TEMPORAL REASONING: A RELATIVISTIC MODEL

by

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Temporal Reasoning: A Relativistic Model

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In 1983, Allen presented an ingenious method for the representation and maintenance of temporal information in the presence of imprecise, uncertain, and relative knowledge about time of occurrence. He introduced 13 relations between his primitive "temporal intervals," providing for the expression of "any relationship which can hold between two intervals." The model, however, did not address the problem of temporally incomparable events, such as events occurring in a distributed system without a common clock. Lamport's interprocessor communication model furnishes an axiomatic system for describing such events and their possible relationships. This article demonstrates that Allen's temporal model can be subsumed in a more general model based on Lamport's axiomatics. It is further suggested that this extended model can provide the underpinnings of a temporal knowledge base containing time-dependent information measured by unsynchronized clocks or in relativistic space-time. In this model, the number of relations between intervals increases dramatically from Allen's 13 or Lamport's 2 or 3 to over 80. Within this context, a modification of Allen's algorithm for the maintenance of a temporal reasoning system is presented, thus permitting the advantages of such a system to extend to reasoning about a wider range of phenomena.

I. INTRODUCTION

The difficulty of representing and reasoning about information with a temporal aspect has long been a central problem in several disciplines, including philosophy, physics, psychology, and computer science. In computer science, temporal issues are of paramount importance both in artificial intelligence and in the description of distributed systems. As modern computer systems move from the linear, single-CPU machines of the past decades to systems in which control and activities are concurrent and distributed, Newtonian models and

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classical logics become ever less capable of accurately capturing and describing them. Consequently, nonclassical temporal logics and modal logics both extensional and intensional, have been developed to provide adequate descriptions.\(^1\) Rather than exotic logics, several researchers have introduced new temporal models.\(^5\)-\(^40\) Of these models, only Lamport's and Winskel's are applicable to asynchronous distributed events, allowing for temporally incomparable or concurrent events and for complex, nonatomic events. Most recently, Alpern and Schneider have used yet another approach: Buchi automata.\(^11\)

This article outlines the creation of a temporal model and an intelligent system capable of reasoning about temporal events in that model, accommodating the difficult temporal problems described above. Buttressed by both the temporal model of Lamport and the knowledge-based approach of Allen, the envisioned system would provide a robust and useful tool for reasoning about distributed events and algorithms. Other researchers after Allen have proposed interesting extensions,\(^12\) analysis,\(^13\),\(^14\) or related algorithms for knowledge maintenance,\(^15\),\(^16\) but none of these efforts deals with true concurrency. Leban, McDonald, and Forster\(^17\) have considered representing identifiable collections of related intervals, while Chun\(^18\) has been concerned with representations causing massively parallel computer systems to obey temporal constraints. The model to be presented exploits Lamport's axiomatic model by showing that it implies an extension of Allen's classification of temporal events which can serve as the basis of a temporal reasoning system capable of handling concurrent events.

In the first paragraph, we mentioned that numerous situations, both in and out of artificial intelligence, require a consistent treatment of time. Most frequently, the linear time of classical physics or the set of natural numbers of mathematics has served as an adequate method of representation, but several application areas exist for which such a conceptual view of time is not appropriate. Consider, for example, two or more independent and spatially remote robots attempting to coordinate in a task, such as focusing a beam on a particular target for a very brief time interval so that all beams arrive at the target at the same moment. Even if all robots are at the same distance from the target, stating that they must fire simultaneously is not an appropriate representation for any of the individual robots, each of which would witness the actions as if the others had fired after it had fired. Thus, a more appropriate expression for the relation between two firing events is that each occurred after the other in some sense. This, of course, is not possible in a linear model of time.

As with the cooperating robots, if independent agents must plan to achieve some goal in the face of incomplete communication with one another, plans must be constructed admitting the possibility of incomparable events—those which are performed by different agents without any communication immediately before, during, or after their execution. Reducing time to a single, totally ordered sequence would require resolution of the relationship between these events. Another apparently quite different application, in which the resolution of temporal order is not always possible, is program verification in distributed systems. Originally intended for just this purpose, Lamport's axiomatic scheme was, nonetheless, not concerned with the automation of reasoning.
Although not a traditional AI problem, when a computer is required to carry out checking of the temporal validity of a distributed algorithm, a reasonable heuristic is needed. The following sections develop Lamport's temporal model in a way that allows the application of constraint propagation methods for checking consistency and deriving unknown temporal relations among events, such as the execution of instructions in a distributed system.

Another related application arises from natural language specifications of a complex real-time distributed system that may contain considerable information about the order of certain events and timing requirements on the performance of particular operations. Together with certain performance characteristics of the intended hardware, the information can be viewed as a temporal constraint satisfaction problem which can be analyzed to determine its consistency. Once again, a linear time model would be inappropriate as a basis for an intelligent knowledge base for reasoning about such problems.

