Deep-Value Investing, Fundamental Risks, and the Margin of Safety

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The margin of safety arises from a deep-value investor's option to defer immediate action. This option is lucidly described by Warren Buffett (Lowe [1997], p. 111):

> In investments, there's no such thing as a called strike. You can stand at the plate and the pitcher can throw a ball right down middle, and if it's General Motors at 47 and you don't know enough to decide on General Motors at 47, you let it go right on by and no one's going to call a strike. The only way you can have a strike is to swing and miss.

Because exercising her option to defer action foregoes a valuable opportunity to wait (indefinitely) for a better price or turn down the opportunity to invest altogether, it is rational for a deep-value investor to demand an investment discount equal to the value of the option to defer action. This discount is the *margin of safety*.

Although academic research on this concept is surprisingly scant, the margin of safety has a special place in the annals of deep-value investing. Benjamin Graham popularized the concept in *Security Analysis* (first published in 1934 with coauthor David Dodd) and *The Intelligent Investor* (first published in 1949). Graham advised that "Confronted with a like challenge to distill the secret of sound investment into three words, we venture the motto, MARGIN OF SAFETY (*sic*). ... to have a true investment, there must be a margin of safety (Grafam [1985], p. 277–283)." Seth Klarman ([1991], p. 89) added that "For a value investor a pitch must not only be in the strike zone, it must be in his 'sweet spot.'" The margin of safety implies that investors should avoid excessive trading. According to Warren Buffett, "My favorite time frame for holding a stock is forever." (Lowe [1997], p. 162).

This article offers a real-options model that shows how much margin of safety a value-oriented investor should demand. The model links the margin of safety to fundamental risks that plague deep-value investors like venture capitalists, entrepreneurs, and fundamentals-oriented asset managers. Although I present the model from the perspective of a representative investor trading in an illiquid market, the model also applies (with suitable reinterpretation) to private-equity settings such as leverage buyouts or that of an entrepreneur taking an innovation to market (e.g., Heaton and Lucas [2000]; Treynor [2005]).

The real-options framework offers a straightforward way to value an investor's not-to-swing option and to derive the margin of safety corresponding to the optimal "swinging" strategy. To proceed, a model of deep-value investment decision making, including an identification of the main risk elements, is required.

I shall first identify the uncertainties confronting a deep-value investor. I shall argue that, in addition to market risk, three types of risks confront a value-oriented decision maker: risk that interim news will prematurely disrupt her private valuation; valuation uncertainty over how reliable and precise her point valuation estimate is; and uncertainty over when the market price will converge to her valuation estimate.

Next, I build a simple model, based on a real options approach, that links these risk factors to the margin of safety. The main result is a formula that links the optimal margin of safety to parameters characterizing the aforelisted risk factors. Applying the formula to S&P500 firms, I find that margins of safety are substantial—typically about 20% to 35% of share prices. Some companies, like Costco and Exxon, have margins of safety below this range while other companies like eBay and Google have margins well above this range.

DEEP-VALUE INVESTING

The first step of deep-value investing is to estimate a business's going concern value based on some fundamental valuation model.[1] This valuation estimate is unavoidably subjective since it requires inputs, like dividend, cash flow, or earnings forecasts, that depend on private information and beliefs (e.g., see Petersen, Plenborg, and Scholer [2006]). Next, one compares this initial valuation estimate, denoted V_0, against the contemporaneous market (or proposed transaction) price S_0. If $V_0 > S_0$, then the business is said to be underpriced by the market. According to Graham ([1985], pp. 281–282), a business is underpriced when there is

> a favorable difference between price on the one hand and indicated or appraised value on the other. That difference is the margin of safety. It is available for absorbing the effect of miscalculations or worse than average luck. The buyer of bargain issues places particular emphasis on the ability of the investment to withstand adverse developments.

If mispricing is sufficiently large, the deep-value investor buys or shorts the mispriced business anticipating that she will eventually be able to unwind her position with abnormal returns.

Deep-value investing relies on the confluence of three basic ingredients:

- *mispricing:* a temporary discrepancy between the market price S_0 and the private valuation V_0;
- *convergence date T:* a (possibly unpredicted) future date $T > 0$ when market price S_T converges (e.g., reconciles) to the projected future value of the valuation estimate;
- *exogenous entry price:* an expectation that the investor is able to establish her desired position at any date τ before the convergence date without destroying the favorable market price S_τ.

I will show that these three ingredients suffice to guarantee the profitability of deep-value investors. Generally, mispricing and convergence are necessary ingredients but an exogenous entry price is not. A responsive entry price reduces, but does not destroy completely, the deep-value investor's abnormal profits.

Mispricing does not require market inefficiency if one accepts that investors have heterogeneous beliefs or long-lived private information. An investor perceives mispricing if she holds private beliefs or information that would cause her to value the business differently than the market price. A mounting body of empirical evidence indicates that mispricing relative to public fundamental information may persist for months or even years (e.g., Ou and Penman [1989]; Lee, Shleifer, and Thaler [1991], and Sloan [1996]).

Arrival of a convergence date requires not only a catalytic event; it also requires that the event is enough to cause the market to converge to the investor's private assessment of value. Convergence and, more specifically, the arrival of convergence is not guaranteed in imperfect markets. Market frictions create limits to arbitrage that could indefinitely delay convergence (Shleifer and Vishny [1997]).

To keep the situation as simple as possible, I shall assume that the investor is a price taker; that is, the market is deep enough that the investor's trades do not affect S_τ. If S_τ would be endogenously affected, deep-value investors can still trade for profit. However, the investor's optimal trading would require additional strategic flourishes to mitigate price response. Admati and Pfleiderer [1988] and Back and Pedersen [1995] discuss optimal informed trading under responsive prices from a game theory perspective. In their models, a price-setting marketmaker reacts to the net market orders and the informed investors make allowances for the price reaction. Accordingly, these models focus on how investors optimally time and distribute their trade volume to mitigate disadvantageous revelation of private information conveyed by their trades.

In contrast, I focus on the limit where the investor is sufficiently small that her trading has no impact on the market price. In this setting, the market price is exogenously determined by the catalytic event and arrival of outside interim news that may alter both the market price and the investor's private assessment of value. Focusing on this setting allows me to link the margin of safety to fundamental risk parameters.

