Moiré Analysis for Assessment of Line Registration Quality

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Abstract. This paper introduces objective macro and micro line registration quality metrics based on Moiré interference patterns generated by superposing a lenticular lens grating over a hardcopy test page consisting of high-frequency Ronchi rulings. Metrics for macro and micro line registration are defined and a measurement procedure is described to enhance the robustness of the metrics computation over reasonable variations in the measurement process. The method analyzes low frequency interference patterns, which can be scanned at low resolutions. Experimental measurements on several printers are presented to demonstrate a comparative quality analysis. The metrics demonstrate robustness to small changes in the lenticular lens and grating superposition angle. For superposition angles varying between 2° and 5°, the coefficients of variance for the two metrics are less than 5%, which is small enough for delineating between test patterns of different print quality.

INTRODUCTION

Image quality analysis is an important component in the development and operation of various digital imaging technologies, such as displays, scanners and printers. To produce visually pleasing images, devices must be designed to minimize defects, such as problems related to color registration and line quality. An efficient way of measuring imaging defects is through the use of special test targets, which are designed to test the limits of the respective imaging technology. Analysis based on test target results can be used to track and minimize image defects during the development phase. This paper presents a method for analyzing printed line quality by analyzing the Moiré patterns resulting from the superposition of a test pattern, consisting of finely spaced lines, and an array of cylindrical lenses of similar spacing.

Other approaches to line quality attributes for hardcopy output include blurriness, raggedness, stroke width, darkness, contrast, fill, and registration.\(^1\)\(^-\)\(^3\) The test targets for these measures consist of a printed black line on a white background. The quality attributes are then quantified through measurements from the printed line. Blurriness measures the average transition length from light to dark, and raggedness measures the geometric distortion of the line's edge from its ideal shape. Line width is the average stroke width measured from either edge along a direction normal to the line under analysis. Line darkness measures the mean line density, which can vary due to voids for example. The contrast attribute captures the relationship between the darkness of the line and that of its surrounding field by measuring the mean reflectance factors. Contrast can vary due to blurring, extraneous marks, haze, or substrate type. Fill refers to the appearance of darkness within the inner boundary of the line. One example of line registration is the color registration of the CMYK components in an inkjet printer. If the same line is printed once with each color, then ideally, all four color lines should collapse into one, and any consistent increase in line width would indicate position errors, or mis-registration, of one or more ink components.\(^3\)

This paper introduces new metrics that differ from previous line quality attribute measures in that they are directly based on the printer's ability to create fine detailed lines. While this metric may be influenced by measures such as raggedness and blur, its use of fine details makes it unique relative to previous measures. The measurement method involves the analysis of low frequency Moiré patterns that change according to small changes in the test patterns. The test pattern consists of finely spaced parallel lines, which an imperfect printer reproduces with some line placement (or registration) errors. The parallel lines are no longer uniformly spaced in this case, and this is reflected in the resulting Moiré line shape. Moiré patterns are used as a nondestructive analysis tool in Moiré interferometry. For this method a photographic grid is printed on the surface of a material under investigation and is irradiated by coherent light. The interfering fringes (Moiré patterns) can indicate the presence of local stress and deformation for in-plane displacement.\(^4\)\(^,\)\(^5\) Moiré interferometry techniques have the advantage of being able to analyze a broad range of engineering materials in small analysis zones at high spatial resolution and sensitivity. This work extends the principles of Moiré interferometry to assess line registration quality by analyzing the Moiré patterns produced by the superposition...
of a lenticular grating on a printed Ronchi-ruling test pattern to characterize underlying printed line registration errors.

The lenticular grating consists of a plastic sheet that is smooth on one side and holds an array of parallel cylindrical lenses or prisms on the other side. The printed test pattern is a Ronchi ruling (a rectangular spatial wave linear grating) with a similar line spacing as the lenticular grating. This quality assessment approach lends itself to automation, since the lenticular grating is thin enough to be fixed to the glass surface of a flatbed scanner and does not interfere with its automatic document feeder mechanism. Since only the shape of the Moiré lines is used for analysis, it is sufficient to use a relatively inexpensive scanner (or scan faster), because high-resolution detail and tone reproduction accuracy are not crucial. This paper presents the underlying equations affecting the critical details of the Moiré patterns, describes a procedure for robust measurement and computation of macro and micro line quality metrics, and presents results for several printers. Measurements are analyzed and compared to a visual assessment of line quality based on a magnified view of the Ronchi pattern created with a high resolution scanner.

