GA-Based Multi-Objective Optimization for Retrofit Design on a Multi-Core PC Cluster

Keunhyoung Park, Byung Kwan Oh & Hyo Seon Park*
Department of Architectural Engineering, Yonsei University, Seoul, Korea
&
Se Woon Choi
Department of Architecture, Catholic University of Daegu, Kyeongsan-si, Korea

Abstract: This article presents a distributed nondominated sorting genetic algorithm II (NSGA-II) for optimal seismic retrofit design using buckling restrained braces (BRBs) on a cluster of multi-core PCs. In the formulation, two conflicting objective functions of the initial BRB installation cost required for seismic retrofitting and damage cost that can be incurred by earthquakes expected during the life cycle of the structure were minimized. Because time-consuming nonlinear structural analyses are required for fitness evaluations of individuals in every generation, parallelism at candidate design level or individual level is exploited by assigning fitness evaluations for individuals to slave core processors evenly. The distributed algorithm is applied to seismic retrofit design of 2D steel frame structure and 3D irregular reinforced concrete structure. The performance of the distributed NSGA-II was assessed based on three criteria: convergence of the distributed algorithm, efficiency of distributed computing, and quality of optimal solutions. Implementation of the distributed algorithm on the multi-core cluster consisting of up to 64 core processors resulted in relatively high speedups or efficiencies of the distributed optimization without deteriorating the quality of the optimal solutions.

1 INTRODUCTION

To improve the seismic capacity of a building structure, a number of retrofitting techniques can be used. Hysteretic dampers (HDs), such as the buckling restrained brace (BRB), are an effective device to minimize damage to the main structural frame by absorbing the input energy from an earthquake (Huang et al., 2000). BRBs are easier to model than general braces, which are affected by buckling, because the compressive and tensile strengths of BRB members are symmetric. Because of the advantages, many studies have been conducted on seismic performance improvement using BRBs, and many of the proposed methods were actually applied to various structures (Sabelli et al., 2003; Kim and Choi, 2004; Xie, 2005; Tremblay et al., 2006; Fahnestock et al., 2007).

For an efficient seismic retrofitting design using BRBs, standard optimization algorithms are not appropriate because the relationship between changes in the values of design variables representing the cross-sectional areas and the seismic performance of the retrofitted building using BRBs is discontinuous and nondifferentiable (Farhat et al., 2009). Because of these reasons, the properties of a BRB are mostly determined through trial-and-error methods. Thus, it is necessary to develop a general and efficient optimization technique that can be applied to retrofitting design using BRBs.

The genetic algorithm (GA) is an alternative to general optimization techniques that can be applied regardless of the characteristics of the problem solution space because its optimization concept is simple and widely applicable (Joly et al, 2014; Reyes et al, 2014; Luna et al, 2014; Pedrino et al, 2013; Menendez et al, 2014; Alexandridis, 2013; Iacca et al, 2014; Cabessa and
Siegelmann, 2014; Hsu, 2013). However, a GA may need a large number of iterations depending on the algorithm characteristics. In addition, a time-consuming nonlinear structural analysis is required for the evaluation of seismic performances of a retrofitting design. Hence, any new GA-based optimization for retrofitting designs using BRBs should reduce the computational time required for optimization.

To solve the excessive computational time for structural optimization, many studies have proposed to improve the time efficiency by using high-performance computational resources (Adeli and Kamal, 1992, 2000, 2002; Park and Sung, 2002; Umesha et al., 2005; Knysh and Kureichik, 2010; Guthier et al., 2014; Distefano and Puliafito, 2014). Nature-based structural optimization approaches for large-scale structures have been developed in recent years (Aldwaik and Adeli, 2014). In particular, parallel computing for structural optimization have been attempted to reduce the computational burden that accompanies structural analysis (Adeli and Kamal, 1993; Adeli and Kumar, 1995a, b; Munir et al., 2014). However, considering the problem size for structural analysis and cost of computational resources, the use of a supercomputer may not be economical, or engineers may have difficulty accessing it. For this reason, many researchers have constructed cluster computers using commercial personal computers (PCs) or workstations to realize high-performance computing (HPC) and utilized them in research and development. As some examples, workstation clusters have been used for finite element analysis (Adeli and Kamal, 1995a; Suzuki and Roux, 2014) and structural optimization (Adeli and Kumar, 1999; Umesha et al., 2005). Many cases of clustering multiple inexpensive PCs for structural optimization are increasingly being reported (Park et al., 2006; Munir et al., 2014).

The computational efficiency and performance of parallel or distributed algorithms in all computers, including PCs, are determined by the performance and structure of the central processing unit (CPU). In its early years, CPUs were single-core CPUs where all tasks are sequentially processed by a single core processor. However, as the core processor clock rate has increased, multi-core CPUs have been developed that are equipped with multiple core processors to overcome physical limitations such as the increased power consumption and heat output. So far, no case studies have been reported on the development or application of distributed optimal seismic retrofit design using PC clusters consisting of multi-core CPUs. The limitation of excessive computational time for a conventional GA-based optimization algorithm for a multi-objective optimization of seismic retrofit design can be overcome by exploitation of multi-core CPUs.