THE FUNDAMENTAL RISKS

Exhibit 1 depicts the timeline of my model. In the exhibit, t_* is the value investor's optimal trading time (to be endogenously determined by the model). Prior to t_*, the investor faces market risk and three fundamental risks,[2] which I define as follows:

- *market risk*: volatility of the market price;
- *news risk*: risk that intervening news between t_* and T disrupts the investor's projected fundamental value estimate;
- *valuation risk*: fear that her estimate of V_t may be systematically biased or imprecise;
- *convergence risk*: uncertainty about date T, when the market price will converge to the projected valuation estimate.

An example where news risk plays an especially important role is during the stub period before an announced merger closes, when the risk of deal-breaking news drives merger-arbitrage spreads. Valuation risk is especially important for stocks with potentially high information asymmetry, such as stocks of firms with low quality or fraudulent accounting (Yee [2006]). For example, even though Worldcom might have looked cheap according to several accounting-based valuation metrics in early 2002, it was a "value trap" (a stock that appears cheap but is not cheap once its hidden problems are recognized). Worldcom's apparent cheapness was a fiction caused by misleading accounting practices that the market had already started to disbelieve. Convergence risk played a central role in the blow up of Long Term Capital Management, which was caused by the failure of statistical-arbitrage trades to convergence as quickly as investors wanted (Lowenstein [2000]).

THE IMPLIED MARGIN OF SAFETY

This section describes how market risk and the three fundamental risks impact the margins of safety a rational investor should demand when buying or shorting.

Net present value analysis suggests buying immediately if one's value estimate exceeds the share price (e.g., $V_t > S_t$), shorting immediately if $V_t < S_t$, and "holding" *only* if $V_t = S_t$. Net present value analysis, however, fails to incorporate the margin of safety concept.

The Appendix shows formally that imposing a margin of safety criterion modifies net present analysis by creating a hold region and shrinking the investor's buy and sell regions. An investor with a margin of safety would:

$$\begin{cases} \textit{buy} & \text{if} \quad V_t > (1+\Delta_+)S_t; \\ \textit{hold} & \text{if} \quad (1-\Delta_-)S_t \le V_t \le (1+\Delta_+)S_t; \\ \textit{short} & \text{if} \quad V_t < (1-\Delta_-)S_t. \end{cases}$$

Positive numbers Δ_+ and Δ_- reflect the size of the investor's margins of safety for, respectively, purchase and shorting. Zero Δ_+ and Δ_- correspond to zero margins of safety. As depicted in Exhibit 2, the width of the hold region is $(\Delta_+ + \Delta_-)S_t$.

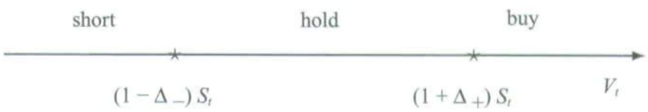
As derived in the Appendix, straightforward formulas link the optimal values of Δ_+ and Δ_- to measures

EXHIBIT 1

At date $t = 0$ the deep-value investor believes a business trading at S_0 dollars per share is worth $V_0 \neq S_0$ dollars per share. She also believes that at some (possibly random) future date T, the stock price S_T will converge to her projected valuation V_T. If S_t and V_t are subject to uncertainty, the investor holds a valuable timing option to wait until some future date $\boldsymbol{t_*}$, when she might transact at an even more favorable price.

EXHIBIT 2

Δ_+ and Δ_- are, respectively, the margins of safety demanded by potential buyers and shorters. The investor buys when $V_t > (1 + \Delta_+)S_t$ and shorts when $V_t < (1 - \Delta_-)S_t$. Otherwise, the investor holds.



of fundamental and market risks. I now describe these formulas. Let

- δ = a positive parameter that reflects the investor's risk adjustment for uncertainty in her fundamental valuation estimate;
- σ_s = prospective volatility of market price;
- σ_v = prospective volatility of the investor's fundamental valuation estimate caused by the arrival of news;
- ρ = correlation between prospective market price and prospective revisions to the investor's fundamental valuation estimate;
- $\overline{T}$ = expected time to when the market price will converge to the investor's valuation estimate.

Then the margin of safety an investor should demand before purchase is given by

$$\Delta_+ = \frac{\delta + \left\{\sqrt{1+2/z}+1\right\}z}{1+\delta} \tag{1}$$

and the margin of safety she should demand before shorting is given by

$$\Delta_- = \frac{\delta + \left\{\sqrt{1+2/z}-1\right\}z}{1+\delta} \tag{2}$$

where

$$z \equiv (\sigma_S^2 - 2\rho\sigma_S\sigma_V + \sigma_V^2)\overline{T}/4$$

Assuming an investor makes her valuation estimates based on moving-average price-to-book ratios, I computed the margins of safety Δ_+ and Δ_- with $\overline{T}$ = 1 year and δ = 0 for all firms in the S&P500 as of May 2007.[3] Exhibit 3 depicts the histograms of my estimates for Δ_+ and Δ_-. As shown, the distributions of Δ_+ and Δ_- are skewed right with median values 33.9% for Δ_+ and 25.3% for Δ_-. If the investor anticipates convergence time is $\overline{T}$ = 2 months, then these distributions, which are not depicted in Exhibit 3, shift to smaller median values 12.7% for Δ_+ and 11.3% for Δ_-.

EXHIBIT 3

Superimposed histograms of margins of safety Δ_+ and Δ_- assuming $\overline{T}$ = 12 months and δ = 0. The histograms are based on the 452 S&P500 firms in Compustat's "Price, Dividends, and Earnings" dataset with 48 uninterrupted months of price and book-value data in the period between June 2003 and May 2007. The 48 excluded firms were missing one or more months of price or book-value data in Wharton's copy of Compustat as of July 2007.

