Analysis of scattering from buried object by using the geometrical optics

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Abstract. This paper is concerned with a theoretical solution to the problem of TE-Scattering radio waves by a buried object in a three-layered medium. The region of interest consists of three homogenous layered medium, the object of an inhomogeneous material with respect to its layer. With the aid of geometrical optics technique, this paper analyzes the influence of the buried object on the scattered magnetic field from the layered medium. Closed-form expressions for the far-field of a VMD, located above the ground surface, are obtained due to the buried object under the ground. This technique takes into consideration all the probabilities of plane waves under and above the ground to calculate the total reflection coefficient. The extra-scattered field due to the perturbation in the earth magnetic field has been obtained by solving Sommerfeld integrals (SI). The solution of SI has been based on the stationary-phase point method. The obtained results are compared with those mentioned in three-layered free buried objects.

1. Introduction

One of the enormous problems around the world is that of detecting and exploring the buried objects. Detection, location, and identification of various objects in a lossy dielectric medium are the important problems in remote sensing. Of particular interest is the detection of landmines and Unexploded Ordnance (UXO) for ultimate removal surface. A special emphasis is given to this problem since it poses serious risk in Egypt and in more than 60 countries in the world.

Mathematicians and engineers had developed mathematical techniques and algorithms for detecting and locating buried objects [1–4]. One of the most typical techniques for scattering radio waves by three-dimensional (3-D) objects is the method of moments (MOM). This method is based on the mixed-potential electric field integral equation (MPIE) that uses the basic functions proposed by Michalski and Zheng [2]. The disadvantage of this scheme is that the Sommerfeld integrals in the Green’s functions are involved and they cannot be directly calculated. Another method to treat the scattering by 2-D case with TM wave incidence has been used to speed up the Sommerfeld-integral evaluation [3]. However, it will lead to large number of unknowns and it is difficult to solve this problem analytically. In order to solve this difficulty, some fast algorithms combine the MOM and modified PO method to determine the EM scattering from buried object [4]. However, all of these methods depend on the tonsorial formulas and dyadic algebra.

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Apart from the full wave numerical method, this paper represents an easy analytical method to evaluate the TE scattering waves of buried object by geometrical optics. We consider the problem of the response of perfectly conducting buried object in lossy dielectric medium characterized by high frequency. This response means the measurements of the scattered magnetic field caused by a perturbation in the earth magnetic field.

Bishay [5] has studied the propagation of radio waves in a three layered media (air, sea and ground). Abo-Seida and Bishay [6] studied the transient electromagnetic field of a pulsed vertical magnetic dipole on a conducting earth. Abo-Seida [7] studied the electromagnetic field due to vertical magnetic dipole buried in stratified media. On the other hand Chew [8] illustrated the geometrical optics series of plane waves, resulting in a consequence of multiple reflections and transmission of rays in three layered medium. He used these series to evaluate the scattered factor in three-layered free buried objects.

The geometrical optics series are used to calculate the extra scattered factor due to a three-layered free buried object. The transmitter and receiver are located in region (1). The former is a horizontal small electric current loop antenna of magnetic moment IA. Therefore, it can be regarded as a vertical magnetic dipole.

The problem is formulated by using a new geometrical optics technique in order to obtain a closed-form expression of the scattered fields involving Sommerfeld integrals. These integrals have been evaluated by using the stationary phase method [9] to derive the far-field approximation. The results have been compared with those mentioned by Chew [8].

2. Geometrical structure and basic equations

This study used a simple three-dimension (3-D) model for the electromagnetic exploration of buried perfect conducting object. The model consists of three-layered medium with a vertical magnetic dipole (VMD) located at the origin, and the receiver at the height “z” in region (1). The permittivity, permeability and conductivity constants are \((\varepsilon_1, \mu_1, \sigma_1)\), \((\varepsilon_2, \mu_2, \sigma_2)\) and \((\varepsilon_3, \mu_3, \sigma_3)\) for the three-layered, respectively.

A metallic object was embedded at height “\(\ell\)” above the lower interface \(z = -d_2\), as illustrated in Fig. 1. The second layer has a finite mean thickness \((h = d_2 - d_1)\), since the upper and lower surfaces are at \((z = -d_1, -d_2)\) respectively. The propagation constant \(k_i (i = 1, 2, 3)\) in the three regions can be determined from the material constants in the usual way [8]. The geometry of the system is described by using the cylindrical co-ordinates \((\rho, \varphi, z)\). The center of the object is \((\rho_0, 0, -d)\) and \((d = d_2 - \ell)\).