### II. ALLEN'S TEMPORAL MODEL

Much of our information about objects and events lacks a unique specifier, that is, a description which uniquely identifies those objects and events. Yet, we succeed in reasoning about those objects and events. James Allen\(^9\) presented a methodology for handling temporal information in the presence of imprecise, uncertain, and relative knowledge about time of occurrence. Based on the 13 relations between his primitive temporal intervals (see Table I), information is represented as a network in which the nodes represent events or facts, and the links represent the possible temporal relationships that can hold between the linked events. In this sense, all of the stored, temporal information is relative. Other approaches have been taken to provide a combination of relative and exact temporal information; for example, Kahn and Gorry\(^9\) offered methods combining timeline specification with relative information. In their approach, however, events were considered to be without duration. Re-

<table>
<thead>
<tr>
<th>Condition</th>
<th>Relation</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>b &lt; c</td>
<td>I BEFORE J</td>
<td>&lt;</td>
</tr>
<tr>
<td>a &gt; d</td>
<td>I AFTER J</td>
<td>&gt;</td>
</tr>
<tr>
<td>b = c</td>
<td>I MEETS J</td>
<td>m</td>
</tr>
<tr>
<td>a = d</td>
<td>I MEET BY J</td>
<td>m̄</td>
</tr>
<tr>
<td>a &lt; c &lt; b</td>
<td>I OVERLAPS J</td>
<td>o</td>
</tr>
<tr>
<td>c &lt; a &lt; b</td>
<td>I OVERLAPPED BY J</td>
<td>ō</td>
</tr>
<tr>
<td>a &gt; c, b = d</td>
<td>I FINISHES J</td>
<td>f</td>
</tr>
<tr>
<td>a &lt; c, b = d</td>
<td>I FINISHED BY J</td>
<td>f̄</td>
</tr>
<tr>
<td>a &gt; c, b &lt; d</td>
<td>I DURING J</td>
<td>d</td>
</tr>
<tr>
<td>a &lt; c, b &gt; d</td>
<td>I CONTAINS J</td>
<td>d̄</td>
</tr>
<tr>
<td>a = c, b &lt; d</td>
<td>I STARTS J</td>
<td>s</td>
</tr>
<tr>
<td>a = c, b &gt; d</td>
<td>I STARTED BY J</td>
<td>s̄</td>
</tr>
<tr>
<td>a = c, b = d</td>
<td>I EQUALS J</td>
<td>=</td>
</tr>
</tbody>
</table>
cently, Pelavin and Allen have assailed the vexing issues of concurrency via the introduction of specialized modal operators within a first order logic. Their approach differs considerably from the one presented here. Finally, Ladkin has proposed extensions of Allen’s model by including arbitrary unions of intervals, creating a substantially more complex situation, but yielding an algebra with far greater expressive power. Allen stated that all possible relationships between two intervals could be expressed in terms of the 13 irreducible relationships. Notwithstanding, his model was based on the assumption of intervals on the real number line for which the trichotomy law holds:

\[ a < b, \quad a = b, \quad \text{or} \quad a > b \quad \text{for all} \quad a \text{ and } b. \]

As elaborated in Section III, this is not an adequate model for reasoning about concurrent events. The approach requires a system to maintain, for each pair of related events, as many of the 13 possible relations that can hold between them. Whenever new information is entered into the system, relations that might be affected are recalculated using a table of “transitivities” (Ref. 19, Fig. 4, p. 514). For example, if it is known that A is BEFORE B and that B is DURING C, then the temporal relation between A and C can be any of the following: A BEFORE C, A OVERLAPS C, A MEETS C, A DURING C, or A STARTS C. Allen presents an efficient algorithm for performing these calculations and also introduces the concept of “reference interval” to induce the clustering of comparable intervals. His algorithm, a type of constraint propagation, derives all logically necessary temporal relations between A and C which result from given relations between A, C, and any third event, B. However, it does not claim to be complete, in the sense that it does not derive all necessary relations implied by the interaction of arbitrary groups of relations.

III. LAMPORT’S TEMPORAL MODEL

Leslie Lamport presented a rather different model of temporal events based on the concurrency in a multiple processor system (or on a relativistic model of events in a space–time continuum). This model is based on only a partial order on the points in space–time:

\[ a < b, \quad a = b, \quad a > b, \quad \text{or} \quad a \text{ incomparable to } b \quad \text{for all} \quad a \text{ and } b. \]

Intervals or events which have duration in such a model have many more interrelationships than those proposed by Allen.

According to the special theory of relativity, all communications and interactions proceed at speeds less or equal to that of the propagation of light. More unsettling still is the claim that no matter what (constant) velocity one physical system has relative to another, they will both obtain the same measurement for the speed of light. The implications of these principles are enormous, but in particular indicate that it is useful to view our universe as a four-dimensional space–time continuum (which in our illustrations will be drawn with only one spatial dimension and one temporal dimension). For a given frame of reference,
each point $P$ of space–time is located at the vertex of an infinite, four-di- mensional cone whose axis extends along the time dimension into the past and future and whose interior represents all the space–time points which could possibly communicate with the given point or be communicated with that point. (See Fig. 1.) Points lying outside the cone are spatially separated but temporally incomparable, meaning that their occurrence is not before, after, or simultaneous with the occurrence of $P$. If events with duration are considered, as does Allen, the possible temporal relationships become even more complicated. For the sake of simplicity of exposition, events will be depicted as horizontal line segments whose relative lengths and positions indicate, respectively, their relative durations and spatial location.