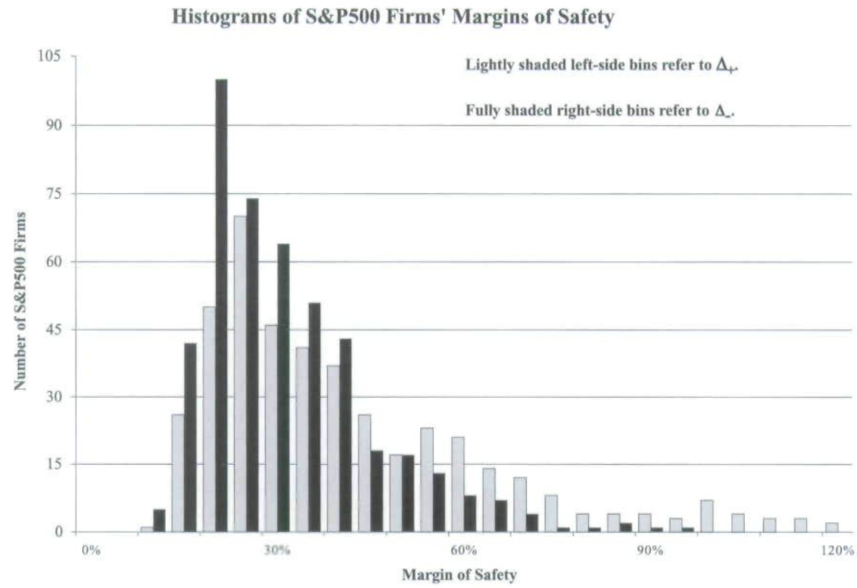


EXHIBIT 4

Margins of safety Δ_+ and Δ_- for selected S&P500 components based on June 2003 through May 2007 prices and book-values and the moving-average price-to-book-value valuation model. I assume no additional valuation risk adjustment ($\delta = 0$), $\bar{T} = 1$ year in the first two columns, and $\bar{T} = 2$ months in the last two columns.

	$\Delta_+^{\bar{T}=12\text{ mnths}}$	$\Delta_-^{\bar{T}=12\text{ mnths}}$	$\Delta_+^{\bar{T}=2\text{ mnths}}$	$\Delta_-^{\bar{T}=2\text{mnths}}$
Apple	95.1%	48.7%	31.9%	24.2%
Chubb	18.4%	15.5%	7.1%	6.7%
Cisco	42.3%	29.7%	15.6%	13.5%
Consolidated Edison	10.4%	9.4%	4.1%	4.0%
Costco Wholesale	13.6%	12.0%	5.4%	5.1%
Disney	24.8%	19.9%	9.5%	8.7%
eBay	63.8%	38.9%	22.5%	18.4%
Exxon Mobil	18.9%	15.9%	7.3%	6.8%
General Electric	23.45%	19.0%	9.0%	8.3%
Google	319.4%	76.2%	87.1%	46.6%
Home Depot	22.2%	18.2%	8.6%	7.9%
Microsoft	73.7%	42.4%	25.6%	20.4%
Time Warner	18.4%	15.5%	7.1%	6.7%
Wal–Mart	19.11%	16.05%	7.4%	6.9%
WholeFoods Market	93.8%	48.4%	31.6%	24.0%

Exhibit 4 lists margins of safety values for 15 widely followed S&P500 firms. The message from Exhibits 3 and 4 is that the magnitudes of the margins of safety are substantial even for large, mature public manufacturers and retailers like Cisco, Costco, and Wal-Mart. Investors with a $\bar{T} = 1$ year anticipated convergence horizon should demand margins of safety of approximately 25% to 35% for most S&P500 companies. Investors who anticipate a shorter convergence horizon would have smaller margins of safety.

Although margins of safety can be surprisingly large, sometimes exceeding 50% of their share prices, $\bar{T} = 1$-year margins exceed 100% only in rare cases. As reflected in Exhibit 4, margins of safety for those firms commonly classified as growth firms, such as Apple, eBay, Microsoft, and WholeFoods Markets, are much larger than those of financial firms (Chubb), utilities (Consolidated Edison), and retailers (Costco and Wal-Mart). This is because fundamentals-based valuation estimates are more volatile and, hence, more risky for growth firms. When valuation estimates are more risky, investors should demand a larger margin of safety to buffer potential valuation errors.

Exceptionally large margins of safety, like that of Google, Microsoft, and WholeFoods in Exhibit 4, are red flags that the fundamental valuation model used to estimate those margins might not apply to those firms. In particular, simple price-to-book valuations are notoriously unreliable for growth firms like Google and Microsoft because much of their price is driven by volatile "off the books" intangible assets not measured by their book values. Microsoft issued a giant special dividend in fiscal 2005 that sharply deflated its book value relative to historical levels. During the 2003–2007 period, WholeFoods experienced a period of unprecedented rapid expansion and acquisitive activity that caused uncommon volatility in its price-to-book ratio.

Equations (1) and (2) imply that investors should demand even greater margins of safety if the convergence date is more than a year away. In particular, suppose $\sigma_v = \sigma_s =$ 30% per annum (a typical number for equity value volatilities) and $\rho = 50\%$. Then

$$z = 0.0225 \times \bar{T}$$

This means, even assuming a precisely credible valuation ($\delta = 0$), the investor who anticipates convergence in one year ($\bar{T} = 1$) would demand margins of safety of $\Delta_+ = 24\%$ and $\Delta_- = 19\%$. However, the investor who does not anticipate convergence for $\bar{T} = 5$ years would demand $\Delta_+ = 60\%$ and $\Delta_- = 38\%$.

ADDITIONAL PROPERTIES OF THE MARGINS OF SAFETY

Equations (1) and (2) encapsulate several basic insights that are summarized as follows:

i) If one anticipates immediate convergence[4] (i.e., $\bar{T} = 0$), then valuation risk is alone responsible for the margins of safety. If one anticipates a waiting period before convergence (i.e., $\bar{T} > 0$), then three fundamental risks all contribute to increasing the demanded margins of safety.
ii) Absent market risk and news risk, the anticipated time $\bar{T}$ to convergence has no impact on the margins of safety.
iii) If S_t and V_t are perfectly correlated ($\rho = 1$) and equivariant ($\sigma_s = \sigma_V$), then the three fundamental risks do not impact the margins of safety.
iv) When all risk factors are present and not degenerately correlated, the demanded margins of safety are strictly increasing functions of σ_s, σ_V, δ, and $\bar{T}$.