Also, the size of the object is small relatively to the volume of the layer; it may be as a small disc. But, the object of large dimensions relative to the electromagnetic wavelength \(\lambda\) which means its radius of curvature large compared to \(\lambda\). The material constants for the buried object are \(\varepsilon_s, \mu_s\) and \(\sigma_s\) with propagation constant \(k_s = k_0 \sqrt{\varepsilon_s + \frac{i\sigma_s}{\varepsilon_0 \omega}}, \) where \(k_0 = \omega \sqrt{\mu_0 \varepsilon_0}, i = \sqrt{-1}\) and \(\omega\) is the angular frequency.

To indicate the theory of this work, we can represent the Chew [8] investigation for the radio waves propagation in three-layered free buried objects. And, the scattered fields in region (1) due to a vertical magnetic dipole (TE-waves) [8], given by

\[
H_{1z}^{\text{super}} = \frac{-i IA}{8\pi} \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}^3} [e^{ik_{1z}z} + R_{12} e^{ik_{1z}(z+2d_1)}] H_0^{(1)}(\lambda \rho) d\lambda,
\]

\[
E_{1z}^{\text{super}} = 0.
\]
(1) Refers to the coefficient \( R_{12} \)
(2) Refers to the coefficient \( T_{12} R_{23} T_{21} e^{2iK_{z2}h} \)
(3) Refers to the coefficient \( T_{12} R_{23} R_{23}^2 T_{21} e^{2iK_{z2}2h} \)
(4) Refers to the coefficient \( T_{12} R_{ob} T_{21} e^{2iK_{z2}(h-\ell)} \)
(5) Refers to the coefficient \( T_{12} R_{23}^2 R_{ob} T_{21} e^{2iK_{z2}(h+\ell)} \)
(6) Refers to the coefficient \( T_{12} R_{23}^2 R_{12} T_{21} e^{2iK_{z2}3h} \)

Fig. 1.

Where \( IA \) is the magnetic moment of a loop of area \( A \) carrying a current \( I \), and \( H_k^0(\lambda \rho) \) is the first kind Hankel function of order zero. Also, the superscript “unper (per)” refers to the unperturbed (perturbed) fields of a system without (with) buried object. The reflection and the transmission coefficients are given by

\[
\overline{R}_{12} = R_{12} + \frac{T_{12} R_{23} T_{21} e^{2iK_{z2}h}}{1 - R_{21} R_{23} e^{2iK_{z2}h}}
\]  

(2)

where,

\[
R_{12} = \frac{\mu_2 k_{1z} - \mu_1 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}, \quad R_{21} = -R_{12}, \quad R_{23} = \frac{\mu_3 k_{2z} - \mu_2 k_{3z}}{\mu_3 k_{2z} + \mu_2 k_{3z}}
\]

\[
T_{12} = \frac{2 \mu_2 k_{1z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}, \quad T_{21} = \left( \frac{\mu_1 k_{2z}}{\mu_2 k_{1z}} \right) T_{12}, \text{ and } k_{iz} = \sqrt{\lambda^2 - k_i^2}, \ i = 1, 2, 3.
\]

Chew has given a physical meaning of \( \overline{R}_{12} \) which is the generalized reflection coefficient for plane waves (TE-waves) propagate in three-layered medium in region (1) as a consequence of multiple reflections and
transmissions of plane waves in regions (1) and (2). In general $\bar{R}_{12}$ may be as ray series or geometrical optics series.

In the next step, we assume the path of the radio wave (TE-waves) propagation as a geometrical optics series of plane waves in three-layered free buried objects. By using Fig. 1, it can be deduce all reflection and scattered rays from the lower interface ($z = -d_2$) and the buried object.

We assume that at the upper interface the 2nd ray is reflected to be factorized into two parts. The first part incidents on the object while the second incidents on the lower interface to be reflected again with coefficient $R_{23}$. Therefore, it is possible to deduce the nth ray in region (1) in the presence of the buried object. Moreover, there exist an infinite number of rays in region (1) from the multiple consequence reflections and transmission of the plane waves in the three-layered medium. We have ray series or geometrical optics series in region (1) which will be given the total reflection coefficient of plane waves scattered from the layered medium and the buried object. Also, neither transmission rays inside the object nor diffraction rays at its surface, since the buried object may be regarded as a perfect conductor. Also, there is no any interface angles since we discuss the probabilities of one plane wave through layer buried object.