The text is organized as follows. The Moiré Model section describes the Moiré line model and discusses normalization techniques and ranges of superposition angles for robust measurements. The Line Registration Quality Measurements and Metrics section describes the measurement procedure and computation of the macro and micro line registration metrics. The Results and Analysis section presents measurement results from three different printers and analyzes measurement variability and quality assessment. Finally, the Conclusion section summarizes results and presents conclusions.

MOIRE MODEL

Figure 1 illustrates the Moiré fringe pattern produced by the superposition of a (printed) linear grid of spacing \( P_0 \) and a lenticular grating of spacing \( P_1 \) at an angle \( \alpha \). The Moiré lines are produced by the lenticular lenses intersecting with the individual lines of the Ronchi ruling. Only two lenses are illustrated in this figure; however, a sheet consisting of many lenticular lenses produces extended patterns of Moiré lines.

The Moiré line spacing, as shown in Fig. 1, is related to the superposition parameters by \(^6,^7\)

\[
P = \frac{P_0 P_1}{\sqrt{P_0^2 + P_1^2 - 2P_0 P_1 \cos(\alpha)}}. \tag{1}
\]

The angle of the Moiré lines with the base of the lenticular sheet \( \phi \) is given by \(^6,^7\)

\[
\tan(\phi) = \frac{P_1 \sin(\alpha)}{P_0 - P_1 \cos(\alpha)}. \tag{2}
\]

An actual Moiré pattern from such a sheet is shown in Figure 2. The Moiré lines deviate from straight lines due to printer imperfections. The Ronchi rule pattern was printed with a 0.4233 mm spacing, and the lenticular lens sheet consisted of lenses with a spacing of 0.630 mm (40 lenses per inch) and the lenses had a magnification factor of 1.505 (making the effective Ronchi line spacing equal 0.637 mm). Fluctuations in the printed line spacing, \( P_0 \), result from line registration errors and create deviations in the Moiré line angle \( \phi \), according to Eq. (2).

The sensitivity of the resulting Moiré line direction angle \( \phi \) to the superposition \( \alpha \) is shown in Figure 3, which plots Eqs. (1) and (2) as functions of \( \alpha \). For the 10° interval shown, the Moiré line spacing decreases from around 80 mm to 2.5 mm. Both \( P \) and \( \phi \) exhibit relatively little change for \( \alpha \) greater than 4°. In practical implementations, the superposition angle cannot be precisely controlled. So ensuring that these changes do not significantly affect the metric is a critical issue to the usefulness of this method. Therefore, selecting an \( \alpha \) around 4° reduces the impact of small changes in the superposition angle. This results in multiple low-frequency Moiré lines over the test pattern for robust analysis.
Also included in the plots of Fig. 3, are actual measurements of the Moiré line spacing and angle for five values of \( \alpha \). The measurements were made by manually setting the superposition angle \( \alpha \) and using a ruler to visually measure the resulting Moiré spacing, and a protractor to measure the Moiré direction \( \phi \). The resulting measurements agreed well with Eqs. (1) and (2) as can be seen by the measurement marker on the graphs of Fig. 3. For the measurement system proposed in this work, the lenticular grating is of high precision, while the actual superposition angle may also be variable depending on the mechanics used to load the test sheet. The following equations show the critical parameters relating the underlying line registration to the Moiré pattern parameters used in the measurement. From this derivation, a normalization step is presented to reduce the sensitivity of metrics computed from the Moiré pattern to variations in parameters of the measurement system.