To capture the essential properties of the temporal relationships between complex and perhaps unspecified activities within any system, Lamport presented a set of axioms. This axiomatic system was used to prove the correctness of sophisticated algorithms for reliable communication and mutual exclusion in systems without shared memory. The rudiments of Lamport’s formalism consist of three primitive concepts and five axioms which collectively provide the framework for temporal descriptions. The primitives are:

1. A set $S$ (finite, or at most countable) of elements called operation executions, which are to be thought of as actions such as the execution of some part of a program. For our purposes, these actions will be understood to be any describable event which could be associated with an interval of time of nonzero length and will be referred to as an event or interval.

   $\overline{A}$

2. A relation, $\rightarrow$, on $S$ called PRECEDES or HAPPENS BEFORE.

   $\overline{A}$

   $\overline{B}$

3. A relation, $\cdot\cdot\cdot\rightarrow$, on $S$ called CAN CAUSALLY AFFECT or just CAN AFFECT. $A \cdot\cdot\cdot\rightarrow B$ can be roughly interpreted as indicating that some portion of the event $A$ takes place before some portion of the event $B$.

   $\overline{A}$

   $\overline{B}$

The notion of time introduced by Lamport is just that imposed by the two relations above, on a system in which the following five axioms hold.

$A1$: $\rightarrow$ is an irreflexive partial order on $S$.

$A2$: $A \rightarrow B$ implies that $A \cdot\cdot\cdot\rightarrow B$ and $B \cdot\cdot\cdot\rightarrow A$ (read "$B$ cannot affect $A$"). Formally, $\rightarrow$ is a subrelation of $\cdot\cdot\cdot\rightarrow$ and $A \rightarrow B \cdot\cdot\cdot\rightarrow A$ is forbidden. Intuitively, preceding is stronger than affecting, and no event can affect the course of any previous event.
A3: $A \longrightarrow B \longrightarrow C$ or $A \longrightarrow B \longrightarrow C$ implies $A \longrightarrow C$.

A4: $A \longrightarrow B \longrightarrow C \longrightarrow D$ implies $A \longrightarrow D$.

A5: For any $A$ in $S$, the set of $B$ such that $A \rightarrow B$ (read "does not precede") is finite.

A triple $(S, \rightarrow, \longrightarrow)$ satisfying A1 through A5 is referred to as a system execution.

In order to describe the possible temporal relations that can hold between two events in such a model, it is sometimes helpful to think more concretely in terms of time intervals made up somehow of atomic events which cannot be further decomposed and which cannot overlap one another. For such atomic events, $\rightarrow$ and $\longrightarrow$ would coincide. Following Lamport, a model is defined as a type of representation of a system execution in terms of such atomic operations.

**Definition 1.** A model of a system execution $(S, \rightarrow, \longrightarrow)$ is a triple $(E, <, \mu)$, where $E$ is a set, $<$ is an irreflexive partial order on $E$, and $\mu : S \rightarrow \{\text{nonempty subsets of } E\}$ is a function satisfying

(a) $A \rightarrow B$ iff for all $x \in \mu(A)$ and all $y \in \mu(B)$, $x < y$ and

(b) $A \longrightarrow B$ iff for some $x \in \mu(A)$ and some $y \in \mu(B)$, $x < y$ or $x = y$.

As will be shown in Section IV, not all system executions have a model. Although the implications of this fact are not established by Lamport, the result demonstrates that his theory is strictly broader than one assuming atomic operations.

An even more restrictive model, one that corresponds to events synchronized by a common clock, is the global time model.

**Definition 2.** A global time model is a model in which $E = \text{the real numbers}$, each $\mu(A) = [A_s, A_f]$ is a closed interval, and $<$ is just the usual order on the real numbers.

The following theorem, which is presented and proved in Lamport and Anger, gives conditions for the existence of a global time model. Under its conditions, the model of Lamport would not differ significantly from the temporal model of Allen.

**Theorem 1.** A system execution $(S, \rightarrow, \longrightarrow)$ has a global time model if and only if for all $A, B$ in $S$, either $A \rightarrow B$ or $B \rightarrow A$.

## IV. THE PROPOSED SYSTEM

That the envisioned system be endowed with the ability to maintain temporal information and reason about true concurrency is a requisite. Toward this end, Allen’s method of temporal representation is extended, adopting a modification of Lamport’s model based on the following theorem of Anger.

**Theorem 2.** A system execution $(S, \rightarrow, \longrightarrow)$ has a model if and only if

M1: $A \longrightarrow A$ for all $A$ in $S$, and

M2: $A \longrightarrow B \longrightarrow C \longrightarrow D$ implies that $A \longrightarrow D$ for any $A, B, C, D$ in $S$. 
**Table II.** Representation of Allen’s relations in an event complex.