We assume that the total reflection coefficient, in the presence of buried object for TE-waves ($E_z = 0$, $H_z \neq 0$) in region (1) of the three-layered medium, is denoted by $\tilde{R}_t$. Therefore, $\tilde{R}_t$ can be derived as the sum of an infinite sequence of all rays in region (1) as in Fig. 1. Then

$$\tilde{R}_t = R_{12} + T_{12} R_{23} T_{21} e^{2ik_{2z}h} + T_{12} R_{21} R_{23}^2 T_{21} e^{2ik_{2z}(2h)} + T_{12} R_{ob} T_{21} e^{2ik_{2z}(h-\ell)}$$

$$+ T_{12} R_{23}^2 R_{ob} T_{21} e^{2ik_{2z}(h+\ell)} + T_{12} R_{21} R_{23}^3 T_{21} e^{2ik_{2z}(3h)} + T_{12} R_{ob} R_{21} R_{23} T_{21} e^{2ik_{2z}(2h-\ell)}$$

$$+ T_{12} R_{ob} R_{21} R_{23}^2 T_{21} e^{2ik_{2z}(2h+\ell)} + \ldots$$

$$\tilde{R}_t$$ can be arranged in the following series formula:

$$\tilde{R}_t = R_{12} + T_{12} R_{23} T_{21} e^{2ik_{2z}h} \left[ \sum_{n=0}^{\infty} (R_{21} R_{23} e^{2ik_{2z}h})^n \right]$$

$$+ T_{12} R_{ob} T_{21} e^{2ik_{2z}(h-\ell)} \left[ \sum_{n=0}^{\infty} (R_{21} R_{23} e^{2ik_{2z}h})^n \right]$$

$$+ T_{12} R_{ob} R_{21} R_{23} T_{21} e^{2ik_{2z}(h+\ell)} \left[ \sum_{n=0}^{\infty} (R_{21} R_{23} e^{2ik_{2z}h})^n \right]$$

By using the following rule,

$$\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \Leftrightarrow |x| < 1,$$

we get:

$$\tilde{R}_t = \left[ R_{12} + \frac{T_{12} R_{23} T_{21} e^{2ik_{2z}h}}{1 - R_{21} R_{23} e^{2ik_{2z}h}} \right] + T_{21} R_{ob} T_{12} \left[ \frac{e^{2ik_{2z}(h-\ell)} + R_{21}^2 e^{2ik_{2z}(h+\ell)}}{1 - R_{21} R_{23} e^{2ik_{2z}h}} \right]$$

$$= \bar{R}_{12} + R^s$$

(5)
Where,

\[ R^* = T_{21} R_{ob} T_{12} \left[ e^{2ik_2z(h-\ell)} + R_{23}^2 e^{2ik_2z(h+\ell)} \right] \frac{1 - R_{21} R_{23} e^{2ik_2z\ell}}{1 - R_{21} R_{23} e^{2ik_2z\ell}} \]  

(6)

\( R^* \) is the extra-scattering factor due to the buried object. It is clear that \( R^* \) is based on the coefficient \( R_{ob} \) which is the reflection coefficient of the buried object in region (2). \( R_{ob} \) can be assumed as

\[ R_{ob} = \frac{\mu_s k^2 z - \mu_k^2 k s z}{\mu_s k^2 z + \mu_k^2 k s z} \]

(7)

Hence, \( R^*(\lambda) = 0 \Leftrightarrow R_{ob} = 0 \), leads to \( \tilde{R}_t = \tilde{R}_{12} \) and this result coincides with the original case [8] when the layer-free buried objects.

3. Evaluation of the scattered perturbed field

In this section we will determine the total scattered magnetic field \( H_{1z}^t \), due to the perturbation in earth magnetic field of region (2) caused by the buried object which will perturb the field in its vicinity. Therefore, the TE scattered fields in region (1), due to a vertical magnetic dipole, in three-layered buried object is given by:

\[ H_{1z}^t = -\frac{i I A}{8\pi} \int_{-\infty}^{\infty} \frac{\lambda_0}{k_{1z}} \left[ e^{ik_{1z}z} \right] + \tilde{R}_t e^{ik_{1z}(z+2d_1)} H_0^{(1)}(\lambda \rho) d\lambda \]

(8)

\[ E_{1z}^t = 0 \]

