Accurate MTF measurement in digital radiography using noise response

Andrew Kuhls-Gilchrist, Amit Jain, Daniel R. Bednarek, Kenneth R. Hoffmann, and Stephen Rudin
Toshiba Stroke Research Center, University at Buffalo, State University of New York, Biomedical Research Building, Room 445, 3435 Main Street, Buffalo, New York 14214

(Received 10 June 2009; revised 26 September 2009; accepted for publication 8 December 2009; published 22 January 2010)

Purpose: The authors describe a new technique to determine the system presampled modulation transfer function (MTF) in digital radiography using only the detector noise response.

Methods: A cascaded-linear systems analysis was used to develop an exact relationship between the two-dimensional noise power spectrum (NPS) and the presampled MTF for a generalized detector system. This relationship was then utilized to determine the two-dimensional presampled MTF. For simplicity, aliasing of the correlated noise component of the NPS was assumed to be negligible. Accuracy of this method was investigated using simulated images from a simple detector model in which the “true” MTF was known exactly. Measurements were also performed on three detector technologies (an x-ray image intensifier, an indirect flat panel detector, and a solid state x-ray image intensifier), and the results were compared using the standard edge-response method. Flat-field and edge images were acquired and analyzed according to guidelines set forth by the International Electrotechnical Commission, using the RQA 5 spectrum.

Results: The presampled MTF determined using the noise-response method for the simulated detector system was in close agreement with the true MTF with an averaged percent difference of 0.3% and a maximum difference of 1.1% observed at the Nyquist frequency \(f_N\). The edge-response method of the simulated detector system also showed very good agreement at lower spatial frequencies (less than 0.5 \(f_N\)) with an averaged percent difference of 1.6% but showed significant discrepancies at higher spatial frequencies (greater than 0.5 \(f_N\)) with an averaged percent difference of 17%. Discrepancies were in part a result of noise in the edge image and phasing errors. For all three detector systems, the MTFs obtained using the two methods were found to be in good agreement at spatial frequencies less than 0.5 \(f_N\) with an averaged percent difference of 3.4%. Above 0.5 \(f_N\), differences increased to an average of 20%. Deviations of the experimental results largely followed the trend seen in the simulation results, suggesting that differences between the two methods could be explained as resulting from the inherent inaccuracies of the edge-response measurement technique used in this study. Aliasing of the correlated noise component was shown to have a minimal effect on the measured MTF for the three detectors studied. Systems with significant aliasing of the correlated noise component (e.g., a-Se based detectors) would likely require a more sophisticated fitting scheme to provide accurate results.

Conclusions: Results indicate that the noise-response method, a simple technique, can be used to accurately measure the MTF of digital x-ray detectors, while alleviating the problems and inaccuracies associated with use of precision test objects, such as a slit or an edge. © 2010 American Association of Physicists in Medicine. [DOI: 10.1118/1.3284376]

Key words: modulation transfer function (MTF), resolution, digital radiography, linear systems analysis, noise power spectrum (NPS), noise, edge, image quality

I. INTRODUCTION

The modulation transfer function (MTF) is the most widely accepted measure of the spatial resolution response of medical x-ray imaging detectors and has proved to be a valuable tool for determining and comparing detector performance.1–4 Although the methods used to measure the MTF have changed slightly over the past several decades,5–14 the underlying premise has remained a constant: A precisely machined object or test device is imaged and the subsequent response of the detector is used to obtain the MTF using several steps, which generally include determination of the line spread function (LSF), taking the Fourier transform of the LSF, and use of various noise reduction and fitting techniques.6,11,12,15–20

The slit11 and edge12 response methods are the two most commonly used and accepted techniques for measurement of the MTF, with the edge-response method being preferred (with adoption by the IEC) (Ref. 21) for various reasons, including simpler construction and less sensitivity to misalignment.12 However, it is well documented that the edge-response method is also vulnerable to a host of potential problems that could influence the measurement and result in inaccuracies. Inaccuracies may result from errors in
the calculated edge angle, noise, influences of scattered radiation, use of finite-element differentiation, profile misregistration and phasing errors, truncation of the LSF tails and incorrect normalization, and windowing and processing. Difficulties also arise when comparing measurements with published results of other investigators, as slight differences in object, acquisition, and/or processing techniques could easily alter the results. Significant variations have even been shown to occur when different individuals measured the MTF of the same data set. The aim of this work is to develop a new method for measurement of the MTF of digital radiographic systems that is less susceptible to such influences, by using the intrinsic noise response of the detector.

Cascaded linear systems methods have been used to accurately predict the signal and noise transfer properties of a wide variety of digital x-ray imaging technologies including x-ray image intensifiers, direct and indirect flat panel detectors (FPD), and CCD/EMCCD-based detectors. Careful inspection of the resultant noise response of such systems from a theoretical framework, as described by the two-dimensional noise power spectrum (NPS), indicates that the resolution response (i.e., the MTF) is inherently included in such information. In this work, we present an exact relationship using a generalized cascaded linear systems analysis. This relationship was then utilized to measure the presampled detector MTF from the noise response alone, without use of a test object.