In this study, to improve the efficiency of retrofit design using BRBs that requires time-consuming iterative nonlinear analysis, a distributed GA-based optimal retrofit design method is proposed that can effectively exploit the computational resources in a cluster of commercial PCs with a multi-core CPU. Each PC runs an i7-2600 quad-core processor (Intel, 2014) and is connected via a NETGEAR switching network (NETGEAR, 2015) that provides gigabit-level communication speed to form a local area network (LAN). The distributed GA was developed through the consideration of the communication characteristics for a PC cluster comprising multi-core CPUs. The developed multi-objective optimization technique was applied to retrofit design of a 2D steel frame structure and a 3D reinforced concrete building structure that required nonlinear static structural analysis and nonlinear dynamic structural analysis. The performance of the distributed algorithm developed in this study was assessed based on criteria such as global convergence, efficiency of distributed computing, and quality of optimal solutions.

### 2 FORMULATION OF A MULTI-OBJECTIVE OPTIMIZATION PROBLEM FOR RETROFIT DESIGN USING BRBs

#### 2.1 Design variables

In this study, two types of BRBs were considered in the retrofit design: circular hollow sections (Di Sarno and Elnashai, 2009; Güneyisi, 2012) and rectangular sections (Güneyisi, 2012) shown in Figure 1. For the retrofitting with BRBs, more effective results can be obtained by considering design variables such as topological issues, cross-sectional areas, and material properties of a BRB. (Adeli, 2002; Farhat et al., 2009). However, the material properties were fixed so that the optimal results could be compared with those in the example (Di Sarno and Elnashai, 2009) for verification. Then, as shown in Figure 1, the BRB section installed at the kth span in a building was determined with the parameters as, diameter $D_k$, width $W_k$, and thickness $T_k$. Thus, in the formulation, the optimal sizes and locations of BRBs are found using the distributed algorithm.

#### 2.2 Multi-objective functions

In this study, the objective functions of the design optimization were set to the initial BRB installation cost required for seismic retrofitting and damage cost that can be incurred by earthquakes expected during the life cycle of the structure retrofitted with BRBs. These two conflicting objective functions were considered for the
2D and 3D building structure examples using nonlinear static analysis and nonlinear dynamic analysis (Adeli et al., 1978; Adeli and Chyou, 1986; Adeli and Mabrouk, 1986).

2.2.1 Objective functions for 2D frames with nonlinear static analysis. The load varying capacity of BRB is determined by the cross-sectional area of the steel core, and the performance and size of the steel case and mortar that surround the BRB steel core are determined by and dependent on the steel core strength. Thus, the cross-sectional area of the steel core is a dominant factor with respect to the total volume needed for the steel core in a BRB. The objective function $f_{2D-1}$ to minimize the BRB weight for 2D frame structures is based on the total volume of the steel cores in the installed BRBs. It can be calculated using Equation (1):

$$
\text{Minimize } f_{2D-1}(\mathbf{x}) = \sum_{i=1}^{n_B} 2A_i l_i
$$

where $A_i$ and $l_i$ are the cross-sectional area and length, respectively, of the BRB installed at the $i$th location for a BRB. The length is determined by the span length and story height. $n_B$ is the total number of possible locations for BRBs. The volume of the steel core is doubled in Equation (1) because two BRBs are installed in the shape of X.

The second objective function $f_{2D-2}$ minimizes the damage caused by an earthquake. This is a damage cost that can be incurred by earthquakes expected during the life cycle of the structure (Wen and Kang, 2001) and can be calculated with Equation (2).

$$
\text{Minimize } f_{2D-2}(\mathbf{x}) = \frac{\nu}{\lambda} \left(1 - e^{-\lambda t}\right) \sum_{i=1}^{k} C_i P_i
$$

$\nu$ is the annual occurrence rate of earthquakes, which is used to model the earthquake occurrence through the Poisson process. $\lambda$ is a lognormal distribution parameter in the seismic hazard distribution, and $t$ is the expected-use period of a retrofitted structure. $k$ represents the seismic damage and is the number of limit states considered. $C_i$ and $P_i$ are the life-cycle cost and probability with respect to a single hazard at the $i$th damage state. $P_i$ is calculated based on the inter-story drift ratio $\Delta$, as defined in Equations (3) and (4).

$$
P_i = P(\Delta > \Delta_i) = -\frac{1}{\nu \times t} \left\{ \ln \left[ 1 - P(t, \Delta > \Delta_i) \right] \right\}
$$

$\Delta_i$ is the inter-story drift ratio, and $P_i(\Delta > \Delta_i)$ is the probability that the $i$th damage state can occur over the period $(0, t)$. In this study, the three different levels of ground shaking were considered to calculate $P_i(\Delta > \Delta_i)$: 50%, 10%, and 2% chance of being exceeded in a 50-year period. To calculate the maximum inter-story drift ratio at each seismic level, nonlinear static structural analysis was performed. Based on the results, a function to represent $P_i(\Delta > \Delta_i)$ was determined through regression analysis. It was assumed that the regression analysis follows the generalized extreme value distribution (Liu et al., 2003) and the damage states based on the inter-story drift ratio (Wen and Kang, 2001; Liu et al., 2003) is given in Table 1.