Elaborating on insight (iv) above, Exhibit 5 depicts the value-to-price ratios the deep-value investor demands before buying or shorting as a function of the aggregate risk measure z. Without the timing option, net present value considerations prescribe buying (shorting) immediately if $V_t/P_t > 1$ ($V_t/P_t < 1$). Accordingly, the margins of safety, Δ_+ and Δ_-, are the respective deviations from $V_t/P_t = 1$ of the two curves. As depicted, both Δ_+ and Δ_- strictly grow with z. Since z monotonically grows with σ_S, σ_V, and $\bar{T}$, this means the margins of safety strictly grow with market risk and with news risk, and also with the anticipated time $\bar{T}$ to convergence.

Finally, the model confirms the common-sense idea that establishing a valuation estimate, even a very inaccurate or imprecise one, is valuable because it provides a baseline to assess whether to exercise an option to buy or short. As suggested by Equations (1) and (2) and spelled out in Appendix B, the value of having a valuation estimate depends on the risk parameters σ_S, σ_V, ρ, $\bar{T}$, and δ. This value strictly decreases with increasing uncertainty in the valuation estimate.

Exhibit 6 depicts the value of the investor's option to buy or short after she establishes a private valuation estimate. Suppose $\sigma_V = \sigma_S = 30\%$ per annum, $\rho = 50\%$, and $\bar{T} = 1$ year. If $\delta = 0$, then even absent any current mispricing, e.g., $V_t = S_t = 1$, the values of the option to buy and short are both $\mathcal{O}(1, 1) = 0.078$. This means even absent contemporaneous mispricing a precise valuation is worth 15.6% of the share price![5] What if the valuation is imprecise so that $\delta = 50\%$? Then the options values are significantly smaller, but still nonzero: $\mathcal{O}(1, 1) = 0.21\%$ of the share price for buying and $\mathcal{O}(1, 1) = 1.4\%$ of the share price for shorting. If the value investor anticipates a longer time to convergence, say, $\bar{T} = 5$ years, these numbers dramatically increase. As depicted in Exhibit 6, if $\delta = 50\%$, the options to buy and short are worth, respectively, $\mathcal{O}(1,1) = 2.7\%$ and $\mathcal{O}(1,1) = 8.7\%$ of the share price.

CONCLUSION

This article presents a model of deep-value investing that links the margin of safety to fundamental risks facing an investor. In addition to market price volatility, the model incorporates three fundamental risks: 1) risk that interim news may unfavorably disrupt the investor's initial valuation estimate before she can realize profits; 2) uncertainty over the reliability of the investor's valuation estimate; and 3) uncertainty over when the market price will converge to the investor's value estimate. The expressions for the margin of safety stated in Equations (1) and (2) indicate that investors should demand substantial margins of safety even for large, mature public companies. While, as indicated in Exhibit 3, margins of safety are typically 20% to 35% of the share price, they could be over 50% for growth companies.

Beyond the quantitative results, this article views fundamental risk as the sum of three constituents: news, valuation, and convergence. Specific elements of fundamental risk (such as information risk or so-called earnings quality risk) cannot be properly appreciated as stand-alone phenomena. Unfortunately, fundamental risk is not that simple. Investors, their information processing behavior, and their trading strategies all contribute to how each risk element affects price formation. Chen, Dhaliwal, and Trombley [2007] report early evidence consistent with this view.

While beyond the scope of this article, the margin of safety concept permeates many other investment settings, such as portfolio choice (Merton [1973], Heaton and Lucas [2000]), predictive hazard models (Yee [2007]), and strategic asset allocation (Campbell and Viceira [2002]). For example, Merton [1973] showed that investors

EXHIBIT 5

Optimal value-to-price ratios to demand before transacting as a function of $z = (\sigma_s^2 - 2\rho\sigma_s\sigma_v + \sigma_V^2)\overline{T}/4$. The top curve refers to the criterion for buying and the bottom curve the criterion for shorting. Raw net present value considerations would indicate buy if $V_t/S_t > 1$ and short if $V_t/S_t < 1$, no margins of safety required. These curves do not meet at $V_t/P_t = 1$ at $z = 0$ due to valuation risk (e.g., $\delta > 0$).

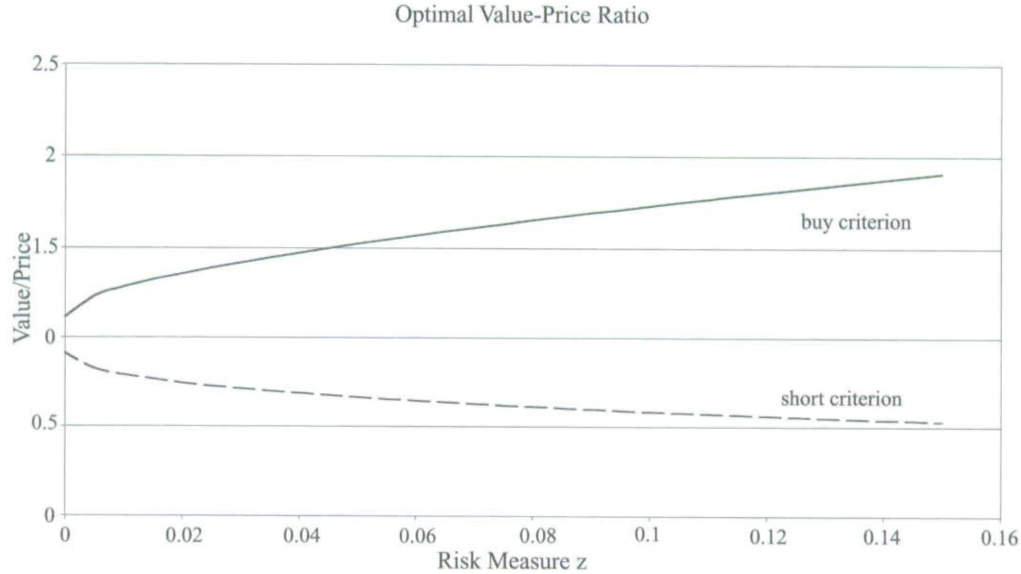
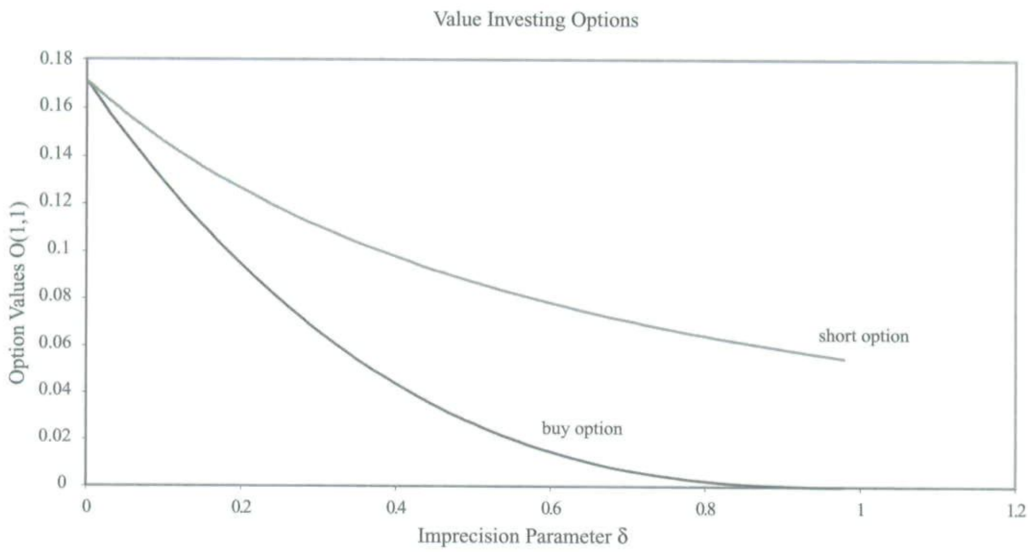