Also, in general for any region, the transverse components \((E_{1\rho}, H_{1\rho})\) can be obtained from z-components Eq. (8) by using the following formulas [11]:

\[ \tilde{E}_\nu = \frac{1}{\lambda^2} \left( \tilde{e}_\rho \frac{\partial}{\partial \rho} + \tilde{e}_\phi \frac{\partial}{\partial \phi} \right) \frac{\partial}{\partial z} E_{1z}^t - i\omega \mu \tilde{e}_z \wedge \left( \tilde{e}_\rho \frac{\partial}{\partial \rho} + \tilde{e}_\phi \frac{\partial}{\partial \phi} \right) H_{1z}^t \]

\[ \tilde{H}_\nu = \frac{1}{\lambda^2} \left( \tilde{e}_\rho \frac{\partial}{\partial \rho} + \tilde{e}_\phi \frac{\partial}{\partial \phi} \right) \frac{\partial}{\partial z} H_{1z}^t - i\omega \varepsilon \tilde{e}_z \wedge \left( \tilde{e}_\rho \frac{\partial}{\partial \rho} + \tilde{e}_\phi \frac{\partial}{\partial \phi} \right) E_{1z}^t \]  

(9)

Since, the total reflection coefficient \( \tilde{R}_t \) can be divided into two terms in Eq. (4), therefore, the corresponding magnetic field Eq. (8) can be divided as follows:

\[ H_{1z}^t = -\frac{i I A}{8\pi} \int_{-\infty}^{\infty} \frac{\lambda_0}{k_{1z}} \left[ e^{ik_{1z}z} \right] \tilde{R}_{12} e^{ik_{1z}(z+2d_1)} H_0^{(1)}(\lambda \rho) d\lambda \]

\[ + \left( -\frac{i I A}{8\pi} \right) \int_{-\infty}^{\infty} \frac{\lambda_0}{k_{1z}} R^* e^{ik_{1z}(z+2d_1)} H_0^{(1)}(\lambda \rho) d\lambda \]

\[ = H_{1z}^{unper} + H_{1z}^* \]  

(10)
where,
\[ H_{1z}^{\text{unper}} = H_{1z}^{d} + I_{1} + I_{2}, \]
\[ H_{1z}^{d} = \frac{-i IA}{8\pi} \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}} e^{ik_{1z}|z|} H_{0}^{(1)}(\lambda\rho) \, d\lambda, \]
\[ I_{1} = \frac{-i IA}{8\pi} \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}} R_{12} e^{ik_{1z}(z+2d_{1})} H_{0}^{(1)}(\lambda\rho) \, d\lambda \]
\[ I_{2} = \frac{-i IA}{8\pi} \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}} \left[ T_{12} R_{23} T_{21} e^{2ik_{21}h} \right] e^{ik_{1z}(z+2d_{1})} H_{0}^{(1)}(\lambda\rho) \, d\lambda \]

And
\[ H_{1z}^{*} = \left( \frac{-i IA}{8\pi} \right) \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}} R^{*}(\lambda) e^{ik_{1z}(z+2d_{1})} H_{0}^{(1)}(\lambda\rho) \, d\lambda \]

Note that: \( H_{1z}^{*} = H_{1z}^{\text{per}} \), where \( H_{1z}^{*} \) is the scattered magnetic field due to the perturbation in the earth magnetic field caused by the buried object.

To calculate the first integrals of \( H_{1z}^{\text{unper}} \), we use Sommerfeld identity:
\[
\left( \frac{i}{2} \right) \int_{-\infty}^{\infty} \frac{\lambda}{k_{z}} e^{ik_{1z}|z|} H_{0}^{(1)}(\lambda\rho) \, d\lambda = \frac{e^{ik_{1z}r}}{r}, r = \sqrt{\rho^2 + z^2}
\]

The direct magnetic field \( H_{1z}^{d} \) can be arranged by using a new technique, introduced by Chew [9] and recently improved by Abo-Seida et al. [12], in the form:
\[ H_{1z}^{d} = \frac{-i IA}{8\pi} \int_{-\infty}^{\infty} \left\{ \lambda^2 \right\} \left\{ \frac{\lambda}{k_{1z}} e^{ik_{1z}|z|} H_{0}^{(1)}(\lambda\rho) \right\} \, d\lambda \]

