PREFERENCE CONSISTENCY IN MULTIATTRIBUTE DECISION MAKING

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ABSTRACT

A number of approaches for multiattribute selection decisions exist, each with certain advantages and disadvantages. One method that has recently been developed, called the Hypothetical Equivalents and Inequivalents Method (HEIM) supports a decision maker (DM) by implicitly determining the importances a DM places on attributes using a series of simple preference statements. In this and other multiattribute selection methods, establishing consistent preferences is critical in order for a DM to be confident in their decision and its validity. In this paper a general consistency check denoted as the Preference Consistency Check (PCC) is presented that ensures a consistent preference structure for a given DM. The PCC is demonstrated as part of the HEIM method, but is generalizable to any cardinal or ordinal preference structures. These structures are important in making selection decisions in engineering design including selecting design concepts, materials, manufacturing processes, and configurations, among others. The effectiveness of the method is demonstrated and the need for consistent preferences is illustrated using a product selection case study where the decision maker expresses inconsistent preferences.

1. INTRODUCTION

There are typically tradeoffs in decision making. Product design requires making a series of tradeoff decisions, including the rigorous evaluation and comparison of design alternatives using multiple, conflicting design criteria or attributes. We can be certain that no one alternative will be best in every attribute. Therefore, how to make the “best” decision when choosing from among a set of alternatives in a design process has been a common problem in research and application in product design and development. While in conceptual design, the objective may be to identify a set of promising concepts, the discussion and context of this work focuses on the decisions in design where a single concept must be selected.

A number of methods have been developed to support this type of decision by capturing and quantifying decision maker preferences, such as the Analytical Hierarchy Process [1], Utility Theory [2], Decision-Based Design [3], and Conjoint Analysis [4]. In general, the multiattribute decision problem can be formulated as follows:

Choose an alternative \( i \),

\[
\text{Maximize } V(j) = \sum_{i=1}^{n} w_i a_{ij},
\]

Subject to \( \sum_{i=1}^{n} w_i = 1 \)

where \( V(j) \) is the value function for alternative \( j \), \( w_i \) is the weight for attribute \( i \), and \( a_{ij} \) is the normalized rating of attribute \( i \) for alternative \( j \). While the \( L_1 \)-norm aggregation function is shown in Eqn. (1), other forms of the aggregation function have also been used [5], including the \( L_2 \)-norm and a parameterized family of aggregations, \( P_s \) [6]. There are many ways to implement and solve this formulation. Most methods focus on formulating the attribute weights, \( w_i \) and/or the alternative scores, \( a_{ij} \), indirectly or directly from the decision maker’s preferences. In new product development, a common challenge in a design process is how to capture the preferences of the end-users while also reflecting the interests of the designer(s) and producer(s). Typically, preferences of end-users are multidimensional and multiattribute in nature. If companies fail to satisfy the preferences of the end-user, the product’s potential in the marketplace will be severely limited. In this paper, we focus on the soundness and consistency of the process of making these decisions. According to Coombs’s condition, when six alternatives are ranked using multiple attributes, the process used to make the decision influences the
outcome over 97% of the time [7, 8]. Since the process by which the decision is made plays such an important role, establishing a theoretically sound decision process is paramount to identifying the correct outcome.

There are a number of approaches to making these general multiattribute selection decisions. Many of the possible methods and their theoretical or empirical limitations are discussed in [9]. Only a subset of the methods and limitations which are relevant to this work are presented in this paper.

The pairwise comparison method takes two alternatives at a time and compares them to each other, in a tournament type of approach until one winning alternative is identified. A pairwise approach is used in the Analytic Hierarchy Process to find relative importances among attributes [1]. Adaptations of AHP and other pair-wise methods are widely used to obtain relative attribute importances [10], to select from competing alternatives [11, 12] and to aggregate individual preferences [13, 14]. There are two fundamental limitations in using pairwise comparisons to make these decisions. First, they ignore the true value functions (also known as utility functions, preference functions, or worth functions) of a decision maker, which are many times nonlinear. Second, when comparing they do not directly account for the relative importance of the attributes. Because of these limitations, there exists some level of uncertainty when evaluating pairs of alternatives. This uncertainty is studied in the context of preference consistency in [15]. While some details exist regarding the theoretical problems with pairwise comparisons [7, 16-17], under certain assumptions, some adaptations of pairwise comparisons can provide effective and practically useful results [18].

Rankings are commonly used to rank order a set of alternatives. U.S. News and World Report annually ranks colleges based upon a number of attributes [18]. The NCAA athletic polls are based on a ranking system. Compared with standard pairwise methods, ranking methods are slightly more elaborate. However, as shown in [20, 22] the ranking procedure violates the Independence of Irrelevant Alternatives (IIA) principle, which states that the option chosen should not be influenced by irrelevant alternatives or clear non-contenders. Ranking approaches are prone to rank reversals when alternatives are dropped from consideration. However, it is noted in [21] that the probability and impact of rank reversals when using ranking systems is quite low. Also, when rankings are used, it is difficult to ascertain the importance of the various attributes that were used to obtain the rankings.