EXHIBIT 6

$\mathcal{O}(V_t, S_t)$ as a function of the valuation precision indicator δ when $V_t = S_t = 1$, $\sigma_V = \sigma_S = 30\%$ per annum, $\rho = 50\%$, and $\overline{T} = 5$ years. The upper graph corresponds to the opportunity to short one share; the lower graph the opportunity to buy one share. As shown, the value of a valuation estimate strictly decreases with increasing valuation uncertainty parametrized by δ.



with sufficient relative risk aversion seek to hold securities that increase in value when global investment opportunities deteriorate. Viewed from the perspective of this article, investors are using Merton's hedging securities, which provide insurance against bad times, to hold capital until their margin of safety clears.

The results of this article highlight what is the most important insight of value investing: *Making a valuation estimate, even an inaccurate or imprecise one, is valuable whether or not it identifies current mispricing.* This is because an in-hand estimate, no matter how uncertain, provides the reference point that enables the investor to assess the margin of safety to demand for proposed transactions.

Accordingly, investors should make a private valuation estimate and demand a margin of safety in accordance with Equations (1) and (2) before trading. Then, guided by these estimates, investors should place limit orders. Investors should not place market orders any more than a batter should commit to swing at the first pitch before it has been thrown.

APPENDIX A

The Model

This appendix section defines the formal model. The next section solves it.

Market risk is the anticipated volatility of market price S_t expected by the investor. Market risk is incorporated by assuming that between $t = 0$ and T

$$dS_t = rS_t dt + \sigma_S S_t dB_t^S \qquad \text{(A.1)}$$

where $E_t[dB_t^S] = 0$. The Brownian motion process B_t^S defines the equivalent risk-neutral Martingale measure and r is the risk-free interest rate. The parameter $\sigma_s > 0$ determines price volatility.

News risk consists of shocks that revise prospective valuation estimates including the future value of V_T that price S_T is projected to converge to. News includes earnings announcements as well as (unexpected) changes in an investor's private sentiment. News risk is incorporated by assuming that, between $t = 0$ and the convergence date T, the investor's projection of her valuation V_t evolves according to

$$dV_t = rV_t dt + \sigma_v V_t dB_t^V \qquad \text{(A.2)}$$

where $E_t[dB_t^V] = 0$. Parameter $\sigma_V > 0$ reflects the anticipated volatility of V_t.

The response of S_t and V_t to news is partially, though not perfectly, correlated. S_t and V_t are partially correlated if the investor revises V_t in response to market price movements or other public information that also affect S_t. S_t and V_t are not *perfectly* correlated because the investor does not evaluate news exactly as equilibrium price does. Accordingly, I assume

$$E_t[dB_t^S dB_t^V] = \rho dt$$

where $\rho \leftrightarrow [-1, 1]$ is the correlation between the Martingales dB_t^S and dB_t^V.

Valuation risk arises because point valuation estimates[6] are inherently imprecise. Moreover, an investor often does not know how imprecise her estimate is (Bewley [1988]; Epstein and Wang [1994]). If an investor recognizes that her best estimate is imprecise and subject to Knightian uncertainty, she knows that the true intrinsic value at date t is not V_t but in fact

$$\tilde{V}_t = V_t + \tilde{\varepsilon}_t$$

where $\tilde{\varepsilon}_t$ is an unobservable mean-zero random variable with unknown distribution. $\tilde{\varepsilon}_t$ would have no impact on the investor's utility if it were diversified away. But deep-value investors, by definition, do not form fully diversified portfolios (Leibowitz [1997]; Heaton and Lucas [2000], and Treynor [2005]). I assume $\tilde{\varepsilon}_t$ is the component of valuation error that is not diversified away. The investor anticipates that, at the convergence date T, S_T converges not to V_T exactly, but to $\tilde{V}_T = V_T + \tilde{\varepsilon}_T$. Accordingly, when she buys or shorts a share at t, she obtains not V_t but V_t *plus a collateral gamble,* $\tilde{\varepsilon}_t$, *with unknown distribution.* If the investor is risk- or ambiguity-averse,[7] she has negative utility for such gambles. Hence, even if $\tilde{\varepsilon}_t$ is mean zero, the investor's certainty equivalent for a share of the business is

$$\hat{V}_t = (1 \mp \delta)V_t \ \text{whereÄ} \begin{cases} - & \text{if buy} \\ + & \text{if short} \end{cases} \qquad \text{(A.3)}$$

where $\delta \in [0, 1)$. Here $V_t \times \delta$ is the certainty equivalent[8] of the collateral gamble undertaken when one relies on V_t to estimate $\tilde{V}_t$.