A relationship between changes in the underlying line spacing and change in the angle of the Moiré pattern can be seen from taking the reciprocal of Eq. (2) and adding deviation terms to the test pattern line spacing and Moiré pattern angles to obtain

\[
\cot(\phi + \varphi(x)) = \frac{[P_0 + e(x)] - P_1 \cos(\alpha)}{P_1 \sin(\alpha)} + \frac{P_0 + e(x)}{P_1 \sin(\alpha)} - \cot(\alpha),
\]

where \( e(x) \) is the additive displacement term for underlying line spacing \( P_0 \), and \( \varphi(x) \) is the deviation from Moiré angle \( \phi \). The angular deviations vary over the printed pattern based on changes in the underlying test pattern. Without loss of generality, this direction is denoted as a function of a single variable \( x \). To separate the deviation terms, a Taylor series can be applied to the cotangent function and expanded about \( \phi \). After higher-order terms are dropped (assuming small deviations), Eq. (3) results in

\[
\varphi(x) = -e(x) \left( \frac{\sin^2(\phi)}{P_1 \sin(\alpha)} + \frac{\cot(\alpha) + \cot(\phi)}{\csc^2(\phi)} - \frac{P_0 \csc(\alpha)}{P_1 \csc^2(\phi)} \right),
\]

where terms in the parenthesis are constant over \( x \) and relate to a constant offset on the Moiré angle \( \phi \). They are subtracted out in the estimation procedure. The effective gain term that scales the line position deviations to Moiré pattern angle deviations is given by

\[
g_m = -\frac{\sin^2(\phi)}{P_1 \sin(\alpha)},
\]

where the gain/sensitivity is determined by the lenticular grid spacing \( P_1 \) and superposition angle \( \alpha \). An alternate derivation of \( g_m \) can be obtained directly through the ratio of the root-mean-square (rms) deviations of \( \varphi \) and \( P_0 \). This would eliminate the offset (zero-order) term of Eq. (4) and allow the gain factor \( g_m \) to be computed directly from the derivatives of Eq. (3) with respect to \( \phi \) and \( P_0 \). The gain factor \( g_m \) in this case is simply \( \partial \varphi/\partial P_0 \).

Since the deviations, \( \varphi(x) \), will be extracted from the Moiré patterns and used for characterization, the sensitivity to \( \alpha \) becomes an issue for consistent measurements (small changes in \( \alpha \), for \( \alpha \) near zero, can result in large changes in the gain). This variability can be significantly reduced by dividing the measured angle deviation by the measured distance between the moiré lines, if the effective Ronchi pattern line spacing is close to that of the lenticular grid. With \( P_1 \) equal to \( P_0 \), Eq. (1) can be simplified using the half angle formula to show the Moiré line spacing is related to \( \alpha \) by

\[
P = \frac{P_1}{2 \sin(\alpha/2)}.
\]

For small \( \alpha \) (as is the case here), \( \sin(\alpha) \) approximately equals \( \alpha \) (in radians). Thus, by applying this approximation to Eqs. (5) and (6), the normalized gain becomes

\[
\tilde{g}_m = \frac{g_m}{P} \approx -\frac{\sin^2(\phi)}{P_1^2}.
\]

This equation shows that the repeatability of the measurement is enhanced through this normalization. The dominant scale factor controlling the gain on the angular displacement is now primarily dependent on the lenticular grid spacing, which can be precisely controlled and does not change with the superposition angle. The next section describes the extraction of \( \varphi(x) \) and \( P \) from the scanned Moiré patterns, and the development of the metrics based on the normalization described by Eq. (7).
LINE REGISTRATION QUALITY MEASUREMENT AND METRICS