In general, determining the relative importance of the attributes in a selection problem is largely an arbitrary process. This arbitrary process can create a number of complications in multiattribute decision making and optimization [23-26]. In previous work, we have developed an approach, called the Hypothetical Equivalents and Inequivalents Method (HEIM), to analytically determine these weights using a decision maker’s stated preferences over a set of hypothetical alternatives.

HEIM is similar to multiattribute approaches described in [2, 27] in that it uses stated equality preferences from the decision maker based on hypothetical alternatives. However, HEIM is unique because it accommodates inequality preference statements and is easily scalable to problems with many attributes because it avoids having to address preferential independence or reduction of dimensionality when there are three or more attributes. While the details of HEIM are given in [22], a brief summary is given here to put the remainder of the paper in proper context.

The basic premise of HEIM is to elicit preferences from a decision maker on pairs of hypothetical alternatives. Hypothetical alternatives have the same set of attributes as the actual alternatives, but differ in the combinations of attribute values. This is done to determine the attribute weights while avoiding any bias the decision maker may have towards the actual alternatives. The “equivalents” part of the method allows a decision maker to make statements like “hypothetical alternatives A and B are equivalent in value to me.” By making this kind of statement, a decision maker is identifying an indifference relationship between A and B. However, finding hypothetical equivalents that are exactly of equivalent value to a decision maker, or “indifference points”, can be a challenging and time-consuming task [28], specifically in the context of constructing utility functions. Therefore, HEIM also accommodates inequivalents in the form of stated preferences such as “I prefer hypothetical alternative A over B.” When a preference is stated, by either equivalence or inequivalence, a constraint is formulated and an optimization problem is constructed to solve for the attribute weights. The weights are solved by formulating the following optimization problem,

\[
\text{Minimize } f(x) = \left(1 - \sum_{i=1}^{n} w_i \right)^2 \tag{2}
\]

Subject to: 

\[
h(x) = 0 \\
g(x) \leq 0
\]

where the objective function ensures that the sum of the weights is equal to one. \(X\) is the vector of attribute weights, \(n\) is the number of attributes, and \(w_i\) is the weight of attribute \(i\).

The constraints are based on a set of stated preferences from the decision maker. The equality constraints are developed based on the stated preference of “I prefer alternatives A and B equally.” In other words, the value of these alternatives is equal, giving the following equality constraint,

\[
h : V(A) = V(B) \text{ or } h : V(A) - V(B) = 0 \tag{3}
\]

The value of an alternative can take a number of forms, including the additive form shown in Eqn. (1), or other forms [5]. We use the additive form in this paper because of its prominence in engineering design applications. The inequality...
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these inconsistencies was “stochastic choice”, or in other
rational decision maker and the case described above may be
rare. However, in [29], eighty decision makers were asked the
100 pairwise comparison questions on 2 separate occasions
with 3-5 days of separation and the consistency rates for each
person ranged from 60-90%. The respondents were certainly
not answering the questions randomly, else the consistency rate
would have been 50%. In another study [30], subjects were
asked the same 42 questions twice, with the second time
coming immediately after the first time. Consistency rates were
well under 100%. The economic choice theory
provide cardinal comparisons of alternatives. HEIM elicits
pairwise comparisons to compare attributes. Ranking methods
identify a preferred alternative with confidence. In this section,
a Preference Consistency Check (PCC) is developed that can be
applied to any ordinal or cardinal set of preferences. In Section
3, the PCC will be applied to HEIM to illustrate its usefulness.

2.1 Fundamental Model
For simplicity and without any loss of generality, we will use a
simple two-alternative preference to illustrate the development
of the PCC. Assume that the DM prefers alternative $B$ over $C$. Using Eqn. (3), this preference can be expressed as shown in
Eqn. (5).

$$V(C) < V(B)$$

$$\sum_{i=1}^{n} w_i a_{iC} < \sum_{i=1}^{n} w_i a_{iB}$$

(5)

Since the value of alternative $B$ is larger than the value of alternative $C$, in order to make the alternatives of equal value, some quantity of value must be added to $C$. This addition of a unitless added value converts Eqn. (5) into an equality where the difference in value between alternative $B$ and $C$ is expressed through the term $Vadd_j$ where $j$ specifies the lower valued alternative, as shown in Eqn. (6).

$$\sum_{i=1}^{n} w_i a_{iC} + Vadd_C = \sum_{i=1}^{n} w_i a_{iB}$$

(6)

