm 3 over iterations for a channel realization of the $(2 \times 4, 1)^4 + 2^4$ system. We observe that $WMSE_{sum}$ values for both algorithms are non increasing over iterations, i.e., the proposed algorithms are convergent. Although the convergence speeds of the proposed algorithms are quite fast for these configurations, they might be slow for networks with large values of $K$ or $d$.

B. Impacts of Transmit Power $P_T$ and $P_X$

We can use the proposed algorithms to obtain insights into the achievable end-to-end sum-rates and multiplexing gains of the AF two-hop interference channel. The achievable end-to-end multiplexing gains are computed as the slope of the curves of the achievable end-to-end sum-rates at high transmit power
values. In the next experiment we investigate the impact of $P_T$ and $P_X$ on the achievable end-to-end multiplexing gains of Algorithm 3 for a $(4 \times 4, 2)^3 + 4^3$ AF relay system. We denote $\Delta P = P_T - P_X$. We plot the achievable end-to-end sum-rates as a function of $P_T$ for different values of $\Delta P$, which are represented by the solid lines in Fig. 4. We observe that Algorithm 2 achieves a multiplexing gain of 6 for all the simulated values of $\Delta P$. Thus, we only need to consider $\Delta P = 0$ in the following experiments. In addition, we observe that when $P_T$ is fixed, the average end-to-end sum-rates is still increasing with $P_X$ but at a sublinear rate. A similar observation can be made when we fix $P_X$ and vary $P_T$. In other words, Algorithm 2 cannot obtain multiplexing gains higher than one if either $P_T$ or $P_X$ is fixed.

![Graph](image)

Fig. 4. Average end-to-end sum-rates of $(4 \times 4, 2)^3 + 4^3$ achieved by Algorithm 3. The solid curves are for different values of $\Delta P$. The dashed line is for $P_X = 30$dB while the dot-dashed line is for $P_X = 25$dB.

C. End-to-End Sum-Rate Comparison of the Proposed Algorithms

In this experiment, we compare the average achievable end-to-end sum-rates of the proposed algorithm for the $(2 \times 4, 1)^4 + 2^4$ system as shown in Fig. 5. We consider a sum relay power constraint and use the same initial condition for all the algorithms. Thanks to power control, Algorithm 3 always outperforms Algorithm 2 in this experiment. Both Algorithm 2 and Algorithm 3 perform much better than Algorithm 1 at low-to-medium SNR values. The reason is that Algorithm 1 does not take into account the desired signal power and the noise power at the receivers, while the other do. Interestingly, at high SNR values, Algorithm 1 outperforms both Algorithm 2 and Algorithm 3. Especially, Algorithm 1 can achieve a higher multiplexing gain than do the other. Zooming in on per user achievable end-to-end rates, we find that for Algorithm 2 and Algorithm 3, some users have much smaller rates than do the others. Algorithm 3
even turns off the data streams associated with some users. It is the unfairness that limits the maximum end-to-end multiplexing gains achievable by the two algorithms. Thus, Algorithm 1 is more suitable than the others for the purpose of investigating the maximum achievable end-to-end multiplexing gains of MIMO AF relay networks.

D. Comparison with DF Relaying and Direct Transmission

To obtain insights into relay functionality selection in the presence of interference, we simulate dedicated DF relay interference channel where one DF relay is dedicated to aid one and only one transmitter-receiver pair, i.e., $K = M$. Using equal time-sharing, the end-to-end achievable rate of a pair is defined as half of the minimum between the achievable rate from the transmitter to the associate DF relay and that from the relay to the receiver. Based on [10], we derive an upper-bound on the achievable end-to-end multiplexing gain $d_{\Sigma}^{DF}$ of the dedicated DF relay interference channel $(N_R \times N_T, d)^K + N_X^K$ as follows

$$d_{\Sigma}^{DF} \leq d_{\Sigma}^{DF,\text{max}} = 0.5 \times \min \left\{ \left\lfloor \frac{K(N_X+N_T)}{K+1} \right\rfloor, \left\lfloor \frac{K(N_R+N_X)}{K+1} \right\rfloor \right\}.$$ 

We also assume that both the transmitters and the DF relays are subject to individual power constraints with the same average maximum transmit power per node of $P_T$ and $P_X$.