The accuracy of the noise-response method was investigated using image simulations of a modeled detector system, in which the “true” MTF was known exactly. Measurements were also performed on several different clinical imaging technologies, including an x-ray image intensifier (XII) and a FPD, as well as a custom-built solid-state x-ray image intensifier (SSXII), which is an electron-multiplying CCD (EMCCD)-based detector with very high-resolution capabilities (greater than 10 lp/mm). Results from the noise-response method were compared with those obtained using the standard edge-response method.

Potential benefits of this method, including elimination of the requirement for a test object (which requires careful construction and precise alignment) and the ability to acquire the MTF in all directions simultaneously, will be discussed. Effects of potential noise sources that have not been considered in the theoretical development of the noise-response method and aliasing will also be discussed.

II. METHODS

II.A. Theoretical development of the noise-response method

Cascaded linear systems theory models the overall response of linear and shift-invariant detector systems as a contributing sum of individual stages, each of which consists of a single interaction process. Progression of quanta through the imaging chain is generally serial in nature, in which the output of one stage is the subsequent input of the next. Parallel processes occur when x-ray converting pho-

phors are implemented, resulting from the emission and re-absorption of secondary x rays. Parallel processes also occur in photoconductive (direct) x-ray detectors. Cascaded linear systems theory has been an extremely useful tool for characterizing and optimizing detector performance and has been used to accurately predict the signal and noise transfer properties of a wide variety of digital x-ray imaging technologies. Excellent agreement has been observed between theoretical calculations and measured results, even with the required estimation of many system performance parameters.

Parallel cascade theory was used to generalize the response from digital x-ray detectors, which were considered to consist of four fundamental stages: X-ray absorption, x-ray-to-light conversion, light-to-electron conversion, and digitization. Each stage consists of several interaction sub-stages, where the quanta undergo one of three processes: Amplification, blurring, or addition of noise. A brief summary of the parallel cascade model analysis is presented below. Additional details can be seen in a number of works.

Amplification processes may involve binary selection or conversion of quanta from one form to another. The signal at the output of stage $i$ of an amplification process is described by

$$\Phi_i(u,v) = g_i\Phi_{i-1}(u,v),$$

where $g_i$ is the average gain of stage $i$ and $\Phi_{i-1}(u,v)$ is the signal in the spatial frequency coordinates $(u,v)$ at stage $i-1$ (i.e., at the input of stage $i$). The NPS at stage $i$ is given by

$$\text{NPS}_i(u,v) = g_i^2\text{NPS}_{i-1}(u,v) + \sigma_{g_i}^2\Phi_{i-1},$$

where $\sigma_{g_i}^2$ is the gain variance of stage $i$, $\Phi_{i-1}$ is the average signal at stage $i-1$, and $\text{NPS}_{i-1}(u,v)$ is the NPS at stage $i-1$ in the spatial frequency coordinates $(u,v)$. It follows that consecutive binomial processes can be combined into a single binomial process with a probability equal to that of the product of the probabilities of each individual stage. Blurring processes may be either stochastic (caused by the spatial spreading or scattering of stochastic quanta) or deterministic (e.g., pixelization resulting from digital sampling). The signal at the output of stage $i$ of a blurring process is given by

$$\Phi_i(u,v) = T_i(u,v)\Phi_{i-1}(u,v),$$

where $T_i$ is the MTF at stage $i$. The NPS after a stochastic blurring stage can be written as

$$\text{NPS}_i(u,v) = [\text{NPS}_{i-1}(u,v) - \Phi_{i-1}]|T_i(u,v)|^2 + \Phi_{i-1}$$

and for a deterministic blurring stage

$$\text{NPS}_i(u,v) = \text{NPS}_{i-1}(u,v)|T_i(u,v)|^2.$$  

An additive noise stage does not change the signal as the images are assumed to have been offset corrected. The NPS after an additive noise stage is given by
II.A.1. Stage 1. X-ray absorption

The first stage in any x-ray detector is the absorption of incident x-rays, which consists of a single binomial-gain substage, in which the x-rays either are or are not absorbed. The signal and NPS at the output of stage 1 are

\[
\Phi_1 = g_1 \Phi_0 \quad \text{(4a)}
\]

and

\[
\text{NPS}_1(u,v) = g_1^2 \Phi_0 \quad \text{(4b)}
\]

where \(g_1\) is the quantum detection efficiency of the absorber, and \(\Phi_0\) is the incident x-ray fluence.

II.A.2. Stage 2. X-ray-to-electron conversion

For indirect detectors, there is an additional stage in which the x-rays are intermittently converted to light via a phosphorescent material. This process involves amplification from the conversion of x-ray photons to light photons and stochastic blurring processes from the light spreading in the phosphor. Parallel events may occur (depending on the x-ray energy and phosphor) as a result of the production of secondary x-rays.\(^{46,47}\) The signal and NPS at the output of stage 2 can be written as\(^{38}\)

\[
\Phi_2(u,v) = g_1 \Phi_0 T_2(u,v) W^{-1} \{E - E_k P_k \omega_k [1 - f_k T_k(u,v)]\}, \quad \text{(5a)}
\]

\[
\text{NPS}_2(u,v) = g_1^2 \Phi_0 T_2^2(u,v) W^{-2} \times \{E - 2E_k P_k \omega_k [1 - f_k T_k(u,v)]\} + g_1^2 \Phi_0 W^{-1}
\]