According to the preference structure of the DM $Vadd_C$ as defined has to be positive which is critical in the development of the consistency check. Since there are $n$ attributes in this selection problem, the increase of value of alternative $C$ can come from an increase in any of the attributes or some combination of the attributes. We assume that the additional value can be expressed by an increase in only one attribute to simplify the derivation and approach. This additional value added to alternative $j$, $Vadd_j$, can be expressed by an increase in attribute $i$ multiplied by the attribute weight $w_i$ as shown in Eqn. (7). This increase in the attribute is denoted as $k_{ij}$, or the consistency indicator.

$$Vadd_j = k_{ij} \cdot w_i$$

(7)

For the example in Eqn. (6), this additional value $Vadd_C$ is then expressed by an increase in attribute $i$, $k_{iC}$, multiplied by the attribute weight $w_i$ and shown in Eqn. (8).

$$\sum_{i=1}^{n} w_i a_{iC} + k_{iC} \cdot w_i = \sum_{i=1}^{n} w_i a_{iB}$$

(8)

Since $Vadd_C > 0$, and by definition all attribute weights are non-negative, $k_{iC}$ must also be positive. If $k_{iC}$ was negative, this

2. THE PREFERENCE CONSISTENCY CHECK (PCC)
As demonstrated in the previous section, a number of
multiattribute decision methods are based on evaluating the
relative value of alternatives or attributes. Pairwise
comparisons compare two alternatives at a time. AHP uses
pairwise comparisons to compare attributes. Ranking methods
provide cardinal comparisons of alternatives. HEIM elicits
preferences from a decision maker on hypothetical alternatives.
Each in these methods, consistency is critical to being able to
identify a preferred alternative with confidence. In this section,
a Preference Consistency Check (PCC) is developed that can be
applied to any ordinal or cardinal set of preferences. In Section
3, the PCC will be applied to HEIM to illustrate its usefulness.

An important facet of the HEIM approach and other decision
making approaches is the assumption of consistent preferences on the behalf of the decision maker. For instance, if a decision maker states that they prefer $A$ over $B$, $B$ over $C$, and $C$ over $A$, they are not consistent, since the preferences structure would be $A > B > C > A$ where “$>$” means “preferred to”. This structure would lead to the following set of constraints:

$$V(A) > V(B) \quad V(B) > V(A)$$

$$V(A) > V(C) \quad V(C) > V(A)$$

$$V(B) > V(C)$$

Since the first two sets of constraints are inconsistent, this structure leaves most decision making approaches in a situation where no solution can be found and no decision can be made. It may appear that developing a set of inconsistent preferences would be rather difficult for a rational decision maker and the case described above may be rare. However, in [29], eighty decision makers were asked the 100 pairwise comparison questions on 2 separate occasions with 3-5 days of separation and the consistency rates for each person ranged from 60-90%. The respondents were certainly not answering the questions randomly, else the consistency rate would have been 50%. In another study [30], subjects were asked the same 42 questions twice, with the second time coming immediately after the first time. Consistency rates were well under 100%. The economic choice theory
researchers concluded in [31] that the most likely source of these inconsistencies was “stochastic choice”, or in other words, mistakes on the part of the decision makers. Identifying these mistakes, or inconsistencies, and correcting them is one of the primary objectives of the work in this paper.

In the method described in [32], a least-distance approximation method using pairwise preference information, attempts to account for inconsistent preferences by introducing "slack" variables to represent the inconsistency. The inconsistency is minimized, however, it may not be eliminated. We take a different approach in this paper and develop a method that identifies and eliminates the inconsistency before a decision is made. In the next section, the fundamentals of the preference consistency check are presented. Then, the approach is applied to a multiattribute selection problem where an actual professional construction contractor is used as the decision maker to exercise the consistency check procedure.

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would indicate that \( \text{Vadd}_c < 0 \), and subsequently, \( V(C) > V(B) \), which violates the DM stated preference of alternative \( B \) over \( C \). Therefore, determining the sign of the \( k_{ij} \) consistency indicator terms is the focus of the remaining part of the derivation.

2.2 Selection of \( k_{ij} \)

To determine \( k_{ij} \), one attribute must be selected. Since \( k_{ij} \) indicates an increase in an attribute for a given alternative, the selected attribute should not already have a high satisfaction. As a general rule, the attribute with the lowest satisfaction in the lower valued alternative should be chosen as the one to determine \( k_{ij} \). If there are two attributes with the same value, then either one can be chosen. In the general problem of Eqn. (8), assume that attribute 1 has the lowest value and is chosen as the attribute to add value to for alternative \( C \). Eqn. (8) can now be written as:

\[
\sum_{i=1}^{n} w_i a_{ic} + k_{1c} \cdot w_1 = \sum_{i=1}^{n} w_i a_{ib}
\]

(9)

Since the \( w_i \)'s are still unknown \( k_{1c} \) can not be directly solved for yet.