An example of a Moiré pattern used to extract parameters for quality metrics is shown in Fig. 2. The pattern was created with a five inch square printed test ruling and a lenticular grating at a superposition angle indicated by the line at the top of the figure. The Moiré patterns result in wavy lines along an angled path. Their deviation from a straight path indicates a faulty printed test pattern that is due to the perturbations of Eq. (3). For example, the lines in the top half of the figure deviate to the left of the expected straight path and then back again. This corresponds to an increase, and then a decrease of angle \( \phi(x) \), which corresponds to changes in \( \varepsilon(x) \) according to Eq. (4). This section describes how these changes can be extracted, characterized, and used to form line quality metrics. The underlying line imperfections for two laser printers are illustrated in Figure 4. This figure compares two portions of the printed Ronchi ruling test pattern. The slices are 8.5 mm long and contain the top 20 printed lines. The figure contains regular tick marks to indicate line numbers and their expected locations. Observe at the junction of the line sets that the line spacing is not consistent between the two printers. Line 1 is aligned for both prints; however, lines between 6 and 19 do not align, and lines from printer LP1 (top line set) deviate from the ideal locations from line 6 onward. The printed lines from LP2 (bottom line set) also deviate from the ideal locations, but the deviations are less pronounced and only start to become large from line 14 onward, indicating the line registration quality of printer LP2 is higher than that of printer LP1. The metrics described in this section will correctly assess this difference from values extracted over the whole printed line pattern.

The printed test pattern used for the results presented in this paper is a 5 × 5 inch square Ronchi ruling. A lenticular lens sheet is superimposed at an angle of around 4° and the resulting pattern is scanned at 600 dpi on an HP ScanJet C7710A flatbed scanner. The scanned image results in a 3000 by 3000 pixel image, which yields 10 pixels per black-white line pair (corresponds to a density of 60 line pairs per inch or a line spacing of 0.4233 mm). The targets are scanned in a monochrome setting and cropped to 2048 by 2048 pixels to limit scan edge effects. The lenticular lens sheet is a Pacur LENSTAR large format polyester sheet with 40 lenticules per inch. The sheet is 0.033 in. thick, which also corresponds to the lenticular focal length. The lenticules have a radius of 0.0146, and a width of 0.0251 inches. For the analysis, the scan is low-pass filtered to emphasize the lower frequency Moiré patterns of interest, using a rotationally-symmetric two-dimensional Gaussian correlation kernel of size 8 and standard deviation parameter of 8. Luminance variability from the scanner, which often affects banding metrics, for example, is mitigated using this approach because only the shape of the Moiré lines are used in the metric and not their intensity. The angle \( \alpha \) between the test pattern and lenticular grating was determined (near 4°) by eye to produce Moiré patterns of good visibility and measurability after scanning for analysis purposes.

The analysis program extracts the contiguous pixel locations of the local minima (or constant gray-level) forming a pattern vertically oriented over the page. The groups of pixel locations associated with the Moiré patterns are characterized by a best-fit (least squares) line to pixel minima to obtain an estimate of the Moiré line corresponding to a perfect line pattern. The groups of pixels near the line corresponding to the actual patterns are identified and smoothed using a higher-order polynomial (order 32). Since multiple lines exist over the page, a search for local minima is performed with a best-fit line to identify each Moiré pattern. To describe this process, denote the scanned Moiré pattern image as \( I(x_m,y_m) \), where \( x_m \) and \( y_m \) respectively represent the discrete row and column positions of the image matrix. As illustrated in Fig. 2, the origin of this coordinate system is located at the top left pixel. The algorithm searches for Moiré lines by assuming the form

\[
R(x_m,m,b) = mx_n + b, \tag{8}
\]

where \( R(x) \) is the \( y \) coordinate of Moiré line, and \( m \) and \( b \) are the slope and \( y \) intercept, respectively. The line parameters are found through an exhaustive search over a range of \( b \) and \( m \) values in order to minimize the following cost function:

![Figure 4. Line misregistration in 8.5 mm long vertical slices of the top twenty lines of the actual printed test patterns of printers LP1 (top) and LP2 (bottom). Rulers show ideal line locations.](image-url)
where \( N \) is the total number of pixel rows in the scanned image. The parameters \( b \) and \( m \) associated with the best-fit line can be determined by

\[
[m_0, b_0] = \arg \min_{m, b} \{C(m, b)\}. \tag{10}
\]