<table>
<thead>
<tr>
<th>Performance level</th>
<th>Damage state</th>
<th>Inter-story drift ratios (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>None</td>
<td>$\Delta &lt; 0.2$</td>
</tr>
<tr>
<td>II</td>
<td>Slight</td>
<td>$0.2 &lt; \Delta &lt; 0.5$</td>
</tr>
<tr>
<td>III</td>
<td>Light</td>
<td>$0.5 &lt; \Delta &lt; 0.7$</td>
</tr>
<tr>
<td>IV</td>
<td>Moderate</td>
<td>$0.7 &lt; \Delta &lt; 1.5$</td>
</tr>
<tr>
<td>V</td>
<td>Heavy</td>
<td>$1.5 &lt; \Delta &lt; 2.5$</td>
</tr>
<tr>
<td>VI</td>
<td>Major</td>
<td>$2.5 &lt; \Delta &lt; 5.0$</td>
</tr>
<tr>
<td>VII</td>
<td>Destroyed</td>
<td>$5.0 &lt; \Delta$</td>
</tr>
</tbody>
</table>
2.2.2 Objective functions for 3D frames with nonlinear dynamic analysis.

Minimize \[ f_{3D-2}(\mathbf{X}) = \sum_{i=1}^{M} \int (V_i \delta_i + M_i \theta_i) dt \]
\[ = \sum_{i=1}^{M} \sum_{t=1}^{N} \left[ \frac{(V_i(t) + V_i(t+1))}{2} \times (\delta_i(t+1) - \delta_i(t)) + \frac{(M_i(t+1) + M_i(t))}{2} \times (\theta_i(t+1) - \theta_i(t)) \right] \]
\[
(5)
\]

For 3D frame structures, the objective function that minimizes the BRB weight is based on the steel core volume inside the BRBs and calculated via Equation (1) in the same manner as in the 2D frame example. For 3D frames, if Equation (2) is set to a structural performance-related objective function, nonlinear dynamic structural analysis should be performed multiple times. Then, the computational time for the design optimization is considerably increased compared to the 2D frames. The reason for the significant increase in computational time is because structural analysis has to be performed using seismic ground motions that correspond to three levels of ground shaking depending on the probability of occurrence to calculate Equation (2). A single ground motion cannot represent a seismic level, so multiple ground motions should be selected for each seismic level in nonlinear dynamic analysis. For each seismic level, 3 to 10 ground motions need to be considered to evaluate the fitness value of an individual for every generation during the optimization by a GA. For every iteration or generation of GA-based optimization, it is required to evaluate the fitness of all individuals in a population. Thus, to avoid the excessive computational time for numerous 3D nonlinear dynamic analyses for Equation (2), the objective function using the dissipated energy in Equation (5) is set to the second objective function \( f_{3D-2} \). To increase the hysteretic energy dissipated by BRBs (Chopra, 1995; Choi and Kim, 2006; Farhat et al., 2009), the energy dissipation of a 3D frame structure during nonlinear dynamic analysis in Equation (5) is minimized.

\( V_i(t) \) and \( M_i(t) \) refer to the shear force and bending moment of the \( i \)th element at the time step \( t \) during nonlinear dynamic analysis. \( d_i(t) \) and \( \theta_i(t) \) refer to the deformation and rotation of the \( i \)th element at the time step \( t \). \( M \) is the number of members and \( N \) is the number of points of ground motion data.

2.3 Constraints

In this study, an inter-story drift ratio can be used as an index to evaluate the performance level of structures employed in previous studies (Liu et al., 2003; Zou and Chan, 2005). A performance-based constraint is set to limit the maximum inter-story drift to below the allowable inter-story drift ratio as given in Equation (6).

\[ c = \frac{\Delta_{\text{max}}}{\Delta_a} \leq 1.0 \]
\[
(6)
\]

\( \Delta_{\text{max}} \) is the maximum inter-story drift ratio from the analysis, and \( \Delta_a \) is the allowable inter-story drift ratio. The ratio was set to 2% based on the drift limit in Table C1-3 of FEMA 356 (American Society of Civil Engineers 356, 2000). For the 3D structures, the drifts in both of the horizontal directions are considered in the maximum inter-story drift ratio, as shown in Equation (7).

\[ \Delta_{\text{max}} = \sqrt{\frac{\Delta_{x-\text{max}}^2 + \Delta_{y-\text{max}}^2}{h_{\text{story}}}} \]
\[
(7)
\]

\( \Delta_{x-\text{max}} \) and \( \Delta_{y-\text{max}} \) are the maximum displacements in the \( x \) and \( y \) directions, and \( h_{\text{story}} \) is the height of the corresponding story.

2.4 Nondominated sorting genetic algorithm II (NSGA-II)

Because the relationship between the design variation and structural performance is nonlinear and discrete, it is difficult to apply existing standard optimization algorithms. Instead, an effective alternative is to apply a GA, which is a type of evolution algorithm (EA) (Farhat et al., 2009; Tan et al., 2012; Campomanes-Alvarez et al., 2013). Many different types of GAs have been applied to structural optimization (Sarma and Adeli, 2000, 2001; Hejazi et al., 2013). For simultaneous optimization of the two objective functions of retrofitting cost and damage cost, a multiobjective evolutionary algorithm (MOEA) should be used (Deb, 2001; Deb et al., 2002; Marler and Arora, 2004). In this study, the NSGA-II was used (Deb, 2001; Deb et al., 2002; Choi et al., 2014), and a distributed optimal design algorithm was developed using a multicore PC cluster. In NSGA-II, the solutions not satisfying the constraint condition are evaluated on the basis of the proportion violating the constraint condition. However, in the relative evaluation of solutions satisfying the constraint condition, the value of constraint function has no influence, and the relative evaluation of solutions is performed by assigning the domination rank on the basis of whether domination is formed with reference to each objective function.
3 DISTRIBUTED NSGA-II ON A MULTI-CORE PC CLUSTER