2.3 Determining the Sign of \( k_{ij} \)

The consistency indicator, \( k_{ij} \), is determined using the concept of Marginal Rate of Substitution (MRS) [33] which captures how much of one attribute the DM is willing to sacrifice to gain a specific amount of another attribute. The MRS, or \( \lambda_{ij} \), can be expressed in Eqn. (10), where the term \( \Delta a_{ij} \) represents the amount of attribute \( l \) the DM is willing to sacrifice in order to gain a amount of \( \Delta a_{ij} \) on the attribute \( i \) for alternative \( j \) (the negative sign is necessary because one of the \( \Delta a \) terms is always negative since it represents a sacrifice in one attribute for a gain in another). Since a DM usually has nonlinear value functions and MRS construction is typically done using the natural attribute units, we convert the MRS of attributes \( l \) and \( i \) for alternative \( j \) into normalized attribute ratings using Eqn. (10) [2]. From the value function in Eqn. (1), the ratio of partial derivatives can also be expressed as a ratio of only \( w_i \) and \( w_j \):

\[
\lambda_{ij} = -\frac{\Delta a_{ij}}{\Delta a_{ij}} = \frac{\partial V}{\partial a_{ij}} = \frac{w_i}{a_{ij}}
\]

(10)

These MRS’s can be used in Eqn. (9) to reduce the number of unknowns to one, \( k_{ij} \), by substituting the following terms on the left side of the equation (alternative \( C \)),

\[
w_2 = \frac{w_1}{\lambda_{12c}}, ..., w_3 = \frac{w_1}{\lambda_{13c}}
\]

and the following terms on the right side of the equation (alternative \( B \))

\[
w_2 = \frac{w_1}{\lambda_{12b}}, ..., w_3 = \frac{w_1}{\lambda_{13b}}
\]

(11)

(12)

giving Eqn. (13).

\[
a_{ic} w_i + a_{ic} \frac{w_i}{\lambda_{12c}} + ... + a_{ac} \frac{w_i}{\lambda_{14c}} + k_{1c} w_1 = a_{ib} w_i + a_{ib} \frac{w_i}{\lambda_{12b}} + ... + a_{ab} \frac{w_i}{\lambda_{13b}}
\]

(13)

Now \( w_1 \) can be canceled reducing Eqn. (13) to Eqn. (14) where the only unknown is \( k_{1c} \).

\[
k_{1c} = a_{ib} + a_{ib} \frac{1}{\lambda_{12b}} + ... + a_{ab} \frac{1}{\lambda_{13b}} - a_{ic} - a_{ic} \frac{1}{\lambda_{12c}} - ... - a_{ac} \frac{1}{\lambda_{13c}}
\]

(14)

After solving for \( k_{1c} \), there are three possible cases to consider.

\begin{itemize}
  \item If \( k_{1c} > 0 \), the DM is consistent since this indicates that the indicated lower valued alternatives is indeed lower valued.
  \item If \( k_{1c} < 0 \) it indicates the DM is not consistent and this preference statement is not correct since the indicated lower valued alternative is actually the higher valued alternative.
  \item If \( k_{1c} = 0 \), it indicates that the DM is not consistent and that the alternatives are actually of equal value. In this case, the preference state should be one of equality (an equality constraint).
\end{itemize}

When the preference statement is changed in the last two cases, it should be evaluated by the DM and then the PCC must be run again to ensure that the new preference has not created any other inconsistencies.

This conclusion is also based on the assumption that the value functions are correct and that the MRS evaluations are correct. There are some general guidelines to ensure that the MRS evaluations were performed correctly including:

1) \( \lambda_{ij} = 1/\lambda_{ij} \): If done correctly and consistently, the amount the DM is willing to sacrifice on attribute \( i \) for a gain in attribute \( l \), should be the reciprocal of the amount the DM is willing to sacrifice on attribute \( l \) for a gain in attribute \( i \).

2) If the DM can rank order the attributes and if attribute \( i \) is more important than attribute \( l \), then \( \lambda_{il} = w_i/w_l > 1 \). For all pairs of attributes where one is more important than another, this relationship should hold.

3) More formal evaluation and convergence routines can be used, as described in [34].

In the next section, the complete HEIM approach including the PCC method is demonstrated on a drill selection problem. A lead construction engineer from a local construction firm is used as the DM and to provide insight into the final selected drill and its implications.

3. DRILL SELECTION PROBLEM

3.1 Problem Setup

Although the PCC is discussed in this section in the context of the HEIM approach, it can be applied to any ordinal or cardinal set of preferences. To illustrate the application of the PCC, assume a decision maker has to select a drill from the following...
set of cordless drills shown in Table 1 [35]. The attributes of interest are the Number of Operations, the Price, and the Weight. A construction engineer is used as the decision maker.