\[
\times \{E - E_k P_k \omega_k [1 - f_k T_k(u,v)]\}, \quad \text{(5b)}
\]

where \(T_2(u,v)\) is the MTF associated with the stochastic spreading of light in the phosphor, \(W\) is the average work energy required to liberate one image forming quanta, \(E\) is the incident x-ray photon energy, \(E_k\) is the energy of the characteristic x-ray energy produced in the phosphor, \(P_k\) is the electron-shell participation fraction, \(\omega_k\) is the electron-shell fluorescent yield, \(f_k\) is the absorption probability from the electron-shell transition, and \(T_k\) is the MTF associated with the secondary x-ray reabsorption at a remote location. For clarity, the energy response was averaged both for the incident polychromatic x-ray spectra and for the atomic response of the x-ray converting phosphor. However, this is generally not required, and in practice energy differences are considered.

Using the following substitutions:

\[
g_{\text{PE}} = W^{-1} \{E - E_k P_k \omega_k [1 - f_k T_k(u,v)]\}, \quad \text{(6)}
\]

\[
T_{\text{PE}}(u,v) = \{E - E_k P_k \omega_k [1 - f_k T_k(u,v)]\}
\times \{E - E_k P_k \omega_k [1 - f_k T_k(u,v)]\}^{-1}, \quad \text{(7)}
\]

\[
A_3(u,v) = \{E - E_k P_k \omega_k [1 - f_k T_k(u,v)]\}^2
\times \{E^2 - 2EE_k P_k \omega_k [1 - f_k T_k(u,v)]
\]

\[
+ E_k^2 P_k \omega_k [1 + f_k - 2f_k T_k(u,v)]\}^{-1}, \quad \text{(8)}
\]

where \(g_{\text{PE}}\) is the mean gain of the photoelectric process, \(T_{\text{PE}}(u,v)\) is the photoelectric MTF, and \(A_3(u,v)\) is the frequency-dependent Swank noise. Eqs. (5a) and (5b) can be simplified to

\[
\Phi_2(u,v) = g_{\text{PE}} g_1 \Phi_0 T_{\text{PE}}(u,v) T_2(u,v), \quad \text{(9a)}
\]

\[
\text{NPS}_2(u,v) = g_{\text{PE}}^2 g_1^2 \Phi_0 A_3(u,v) T_{\text{PE}}^2(u,v) T_2^2(u,v) + g_{\text{PE}} g_1 \Phi_0. \quad \text{(9b)}
\]

Eqs. (6)–(8) are consistent with those described by Hajdok et al.\(^{39}\)

II.A.3. Stage 3. Light-to-electron conversion

Prior to digitization, light photons (or x-rays in the case of direct detectors) are converted into electrons. Light photons undergo a binomial selection process, in which they either are or are not converted into an electron. This conversion may also include stochastic blurring processes, for instance, light spreading in fiber optic components. The signal and NPS at the output of stage 3 are described by

\[
\Phi_3(u,v) = g_3 g_{\text{PE}} g_1 \Phi_0 T_3(u,v) T_{\text{PE}}(u,v) T_2(u,v) \quad \text{(10a)}
\]

\[
\text{NPS}_3(u,v) = g_3^2 g_{\text{PE}}^2 g_1^2 \Phi_0 A_3(u,v) T_{\text{PE}}^2(u,v) T_2^2(u,v) + g_3 g_{\text{PE}} g_1 \Phi_0 \quad \text{(10b)}
\]

where \(g_3\) is the light-to-electron conversion factor (with consecutive binomial processes combined into one binomial process) and \(T_3(u,v)\) is the MTF associated with stochastic spreading of light in the conversion process.

II.A.4. Stage 4. Digitization

The final stage for any digital x-ray detector is digitization. Digitization occurs with the conversion of electrons to digital numbers (DNs), an amplification process. The signal is accumulated in pixel elements, resulting in deterministic blurring. Noise extending beyond the Nyquist frequency is aliased to lower frequencies. Digitization also involves the addition of electronic noise resulting from the digital readout process. The signal and NPS at the output of stage 4 are

\[
\Phi_4(u,v) = \Delta x \Delta y g_4 g_{\text{PE}} g_1 \Phi_0 T_4(u,v) T_3(u,v) T_2(u,v) \quad \text{(11a)}
\]

\[
\times T_{\text{PE}}(u,v) T_2(u,v),
\]

\[
\text{NPS}_4(u,v) = \Delta x \Delta y g_4^2 g_{\text{PE}}^2 g_1^2 \Phi_0 A_3(u,v) T_{\text{PE}}^2(u,v) T_2^2(u,v) + \Delta x \Delta y g_4 g_{\text{PE}} g_1 \Phi_0 \quad \text{(11b)}
\]
\[
\text{NPS}_4(u,v) = \text{NPS}_{\text{PRE}}(u,v) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \text{NPS}_{\text{PRE}}(u, v \pm \Delta x, n \pm \Delta y) + \text{NPS}_{\text{Add}}(u,v),
\]

where

\[
\text{NPS}_{\text{PRE}}(u,v) = \Delta x \Delta y g_4 g_3 \text{NPS}_{\text{Add}}(u,v) \phi_0 T_2^2(u,v) + \Delta x \Delta y g_2 g_3 \text{NPS}_{\text{Add}}(u,v)
\]

is the presampled NPS. \(\Delta x\) and \(\Delta y\) are the pixel width in the horizontal and vertical directions, \(n\) and \(m\) are the number of replicates, \(g_4\) is the electron-to-digital-number conversion factor which is assumed to have no variation, \(T_3(u,v)\) is the aperture MTF associated with integration of the signal over a finite region (e.g., the sinc function of the detector element size), and \(\text{NPS}_{\text{Add}}(u,v)\) is the additive electronic noise.