3.1 System architecture for a PC cluster with multi-core CPUs

Recent commercially available multi-core PCs have a multiple-instruction, multiple-data (MIMD) structure to form a tightly coupled system, where the CPU that performs the computation consists of multiple core processors, but other components are shared (Flynn, 1972). The multi-core PC used in this study consisted of four core processors in the CPU, as shown in Figure 2 (Intel, 2014). However, other components such as the cache memory, random-access memory (RAM), hard disk drive (HDD), and LAN card for communication were shared by all of the core processors. PCs in the cluster all provided the same performance: the CPU was an Intel Core i7 2600 K (Intel, 2014) with a 3.4 GHz clock rate, and 8 GB of RAM was installed in each PC.

When clustering multi-core CPUs, scaling them up on the basis of the core processor unit is impossible because of the strong communication bond between core processors inside a CPU. Thus, scaling on the basis of the CPU or PC unit is more realistic, as shown in Figure 3. In this study, the cluster had 16 PCs, and the CPU of each PC had four core processors. As shown in Figures 2 and 3, the core processors access data in the cache memory, RAM, and HDD in order of fast access or can use direct memory access to each level to read or write data.

The scheduler used for parallelization of the optimal design algorithm in this study is a function provided by the MATLAB Distributed Computing Server (MDCS) (MATHWORKS, 2015), which does not support direct data exchange between core processors through the internal CPU cache memory, RAM, or HDD. Thus, the $C_{ij}$ core processor acts as a master node and slave node at the same time, and other core processors act as slave nodes to exchange data through a network switch. All PCs in the cluster are connected via a NETGEAR gigabit network switch (NETGEAR, 2015) with a transmission rate of 1.0 Gbit/s to communicate with each other. To transfer data from the slave core processors to the master node located in the same PC or other PCs, external communication is performed via a single LAN card inside the PC. Thus, the communication route is shared. In other words, a bottleneck can occur in the LAN card of the PC as communication increases.
between the master node and slave nodes. This article proposes a distributed algorithm to minimize the communication between nodes in the PC cluster.

3.2 Distributed NSGA-II

A GA-based optimization for seismic retrofit design generally requires iterative nonlinear analysis. The computational time required for genetic operations such as mutation or crossover in a GA is much less than the computational time required in nonlinear analysis for each candidate solution (individual in a population). Based on the characteristics of a GA, to effectively exploit the distributed computational capacity of multi-core CPUs, the fitness evaluation requiring a relatively long computational time for nonlinear analysis was assigned to multiple core processors, as shown in Figure 4. As shown in Figure 4, one of the core processors (C$_{1,1}$ in Figures 3 and 4) in the cluster is designated as a master node in a master–slave programming model to perform genetic operations. The fitness evaluation of $m$ candidate solutions, which corresponds to the population size for NSGA-II, is performed by the master core possessor and $4n - 1$ slave processors ($4n$ core processors in total for the case of $n$ PCs). Then, a core processor performs $m/4n$ (rounded up to the nearest integer) nonlinear structural analyses in every generation, and the results are sent to the master node. Therefore, in this article, parallelism at candidate design level or individual level is exploited for development of distributed NSGA-II on a multi-core PC cluster in Figure 3.

3.3 Performance of distributed NSGA-II

Windows 7 is chosen as the PC OS in this study. The toolbox functions of the MATLAB Distributed Computing Server (MDCS) (MATHWORKS, 2015), which is based on the MPI, were used as the message passing program. OpenSees (Mazzoni et al., 2006) was used to evaluate the fitness of $m$ candidate designs required in every generation for static nonlinear and dynamic nonlinear structural analyses.

The performance of the distributed NSGA-II developed in this study was assessed based on three criteria: (1) global convergence, indicating multiple iterative optimal design results converged to a similar optimal result; (2) efficiency of computational time based on number of core processors used; and (3) quality of optimal solutions.

For the global convergence test, which evaluated whether the design optimization results were produced consistently, optimal solutions were obtained from three or five independent distributed runs for each cluster configuration. NSGA-II obtains a set of feasible solutions known as Pareto optimal solutions—in contrast to general optimization techniques that present a single optimal design according to a single objective function—because it is a multi-objective optimization.
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4 APPLICATION TO OPTIMAL SEISMIC RETROFITTING USING BRBs

In this study, the structural responses of frames retrofitted with BRBs were evaluated through nonlinear static structural analysis for the 2D steel frame structure and nonlinear dynamic structural analysis for the 3D reinforced concrete frame structure. Tables 2 and 3 present the design variable ranges used in the 2D steel frame structure and 3D reinforced concrete frame structure. For distributed implementations, the sizes of population for NSGA-II are set to 40 and 64 for the 2D and 3D examples, respectively.

4.1 2D steel frame structure with nonlinear static analysis

The 2D frame example is taken from the literature (Di Sarno and Elnashai, 2009) for the sake of comparison. As shown in Figure 5, three different cross-sections were used in the retrofitting design scheme using BRBs in the reference and the total volume of the steel cores in the BRBs was 1473.8 m$^3$. The maximum inter-story drift and natural frequency of the retrofitted structure were reduced from 3.96% to 1.35% and from 2.05 s to 1.08 s, respectively.