Convergence risk is the risk that the stock price will not converge to the investor's valuation projection for an unacceptably long time. A Poisson process governs arrival of the convergence date T. At any time interval dt, convergence occurs with a chance of $1dt$. This means at date t the expected convergence date is[9]

$$\bar{T} \equiv E_t[T] = 1/1$$

At convergence, the investor who has purchased (shorted) one share obtains (is liable for) a share worth $\tilde{V}_T = V_T + \tilde{\varepsilon}_T$. The $\lambda \to \infty$ limit recovers an efficient market with homogeneous beliefs, in which case $\bar{T} = 0$ and $V_t = S_t - \tilde{\varepsilon}_t$ for all t.

Let $\mathcal{O}(V_t, S_t)$ denote the value of the investor's option to trade one share of the business at time t given private valuation V_t and existing market price S_t. $\mathcal{O}(V_t, S_t)$ equals the expected present value of the difference between V_t and S_t at the optimal time $t = t_*$ for the investor to establish her position:

$$\mathcal{O}(V_t, S_t) = \pm E_t\left[\left\{(1 \mp \delta)V_{t_*} - S_{t_*}\right\}e^{-r(t_*-t)}\right]$$

where the upper (lower) signs apply when the investor is looking to buy (short). The corresponding optimal time to buy (short) is given by

$$t_* = \text{argmax}_{\tau\in[t,T]}\left\{\pm E_t\left[\left\{(1 \mp \delta)V_\tau - S_\tau\right\}e^{-r(\tau-t)}\right]\right\}$$

$\mathcal{O}(V_t, S_t)$ satisfies the boundary conditions

$$\mathcal{O}(V_t, St) = \begin{cases} (1-\delta)V_{t_*} - S_{t_*} & \text{if } \tau = t_* \text{ \& buying;} \\ S_{t_*} - (1+\delta)V_{t_*} & \text{if } \tau = t_* \text{ \& shorting;} \\ 0 & \text{if } \tau \geq T. \end{cases} \quad \text{(A.4)}$$

The first two boundary conditions refer to the investor's profit when she establishes her position. The third condition imposes expiration of the option at the convergence date T.

$\mathcal{O}(V_t, S_t)$ may be interpreted as a perpetual American option to exchange an asset of value V_t for an asset of value S_t (Margrabe [1978]). Alternatively, it can be viewed as an arbitrage timing option (MacDonald and Siegel [1986]; Brennan and Schwartz [1990]; Ingersoll and Ross [1992]). What is new here is that V_t is a *private* valuation (a belief) and the recognition that *holding a private valuation or belief has option value in itself.* In other words, $\mathcal{O}(V_t, S_t)$ is a real option to exchange a private belief about a future market price for the instant market price.

Since the option to transact is never an obligation to do so, the value of an option to buy or short is strictly positive:[10]

$$\mathcal{O}(V_t, S_t) > 0 \;\forall\; V_t > 0 \;\&\; S_t > 0$$

This means that forecasting a valuation path, V_t, gives the investor a valuable option: the option to transact based on that forecast. Therefore, *every forecast has value*: the value of the opportunity to make a forecast-based investment.

APPENDIX B

Formal Results

Lemma 1 If Equations (A.1), (A.2), and (A.3) govern S_t, V_t, and $\hat{V}_t$, and the convergence date T arrives according to a Poisson process with expectation $\bar{T}$, then the optimal margins of safety to demand before purchasing and shorting are given respectively by Equations (1) and (2).

Proof of lemma 1 Let $\mathcal{F}(V_t, S_t, q_t)$ be a function of V_t and S_t defined by Equations (A.2) and (A.1), and q_t be a Poisson "jump" process. Assume initially $q_0 = 0$ and, in any instant dt, q_t, has a chance $1 > 0$ of jumping up by one unit. This jump process corresponds to the arrival of the convergence date T: when $q_t = 0$, the investor sees mispricing ($V_t \neq S_t$) and, when q_t jumps to 1 the very first time, T has arrived. Accordingly, identify

$$F(V_t, S_t, q_t) = \begin{cases} \mathcal{O}(V_t, S_t) & \text{if } q_t = 0 \\ 0 & \text{if } q_t \geq 1. \end{cases}$$

This identify imposes on $\mathcal{F}$ the third boundary condition of Equation (A.4).

Let's focus now on the $q_t = 0$ region where $\mathcal{F} = \mathcal{O}$ until a jump occurs. By standard arguments (Dixit and Pindyck [994]) $\mathcal{O}$ satisfies the Bellman condition

$$r\mathcal{O}\,dt = c\,dt + E_t[d\mathcal{O}]$$

where $E_t[d\mathcal{O}]$ is the expected prospective change of $\mathcal{O}$ in the risk-neutral measure during time interval $[t, t + dt]$. The other term in the RHS, $c\,dt$, represents any cash dividends the option holder receives in the same period. Since the model assumes no dividends are paid before convergence,[11] $c = 0$.

Ito's Lemma implies

$$E[d\mathcal{O}] = \left\{\frac{1}{2}\sigma_S^2 S_t^2 \mathcal{O}_{ss} + \sigma_S\sigma_V S_t V_t \mathcal{O}_{sv} + \frac{1}{2}\sigma_V^2 V_t^2 \mathcal{O}_{vv} + rS_t\mathcal{O}_s + rV_t\mathcal{O}_v + \lambda[\underbrace{F_{|q_t=1}}_{0} - \mathcal{O}]\right\}dt$$

where $\mathcal{O}_s \equiv \frac{\partial\mathcal{O}}{\partial S_t}$ and $\mathcal{O}_v \equiv \frac{\partial\mathcal{O}}{\partial V_t}$. Putting this together with the Bellman equation yields

$$\frac{1}{2}\sigma_S^2 S_t^2 \mathcal{O}_{ss} + \sigma_S\sigma_V S_t V_t \mathcal{O}_{sv} + \frac{1}{2}\sigma_V^2 V_t^2 \mathcal{O}_{vv} + rS_t\mathcal{O}_s + rV_t\mathcal{O}_v - (r+\lambda)\mathcal{O} = 0 \quad \text{(B.1)}$$

which is the fundamental PDE to be solved. $\mathcal{O}$ must satisfy the boundary conditions of Equation (A.4).