Using a midvalue splitting technique [2], value functions for each attribute across each attribute range were developed with a manager/owner of a construction contracting company as the primary DM and are shown in Figure 1. As part of the HEIM approach, a set of hypothetical alternatives (hypothetical drills) are developed and are shown in Table 2. These hypothetical alternatives are developed in such a way to sample the complete attribute space in an ordered, balanced way using an L9 Orthogonal Array [36]. The attribute space is defined by the maximum and minimum attribute values of the actual alternatives in Table 1 (350-630 for no. of operations, $70-$100 for price, and 5.5 lb-9.4 lb for weight). The L9 array results in nine alternatives with the normalized attribute combinations, where 0 indicates the lowest value for the attribute and 1 indicates the highest value for the attribute, shown in Table 2. The last column represents the value of the alternative using the additive value function, as given in Eqn. (1).

<table>
<thead>
<tr>
<th>Drill</th>
<th>No. of oper.</th>
<th>Price [$]</th>
<th>Weight [lb]</th>
<th>Value of alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>350</td>
<td>70</td>
<td>6</td>
<td>0.5w₁+0.5w₂+0.5w₃</td>
</tr>
<tr>
<td>#2</td>
<td>370</td>
<td>80</td>
<td>5.7</td>
<td>w₁+w₂+0.5w₃</td>
</tr>
<tr>
<td>#3</td>
<td>380</td>
<td>85</td>
<td>5.5</td>
<td>w₁+w₂</td>
</tr>
<tr>
<td>#4</td>
<td>400</td>
<td>72</td>
<td>6.5</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#5</td>
<td>420</td>
<td>82</td>
<td>6.1</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#6</td>
<td>430</td>
<td>88</td>
<td>5.8</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#7</td>
<td>450</td>
<td>74</td>
<td>6.9</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#8</td>
<td>470</td>
<td>85</td>
<td>6.5</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#9</td>
<td>480</td>
<td>91</td>
<td>6.1</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#10</td>
<td>500</td>
<td>79</td>
<td>7.2</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#11</td>
<td>520</td>
<td>89</td>
<td>6.9</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#12</td>
<td>530</td>
<td>94</td>
<td>9.4</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#13</td>
<td>550</td>
<td>84</td>
<td>7.5</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#14</td>
<td>570</td>
<td>93</td>
<td>7.2</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#15</td>
<td>580</td>
<td>97</td>
<td>6.7</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#16</td>
<td>600</td>
<td>90</td>
<td>7.8</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#17</td>
<td>620</td>
<td>98</td>
<td>7.5</td>
<td>w₁+w₂+w₃</td>
</tr>
<tr>
<td>#18</td>
<td>630</td>
<td>100</td>
<td>7</td>
<td>w₁+w₂+w₃</td>
</tr>
</tbody>
</table>

Table 2 Hypothetical Alternatives

The hypothetical alternatives of Table 2 are converted into natural unit alternatives using the value functions in Figure 1. For instance, for hypothetical alternative D, the number of operations is 0, indicating the lowest value for the number of operations, or 350. The price is set at 0.5, and from the middle graph of Figure 1, this corresponds to a price of $90. The weight is set at 0.5 and from the graph on the right-hand side of Figure 1, this corresponds to a weight of 7 lb. The other hypothetical alternatives are developed in a similar fashion and are partitioned into three sets of three alternatives in Table 3, to make it easier for the DM to express preferences over a set of three alternatives instead of all nine alternatives.

An important issue to consider when partitioning the alternatives is to make sure that useful comparisons can be made between alternatives. For instance, it would not be useful to develop a subset where one alternative is clearly better than the other alternatives across all of the alternatives. Different possible partitions are certainly possible, but as long as decision maker consistency is maintained, different partitions should lead to the same solution. The partitioning issue is expanded upon in [36] and continues to be an area of current study.

3.2 Developing Preference Structures

In the next step the preference statements of the DM for each set have to be determined which are given in Table 4. The preference statements B ≻ A and C ≻ A do not add any valuable
information, since alternative A is dominated by every other alternative. Therefore, the statements with alternative A are not included in the analysis. From Table 4 five unique preference statements can be derived which are listed in Table 5 including their representation in terms of values and the resulting inequality constraints. All the other constraints from Table 4 are redundant to those listed in Table 5.