The system MTF \(T_{\text{SYS}}(u,v)\) and the effective system gain \(\bar{g}\) in units of DN per absorbed x-ray photon can be written as

\[
T_{\text{SYS}}(u,v) = T_3(u,v) T_{\text{PE}}(u,v) T_2(u,v)
\]

and

\[
\bar{g} = g_4 g_3 \text{PRE}.
\]

Combining Eqs. (11a)–(11c), (12), and (13) we can then simplify to the following expressions:

\[
\text{NPS}_4(u,v) = \Delta x \Delta y \left[ \frac{T_{\text{SYS}}^2(u,v)}{A_5(u,v)} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{T_{\text{SYS}}^2(u, n \pm \Delta y, m \pm \Delta x)}{A_5(u,v)} \right] \bar{g} + \text{NPS}_{\text{Add}}(u,v),
\]

where \(F_P\) is the pixel fill factor. If the aliasing of the correlated noise is negligible, or, similarly, if

\[
T_{\text{SYS}}^2(f_N) \ll 1,
\]

which is a reasonable assumption for indirect detectors,\(^{54,55}\) Eq. (16) simplifies to\(^ {53}\)

\[
\text{NPS}_4(u,v) = \Delta x \Delta y \left[ \frac{T_{\text{SYS}}^2(u,v)}{A_5(u,v)} + \frac{g_4}{F_P} \right] \Phi_4 + \text{NPS}_{\text{Add}}(u,v).
\]

Potential errors introduced by making this assumption will be discussed further in the results section.

Equation (18) is the output digital NPS, the form of which is generic for a wide range of detector technologies. The NPS consists of additive contributions from primary quantum noise, Poisson excess noise, secondary quantum noise, and additive electronic noise.\(^ {53}\) In general, a structure noise term which scales proportionally to \(\Phi_4^2\) should also be included, as a result of variations in detector sensitivity. However, all images used in this analysis were corrected for gain variations using an average of 30 flat-field images at each exposure level, resulting in a negligible structure noise component. Hence, this term was omitted. The slope of a linear fit of the NPS plotted versus mean signal \(\Phi_4\) then goes as

\[
\text{Slope}(u,v) = \frac{\Delta x \Delta y \bar{g}}{A_5(u,v)} T_{\text{SYS}}^2(u,v) + \frac{\Delta x \Delta y g_4}{F_P}
\]

and can be used to separate the additive instrumentation noise from the quantum noise sources, which scale proportionally with the detector entrance exposure. Equation (19) can then be used to determine the system MTF \(T_{\text{SYS}}\), as described below in Sec. II B.
II.B. Experimental procedure

The procedure used for determining the presampling MTF from the detector noise response is described below.

II.B.1. Step 1: Preparation of the imaging setup

A standardized x-ray spectrum \(^{56}\) was used to facilitate comparisons as the response of digital x-ray detectors has been shown to have a significant dependence on the energy of the incident x-ray beam. \(^{57}\) The desired spectra were achieved using a specified thickness of added aluminum filtration (Alloy 1100) (Ref. 58) and by adjusting the kVp of the generator to achieve a specific half value layer. A standard measurement geometry was also used, as defined by the IEC (Ref. 21) to minimize the effects of scatter, with as large a source-to-image distance as the imaging system would allow (greater than 100 cm) and with the added filtration placed as close as possible to the source. To simplify the measurement, the x-ray scatter reduction grid was removed (when appropriate). Clinical digital x-ray detectors typically implement highly nonlinear image processing in an attempt to improve perceived image quality. This image processing could potentially affect the measurement and was disabled to avoid any complications.

II.B.2. Step 2: Measurement of the NPS

NPS measurements were also done as prescribed by the IEC. \(^{21}\) Flat-field images were acquired using the standard spectrum and geometry with no test object in the beam. Responses of the detectors used in this study were linear relative to the number of input quanta, so the implementation of a conversion function (for linearization) was unnecessary. A central region of interest (ROI) was selected in the image for analysis and broken into 25 half-overlapping 256 \(\times\) 256 pixel regions. To improve statistics of the measurement, 30 flat field images (providing more than \(17 \times 10^6\) independent image pixels) were used for analysis. The NPS was then calculated using \(^{59}\)

\[
\text{NPS}(u_v, v_k) = \frac{\Delta x \Delta y}{M \cdot 256 \cdot 256} \sum_{i=1}^{256} \sum_{j=1}^{256} [I(x_i, y_j) - \bar{I}(x_i, y_j)] \exp[-2\pi i(u_x x_i + v_y y_j)]^2 ,
\]

(20)

where \(M\) is the number of ROIs analyzed, \(I(x_i, y_j)\) is the signal at pixel location \((x_i, y_j)\), \(\bar{I}(x_i, y_j)\) is the average signal at pixel location \((x_i, y_j)\), and \(u_v\) and \(v_k\) are the spatial frequency values in the horizontal and vertical directions, respectively, which are sampled at an interval of \(f_{Nyquist}/128\).