To solve the Equation (B.1), note that if S_t and V_t are halved with everything else held fixed (say by instantaneous conversion of the units from dollars to pounds), the value $\mathcal{O}$ of the option should also be halved. Hence, $\mathcal{O}$ must be linear homogeneous of degree one:

$$\mathcal{O}(\alpha_t V_t,\ \alpha_t S_t) = \alpha_t \mathcal{O}(V_t,\ S_t)$$

where α_t is any positive time-dependent parameter. Choosing $\alpha_t = 1/S_t$ implies

$$\mathcal{O}(V_t,\ S_t) = S_t \times u(x_t)$$

where $x_t \equiv V_t/S_t$ is the value-to-price ratio and $u(x_t) \equiv O(x_t,1)$. In terms of the new variable x_t, Equation (B.1) is

$$\frac{1}{2}\Sigma^2 x_t^2 u_{xx} - \lambda u = 0$$

where $u_{xx} \equiv \frac{\partial^2 u}{\partial x^2}$ and $\Sigma^2 = \sigma_S^2 - 2\rho\sigma_S\sigma_V + \sigma_V^2$. This partial differential equation is solved by $u = Cx^{\gamma\pm}$ where

$$\gamma_\pm = \frac{1 \pm \sqrt{1 + 2/z}}{2}$$

and $z \equiv \overline{T}\Sigma^2/4$.

Constant C will be determined by the boundary condition as follows. If the investor is seeking to purchase, the value of her option must strictly grow with increasing x_t. Since $\gamma_+ > 1$ and $\gamma\_ < 0$, this means $u \propto x_t^{\gamma_+}$ if the investor is looking to purchase. Likewise, if an investor is looking to short, her option must strictly drop in value with the value-to-price ratio so $u \propto x_t^{\gamma_-}$. Let us focus exclusively on the purchase problem. Boundary condition Equation (A.4) requires

$$u(x_*) = (1-\delta)x_* - 1$$

if the investor purchases when $x_t = x_\star$, which implies

$$u(x_t;\ x_*) = \{(1-\delta)x_* - 1\}\left(\frac{x_t}{x_*}\right)^{\gamma_+}.$$

The second argument of $u(\cdot;\cdot)$ has been appended to emphasize its value depends on $x_\star$ as well as the current value-to-price ratio x_t.

The purchasing investor will choose the margin of safety to maximize her option value. Maximizing $u(x_t;\ x_*)$ with respect to x_* yields

$$x_{t_*} \equiv \text{argmax}_{x_* \in R^+} u(x_t,\ x_*) = \frac{1}{1-\delta} \times \frac{\gamma_+}{\gamma_+ - 1}.$$

Noting that

$$\frac{\gamma_\pm}{\gamma_\pm - 1} = 1 + 2z\gamma_\pm$$

yields

$$\frac{V_{t_*}}{S_{t_*}} = \frac{1 + \left\{\{\sqrt{1+2/z} + 1\}\right\}z}{1-\delta} > 1$$

for the minimum value-to-price ratio to demand before purchase. The corresponding margin of safety is $\Delta_+ = V_{t_*}/S_{t_*} - 1$.

The optimal shorting problem is solved similarly. The value-to-price ratio must fall below

$$\frac{V_{t_*}}{S_{t_*}} = \frac{1 - \left\{\sqrt{1+2/z} - 1\right\}z}{1+\delta} < 1$$

before shorting.

Proposition 1. *The value of a valuation estimate strictly decreases with increasing investor uncertainty in its accuracy.*

Proof: A valuation V_t bestows on the cognoscienti a valuable option: the option to transact using V_t as an anchor. $\mathcal{O}(V_t,\ S_t)$ is the value to an investor of having value estimate V_t conditional on market price S_t. This implies the investor who wants to trade N shares should be willing to spend up to $N \times \mathcal{O}(V_t,\ S_t)$ dollars to obtain value estimate V_t.

The option value $\mathcal{O}(V_t, S_t)$ is given by the formulas stated in the proof of Lemma. Define z as in Lemma 1 and denote $x_t \equiv V_t/S_t$. Let

$$\gamma_\pm \equiv \frac{1 \pm \sqrt{1+2/z}}{2}$$

Then the value of knowing valuation V_t conditional on market price S_t and an intention to purchase at the optimal time t_* is

$$\mathcal{O}(V_t,\ S_t) = \{(1-\delta)x_{t_*} - 1\}\left(\frac{x_t}{x_{t_*}}\right)^{\gamma+} S_t$$

where

$$x_{t_*} \equiv \frac{1 + \left\{\sqrt{1+2/z} + 1\right\}z}{1-\delta}.$$

Similarly, the value of knowing V_t conditional on S_t and an intention to short at the optimal time is

$$O(V_t, S_t) = \{1 - (1+\delta)x_{t_*}\}\left(\frac{x_t}{x_{t_*}}\right)^{\gamma-} S_t$$

where

$$x_{t_*} \equiv \frac{1 - \left\{\sqrt{1 + 2/z} - 1\right\}z}{1+\delta}.$$

The proposition follows from inspection of these formulas.

ENDNOTES

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[1]Leibowitz [1997], Copeland, Koller, and Murrin [2000], and Yee [2005] survey popular fundamental valuation models.

[2]Another fundamental risk is the risk of being forced to forego a superior future opportunity because the investor ties up her capital in the current investment. I do not build this risk into my model because a successful deep-value investor can circumvent capital constraints by raising additional capital.

[3]Due to missing Compustat data, I computed Δ_+ and Δ_- values for only 458 of the 500 index firms. In my calculation, an investor estimates the value of firm f as $V_t = \beta\ B_t$, where B_t is f's most recent reported book value and β is the 12-month moving average of f's price-to-book ratio. I assume that σ_S^2 (σ_S^2) is the 24-month moving-variance of f's month-end log prices (log value estimates), and that ρ is the 24-month moving-correlation between $\log(S_t)$ and $\log(V_t)$. Where available, one could also use implied volatilities from options prices.

[4]This is the case under homogeneous beliefs and efficient markets.

[5]15.6% = 7.8% + 7.8%, the value of the option to buy plus the value of the option to short. Knowing the valuation gives the investor both options.