<table>
<thead>
<tr>
<th>Relation</th>
<th>Abbr</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CAN AFFECT</td>
<td>---&gt;</td>
<td>A ---&gt; B (primitive term)</td>
</tr>
<tr>
<td>2. PRECEDES</td>
<td>--&gt;</td>
<td>A --&gt; B (primitive term)</td>
</tr>
<tr>
<td>3. STARTS BEFORE</td>
<td>sb</td>
<td>X, A --&gt; C --&gt; B</td>
</tr>
<tr>
<td>4. FINISHES BEFORE</td>
<td>fb</td>
<td>X, A --&gt; C ---&gt; B</td>
</tr>
<tr>
<td>5. STARTS LATER</td>
<td>sl</td>
<td>X, B ---&gt; C &amp; A --&gt; C</td>
</tr>
<tr>
<td>6. FINISHES LATER</td>
<td>fl</td>
<td>X, C, B --&gt; C &amp; A --&gt; C</td>
</tr>
<tr>
<td>7. CO-ORIGINAL WITH</td>
<td>co</td>
<td>not sb &amp; not sl</td>
</tr>
<tr>
<td>8. CO-TERMINAL WITH</td>
<td>ct</td>
<td>not fb &amp; not fl</td>
</tr>
<tr>
<td>9. BEFORE</td>
<td>&lt;</td>
<td>X, A --&gt; C --&gt; B</td>
</tr>
<tr>
<td>10. MEETS</td>
<td>m</td>
<td>--&gt; &amp; not &lt;</td>
</tr>
<tr>
<td>11. OVERLAPS</td>
<td>o</td>
<td>sb &amp; fb &amp; not A --&gt; B</td>
</tr>
<tr>
<td>12. DURING</td>
<td>d</td>
<td>sl &amp; fb</td>
</tr>
<tr>
<td>13. STARTS</td>
<td>s</td>
<td>co &amp; fb</td>
</tr>
<tr>
<td>14. FINISHES</td>
<td>f</td>
<td>ct &amp; sl</td>
</tr>
<tr>
<td>15. PROJECTS TO</td>
<td>p</td>
<td>co &amp; ct</td>
</tr>
<tr>
<td>16. inverse REL</td>
<td>RELj</td>
<td>B REL A</td>
</tr>
</tbody>
</table>

Moreover, if M1 and M2 hold, a model can be found such that the mapping μ is one to one.

Predicated on this result, Definition 3 provides the necessary setting.

**Definition 3.** A triple (S, -->, --->), S a finite (or at most countable) set and satisfying A1 through A4, M1 and M2 is called an event complex.

From the foregoing discussion, it may be noted that an event complex always has a model. An event complex can be thought of as a description of some related set of occurrences in time and space, such as all the information available about a historical event, a medical history, the required motions of a robot, etc. Ben-David has also given a class of sound and complete models for this set of axioms.²³

Table II represents all of the 13 temporal relations given by Allen in terms of the primitives of event complexes. In Table II, item 1 is the primitive CAN CAUSALLY AFFECT arrow, which maps, in the case of a global time model, into “not AFTER nor MET BY.” Moreover, items 1 through 8 are not disjoint relations, but are necessary to facilitate defining and comprehending the subsequent relations. Thus, Allen’s relations are those corresponding—under a global time mapping—to 9 through 15 and their inverses. (PROJECTS TO is self-inverse in the presence of a global time model.)

It is significant that, whereas in the presence of a global time model A sb B is equivalent to B sl A, so that sl and sb are inverses (sb = slj and sl = sbj), this is not true in general. Indeed, A sb B does imply A slj B, but not conversely. It is, moreover, possible to have A sl B and B sl A both true! Consequently, appropriate English phrases have been chosen for describing, in essence, all ways in which A can affect B. The inverse relations, describing how B can effect A, are simply referred to as the inverses.

Table III extends Table II to the complete scheme for naming temporal relations between two events in an event complex. An entire collection of new
### Table III. Classification of event relations in an event complex.

<table>
<thead>
<tr>
<th>Relation</th>
<th>Abbr</th>
<th>Definition</th>
</tr>
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<tr>
<td>2. PRECEDES</td>
<td>--&gt;</td>
<td>A --&gt; B (primitive term)</td>
</tr>
<tr>
<td>3. STARTS BEFORE</td>
<td>sb</td>
<td>\exists C, A ---&gt; C B</td>
</tr>
<tr>
<td>4. FINISHES BEFORE</td>
<td>fb</td>
<td>\exists C, A --&gt; C --&gt; B</td>
</tr>
<tr>
<td>5. STARTS LATER</td>
<td>sl</td>
<td>\exists C, B --&gt; C &amp; A --&gt; C</td>
</tr>
<tr>
<td>6. FINISHES LATER</td>
<td>fl</td>
<td>\exists C, B --&gt; C &amp; A --&gt; C</td>
</tr>
<tr>
<td>7. CO-ORIGINAL WITH</td>
<td>co</td>
<td>not sb &amp; not sl</td>
</tr>
<tr>
<td>8. CO-TERMINAL WITH</td>
<td>ct</td>
<td>not fb &amp; not fl</td>
</tr>
<tr>
<td>9. BEFORE</td>
<td>&lt;</td>
<td>\exists C, A --&gt; C B</td>
</tr>
<tr>
<td>10. MEETS</td>
<td>m</td>
<td>--&gt; &amp; not &lt;</td>
</tr>
<tr>
<td>11. OVERLAPS</td>
<td>o</td>
<td>sb &amp; fb &amp; not A --&gt; B</td>
</tr>
<tr>
<td>12. DURING</td>
<td>d</td>
<td>sl &amp; fb</td>
</tr>
<tr>
<td>13. CONTAINS</td>
<td>cn</td>
<td>sb &amp; fl</td>
</tr>
<tr>
<td>14. SURPASSEES</td>
<td>su</td>
<td>sl &amp; fl &amp; ---&gt;</td>
</tr>
<tr>
<td>15. STARTS</td>
<td>s</td>
<td>co &amp; fb</td>
</tr>
<tr>
<td>16. FINISHES</td>
<td>f</td>
<td>ct &amp; sl</td>
</tr>
<tr>
<td>17. PROJECTS TO</td>
<td>p</td>
<td>co &amp; ct</td>
</tr>
<tr>
<td>18. EXTENDS</td>
<td>e</td>
<td>co &amp; fl</td>
</tr>
<tr>
<td>19. PRE-EXTENDS</td>
<td>pe</td>
<td>ct &amp; sb</td>
</tr>
<tr>
<td>20. AFTER</td>
<td>&gt;</td>
<td>\exists C, B --&gt; C &amp; A --&gt; C</td>
</tr>
<tr>
<td>21. FOLLOWS</td>
<td>fol</td>
<td>--&gt; &amp; not &gt;</td>
</tr>
<tr>
<td>22. inverse REL</td>
<td>RELi</td>
<td>REL1 REL2 [for legal pairs]</td>
</tr>
<tr>
<td>23. REL1 &amp; REL2</td>
<td>REL1 REL2</td>
<td>A REL1 B &amp; B REL2 A (for legal pairs)</td>
</tr>
</tbody>
</table>