As the 2D frame is a nine-story and five-span building, as shown in Figure 5, there are 45 possible locations for installation of BRBs. In this study, for symmetry in lateral stiffness of the structure, the locations and shapes of BRBs were set to be symmetrical about the middle span in design strategy, which is as follows:

$$
\text{Section}(i,j) = \text{Section}(i, N_B + 1 - j), i = 1, 2, \ldots N_S,
$$

$$
\text{Section}(i,j) = \text{Section}(i, N_B + j), i = 1, 2, \ldots N_S,
$$

$$
\text{Section}(i,j) = \text{Section}(N_S - i + 1, N_B + j), i = 1, 2, \ldots N_S.
$$

where $i$ is the index of the resource scale case with $2^{i-1}$ core processors that is distinguished by the size of the computational resources. $T_i$ is the time taken to calculate a single generation in a GA when optimization is performed at the $i$th resource scale. $T_1$ is the time taken to perform the design optimization using a serial version. $T_{i,j}$ is the arithmetic mean value of the times taken for the $j$th generation at the $i$th resource scale case, and $n_g$ is the total number of generations for a distributed run for the $2^{i-1}$-core configuration.

The improvement in quality of an optimal solution is based on the values of the objective functions, which are calculated using the structural responses of the retrofitted structure using BRBs. Changes in the values of the objective functions before and after the retrofitting are compared to evaluate the efficiency of the design optimization.

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Area (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>15</td>
<td>70,509.12</td>
</tr>
<tr>
<td>350</td>
<td>17.5</td>
<td>95,970.75</td>
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<tr>
<td>400</td>
<td>20</td>
<td>125,349.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
<th>Area (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>15</td>
<td>5,575</td>
</tr>
<tr>
<td>300</td>
<td>15</td>
<td>8,775</td>
</tr>
<tr>
<td>400</td>
<td>15</td>
<td>11,775</td>
</tr>
</tbody>
</table>
Section(i, j) is the cross-section of the BRB installed for the jth span at the ith floor. \( N_S \) is the total number of stories and \( N_B \) is the number of spans where BRB can be installed in a single floor. In the 2D example with \( N_S = 9 \) and \( N_B = 5 \), the cross-sectional areas of BRBs for each story level and span can be set independently under the condition of the symmetry.

For the nonlinear static analysis of the 2D frame structure using the response spectrum of the rare earthquake (2%/50 years), pushover analysis was performed until the displacement at the top floor became the target displacement. The target displacement was calculated using Equation 10) (American Society of Civil Engineers, 2000):

\[
\delta_i = C_0 C_1 C_2 C_3 S_o \frac{T_e^2}{4\pi^2 g} \tag{10}
\]

\( T_e \) is the effective period of the building, \( C_0 \) is the modification factor related to the response displacement with respect to the modal participation factor and 1.48 was used as the value of \( C_0 \), \( C_1 \) is the modification factor to relate the expected maximum nonlinear displacement to the displacement calculated for linear elastic response, \( C_2 \) is the modification factor to represent the effect of pinched hysteretic shape, stiffness degradation and strength deterioration on maximum displacement response. \( C_3 \) is the modification factor that represents the increased displacement due to dynamic P–Δ effect, and \( S_o \) is the response spectrum acceleration.

4.1.1 Convergence based on Pareto optimal solutions. The number of core processors in the computing system used in this article was increased by 1, 2, 4, 8, 16, 32, and 64 (1/4, 1/2, 1, 2, 4, 8, and 16 quad-core CPUs) in order. To evaluate the global convergence of the distributed NSGA-II algorithm, Pareto optimal solutions obtained from five independent distributed runs for the 2-, 4-, 8-, 16-, 32-, and 64-core configurations and five serial runs were used. The values of the objective functions for the retrofitting design proposed in the reference (Di Sarno and Elnashai, 2009) and Pareto front lines based on the Pareto optimal solutions obtained from 30 distributed runs and 5 serial runs are compared in Figure 6. For the distributed implementation, 30 independent runs with different random seeds showed a similar tendency in the relationship between the two objective functions. When the total volumes of steel cores of the Pareto optimal solutions were around 500 and 1,500 m³, the changes in the lifetime seismic damage costs were larger than the changes in the total volume of steel cores. In contrast, the distribution of the lifetime seismic damage costs in the Pareto optimal solutions where the total volume of steel cores was above 1,500 m³ was denser than in the other parts of the solutions.
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4.1.2 Time efficiency of the distributed optimization. As mentioned in section 3.3, the speedup was calculated as the average computational time per generation. Wall-clock times for complete optimization for 2D example and 3D example of the 2-, 4-, 8-, 16-, 32, and 64-core configurations are shown in Figure 7. In Table 4, average computational time of five serial runs is compared with the average computational time of five distributed runs for the 2-, 4-, 8-, 16-, 32, and 64-core configurations. It shows that the distributed implementation of NSGA-II can drastically reduce the computational effort. Figure 8 shows the speedup curves for the example of the 2-, 4-, 8-, 16-, 32-, and 64-core configurations relative to the serial runs. In the distributed algorithm shown in Figure 4, the fitness of 40 candidate solutions for 2D example is evaluated concurrently by assigning of \( m/4n \) (rounded up to the nearest integer) candidate solutions to a core processor. Then, the number of candidate designs assigned to a core processor is 20, 10, and 5 for the 2-, 4-, and 8-core configurations, respectively. As shown in Table 4, for the 2-, 4-, and 8-core configurations, the even distribution of workloads across 4\( n \) core processors resulted in efficiencies of over 80%. The efficiency is defined as the speedup divided by the number of core processors. However, the efficiencies of the distributed algorithm on the 16-, 32-, and 64-core configurations are found to be less than 80% due to uneven load balancing across core processors. It is also notable that the size of population or number of candidate design for the 2D example is less than the number of core processes in the 64-core configuration (16 quad-core CPUs).