II.B.3. Step 3: Separation of additive instrumentation noise

Additive instrumentation noise is generally “white” (i.e., a constant as a function of spatial frequency) and does not contain information of the detector MTF. The additive instrumentation noise is constant as a function of exposure, whereas the quantum noise scales proportionally with the detector entrance exposure. A linear regression fitted to a plot of NPS versus signal was used to separate the two noise components [Eq. (18)] with the slope representing the quantum noise per unit signal. To provide a good estimate of regression coefficients, the NPS was measured across a range of exposures, which encompassed a majority of the dynamic range of the detector.

II.B.4. Step 4: Determination of the MTF

The system MTF \(T_{\text{Sys}}(u, v)\) was then determined from the quantum noise component of the NPS, using Eq. (19). A functional form of \(T_{\text{Sys}}(u, v)\), based on underlying physical principles, was used in the fit of the slope or quantum noise per unit signal as a function of spatial frequency. Fitting functions used for this analysis were based on a Gaussian mixture model \(^{60}\) and the complementary error function (ERFC) (Ref. 61) and are shown below, in one dimension for simplicity

\[
F_{\text{Gaussian Mixuture}}(f) = \frac{h_1}{A_3(f)} \left[ \sum_{i=1,2,3} h_{2,i} \times \exp \left( -\frac{(f-h_3^i)^2}{h_4^i} \right) \right] + h_5 ,
\]

(21)

and

\[
F_{\text{ERFC}}(f) = \frac{h_1}{A_3(f)} \left[ \frac{1}{2} \text{ERFC} \left( \frac{f-h_3}{h_5} \right) \right] + h_5 ,
\]

(22)

where \(A_3(f)\) is the calculated frequency-dependent Swank factor, \(h_i\) corresponds to the \(i\)th fitting parameter, and \(N\) is the number of Gaussian forms in the Gaussian mixture model. Derivation and calculation of \(A_3(u, v)\) has been described by Hadjok et al., \(^{49,62,63}\) demonstrating good agreement with Monte Carlo and measured results. A total of four different fitting functions [Eq. (21) with \(N=1, 2, 3\) and Eq. (22)] were used and the one providing the best fit was used for subsequent analysis. Different fitting functions were used to ensure very high precision in the fitting technique. Frequencies were sampled at an interval spacing of \(f_{Nyquist}/128\). The MTF was determined directly from the fitting function

\[
T_{\text{Sys}}(f) = \sum_{i=1}^{N} h_{2,i} \exp \left( -\frac{(f-h_3^i)^2}{h_4^i} \right) \text{ or }
\]

\[
T_{\text{Sys}}(f) = \frac{1}{2} \text{ERFC} \left( \frac{f-h_3}{h_5} \right)
\]

(23)

but could also be determined indirectly using the slope data and fit parameters \(h_1\) and \(h_5\)

\[
T_{\text{Sys}}(f) = \sqrt{\frac{A_3(f)(\text{slope}(f)-h_5)}{h_1}} ,
\]

(24)

which is more robust in allowing the data to deviate somewhat from the fitted result. If results provided by Eqs. (23) and (24) were to show a significant discrepancy, it would be...
indicative that a more appropriate functional form should be sought. The fit parameters \( h_1 \) and \( h_2 \) correspond to \( \Delta x \Delta y g \) and \( \Delta x \Delta y g / F_r \), respectively, and may be used to determine the system gain \( (g) \) and the electron-to-digital-number conversion factor \( (g_4) \) if the pixel size is known. More advanced methods could also be used to extract \( T_{sys} \), including quadratic optimization. However, for this analysis the use of a fitting technique proved sufficient.

II.C. Image simulations

To evaluate the accuracy of the noise-response method, images were simulated using MATLAB (version 7.0.1, MathWorks, Natick, MA) for a simplified detector model in which the MTF was known exactly. The detector model was based on a high-resolution, high-sensitivity EMCCD-based SSXII and is described further below.