All represent relations between A and B, to be read A REL B. 9 through 21 give Allen's 13 relations in any global time model.

relations arise of the form shown in item 23. They are specified by a pair of relations: the first being one of the relations 1–21 and the other being an inverse of one of these relations.

The composition of two relations, REL1 and REL2, is defined to be that relation such that A REL1 ∘ REL2 B if and only if there exists a C such that A REL1 C and C REL2 B. This definition allows the rephrasing of the definitions of a number of the relations in Table III:

\[
\begin{align*}
    sb & ::= \cdots \circ \rightarrow \\
    fb & ::= \rightarrow \circ \cdots \\
    sl & ::= \rightarrow \not \circ \rightarrow i \\
    fl & ::= \not \rightarrow \circ \rightarrow i \\
    < & ::= \circ \rightarrow i \\
    > & ::= \not \circ \rightarrow i \\
    m & ::= \rightarrow \& \not (\not \circ \rightarrow) \\
    fol & ::= \not \circ \rightarrow \& \not (\circ \rightarrow i)
\end{align*}
\]

The remaining relations in Table II are conjunctions, negations, and inverses of these.

Starting with Lamport's two arrows, the algebra LA*, consisting of all relations obtainable by repeated application of disjunction, conjunction, negation.
tion, inverse, and composition, is generated. Table III provides the necessary terminology to allow a complete description of this algebra. For example, the relations 9 through 21 are disjoint, and the remaining relations in the table are obtainable from them through conjunction, disjunction, and inverse. It will be shown that all of the relations in $L^*$ can be written as disjunctions of the relations of type 23: REL1/REL2, with REL1 and REL2 chosen from types 9 through 21.

The final classification of relations is obtained from the trees depicted in Figures 2 through 4. In Figure 2, each set of siblings corresponds to a single

\[ \text{Figure 2. Top of tree of possible temporal relations (TPTR).} \]
additional choice of A REL B from a given set of (2 or 3) all inclusive and mutually exclusive alternatives. The arcs are labeled with the alternative relations, whereas the nodes are labeled in the figures either by @ (no special name) or by (REL), if a particular name REL applies. The relation represented by a node is the conjunction of all choices from the root to the node. Observe that the leaves of Figure 2 are just the relations 9 through 21 of Table III. Figure 3 is the tree of inverse relations and has been presented as a mirror image of Figure 2. The complete Tree of Possible Temporal Relations (TFTFR) is obtained by grafting onto each leaf of Figure 2 the maximal subtree of Figure 3, which does not violate (by conjoining along a path two incompatible relations) any of the conditions 1 through 7 of the following proposition.

**Proposition 1.** In any event complex (S, →, ↔→), these seven implications hold:

1. $1 \rightarrow$ implies $\neg\neg\neg\neg\neg\neg\neg\rightarrow i$
2. $2\rightarrow$ implies $\neg\neg\rightarrow i$
3. $3 \leftarrow$ implies $\neg\neg\rightarrow i$
4. $sb \rightarrow$ implies $\neg\neg\rightarrow i$
5. $co \rightarrow$ implies not $sb$
6. $fb \rightarrow$ implies $\neg\neg\rightarrow i$
7. $ct \rightarrow$ implies not $fb$

Seven others, I1 through 7I, are obtained by replacing each relation by its inverse in 1 through 7.

**Proof:**

(1) and (2) follow directly from Axiom A2.

(2) $A < B \implies A \rightarrow C \rightarrow B \implies A \rightarrow C & B \neg\neg\neg\neg\neg\neg\neg\rightarrow C$ (Axiom A2)

$\implies B > A \implies A > iB$. 

(4) \( A \rightarrow B \implies A \rightarrow C \rightarrow B \), so \( A \rightarrow C \rightarrow B \rightarrow C \) (Axiom A2) \( \implies B \rightarrow A \)
\( \implies A \rightarrow B \).
(5) \( co \implies \) not \( sl \) (by definition) \( \implies \) not \( sbi \) (by 4i).
(6) and (7) are similar to (4) and (5), respectively.