4.1.3 Seismic capacity of retrofitted structure. In Figure 6, the values of two objective functions in Equations (1) and (2) for the retrofitting design in the reference (Di Sarno and Elnashai, 2009) are compared to values of objective functions for optimal solutions obtained from 30 distributed runs and 5 serial runs. Compared to the retrofitting design in the reference (Di Sarno and Elnashai, 2009), optimal retrofit solutions, having

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Table 4

<table>
<thead>
<tr>
<th></th>
<th><strong>2D example</strong></th>
<th></th>
<th><strong>3D example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time (s)</td>
<td>Efficiency (%)</td>
<td>Time (s)</td>
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<td>199.9</td>
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<td>2-core</td>
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<td>60.6</td>
<td>3.30</td>
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<td>30.9</td>
<td>6.47</td>
<td>192.7</td>
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<tr>
<td>16-core</td>
<td>17.6</td>
<td>11.39</td>
<td>99.8</td>
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<td>11.5</td>
<td>17.37</td>
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<tr>
<td>64-core</td>
<td>7.3</td>
<td>27.29</td>
<td>27.8</td>
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</tbody>
</table>

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Fig. 6. Pareto front lines for 2D example from 30 independent distributed and 5 serial runs.

Fig. 7. Wall-clock times for complete optimization of 2D steel frame and 3D irregular RC frame structures.

Fig. 8. Relative speedup curves for examples.
Table 5
Comparison of the values for the two objective functions obtained from serial and distributed runs for the 2D example

<table>
<thead>
<tr>
<th>Case</th>
<th>Optimal solutions based on the serial configuration</th>
<th>Optimal solutions distributed runs based on the 64-core configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total volume of steel cores (cm$^3$) (%)</td>
<td>Lifetime seismic damage costs ($) (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total volume of steel cores (cm$^3$) (%)</td>
</tr>
<tr>
<td>Sarno and Elanshai (2009)</td>
<td>1,473.8 (100)</td>
<td>52,407.6 (100)</td>
</tr>
<tr>
<td>1st trial case</td>
<td>1,504.41 (102.1)</td>
<td>5,988.16 (11.4)</td>
</tr>
<tr>
<td>2nd trial case</td>
<td>1,470.87 (99.8)</td>
<td>4,976.17 (9.5)</td>
</tr>
<tr>
<td>3rd trial case</td>
<td>1,492.39 (101.3)</td>
<td>7,326.75 (14.0)</td>
</tr>
<tr>
<td>4th trial case</td>
<td>1,456.16 (98.8)</td>
<td>5,893.43 (11.3)</td>
</tr>
<tr>
<td>5th trial case</td>
<td>1,516.70 (102.9)</td>
<td>6,479.12 (12.4)</td>
</tr>
</tbody>
</table>

the reduced lifetime seismic damage costs with similar volumes of steel cores or the reduced volumes of steel cores with similar lifetime seismic damage costs were found by the serial and distributed runs. In Table 5, the values of two objective functions for optimal solutions having the reduced lifetime seismic damage costs with similar volumes of steel cores obtained from serial and distributed run for the 62-core configuration are compared with those for the retrofitting design in the reference (Di Sarno and Elnashai, 2009). For the serial runs, the objective function value with respect to the total volume of steel cores fluctuated within 98.8%–102.9% compared to the value of the objective function in the reference (Di Sarno and Elnashai, 2009); meanwhile, the lifetime seismic damage costs were reduced to 9.5%–10.4%. For candidate optimal solutions using 64 core processors (16 quad-core CPUs), while the total volume of steel cores of the candidate solutions fluctuated within 97.1%–103.0%, the life seismic damage costs were reduced to 8.2%–13.1%. In addition, as shown in Figure 9, the pushover curves of five candidate solutions from the 64-core configuration are compared to the curves for the retrofitting design in the reference (Di Sarno and Elnashai, 2009).

4.2 3D irregular reinforced concrete frame structure with nonlinear dynamic analysis

This example is a three-story reinforced concrete (RC) frame building with an asymmetric plan as shown in Figure 10a. Since the SPEAR building in Figure 10 was constructed with the materials and practices used in Greece in the early 1970s, it represents older construction in Greece and elsewhere in the Mediterranean region without engineered earthquake resistance (Di Sarno and Elnashai, 2002; Dolek and Fajfar, 2002; Stratan and Fajfar, 2002; Reynouard et al., 2010).