1) Maximum Achievable Multiplexing Gains: We fix $N_R = N_T = 2$ and $d = 1$, and use Algorithm 1 to evaluate the achievable end-to-end multiplexing gains as a function of $K$ for $N_X = 3$ and $N_X = 5$ for both AF relays, DF relays, and direct transmission, as shown in Fig. 6. We notice that with these values
of $N_X$ and when $K$ is small, due to the half-duplex loss, both the AF relay and DF relay cases achieve lower multiplexing gains than does the direct transmission. While the DF relay case cannot outperform the direct transmission even when $K$ is increased, the AF relay case can achieve higher multiplexing gains when there are more than 6 users. Thus, we can claim that AF relays help increase the achievable end-to-end multiplexing gains of interference channels. In addition, we observe that there exist upper-bounds on the achievable end-to-end multiplexing gains for all the simulated cases - AF relays, DF relays, and direct transmission. Finding closed-form expressions of such bounds in the MIMO AF relay interference channel is left for future work.

2) Achievable End-to-End Sum-Rates: We compare the end-to-end sum-rate performance of Algorithm 2 for given relay systems with other existing transceiver design strategies for the two-hop interference channel. Specifically, we simulate two strategies for the AF relay case: i) time division multiple access (TDMA) distributed beamforming (BF) and ii) dedicated relay BF. In the AF TDMA distributed BF, all the relays help only one transmitter-receiver pair at a time (which is an extension of the design in [37] for multiple-antenna receivers). In the dedicated relay BF, we assume that each AF relay is dedicated to aid one and only one transmitter-receiver pair. This means that interference is ignored and we apply the joint source-relay design in [49], [50] independently for the two-hop channels from the transmitters to their associated receivers. We also simulate different strategies for the MIMO single-hop interference channels on the two hops, including selfish (SF) beamforming (i.e., each transmitter aims at maximizing the achievable rate to its associated receiver), interference alignment strategy based on total leakage.
(TL) minimization [28], and the iteratively weighted MSE sum-rate (SR) maximization strategy [32]. We assume the same strategy is used for both hops.

![Graph](image)

Fig. 7. Achievable end-to-end sum-rates for the \((2 \times 2, 1) + 2^4\) system.

Fig. 7 shows the results for the \((2 \times 2, 1) + 2^4\) system. Note that Algorithm 3 outperforms all the other in all regions. It achieves an end-to-end multiplexing gain of 2 (which is equal to half of the total number of data streams). Unaware of interference, the dedicated relay strategies for both AF relay and DF relay cases achieve zero multiplexing gains. While the multiplexing gain achieved by the DF TL & TL strategy is zero, that by the DF SR & SR strategy is nonzero. The reason is that interference alignment is not feasible for the configuration on the two hops, interference cannot be completely eliminated using the TL algorithm. Although the SR algorithm is able to turn off some data streams, one data stream on each hop in this case, to make interference alignment feasible. Note, however, that it may turn off data streams of different pairs on two hops. Thus, on average the DF SR & SR strategy achieves an end-to-end multiplexing gain less than 1.5 (half of the number of remaining data streams when interference alignment is feasible). Finally, thanks to orthogonalization transmission, the AF TDMA distributed BF can achieve an end-to-end multiplexing gain of 0.5.

VII. CONCLUSIONS AND FUTURE WORK

We developed three cooperative algorithms for joint designs of the transmitters, the relays, and the receivers of the MIMO AF two-hop interference channel with constant channel coefficients. Algorithm 1 aims at minimizing the sum power of the interference signals and the enhanced noise from the relays. Based on a relationship between MSE and mutual information, Algorithm 2 (Algorithm 3) is able to
find a stationary point of the end-to-end sum-rate maximization problems with equality (inequality) power constraints. All the proposed algorithms are guaranteed to converge to stationary points of the corresponding optimization problems but not their global optima. Our simulations show that thanks to the consideration of the desired signal power and the noise power at the receivers, Algorithm 2 and Algorithm 3 outperform Algorithm 1 at low-to-medium SNR in terms of average end-to-end sum-rates and multiplexing gains. Nevertheless, they perform worse than Algorithm 1 at high SNR due to unfairness in achievable rates among users. The multiplexing gains achievable by the proposed algorithms provide lower bounds on the total number of degrees of freedom in MIMO AF relay networks, which remains unknown. It is also shown that AF relays enhance the feasibility of interference alignment at the receivers, leading to higher multiplexing gains than both DF relays and direct transmission. Future work will focus on developing cooperative algorithms that require less overhead, have faster convergence speed and allow for lower implementation complexity.

REFERENCES