Once the best fit is found over the image, the slope \( m_0 \) is fixed and \( b \) is incremented over the column pixels of \( I(x, y) \) and local minima detected to find the other patterns. Since the images are relatively simple, the minima appear with good distinction, and a threshold can be set to ignore insignificant minimum peaks and collect the set of \( b \) values corresponding to local minima denoted by

\[
B = [b_1, b_2, b_3, b_4, b_5, \ldots, b_Y], \tag{11}
\]

such that \( B \) is a vector, of length \( Y \), containing the intercept values of the lines that are the best linear fits to the actual Moiré lines. The average distance between the minima is taken as an estimate of the Moiré line spacing given by

\[
P = \frac{1}{Y-1} \sum_{i=1}^{Y-1} |b_{i+1} - b_i|. \tag{12}
\]

The actual curved Moiré pattern can be found by locating the local minimum for each \( x \) coordinate in the neighborhood of each fitted line. For some lenticular grids; however, the locally dark image points appear at the lens intersections, creating a regular discontinuity over the pattern. To improve the detection of the Moiré pattern pixel, a midluminance gray level was used. Therefore, the \( y \) coordinates of the actual Moiré patterns were determined by the pixels closest to the Moiré pattern gray level \( I_m \) in the neighborhood of the fitted line. The collection of points for the \( i \)th Moiré pattern is denoted as

\[
S_i(x) = \left\{ \arg \min_y [I(x, y) - I_m] \right\} \left( R(x; m_0, b_i) - \frac{\hat{p}}{2} \right), \tag{13}
\]

where \( I_m \) is the mean luminance of the Moiré patterns. Figure 5 illustrates the results of this extraction process for two sample laser printer outputs. A 32-order polynomial was fitted to the locus of points from Eq. (13) in order to smooth and overlay the Moiré patterns, along with the best fit lines for visual inspection, on the actual scanned image. With the approach described above the need for smoothing is important because of the periodic dark bands of the lens intersections cause regular glitches in the points. While other methods can be used for smoothing, such as the median filter, this work uses the 32-order polynomial fitted to the points identified by Eq. (13). The results observed in Fig. 5 demonstrate that the extraction procedure is indeed capturing the basic elements of the Moiré patterns.

The deviation from all lines is characterized by a mean deviate at each row, given by

\[
L(x_n) = \frac{1}{Y} \sum_{y=1}^{Y} [\hat{S}_i(x_n) - R(x_n; m_0, b_i)], \tag{14}
\]

where \( \hat{S}_i \) is the resulting polynomial fit to the points of \( S_i \) in Eq. (13). The derivative of \( L(x) \) is equal to the tangent of angle \( \varphi(x) \); and for small values of \( \varphi(x) \), can be estimated with a numerical gradient as follows:

\[
\]
given by

\[ \varphi(x_n) = \tan[\varphi(x_n)] = \frac{1}{2\Delta}[L(x_{n+1}) - L(x_{n-1})]. \]  

The estimate of the Moiré angle can be used to compute the macro quality metric referred to as the normalized average variance (NAV), given by

\[ \sigma^2_L = \frac{1}{\hat{p}^2(N-1)} \sum_{n=1}^{N} \varphi(x_n)^2. \]  

Note this metric reflects the average line registration error over the whole test pattern. A micro quality line metric can be taken over local portions of the test pattern and involve the row corresponding to the worst deviation. This metric is called the normalized maximum deviation (NMD), and is given by

\[ \bar{\varphi}_L = \frac{1}{\hat{p}} \max_{n}[\varphi(x_n)]. \]  

RESULTS AND ANALYSIS

To demonstrate the robustness of the metrics to superposition angle variation, a pattern from laser printer LP1 was scanned for four different \( \alpha \) values, and the resulting NAV and NMD quality metrics are presented in Table I. It can be seen that the spacing decreases with increasing angle, while the NAV and NMD measures stay relatively constant. Quantitatively, the coefficients of variances (CV) for the metrics over the \( \alpha \) variations are 4.9% and 3.3% for the NAV and NMD, respectively. The CV is the ratio of the standard deviation to the mean of a data set, and it provides a quantity related to the measurement resolution, which is affected by factors such as printer and scanner settings, as well as, properties of the lenticular lenses used, such as lens spacing and precision.