4.2.1 Structural modeling for nonlinear dynamic analysis of 3D frame structure. At the European Laboratory for Structural Assessment (ELSA), full-scale pseudodynamic tests on the SPEAR building were performed (Cosenza et al., 2005; Fajfar et al., 2006; Reynouard et al., 2010). In this study, based on the behavior of the test model proposed by Strantan and Fajfa (2002), a modified post-test model (Fajfar, 2004; Dolšek and Fajfar, 2005) was used for the design optimization. The elements in the post-test model considered the inelastic rotational behavior due to the moment only. Inelastic behavior due to the shear and axial forces was considered by moment-rotation behavior. The post-test model considered the strength degradation in the envelope of the element moment-rotation relation and P-Δ effect at the global level (Fajfar et al., 2006).

“Zero-length section elements” with “uniaxial hysteretic material” were used as plastic hinges at both
ends of the beams and columns in the modeling of the SPEAR building, and “elastic beam-column elements” in OpenSees (Mazzoni et al., 2006) was used as the elastic part in between the plastic hinges.

As shown in Figure 11, the element moment-rotation relation for plastic hinges were modified to tri-linear curves through parameter study using the displacement-controlled analysis developed by Dolsek and Fajfar (Dolsek and Fajfar, 2002), and the positive moment and negative moment behaviors are symmetrical. In the first point of Figure 11, the yield moment $M_y$ and the maximum moment $M_{\text{max}}$ were determined from cross-section analysis. The effective yield rotation was calculated using the formula $\theta_y = M_y L/6EI_{\text{eff}}$, where $L$ is the length and height of a beam and column, $E$ is the modulus of elasticity, and $I_{\text{eff}}$ is the effective moment of inertia of the element. The rotations $\theta_{\text{max}}$ and $\theta_u$, the latter corresponds to a 20% drop in strenth and is considered representative for the “Near Collapse” limit state, were estimated by the CAE method (Peruš et al., 2006), using experimental databases for columns (Fajfar et al., 2006).

The same properties of the concrete ($f_{c}'=25 \text{ MPa}$) were assumed for the whole structure. For the steel, two different values of $f_y=495 \text{ MPa}$ and $f_y=377 \text{ MPa}$ were used for $\phi_{12}$ bars and $\phi_{20}$ bars, respectively (Fajfar et al., 2006).

More details on the experiment and posttest model can be found in Stratan and Fajfar (2002) and Fajfar et al. (2006).

As the SPEAR building has an asymmetric plan, the BRB locations were set up without considering the symmetry in lateral stiffness. As shown in Figure 10, the locations where BRB can be installed were the spans between columns C1–C5, C1–C2, C4–C7, C5–C9, and C8–C9. Internal spans (C1–C3, C3–C4, C3–C6, and C3–C9) were removed from consideration. In addition, locations where torsion could be generated in columns, such as the spans between columns C8–C6 and C6–C7, and locations where the external walls were out-of-plane because the BRB should be installed.
### Table 6
Dissipated energy by SPEAR building for the candidate optimal solutions

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Total volume of steel cores in BRBs</th>
<th>Dissipated energy by SPEAR building</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0 m³</td>
<td>1.5 m³</td>
</tr>
<tr>
<td>Before retrofitting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st trial</td>
<td>54.25 (100%)</td>
<td></td>
</tr>
<tr>
<td>2nd trial</td>
<td>6.60 (13.0%)</td>
<td>4.79 (8.8%)</td>
</tr>
<tr>
<td>3rd trial</td>
<td>6.76 (12.5%)</td>
<td>4.49 (8.3%)</td>
</tr>
<tr>
<td>4th trial</td>
<td>8.86 (16.3%)</td>
<td>5.66 (10.4%)</td>
</tr>
<tr>
<td>5th trial</td>
<td>7.03 (13.0%)</td>
<td>6.94 (12.8%)</td>
</tr>
<tr>
<td>6th trial</td>
<td>6.12 (11.3%)</td>
<td>6.08 (11.2%)</td>
</tr>
<tr>
<td>7th trial</td>
<td>7.50 (13.8%)</td>
<td>6.00 (11.1%)</td>
</tr>
<tr>
<td>Serial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st trial</td>
<td>54.25 (100%)</td>
<td></td>
</tr>
<tr>
<td>2nd trial</td>
<td>6.60 (13.0%)</td>
<td>4.79 (8.8%)</td>
</tr>
<tr>
<td>3rd trial</td>
<td>6.76 (12.5%)</td>
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<td>5th trial</td>
<td>7.03 (13.0%)</td>
<td>6.94 (12.8%)</td>
</tr>
<tr>
<td>6th trial</td>
<td>6.12 (11.3%)</td>
<td>6.08 (11.2%)</td>
</tr>
<tr>
<td>7th trial</td>
<td>7.50 (13.8%)</td>
<td>6.00 (11.1%)</td>
</tr>
<tr>
<td>64-core</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st trial</td>
<td>54.25 (100%)</td>
<td></td>
</tr>
<tr>
<td>2nd trial</td>
<td>6.60 (13.0%)</td>
<td>4.79 (8.8%)</td>
</tr>
<tr>
<td>3rd trial</td>
<td>6.76 (12.5%)</td>
<td>4.49 (8.3%)</td>
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<td>7th trial</td>
<td>7.50 (13.8%)</td>
<td>6.00 (11.1%)</td>
</tr>
</tbody>
</table>

Fig. 13. Pareto front lines for 3D example from 18 independent distributed and 3 serial runs.

in the center of the column, such as the span between columns C2–C4, were not considered.