Table 4 Preference Statements for Each Set

<table>
<thead>
<tr>
<th>Preference statements</th>
<th>Preference statements in value levels</th>
<th>Inequality constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>C &lt; B</td>
<td>( w_f + w_y + 0.5 w_z &lt; 0 )</td>
<td>( 0.5 w_f + 0.5 w_y - 0.5 w_z &lt; 0 )</td>
</tr>
<tr>
<td>D &lt; E</td>
<td>( 0.5 w_f + 0.5 w_y &lt; 0 )</td>
<td>( -0.5 w_f - 0.5 w_y + 0.5 w_z &lt; 0 )</td>
</tr>
<tr>
<td>E &lt; F</td>
<td>( 0.5 w_f + w_y &lt; w_f + 0.5 w_z )</td>
<td>( -0.5 w_f + w_y - w_z &lt; 0 )</td>
</tr>
<tr>
<td>G &lt; I</td>
<td>( w_f + w_y &lt; w_f + 0.5 w_z )</td>
<td>( -w_f + 0.5 w_y - 0.5 w_z &lt; 0 )</td>
</tr>
<tr>
<td>I &lt; H</td>
<td>( 0.5 w_f + 0.5 w_y &lt; 0 )</td>
<td>( 0.5 w_f + 0.5 w_y - 0.5 w_z &lt; 0 )</td>
</tr>
</tbody>
</table>

Table 5 Different Representations of the Preferences

3.3 Application of the PCC

From the value equations in Table 5 (second column) the \( k_{ij} \)’s shown in Table 6 are chosen to represent the value increases based on the worse attribute of the lower-valued alternative. For example, for the preference, C < B, the first and second attributes for alternative C are already at their highest satisfaction level (coefficient of 1 for \( w_f \) and \( w_y \)), and so attribute three is chosen (\( k_{13C} \)).

In the next step the MRS’s have to be determined to calculate the consistency indicators in Table 6. The DM was asked for a series of MRS values, similar to the example form shown in Table 7. In this form, the DM is asked to determine for alternative B how many operations he is willing to sacrifice for a 1 lb decrease in weight, and how much more he is willing to pay for the same 1 lb decrease in weight. Note that this is a cross-attribute preference assessment, as opposed to the value functions shown in Figure 1 which only capture preferences on each attribute individually. This cross-attribute tradeoff assessment is critical to determine the consistency of the statements in Table 4 since the DM implicitly processes attribute tradeoffs when stating their preferences over the hypothetical alternatives. Using the value functions of Figure 1, and Eqn. (10), the MRS responses are converted to unitless MRS values as shown in Table 8.

Using these MRS values in a similar manner to the approach outlined in Section 2.3, the consistency indicators are solved for and shown in Table 9.

Table 6 The Consistency Indicators, \( k_{ij} \)’s, for Each Preference

Table 7 Example MRS Form

Table 8 MRS Values for Each Alternative

Table 9 Initial Values of \( k_{ij} \)
From the data in Table 9 it can be concluded that the preference statements $C < B$ and $I < H$ are incorrect, since they are negative. This is not surprising, since the constraints for these preferences in Table 9 are exactly the same. If one were wrong, the other certainly must also be wrong. This means that these two preference statements have to be switched to $C > B$ and $I > H$. With these new preferences, two new consistency indicators are calculated using the MRS’s from Table 8 ($k_{1B} = 0.47$, $k_{1H} = 0.42$) which indicates that the preference statements $C > B$ and $I > H$ are correct. Since the original preference statement, $H > I > G$ from Table 4 has been corrected to $I > H > G$ it also has to be examined if the preference statement $H > G$ is correct. Using the same procedure, it was determined that this statement was incorrect as well and was switched to $H < G$. At this point all the consistency indicators are positive as shown in Table 10, indicating a set of consistent preferences.

<table>
<thead>
<tr>
<th>Preference statements</th>
<th>$k_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C &lt; B$</td>
<td>$k_{3C} = 0.47$</td>
</tr>
<tr>
<td>$D &lt; E$</td>
<td>$k_{1D} = 0.43$</td>
</tr>
<tr>
<td>$E &lt; F$</td>
<td>$k_{3E} = 0.97$</td>
</tr>
<tr>
<td>$G &lt; I$</td>
<td>$k_{1G} = 0.09$</td>
</tr>
<tr>
<td>$I &lt; H$</td>
<td>$k_{4I} = 0.63$</td>
</tr>
</tbody>
</table>

Table 10 Final Values of the $k_{ij}$’s

While it may seem that like these preference changes are being made without input or confirmation from the DM, this is not the case. The new preferences were developed using the input from the DM and were confirmed by the DM when we showed him the incorrect preferences. In fact, he stated that he was not quite sure why he initially stated his preferences that way, but agreed with the changes and corrections. This provides anecdotal evidence to the research studies on the mistakes that rational decision makers can make as discussed in [29-31]. The final preference statements and corresponding inequality constraints are given in Table 11.