Thirty flat-field images were simulated at six different exposure levels [17, 27, 43, 53, 67, and 83 \( \mu R \) (4.4, 7.0, 11, 14, 17, and 21 nC/kg)] according to the following prescription. First, a number of incident x-ray photons, based on Poisson statistics, were randomly generated per pixel (32 \( \times \) 32 \( \mu m^2 \)) for a 1000 \( \times \) 1000 image matrix. The number of x-ray photons absorbed in the phosphor was then determined using a binomial selection process with a success rate of 0.77. A conversion gain of 500 was used to determine the subsequent number of light photons generated per absorbed x-ray photon. The blur associated with the conversion process was assumed to be a single Gaussian with a standard deviation of 28 \( \mu m \) (full width half maximum of 66 \( \mu m \)). Light photons were then converted to electrons using binomial selection with a success rate of 0.5. Electrons were in turn “digitized” using an electron-to-digital-number conversion factor of 0.8 e\(^{-} / \)DN and electronic noise was added (Gaussian with zero-mean) with a standard deviation of 10 DN. The true MTF of this simulated detector system is given by

\[
T_{true}(u,v) = \exp \left[ -\frac{1}{2} \left( \left( 2\pi \sigma \Delta x u \right)^2 + \left( 2\pi \sigma \Delta y v \right)^2 \right) \right] \\
\times \frac{\sin(\pi \Delta xu)}{\pi \Delta xu} \frac{\sin(\pi \Delta yv)}{\pi \Delta yv},
\]

where \( \sigma \) is the standard deviation of the Gaussian blurring function.

Edge images were also simulated for the detector model described above. Images of an edge test object were generated in a manner similar to that described by Carton et al.\(^{24} \)

The edge was centered in the image, oriented at an angle of 2\(^{\circ} \) relative to the pixel rows, and assumed to absorb 95\% of the incident x rays. An average of ten frames was used to reduce the noise content in the images.

II.D. Experimental measurements

For further validation of the method, a comparison was made between the MTFs determined for digital x-ray detectors by the noise-response method and an established measurement technique. Images were acquired for three different detectors, covering the gamut of what we believe to be the past, present, and future of medical x-ray imaging technologies: An XII, an indirect FPD, and a SSXII. The XII was a model CAS-8000 V (Toshiba Medical Systems Corp., Tustin, CA) with an estimated CsI:Na phosphor thickness of 500 \( \mu m \). The XII was operated in 4.5 in. magnification mode with an effective pixel size of 119 \( \mu m \). A Varian Paxscan 2020+FPD (Palo Alto, CA) was used with a CsI:Tl phosphor thickness of 600 \( \mu m \) and a pixel pitch of 194 \( \mu m \). The SSXII\(^{40,43–45} \) was a prototype developed by our laboratory for high-resolution, high-sensitivity imaging, and had a 375 \( \mu m \) thick CsI:Tl phosphor and an effective pixel size of 32 \( \mu m \). Each imaging system was prepared as described in Sec. II B. The RQA5 spectrum was used for all measurements by adding 21 mm of aluminum (alloy 1100) to a 76 kVp x-ray beam, providing an approximate half value layer of 7.1 mm of Al.

For the noise-response analysis, 30 flat-field images were acquired at six different mA s values spanning a majority of the 12 bit dynamic range of the detectors. Images were also acquired of an opaque edge, slightly angulated at 1\(^{\circ}–3^{\circ} \) relative to both the horizontal and vertical directions of the pixel matrix. All images were gain and offset corrected. MTF measurements for both the noise-response and edge-response methods were done on three separate occasions to gauge the uncertainty of the measurements.

III. RESULTS AND DISCUSSION

III.A. Image simulations

The NPS was determined for the simulated flat-field images in a central 768 \( \times \) 768 region, providing 25 overlapping ROIs per image or 750 overlapping ROIs for the entire 30 image sequence. The combined horizontal and vertical response was taken by averaging along the zero frequency axes and the three adjacent rows/columns. This averaged NPS is shown in Fig. 1 for the six different incident x-ray exposures used.
The NPS was plotted as a function of signal at each spatial frequency (with a sampling interval of $f_N/128$ or 0.12 cycles/mm), and the data were fitted with a linear regression to separate the quantum noise and the additive electronic noise, as shown in Fig. 2 for four representative frequencies. In total, 128 frequencies were fitted (from 0 to 15.6 cycles/mm). Overall, the fit was very good with an average $r^2$ value of 0.996. The 95% confidence intervals of the regression coefficients, taken as a percentage of the regression coefficients, were also averaged for all spatial frequencies, and the average was 5.6%. This indicates that the use of 30 flat-field images acquired at six different exposures should provide sufficiently high precision for the subsequent analysis.

In this manner, the quantum noise per unit signal was obtained as a function of spatial frequency. Four functions were then fit to the quantum noise distribution, as described by Eq. (21) (with $N=1$, 2, and 3) and Eq. (22). Figure 3 shows the quantum noise per signal level with the best fit which was obtained using a single Gaussian ($N=1$) functional form for $T_{Sys}$. Good agreement was observed at all spatial frequencies and the fit was shown to largely overlay the simulation data. Averaged percent differences were determined to be 1.9%, 5.2%, and 4.3% for spatial frequencies ranging from 0 to 5, 5 to 10, and 10 to 15 cycles/mm, respectively. The system MTF obtained from this fit is shown on a semilogarithmic plot in Fig. 4. Also shown in this plot is the true MTF, which was known exactly [Eq. (25)]. The two curves are in very good agreement with an averaged percent deviation of 0.3% (mean difference of 0.001) and a maximum of 1.1% at the Nyquist frequency.