As an example, \( A > B \) implies nothing about the inverse relations, and hence the whole inverse tree of Figure 3 can be appended below the node \( (>) \) in Figure 2. On the other hand, the node \( (o) \) is equivalent to \( --> \& --> \& sb \& fb \), so applying implications 2, 4, and 6 asserts that only the part of the inverse tree which contains \( sl \& fl \) but not \( --> i \) can be grafted at the node, leaving only the tree shown in Figure 4. There are thus only three leaf relations involving \( OVERLAPS: ol' > i, olfoli, \) and \( ol'sui. \) Completing the tree TPTR in this manner yields 82 leaf nodes corresponding to the 82 possible irreducible (no strict subrelations) disjoint ways in which this model allows describing the temporal relationship of two events in an event complex.

Recalling that the choices at each level of the TPTR, in Figures 2 through 4, is between all inclusive and mutually exclusive alternatives, and by studying the table of compositions (Table 4), we may determine that these 82 relations, constituting the leaves of the TPTR, are the unique generators of \( LA^* \) satisfying that they are:

(1) \textit{Mutually exclusive:} no two events can be in more than one of these relationships to one another. (In other words, the conjunction of any two of them is the empty relation.)
(2) \textit{All inclusive:} every pair of events must be in one of these relationships to one another. (In other words, the disjunction of all of them is the universal relation, 1, or no information.)
(3) \textit{Atomic:} none of these relations can be written as a proper disjunction of other relations in \( LA^* \).

There are 64 different compositions of pairs of the eight basic arrows given by \( -->, ->, \), their inverses and the negations of these four. Twelve of them are the first six shown above and their inverses. A large number of the rest, such as \( ---> o --> \), give the universal relation (1 or no information). The remainder

![Figure 4. Bottom of TPTR starting with (o).](image-url)


give further, unnamed relations which can, nonetheless, be expressed as disjunctions of the 82 irreducible relations.

Corollary to the Proposition. If there exists a global time model for the given event complex, then the seven implications of the Proposition become seven if-and-only-if statements. In this case, the two trees of Figures 2 and 3 are, in fact, the same tree, and there are only 13 leaves in TPTR.

It is, of course, more than mere coincidence that there are just 13 leaves in the TPTR in the presence of a global clock. It will now be shown that in this case, the 13 irreducible relations obtained correspond semantically to their obvious counterpart in Allen’s approach. In order to do so, the definitions of relations 3 through 21 of Table III are mapped by the global time mapping μ to intervals on the real line, and the consequences of these definitions interpreted there.

Suppose there exists a global time mapping μ. Consider first the relations sb and fb:

\[ A \text{ sb } B \text{ if and only if } \exists C \text{ with } A \rightarrow C \rightarrow B \]  
(definition)

iff exists C and exists \( a \in \mu(A) \), \( c \in \mu(C) \) with

\[ a \leq c \text{ and for all } z \in \mu(C) \text{ and } b \in \mu(B) \text{ } z < b \text{ holds} \]

iff some \( a \in \mu(A) \) is less than all \( b \in \mu(B) \)

iff the start point of \( \mu(A) \) is less than the start point of \( \mu(B) \)

iff \( \mu(A) \) (< or m or o or f or d) \( \mu(B) \)  
(by Table I)

Similarly,

\[ A \text{ fb } B \text{ if and only if } \exists C \text{ with } A \rightarrow C \rightarrow B \]  
(definition)

iff exists C and for all \( a \in \mu(A) \), \( c \in \mu(C) \) \( a < c \) and

there exists \( z \in \mu(C) \) and \( b \in \mu(B) \) with \( z \leq b \)

iff every \( a \in \mu(A) \) is < some \( b \in \mu(B) \)

iff the end point of \( \mu(A) \) is < the end point of \( \mu(B) \)

iff \( \mu(A) \) (< or m or o or s or d) \( \mu(B) \)  
(by Table I)

Consider now the relations sl and ft:

\[ A \text{ sl } B \text{ if and only if } \exists C \text{ with } B \rightarrow C \text{ and } A \rightarrow C \rightarrow B \]  
(definition)

iff exists \( C \) and exists \( b \in \mu(B) \), \( c \in \mu(C) \) with

\[ b \leq c \text{ and no } a \in \mu(A) \text{ is } \leq \text{ any } z \in \mu(C) \]
TEMPORAL REASONING

iff exists \( b \in \mu(B) \) with \( b \leq \) all \( a \in \mu(A) \)
iff start point of \( \mu(B) \) \( \leq \) start point of \( \mu(A) \)
iff \( \mu(A) \) \((d \text{ or } f \text{ or } oi \text{ or } mi \text{ or } < i) \) \( \mu(B) \)

Similarly,

A \( fl \) B iff end point of \( \mu(A) \) > end point of \( \mu(B) \)
iff \( \mu(A) \) \((di \text{ or } si \text{ or } oi \text{ or } mi \text{ or } < i) \) \( \mu(B) \)

Since \( co \) and \( ct \) are defined in terms of the preceding four relations, it is an easy matter to see they correspond under \( \mu \) to coincident start points and coincident end points, respectively. We can now take up in turn the seven basic relations 9 through 12 and 15 through 17 of Table III. The remaining six—CONTAINS, SURPASSES, EXTENDS, PRE-EXTENDS, AFTER, and FOLLOWS—are, in light of the Corollary to Proposition 1, just the inverses of the preceding six (PROJECTS TO is self-inverse). The global time mapping, \( \mu \), preserves inverses, thereby completing the proof. For clarity, we shall use the full names of the relations when referring to events in the event complex and use Allen’s abbreviations for relations in the global time model on the real line.