<table>
<thead>
<tr>
<th>Preference statements</th>
<th>Inequality constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B &lt; C$</td>
<td>$-0.5w_1 - 0.5w_2 + 0.5w_3 &lt; 0$</td>
</tr>
<tr>
<td>$D &lt; E$</td>
<td>$-0.5w_1 - 0.5w_2 + 0.5w_3 &lt; 0$</td>
</tr>
<tr>
<td>$E &lt; F$</td>
<td>$-0.5w_1 + w_2 - w_3 &lt; 0$</td>
</tr>
<tr>
<td>$G &lt; I$</td>
<td>$-w_2 + 0.5w_1 + w_3 &lt; 0$</td>
</tr>
<tr>
<td>$H &lt; G$</td>
<td>$0.5w_2 - w_3 - 0.5w_1 &lt; 0$</td>
</tr>
</tbody>
</table>

Table 11 Final Preference Structure and Inequality Constraints

3.4 Identifying the Best Alternative

Using the inequality constraints from Table 11 the following optimization formulation in Eqn. (15) can be set up in order to determine the attribute weights and the most preferred drill for the DM. There are only four inequality constraints as the ones resulting from the preference statements $B < C$ and $E < D$ are redundant.

$$\text{Min} \quad (1 - (w_1 + w_2 + w_3))^2$$

$$\text{s.t.:} \quad g_1 : -0.5w_1 - 0.5w_2 + 0.5w_3 < 0$$
$$g_2 : -0.5w_1 + w_2 - w_3 < 0$$
$$g_3 : w_1 + 0.5w_2 + w_3 < 0$$
$$g_4 : 0.5w_2 - w_3 - 0.5w_1 < 0$$
$$0 \leq w_1, w_2, w_3 \leq 1$$

In Eqn. (15), we are trying to find a set of weights that sum to one and that satisfy the constraints. However, there could exist multiple sets of weights that satisfy these criteria and are both feasible and optimal. In addition, these multiple sets of weights could result in a different drill being selected as the most preferred drill, as shown for other problems in another study [36]. This is not an advantageous consequence, as it leaves the DM without having identified a single, preferred choice. Therefore, the basic concepts of the PCC, that a quantifiable difference between two alternatives exists (Eqn. (6)), and that this difference can be formulated as an increase in a single attribute (Eqn. (7)), are used to identify the single most preferred drill, as described in the following discussion.

In order to determine all the possible sets of feasible and optimal weights and their corresponding winnings drills, many combinations of weights are generated in the feasible space, as defined by the preference constraints and the constraint that the sum of the weights must equal one. A fine grid technique is used to generate the points. For each combination of weights, the value functions of all 18 drills are calculated and the winning drill is determined. As a result, unless all feasible and optimal combination of weights result in the same winning drill, there will be more than one winning drill. In Figure 2, the feasible and optimal design space is shown where the large blue triangle represents the $w_1 + w_2 + w_3 = 1$ plane and all the points lie on this plane (the axes are shifted slightly to aid in visualizing the plot). The smaller triangular region of colored points represents the feasible design space, where only three of the four inequality constraints are active ($g_2$, $g_3$ and $g_4$). The five colored regions in the feasible space correspond to five different drills that have the highest values using the combinations of weights in that region.

At this point, the DM has reduced the number of possible alternatives from eighteen to five, but a single best drill has not been determined. However, from the boundaries of the feasible region the max and min values of each weight can be found using the set of generated weight combinations. These ranges are shown in Table 2. While these ranges are visually apparent from the feasible space in Figure 2, for problems with more than three attributes these ranges are simply found from the set of feasible weight combinations. Using the lower and upper bounds of the weights in Table 12, the $k_{ij}$ values of Table 10,
and Eqn. (7) the minimum and maximum value differences can be determined for each preference statement (shown in Table 13).

\[ V(D) < V(E) \]
\[ V(D) + V\text{add}_D = V(E) \]  \hspace{1cm} (16)
\[ V(E) - V(D) = V\text{add}_D \]
\[ V(E) - V(D) \geq V\text{add}_{D\text{min}} \]  \hspace{1cm} (17)
\[ V(D) - V(E) + V\text{add}_{D\text{min}} \leq 0 \]
\[ V(E) - V(D) \leq V\text{add}_{D\text{max}} \]  \hspace{1cm} (18)

These two new constraints are shown as \( g_{11} \) and \( g_{12} \), respectively, in Eqn. (19). The constraints, \( g_{2-g4} \), are also converted to two constraints each.

Figure 3(a) shows the winning alternatives within the feasible design space resulting from Eqn. (19). It can be recognized that the number of feasible winning solutions dropped from five to two. As there are still two feasible winning alternatives left the process is applied again. However this time the new lower and upper bounds of the weights resulting from Figure 3(a) are used to calculate the \( V\text{add}_{min} \) and \( V\text{add}_{max} \) values while the \( k_{ij} \) values remain the same. This leads to the feasible design space shown in Figure 3(b).