For comparison, the MTF determined using the simulated edge images is also shown in Fig. 4. The edge-response MTF was determined using standard methods with no image processing [i.e., no smoothing of the edge spread function (ESF) and subsequent LSF and no use of a windowing function] as such processing would inherently affect the results. Good agreement was observed up to 12.5 cycles/mm with an averaged percent deviation of 2.7% (mean difference of 0.016). The ROI selected around the edge was 3.2 cm wide (i.e., the entire FOV), which resulted in an unavoidable truncation of the LSF tails, which is an inherent problem of the edge-response method, especially for small FOV detectors. However, because no low-frequency effects were modeled, this truncation did not significantly affect the accuracy of the edge-response method. The noise-response method does not rely on the intermediate determination of the LSF and therefore is not susceptible to the use of a finite ROI size. Above 12.5 cycles/mm, the edge-response MTF was shown to diverge from the true MTF with an averaged percent deviation of 35% and a maximum of over 100%. This divergence was found to result from both noise in the ESF and the LSF and from phase errors (which result from the resampling of the ESF at regular intervals, which is a requirement of the fast
Fourier transform)\(^{20}\) and is indicative of the inherent error in the edge-response MTF procedure used for this analysis.\(^{22}\)

### III.B. Experimental measurements

A similar analysis was also done for the three actual detector systems as described in Secs. II B and III A. Zero-frequency NPS values were omitted from the fitting procedure due to the potential for a sharp increase at very low spatial frequencies as a result of artifacts.\(^{64}\) The combined horizontal and vertical response was taken by averaging along the zero frequency axes and the three adjacent rows/columns. For each detector, the quantum NPS per unit signal was fitted with Eqs. (21) and (22). The fitting function that provided the best fit was used for further analysis. Careful attention was paid to ensure that a good fit was obtained at all spatial frequency values which were sampled at an interval of \(f_N/128\) in order to avoid potential systematic errors resulting from poor fitting techniques. Table I summarizes the goodness of fits for each of the three detectors. The 95% confidence intervals of the regression coefficients, obtained by fitting the NPS versus signal to a linear curve at each spatial frequency, are given in the second column. Four different fitting functions were then used to fit the resulting slope data as a function of spatial frequency and the function giving the best fit that was used to provide the presampled MTF is indicated in the third column. The average percent deviation between the slope data and the best fit function is given in the last four columns for four frequency intervals.

<table>
<thead>
<tr>
<th>Detector</th>
<th>95% confidence intervals of regression coefficients (percent of coefficient)</th>
<th>Best fit function</th>
<th>Fitting function agreement with slope data (Averaged % deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XII</td>
<td>3.1</td>
<td>Gaussian</td>
<td>(-0.59 \quad 0.75 \quad -0.95 \quad 0.18)</td>
</tr>
<tr>
<td>FPD</td>
<td>2.0</td>
<td>Gaussian</td>
<td>(-0.72 \quad 0.55 \quad 0.21 \quad 0.04)</td>
</tr>
<tr>
<td>SSXII</td>
<td>3.6</td>
<td>ERFC</td>
<td>(-0.08 \quad -0.2 \quad 0.01 \quad 0.39)</td>
</tr>
</tbody>
</table>

![Fig. 5. The XII presampled MTF measured using the noise and edge-response methods plotted up to the Nyquist frequency of 4.2 cycles/mm.](image)

![Fig. 6. The FPD presampled MTF measured using the noise and edge-response methods plotted up to the Nyquist frequency of 2.5 cycles/mm.](image)
Measurements on the three detectors show very good agreement, with the noise-response MTF shown to deviate by more than 50% of the true MTF. Also shown is the percent difference of the MTF determined using the noise-response and edge-response methods plotted up to the Nyquist frequency of 15.6 cycles/mm.

Figure 7: The SSXII presampled MTF measured using the noise and edge-response methods plotted up to the Nyquist frequency of 15.6 cycles/mm. Significantly larger than that of the noise-response method (the error bars of which were generally less than the thickness of the curve), perhaps in part due to slight variations in the orientation of the edge test object and differences in the response of the procedure itself (as mentioned in Sec. I). The two methods were shown to largely agree within experimental uncertainty for all three detectors; however, systematic differences were observed.

Figure 8 shows the percent difference between the measured MTF from the edge-response method and the noise-response method for the simulated detector system and the XII, FPD, and SSXII. Also shown is the percent difference between the noise-response MTF of the simulated detector compared with the true MTF. To facilitate comparisons between detectors, the spatial frequencies were normalized to the appropriate \( f_N \). Close agreement with the true MTF was observed when using the noise-response method, with a maximum percent difference of 1.1%, whereas the edge-response MTF was shown to deviate by more than 50%. Measurements on the three detectors show very good agreement between the two methods up to one half of the Nyquist frequency \( f_N \), with an averaged percent difference of 3.4%. Above \( f_N/2 \), the edge-response method increasingly diverges from the noise-response method, with an average percent difference of 20%. Deviations of the experimental results largely follow the trend of the simulation results, suggesting that the discrepancies between the two methods are due to the inaccuracies of the edge-response procedure implemented for this study. Difficulty arises when attempting to correct for this error as it is inherently dependent on the particular images used in the analysis. Simulation images, in which the true MTF is known, provide the only scenario where the error in the MTF measurement can be accurately assessed. However, simulation images are significantly simpler than actual images which include scatter and variations in object uniformity in addition to many other physical effects (as described in Sec. I). Also, the edge-response method for the SSXII was shown to exhibit oscillatory behavior at higher spatial frequencies. Others have shown that when the presampled MTF is negligible for frequencies above \( 2f_N \), phasing errors in the MTF may become sinusoidal in nature, which could account for these oscillations. Oscillatory behavior also arises from truncation of the LSF tails, which results from use of a finite ROI.