1. A BEFORE B
   iff exists C with A \( \rightarrow \) C \( \rightarrow \) B (definition)
   iff \( \mu(A) \) \((< \text{ or } m) \) \( \mu(C) \) \((< \text{ or } m) \) \( \mu(B) \)
   (definition of global time model)
   iff \( \mu(A) < \mu(B) \) (Allen’s table)

2. A MEETS B
   iff A \( \rightarrow \) B and not A \( < \) B
   iff \( \mu(A) \) \((< \text{ or } m) \) \( \mu(B) \) and not \( \mu(A) < \mu(B) \)
   iff \( \mu(A) \) \( m \mu(B) \)

3. A OVERLAPS B
   iff A \( sb \) B and A \( fb \) B and A \( \rightarrow \) B
   iff \( \mu(A) \) \((< \text{ or } m \text{ or } o \text{ or } f \text{ or } di) \) \( \mu(B) \) and
   \( \mu(A) \) \((< \text{ or } m \text{ or } o \text{ or } s \text{ or } d) \) \( \mu(B) \) and
   not \( \mu(A) \) \((< \text{ or } m) \) \( \mu(B) \) (by analysis of \( sb \) and \( fb \))
   iff \( \mu(A) \) \( o \mu(B) \) (by atomicity of relations)

4. A DURING B
   iff A \( sl \) B and A \( fb \) B
   iff \( \mu(A) \) \((d \text{ or } f \text{ or } oi \text{ or } mi \text{ or } < i) \) \( \mu(B) \) and
   \( \mu(A) \) \((< \text{ or } m \text{ or } o \text{ or } s \text{ or } d) \) \( \mu(B) \)
   iff \( \mu(A) \) \( d \mu(B) \) (by atomicity of relations)

5. A STARTS B
   iff A \( co \) B and A \( fb \) B
   iff not (A \( sb \) B) and not (A \( sl \) B) and A \( fb \) B
   iff not \( \mu(A) \) \((< \text{ or } m \text{ or } o \text{ or } f \text{ or } di) \) \( \mu(B) \) and
   not \( \mu(A) \) \((< \text{ or } m \text{ or } o \text{ or } s \text{ or } d \text{ or } < i) \) \( \mu(B) \) and
   \( \mu(A) \) \((< \text{ or } m \text{ or } o \text{ or } s \text{ or } d) \) \( \mu(B) \)
   iff \( \mu(A) \) \( s \mu(B) \)
(6) $A$ FINISHES $B$
   iff $A \text{ ct } B$ and $A \text{ sl } B$
   iff $\mu(A) f \mu(B)$ by very similar reasoning

(7) $A$ PROJECTS TO $B$
   iff $A \text{ co } B$ and $A \text{ ct } B$, which by similar arguments
   rules out all possibilities except $\mu(A) = \mu(B)$.

For a temporal reasoning system to be able to argue about the temporal
relations, it can either maintain a table of possible compositions (e.g., the
transitivity table of Allen) or learn some general rules about combining
relations. For example, $A \circ B$ and $B \circ C$ implies that $A \circ C$, whereas $A \text{ su } B$ and $B \text{ su } C$ implies nothing about the relation of $A$ and $C$. The proposed system
maintains a partial table while learning the rules for the composition of conjunctions. Table IV shows the compositions of the eight intermediate relations (1–8)
of Table III. The complete table needed by the proposed system is 32 by 32,
containing Table IV plus the inverse relations and all the negations of these
relations.

For example, given that $A \circ B$ is in the network and $B \circ f \circ C$ is added, the
algorithm must calculate $A \text{ REL } C = A \circ (o \circ f) \circ C$. Figure 5 shows the calculation
of $o \circ f$, writing 1 for no information. Note that 1 & REL = REL for any relation
REL.

V. KNOWLEDGE MAINTENANCE

Allen gives an algorithm for propagating new temporal information
throughout the network of related time intervals by means of a mechanism
similar to Watz's constraint propagation algorithm. Whenever new information
is added in the form of a new node (interval) or new relations between
existing nodes, the transitivity table is used to combine this information with
that of neighboring links to further constrain the possible link labels.

![Diagram](image-url)

\[ o \circ f = (\leftrightarrow & \rightarrow \rightarrow & \rightarrow \rightarrow \rightarrow & \rightarrow \rightarrow \rightarrow & \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow 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The method presented in this article is essentially the same as the scheme just described, but modification to the logic and to the stored information is necessary for efficiency due to the large number of relations possible in an event complex. Although any two events A and B must be related in exactly 1 of the 82 ways given by the leaves of the tree of possible temporal relations (TPTP) described in the previous section, in the presence of incomplete knowledge such as that represented by "A starts later than B," this model yields $2^{82}$ different possible relations indicated by all possible subsets of the 82 leaves. Greater economy of expression is gained by thinking of the possible relations between two events, A and B, as a subset of the paths of the TPTP. Once again, the nodes of the TPTP are interpreted to represent the conjunction of all the conditions given on the path from the root to that node. For example, the leaf node (cnff) corresponds to

$$\neg \neg \& \neg \neg \& sb \& fl \& \rightarrow \& i \& \rightarrow \& i \& sli \& cti.$$

Consequently, if it is only known that A fl B, then this corresponds to the set of paths from the root ending in an arc labeled fl in the TPTP, of which there happen to be three. It must then be assumed that any relations represented by the paths containing fl are logically possible, giving in this case 24 irreducible relations (corresponding to 24 leaves).