At \( \rho \to \infty \) there is an approximate formula for Hankel function as
\[ H_{0}^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi \lambda \rho}} e^{i(\lambda \rho - \frac{\pi}{4})}. \]

The first bracket in Eq. (14) is a slowly varying part, while the second bracket is rapidly varying as \( \rho \to \infty \). The location of the stationary phase point is given by:
\[
\frac{d}{d\lambda} \left\{ \lambda \rho + |z| \sqrt{\lambda^2 - k_{1z}^2} \right\}_{\lambda=\lambda_{0}} = 0, z > 0
\]

The solution of Eq. (15) is given by:
\[ \lambda_{0} = \frac{k_{1} \rho}{r} = k_{1} \sin \alpha \]
where, \( r = \sqrt{\rho^2 + z^2} \). Then Eq. (14) with the aid of Sommerfeld identity takes the following form:

\[
H_{1x}^d = \frac{-iA}{4\pi} \left( k_1 \sin \alpha \right)^2 \left( \frac{e^{ik_1r}}{r} \right)
\]

(17)

The solution of \( I_1 \) in Eq. (12) can easily be obtained by using the previous technique. Then

\[
I_1 = \frac{-iA}{4\pi} \left( k_1 \sin \theta \right)^2 \left\{ \frac{k_{1x} - k_{2x}}{k_{1x} + k_{2x}} \right\}_{\lambda = \lambda_1} \left( \frac{e^{ik_1r}}{r} \right)
\]

(18)

where,

\[
\lambda_1 = k_1 \frac{\rho}{R} = k_1 \sin \theta , , R = \sqrt{\rho^2 + (z + 2d_1)^2}
\]

Furthermore, the denominator in \( I_2 \) can be expanded as an infinite series, since \( |R_{21} R_{23} e^{2ik_2 h}| < 1 \). Therefore, \( I_2 \) can be arranged as:

\[
I_2 = \frac{-iA}{8\pi} \sum_{m=1}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{\lambda}{k_{2x}} e^{2imk_2 h} H_0^{(1)}(\lambda \rho) \right\} d\lambda
\]

To find the asymptotic expansion of the multiple reflected fields \( I_2 \), we assume that both the source and the observation point are close to the top interface, but the receiver (the observation point) is in far region from the system of study. That is “z” and “d_1” are small and high frequency being applied on the system leading to \( |k_1| \ll |k_2| \). The second bracket in \( I_2 \) is the rapidly oscillating part of its integrand. Then, by using the previous technique we obtain:

\[
I_2 = \frac{-iA k_2^2}{4\pi} \sum_{m=1}^{\infty} \left( \sin \theta_m \right)^2 \left\{ \frac{k_{2x}}{k_{1x}} \right\}_{\lambda = \lambda_m} \left( \frac{e^{ik_2 R_m}}{R_m} \right)
\]

(19)

where,

\[
\lambda_m = k_2 \frac{\rho}{R_m} = k_2 \sin \theta_m , , R_m = \sqrt{\rho^2 + (2mh)^2}
\]

Physically, the stationary phase-point contribution of m-th term in the series Eq. (19) can be as the contribution from the spherical wave with propagation constant \( k_2 \) emanating from m-th image point in region (2).

To complete the evaluation of the total magnetic field \( H_{1x}^t \), we have to calculate \( H_{1x}^r \) in Eq. (13) by using the same way, then:

\[
H_{1x}^t = \frac{-iA}{8\pi} \left[ \sum_{m=1}^{\infty} \Phi_1(m) + \sum_{m=1}^{\infty} \Psi_1(m) \right]
\]

(20)
where,
\[
\Phi_1(m) = \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}} T_{21} R_{ob} T_{12} e^{2ik_{2z}(h-\ell)} [R_{21}^{m-1} R_{23}^{m} e^{2imk_{2z}(h)}] e^{ik_1(x+z+2d_1)} H_0^{(1)}(\lambda \rho) d\lambda,
\]
\[
\Psi_1(m) = \int_{-\infty}^{\infty} \frac{\lambda^3}{k_{1z}} T_{21} R_{ob} T_{12} e^{2ik_{2z}(h+\ell)} [R_{21}^{m-1} R_{23}^{m+2} e^{2imk_{2z}(h)}] e^{ik_1(x+z+2d_1)} H_0^{(1)}(\lambda \rho) d\lambda,
\]