[6]I use the term valuation estimate broadly to refer to implementations of anything from discounted cash flow to the method of comparables or accounting-based techniques like residual income valuation.

[7]While ambiguity aversion would modify representations of the investor's expected utility for $\tilde{V}_T$ (e.g., Gilboa and Schmeidler [1989]), the notion of certainty equivalent still applies.

[8]For instance, if $\tilde{\varepsilon}_t \sim N(0, \sqrt{V_t}\sigma)$ and heteroskedatic, under pure risk-aversion the investor's utility function might be $u(x) = e^{-\gamma x}$, in which case $\delta = \gamma\ \sigma^2/2$. To capture ambiguity aversion, one might posit that $e^{-\gamma(1\mp\delta)V_t} = (1-\kappa)E[u(\tilde{V}_t)] + \kappa u(V_t \mp \underline{\varepsilon})$, where $\underline{\varepsilon}$ is the most-feared plausible outcome and k reflects the degree of ambiguity aversion. Then $\delta = \mp \log[(1-\kappa)e^{\frac{V_t\sigma^2\gamma^2}{2}} + \kappa e^{\pm\gamma\underline{\varepsilon}}]^{\frac{1}{V_t\gamma}}$. In this case δ picks up a t dependence via V_t.

[9]For a Poisson process, the probability the first arrival occurs at time s units in the future is $\lambda e^{-\lambda s}$. Hence, $E_t[T] = \int_0^\infty ds\ \lambda e^{-\lambda s} s = 1/\lambda$.

[10]This is not to say the trade cannot result in a loss for the investor. Positivity is guaranteed only in *ex ante* expectation and under optimal investor behavior.

[11]If the business pays dividends, then S_t in Equation (A.1) would appreciate at less than the risk-free rate. Moreover, if the investor anticipates dividends, she must also adjust Equation (A.2) accordingly.

REFERENCES

Admati, A., and P. Pfleiderer. "A Theory of Intraday Trading Patterns." *The Review of Financial Studies*, 1 (1988), pp. 3–40.

Back, K., and Hal Pedersen. "Long-Lived Information and Intraday Paterns." *Journal of Financial Markets*, 1 (1995), pp. 385–402.

Bewley, T. "Knightian Uncertainty." In *Frontiers of Research in Economic Theory*, eds. D. Jacobs, E. Kalai, and M. Kamien. United Kingdom: Cambridge University Press, 1988.

Brennan, M., and E. Schwartz. "Arbitrage in Stock Index Futures." *Journal of Business*, 63 (1990), pp. 7–31.

Campbell, J., and L. Viceira. *Strategic Asset Allocation: Portfolio Choice for Long Term Investors*. Oxford University Press, 2002.

Chen, L., D. Dhaliwal, and M. Trombley. "The Effect of Fundamental Risk on the Market Pricing of Accruals Quality." University of Arizona Working paper, 2007.

Copeland, T., T. Koller, and J. Murrin. *Valuation: Measuring and Managing the Value of Companies*. John Wiley and Sons, 2000, 3rd edition.

Epstein, L., and T. Wang. "Intertemporal Asset Pricing under Knightian Uncertainty." *Econometrica*, Vol. 62, No. 2 (1994), pp. 283–322.

Gilboa, L., and D. Schmeidler. "Maximin Expected Utility with a Non-Unique Prior." *Journal of Mathematical Economics*, 1989, pp. 141–153.

Graham, B. *The Intelligent Investor: a Book of Practical Counsel*. New York: Harper & Row, 1985.

Heaton, J., and D. Lucas. "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk." *Journal of Finance*, Vol. 55, No. 3 (2000), pp. 1163–1198.

Ingersoll, J., and S. Ross. "Waiting to Invest: Investment and Uncertainty." *Journal of Business*, 65 (1992), pp. 1–29.

Klarman, S. *Margin of Safety: Risk-Averse Value Investing Strategies for the Thoughtful Investor*. New York: HarperCollins, 1991.

Lee, C., A. Shleifer, and R. Thaler. "Investor Sentiment and the Closed-End Fund Puzzle." *The Journal of Finance*, Vol. 46, No. 1 (1991), pp. 75–109.

Leibowitz, M. "Franchise Margins and the Sales-Driven Franchise Value." *Financial Analysts Journal*, Vol. 53, No. 6 (1997), pp. 43–53.

Lowe, J. *Warren Buffett Speaks*. New York: John Wiley & Sons, 1997.

Lowenstein, R. *When Genius Failed*. New York: Random House, Inc., 2000.

Margrabe, R. "The Value of an Option to Exchange One Asset for Another." *The Journal of Finance*, Vol. 33, No. 1 (1978), pp. 177–86.

McDonald, R., and D. Siegel. "The Value of Waiting to Invest." *Quarterly Journal of Economics*, 101 (1986), pp. 707–728.

Merton, R. "An Intertemporal Capital Asset Pricing Model." *Econometrica*, 41 (1973), pp. 867–887.

Ou, J., and S. Penman. "Financial Statement Analysis and the Prediction of Stock Returns." *Journal of Accounting and Economics*, 11 (1989), pp. 295–329.

Petersen, C., T. Plenborg, and F. Scholer. "Issues in Valuation of Privately Held Firms." *Journal of Private Equity*, 2006, pp. 33–48.

Shleifer, A., and R. Vishny. "The Limits of Arbitrage." *Journal of Finance*, Vol. 52, No. 1 (1997), pp. 35–55.

Sloan, R. "Do Stock Prices Fully Reflect Information in Accruals and Cash Flows about Future Earnings?" *The Accounting Review*, Vol. 171, No. 3 (1999), pp. 289–316.

Treynor, J. "The Investment Value of an Idea." *Financial Analysts Journal*, Vol. 61, No. 3 (2005), pp. 21–25.

Yee, K. "Earnings Quality and the Equity Risk Premium: A Benchmark Model." *Contemporary Accounting Research*, Vol. 23, No. 3 (2006), pp. 833–877.

——. "Perspectives: Combining Value Estimates to Increase Accuracy." *Financial Analysts Journal*, Vol. 60, No. 4 (2005), pp. 23–28.

——. "Fundamental Analysis in Time." Columbia University work in progress, 2007.