Thus, the temporal representation scheme consists of the network of existing nodes, with each link named by the disjunction of precisely those labels which correspond to the highest levels (closest to the root) in the (TPTP) tree such that all the relations in the subtree below could be true of that link. Besides the network itself, the system stores the TPTP; a list, LPT, of all possible terms (such as sb, co, and sui), and for each, a list of pointers to all occurrences of that term in the TPTP; and the table of intermediate composites, TIC, partially given in this work, Table IV.

Whenever a new temporal relation is to be added to the network, it is first resolved with any pre-existing relations for the same link. (See the algorithm in Fig. 6.) For example, if fb is the label on a link from A to B, the addition of sb is interpreted to mean A fl B & A sb B. Tracking down the tree those paths containing both relations yields o as the new labeling. If, to take a slightly different case, ffl were added to a link labeled sb, the result would have to be stored as olsul, pesul, cnlul because there is no higher node in the tree which has precisely this set of leaves in its subtree. (Notice that the set of labels on a link are in disjunction, whereas adding a new label is taken as a conjunction with pre-existing ones.) If the resulting set of labels is empty, an exception is reported indicating an inconsistency.

Next, the new information is propagated to neighboring links using the TIC. A REL1 B REL2 C becomes A REL1 \& REL2 B. If REL1 and REL2 are in the TIC, then REL1 \& REL2 is calculated and added to the relations on the link from A to C, which must then be placed on a queue of links to be evaluated as in the preceding paragraph. This must be repeated for every pair of relations satisfying A REL1 B REL2 C before moving to the next link. If, however, the relations are not in the TIC, then they must be decomposed into the conjunc-
While ToDo not empty Do
  Begin
    Get (A,B) from ToDo;
    New (A,B) := \{ r \& s : r \in R(A,B) and s \in N(A,B) \};
    If New (A,B) \neq N(A,B) (* Label set has changed *)
    Then
      Begin
        N(A,B) := New(A,B);
        For every node C with N(B,C) not empty Do
          Begin
            R(A,C) := \{ r \circ s : r \in N(A,B) and s \in N(B,C) \};
            If R(A,C) \neq N(A,C)
              Then put (A,C) on ToDo;
          End;
        End;
      End;
    End;
  End;

Figure 6. Temporal update algorithm.

...tion of others along the path of the TPTR leading to each and the compositions of the components found in the TIC. The general rule, for relations r1, r2, r3, and r4 is:

\[(r1 \& r2) \circ (r3 \& r4) = (r1 \circ r3) \& (r1 \circ r4) \& (r2 \circ r3) \& (r2 \circ r4).\]

The algorithm of Figure 6 describes what is done when a new relation, \( r \), is added to the network. Let (A, B) represent an event pair from the event complex, \( N(A, B) \) the set of labels (temporal relations which hold disjunctively for \( (A, B) \) on the link \( (A, B) \) in the networks, \( R(A, B) \) any new relations to be added (conjunctively) to the link, and ToDo a queue of links to be processed. To add A r B to the network, \( R(A, B) \) is set to \( r \) and \( (A, B) \) is placed on the empty queue.

Recall that the calculation of \( r \circ s \) in the inner loops of this algorithm requires tracing up the TPTR unless \( r \) and \( s \) appear directly in the TIC. It is also assumed that \( N(A, B) \) is left in the "reduced" form each time by maintaining in the label sets only the relations which are deepest along any path containing more than one.

The complexity of the foregoing algorithm is the same as that given in Allen\(^{13}\) polynomial time. If table look-up is applied with an 82 by 82 table of compositions, the new algorithm differs little from the previous one except for heavy fixed overhead for the searches. When the compositions from a smaller set are calculated, the look-up is quick, but more look-ups must take place and the TPTR must be traversed. Being a shallow, fixed tree, that time is always less than some fixed worst case; therefore, the time is still polynomial. Al-
though we have carried out a partial implementation with an efficient coding scheme for the TPTR, further analysis and testing for more exact measurements remains.

Just as with the algorithm for the linear time model, the algorithm presented is sound but not complete: any relations which are consistent with the given information will appear in the resulting network, but relations may be represented which, in reality, are not possible. For consistency checking applications, this indicates that if the algorithm finds an inconsistency, the information is indeed inconsistent; however, not all inconsistencies will be detected.

VI. CONCLUSION

Although considerable research effort has been focused on the creation of new logics and the adoption of existing logical methods for temporal reasoning, little progress has been made in AI toward the temporal description of truly concurrent events. The model and methods advanced in this article create both a new descriptive language for imprecise or incomplete temporal information as well as a representation and inferencing mechanism for the manipulation of this information. Whereas the formal temporal model is based on the work of Lamport, the implementation stems from the work of Allen.

The algorithm and data structures produce an inferencing system which maintains a network of temporal information (with reasonable efficiency) to operate in a relativistic space–time environment, or in the description of a distributed system. The temporal network serves as the basis for a system able to assemble a total picture in space–time from scraps of imprecise information. Given known constraints on the behavior of such processes, the temporal system is able to draw new conclusions and answer many questions about possible time-dependent events.

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