As an example of how the quality metrics respond to different printers, the NAV and NMD metrics were computed using the outputs of two laser printers (LP1 and LP2, used in Fig. 5), and an inkjet printer denoted as IP. Table II shows a numerical comparison between these printer outputs, as well as the measurement parameters. The CV values computed from Table I can be used to examine the relative comparison of line registration quality between printers. For example, the difference between the NAV values as a percentage of their mean is 15% for printers LP1 and LP2 in

<table>
<thead>
<tr>
<th>Printer</th>
<th>( \alpha )</th>
<th>( \hat{p} )</th>
<th>NAV</th>
<th>NMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP1</td>
<td>3</td>
<td>4.26</td>
<td>2.11 \times 10^{-9}</td>
<td>13.1 \times 10^{-5}</td>
</tr>
<tr>
<td>LP2</td>
<td>3.8</td>
<td>3.73</td>
<td>1.79 \times 10^{-9}</td>
<td>11.7 \times 10^{-5}</td>
</tr>
<tr>
<td>IP</td>
<td>3.7</td>
<td>3.66</td>
<td>2.37 \times 10^{-9}</td>
<td>11.4 \times 10^{-5}</td>
</tr>
</tbody>
</table>

Table II. This value is greater than the 4.9% variation expected from the measurement variability and thus indicates that the large scale (macro) line registration quality of LP2 is better than that of LP1 (consistent with observations in Figs. 4 and 5). In addition, the NMD measurements differ by 8.3%, which is greater than the 3.3% CV for the NAV measure. Comparing the inkjet printer IP with LP1, it is evident from the NAV values that IP has poorer quality (consistent with examinations of scaled-up observation of the line quality); however, the NMD values differ by 10.8% of their mean, which is greater than the 3.3% CV value. This indicates that even though LP1 has better line registration on a macro scale (on average across the page), it has greater isolated deviations than IP.

These results suggest that the above measures can serve as a quality metric for printed line registration. The NAV measure reflects the average printed line spacing \( P_0 \) constancy over the length of the test page. A quasiperiodic pattern in \( L(x_n) \) reflects banding like intensity variations across the test page as observed in Fig. 4 for LP1. These shape variations reflected periodic fluctuations in the printed line spacing, which are likely due to the same problems causing banding, such as imperfect rollers in the print process direction or gear noise. Moreover, \( L(x_n) \) can isolate process motion-related banding causes from other ones that affect reflectance, such as toner or ink deposition inconsistencies.

CONCLUSION

This work outlines the use of Moiré analysis for the quantification of line registration. The line registration metrics developed here are based on modeling the interference between a lenticular lens sheet and a hardcopy test target containing a Ronchi ruling or linear grating, and they provide examples of how the resulting Moiré patterns can be used to measure line registration quality. There are clearly other metrics that can be derived from the extracted Moiré patterns that can emphasize other issues depending on the application. For instance, if the Moiré line deviations are quasiperiodic, it is likely that these deviations indicate the root cause of banding. Therefore, metrics based on the periodicity of these deviations over macro regions can be used for banding characterization. The work derived general equations to help in designing of metrics that have good repeatability.

The experimental setup presented in this work suggests methods for volume processing of hardcopy samples. This would require a scanner with an automatic document feeder. A lenticular lens sheet could then be embedded into the
scanner glass or fixed on top of the glass where the test sheet could slide over it. The resulting patterns would then be scanned, and software applied to compute the performance analyses as described in this paper. The monochrome and low-resolution scans are relatively easy to produce and analyze on a computer. A potential problem resulting from an automatic document feeder is maintaining a constant superposition angle between test page and lens sheet; however, it has been shown that the proposed metrics are robust to small changes in the angle. A more significant problem would arise from variations in the distance between the test pattern and lenticular sheet, such as might result from trapped air or irregular pressure on the test pattern. In this case the Moiré line will be artificially skewed causing variations from the distance rather than line mis-registration. It would be important in such a system to ensure the automatic feed (or any other system) minimizes this variation for accurate metrics.

REFERENCES