It should be noted that any aliasing not being properly taken into account (as described in Sec. II A) or Lubberts effect would tend to disproportionately increase the NPS at higher spatial frequencies, resulting in a corresponding inflation in the measured MTF at higher spatial frequencies when using the noise-response method, as demonstrated by Eq. (16). However, for each of the three detector systems studied in this analysis, the edge-response MTF was always larger than the noise-response MTF indicating that these effects were likely minimal. To estimate the error resulting from the assumption that the correlated noise alias was negligible, a fit including these effects was performed, using the following equation [which is a combination of Eqs. (16) and (21)]:

\[
F_{\text{Gaussian Mixture}}(f) = \frac{h_1}{A_5(f)} \left[ \sum_{n=-100}^{100} h_2 \exp \left( -\frac{(|f - 2nf_N| - h_3)^2}{h_4} \right) \right]^2 + h_5, \tag{26}
\]

where ±100 frequency replicates were considered. The MTF was then obtained as otherwise described in Sec. II B. The calculated error in the MTF [taken to be the percent difference between the MTF determined using Eqs. (21), (22), and (26)] was found to be 3.5% maximally at the Nyquist frequency for the FPD, and less at lower spatial frequencies and for the other two detectors. Improved precision could result from including the effects of aliased correlated noise. Further investigation is required to determine whether or not the noise-response method can provide an accurate MTF for heavily aliased systems, such as direct detectors.
Accurate MTF measurements were obtained using the noise-response method on three very different detector technologies, using the same generalized cascaded-systems model, supporting the notion that the generalized form of the NPS, as given in Eq. (14b), accurately describes a wide range of detectors. Cascaded-systems models by others, for a wide variety of detector technologies (including direct detectors), also indicate general agreement with Eq. (14b). Therefore, verification of Eq. (14b) appears to be unnecessary for well-behaved systems. In each instance, the NPS contains a correlated and uncorrelated component which scales proportionally with exposure, with the correlated component being passed by the system MTF. Appropriate isolation of this component enables recovery of the system MTF from the NPS. Admittedly, it is difficult to foresee all physical processes and to include each in the model. Other potential effects that have not been explicitly included, such as lag or detector multiplicative noise, would scale the NPS equally at all spatial frequencies, and therefore would not affect the measured MTF, as they would get absorbed in the fitting coefficients $h_1$ and $h_5$ in Eqs. (21) and (22). Afterglow should have the same frequency components as the light (or electrons) emitted “immediately” after x-ray absorption and therefore would also not affect the measurement. Only noise sources that were to vary with spatial frequency would affect the measurement, and it is difficult to envision any such processes.

The two-dimensional NPS provides the noise response in all directions. As such, the MTF measured using the noise-response method can be determined not only in the horizontal and vertical directions (an average of which was used in this analysis for comparison with the edge-response method), but at any arbitrary angle relative to the pixel matrix. This is contrary to standard methods, which are inherently one-dimensional and rely on slight angulations relative to the pixel rows or columns to provide a “finely sampled” response, hence limiting them to the orthogonal or near-orthogonal directions. Accurate characterization of the two-dimensional MTF could prove useful for improved system evaluation and observer performance models.

### IV. CONCLUSIONS

A new method for determination of the presampled MTF, the “noise-response method,” has been described and evaluated. The accuracy of this method was demonstrated using both simulated and experimental data sets. For the simulated image set which used a simple detector model for which the true MTF was known exactly, excellent agreement was obtained with the MTF determined using the noise-response method, with a maximum deviation of 1.1%. Comparison measurements were also made on this simulated data set with the established edge-response method, and these showed deviations greater than 35% from the true MTF. Experimental measurements on a range of detector technologies (including an XII, FPD, and SSXII) demonstrated agreement between the noise-response and edge-response methods within experimental uncertainty, with discrepancies likely resulting from errors inherent in the edge-response MTF procedure.

Compared to current measurement methods, the noise-response method simplifies the MTF determination by eliminating the need for manufacture and alignment of precisely machined test objects, thereby eliminating inaccuracies that result from the use of such objects and subsequent analysis of the resulting images. Further, the two-dimensional MTF is readily obtained with the noise-response method, whereas traditional edge and slit methods are inherently one-dimensional.

### ACKNOWLEDGMENTS

This work was supported in part by the National Institutes of Health under Grant Nos. RO1-EB002873 and RO1-EB008425 and an equipment grant from Toshiba Medical Systems Corp.

---

4Electronic mail: atkuhls@buffalo.edu

64M. B. Williams, P. A. Mangiafico, and P. U. Simoni, “Noise power spec-
tra of images from digital mammography detectors,” Med. Phys. 26,

of methods for image quality characterization. II. Noise power spectrum,”

66S. N. Friedman and I. A. Cunningham, “A moving slanted-edge method
to measure the temporal modulation transfer function of fluoroscopic sys-