These last integrals of \(\Phi_1(m)\) and \(\Psi_1(m)\) can be arranged in slowly and rapidly parts to obtain the leading-order expansions as follows:
\[
\Phi_1(m) = -2i(k_2 \sin \alpha_{1m})^2 \left\{ \frac{k_2}{k_{1z}} \right\} T_{12} R_{21}^{m-1} R_{23}^{m} R_{ob} T_{21} e^{ik_1(x+2d_1)} \frac{e^{ik_2 r_{1m}}}{r_{1m}} \]
\[
\Psi_1(m) = -2i(k_2 \sin \alpha_{2m})^2 \left\{ \frac{k_2}{k_{1z}} \right\} T_{12} R_{21}^{m-1} R_{23}^{m+2} R_{ob} T_{21} e^{ik_1(x+2d_1)} \frac{e^{ik_2 r_{2m}}}{r_{2m}}
\]

where,
\[
\lambda_{1m} = k_2 \frac{\rho}{r_{1m}} = k_2 \sin \alpha_{1m}, \quad \lambda_{2m} = k_2 \frac{\rho}{r_{2m}} = k_2 \sin \alpha_{2m}, \]
\[
r_{1m} = \sqrt{\rho^2 + 4[(h-\ell) + (mh)]^2} \quad \text{and} \quad r_{2m} = \sqrt{\rho^2 + 4[(h+\ell) + (mh)]^2}
\]

4. Numerical results

In order to indicate the efficiency and accuracy of the Geometric Optics (GO) method proposed in this paper, two examples are discussed in this section.

4.1. Example (1)

We consider an object embedded in region (2), as shown in Fig. 1. If the three-layered medium are air, soil, and rocks with the material constants \(\varepsilon_1 = \varepsilon_0, \varepsilon_2 = 10\varepsilon_0, \varepsilon_3 = 20\varepsilon_0\) and \(\varepsilon_s = 0\), \(\varepsilon_0 = 8.854 \times 10^{-12}\) \(\mu_1 = \mu_2 = \mu_3 = \mu_0, \mu_s = 0.04\) and, \(\sigma_1 = 0, \sigma_s = 1000, \sigma_3 = 0.1, \sigma_2 = 0.02.\)

Let \(\rho = 50\ m, \ell = 5\ m, d_1 = 1\ m\) and \(\omega = 4\pi \times 1000\ \text{KHZ}\). The ground is illuminated by a z-polarization plane wave and the incident angle \(\theta = 0\). The z-scattered magnetic field form three-layered free buried objects and in three-layered buried object are obtained, under the above data, relative to the depth \(h\) as in Fig. 2.

4.2. Example (2)

We consider a circular disc buried under a flat ground as shown in Fig. 1, and the three-layered medium are air, sand, and sandstone with the material constants \(\varepsilon_1 = \varepsilon_0, \varepsilon_2 = 4, \varepsilon_3 = 6, \mu_3 = \mu_2 = \mu_1 = \mu_0, \mu_s = 4 \pi \times 10^{-3}\) and \(\sigma_3 = 0.2, \sigma_2 = 0.03, \sigma_1 = 0.\) Let \(h = 10\ m, d_1 = 1\ m, \omega = 4\pi \times 50\ \text{KHZ},\) and \(\ell = 5\ m.\) The z-scattered magnetic field form three-layered free buried objects and in layer buried object are obtained, under the above data, relative to the radial distance \(\rho\) as in Fig. 3.
5. Conclusion

In this paper, the geometric optics (GO) method is proposed to simulate the TE scattering from 3-D object buried under a flat ground. This method takes into account all the probabilities of plane waves under and above the ground in the presence of the buried object. The extra-scattered field has been indicated by the perturbation in the earth field, owing to the buried objects, compared with the earth magnetic field. This paper presents an easy applicable method to check the presence of a metallic buried object under the ground. This method is applicable when the receiver is located at \( \rho \geq 50 \) m outside the system and frequency is more than 2000 KHz. These valid conditions refer to the GO method which is suitable for detecting UXO.
Fig. 3. a). The z-scattered magnetic field $H_{1z}^{\text{unper}}$ from three layered-free buried objects versus the radial distance $\rho$. b) The z-scattered magnetic field $H_{1z}^{\text{per}}$ from three-layered buried objects versus the radial distance $\rho$. c) comparison between the scattered magnetic field for a buried object in three layered medium $H_{1z}^{\text{per}}$ and magnetic field in three layered-free buried objects $H_{1z}^{\text{unper}}$ relative to the radial distance $\rho$.

References