For design optimization of the 3D irregular reinforced concrete frame structure, the width and thickness of the steel panels used in the steel cores were set independent of each other. Thus, a wide range of BRB sections could be selected from the section list in Table 3. For nonlinear dynamic analysis of the 3D example, input signals used in Dolšek and Fajfar (2005) were used in this study, which is presented in Figure 12. The ground motion chosen for the analysis considered two orthogonal semi-artificial ground motions, based on a record in Hercegnovi during the 1979 Montenegro earthquake, and fitted to the EC8 spectrum (type I, Soil C) (Fajfar et al., 2006). The components were applied in the X and Y directions of the model.

4.2.2 Convergence of Pareto optimal solutions. Similar to the nonlinear static analysis of the 2D example, the number of core processors was increased by 1, 2, 4, 8, 16, 32, and 64 in order for the retrofitting design optimization. For this example, to evaluate the global convergence of the distributed NSGA-II algorithm, Pareto optimal solutions obtained from three independent distributed runs for the 2-, 4-, 8-, 16-, 32-, and 64-core configurations and three serial runs were shown in Figure 13. Three serial runs and 18 distributed runs with different random seeds showed a similar tendency in the relationship between the two objective functions. As can be expected, the sum of dissipated energy by the SPEAR building increased rapidly as the total volume of the steel cores for BRBs became smaller.
4.2.3 Time efficiency of the distributed optimization. Wall-clock times for complete optimization for 3D example for the 2-, 4-, 8-, 16-, 32-, and 64-core configurations are shown in Figure 7. In Table 4, average computational time of three serial runs is compared with the average computational time of three distributed runs for the 2-, 4-, 8-, 16-, 32-, and 64-core configurations. It shows that the distributed implementation of NSGA-II can drastically reduce the computational effort. Figure 8 shows the speedup curves for the example of the 2-, 4-, 8-, 16-, 32-, and 64-core configurations relative to the serial runs. In the dynamic structural analysis of the 3D irregular reinforced concrete frame structure, the computational time per generation was approximately 6.7 times that for the nonlinear static structural analysis of the 2D steel frame example. As the time-consuming fitness evaluation of 64 candidate designs for the 3D example is done concurrently by slave processors, the speedups for 3D example are higher than for the 2D example. For this example with 64 individuals, the number of candidate designs assigned to a core processor is 32, 16, 8, 4, 2, and 1 for the 2-, 4-, 8-, 16-, 32, and 64-core configurations, respectively. The even load balancing of the time-consuming fitness evaluation resulted in relatively high speedups or efficiencies of the distributed optimization.

4.2.4 Seismic capacity of retrofitted structure. Improvements of seismic capacity of the example using BRBs were verified through a comparison of the distribution of values for the total dissipated energy by the SPEAR building in the Pareto optimal solutions. The dissipated energy by the SPEAR building is decreased as the total volume of steel cores for BRBs increases. That is, as shown in Figure 13, the dissipated energy by the SPEAR building is generally inversely proportional to the total volume of steel cores for BRBs.

In Table 6, the value of the dissipated energy by the building for the candidate optimal solution where the total volume of steel cores in the BRBs was the closest to about 1.0 or 1.5 m$^3$ is presented. Prior to the retrofitting using BRBs, when the earthquake loading was applied, 54.25 kNm of energy was dissipated by the SPEAR building. For the retrofit design with the total volume of steel cores of 1.0 m$^3$, the values of energy dissipation of the SPEAR building were reduced to an average of 13.31% of the dissipated energy for the building before the retrofitting. In addition, the value of the dissipated energy by the building for the optimal solution where the dissipated energy by the SPEAR building was the closest to about 20 or 30 kNm is
presented in Table 6. For the retrofit design with the dissipated energy of the building of 20 kNm, the value of energy dissipation of the SPEAR building was reduced to an average of 48.35% of the dissipated energy for the building before the retrofitting.

Because multi-objective optimization is performed, a Pareto set is obtained, and thus, it is difficult to directly compare with the value by single objective function. Therefore, to identify the trend of optimal solutions intuitively, the solutions were evaluated through the following process. In the Pareto front of Figure 14, point A-minimum value for the volume of BRB and point B-minimum value for dissipated energy are shown, in which one of objective function values from two objective functions is minimized. The optimal solutions for points A and B are compared in Figure 15.

5 CONCLUSIONS

In this study, a distributed NSGA-II algorithm for a multi-core PC cluster is proposed to reduce the excessive computational time for optimization of the retrofitting design using BRBs. A candidate design level or individual level parallelism is exploited for development of the algorithm on a cluster of 16 PCs with Intel i7 quad-core CPUs. To demonstrate the performance of the proposed algorithm, the distributed NSGA-II is applied to optimal seismic retrofit design of 2D steel frame structure and 3D reinforced concrete structure. For the 2D example, the initial BRB installation cost required for seismic retrofitting and damage cost that can be incurred by earthquakes expected during the life cycle of the structure are minimized simultaneously. The initial BRB installation cost required for the seismic retrofitting and the dissipated energy by the 3D reinforced concrete structure during nonlinear dynamic analysis are minimized for the 3D example. Pareto optimal solutions obtained from five independent implementations for the 2D example and three independent implementations for the 3D example on the serial, 2-, 4-, 8-, 16-, 32-, 64-core configurations show the convergence of the proposed algorithm. Also, the distributed algorithm implemented on a cluster of multi-core PCs can significantly reduce the computational time for the optimization without deteriorating the quality of the optimal solutions.

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