Using the maximum and minimum values of \( V\text{add} \), each constraint in Eqn. (15) can be converted into two more constraints, further reducing the feasible region. For instance, the constraint \( g_1 \) in Eqn. (15) corresponding to \( D \prec E \) is first converted into an equality as shown in Eqn. (16). Then using the \( V\text{add}_{min} \) and \( V\text{add}_{max} \) values in Table 13, the equality is converted into two inequality constraints, as shown in Eqns. (17-18). In Eqn. (17), the difference between alternatives \( E \) and \( D \) must be at least equal to or greater than the minimum value difference. In Eqn. (18), the difference between alternatives \( E \) and \( D \) must be at least equal to or less than the maximum value difference.

![Figure 2 Feasible Design Space](image)

**Table 12 Min. and Max. Weights**

<table>
<thead>
<tr>
<th>Pref. statement</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D \prec E )</td>
<td>0.4</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>( E \prec F )</td>
<td>0.67</td>
<td>0.4</td>
<td>0.48</td>
</tr>
</tbody>
</table>

**Table 13 Min. and Max. Value Differences**

Using the maximum and minimum values of \( V\text{add} \), each constraint in Eqn. (15) can be converted into two more constraints, further reducing the feasible region. For instance, the constraint \( g_1 \) in Eqn. (15) corresponding to \( D \prec E \) is first converted into an equality as shown in Eqn. (16). Then using the \( V\text{add}_{min} \) and \( V\text{add}_{max} \) values in Table 13, the equality is converted into two inequality constraints, as shown in Eqns. (17-18). In Eqn. (17), the difference between alternatives \( E \) and \( D \) must be at least equal to or greater than the minimum value difference. In Eqn. (18), the difference between alternatives \( E \) and \( D \) must be at least equal to or less than the maximum value difference.

![Figure 3 Feasible Design Space Reduction](image)

**a) Two Possible Alternatives**

**b) One Final Alternative**

At this point, there is only one feasible winning alternative and the attribute weights of the final solution are: \( w_1 = 0.46, w_2 = 0.23, w_3 = 0.31 \). From this solution, it is clearly seen that the number of operations (attribute) is most important to the DM.
followed by the weight (attribute 3) and the price (attribute 2). Using these weights, the most preferred drill from Table 1 for the DM is drill #18. This drill has the highest number of operations among all drills but it is also the most expensive while having a moderate weight. The construction engineer who acted as the DM confirmed that this drill is very attractive to him. He added that he typically buys expensive drills because they have the most capability and battery power (reflected in the number of operations). He only mildly considers the weight of the drill when making the decision, and affirmed that number of operations is always his most significant priority when choosing a drill. While the weight and price are not ignored, they are certainly not as important as the number of operations. This validates from an empirical perspective the set of weights found using this method.

4. OBSERVATIONS AND CONCLUSIONS

In this paper, an approach to multiattribute selection decision making is presented with an emphasis on ensuring consistency in decision maker’s preferences. Without consistent preferences, it is not only difficult to handle preference information in a formal manner, but it is also leaves the decision maker with very little support or confidence in their decision. Worse case, it may even leave the decision maker with conflicting decision support information that would lead to no discernable decision. Since being able to effectively make multiattribute decisions marked by tradeoffs between conflicting criteria is critical in many fields, including most notably product and systems design, it is beneficial to have a method that provides sound, consistent, effective decision support. The approach to maintaining consistency is based on simple preference statements, and marginal rates of substitution. The consistency model also is effective at ensuring that the HEIM method identifies one preferred solution. Some observations and conclusions can be made regarding the approach.

· Both in the case study of this paper and in multiple other applications of the approach with actual product design engineers (with problems involving laptop design, and small sedan vehicle design) the method effectively identified areas of inconsistency in the decision maker’s preferences. The decision maker in each case confirmed the corrections to the preferences. The method gives some insight into the source of decision maker mistakes, as defined and studied in [31], and helps to correct these mistakes.

· The decision maker is involved in the application of the method in a number of important steps. First, the decision maker must develop value functions for the attributes that are being considered in the decision. Then, he/she must provide stated preferences between pairs of hypothetical alternatives. Lastly, he/she must confirm any correction to the preferences and needs to ensure that the marginal rates of substitution are consistent and correct. Other than these important interactions with the decision maker, all other steps of the approach can be automated. An Excel program has been developed to automate the other steps and has been used with a number of case studies and over thirty decision makers to validate its usefulness and effectiveness. Current work is focused on integrating the program with other industrial decision support software programs.

· Expanding the approach to group decision making where consistency among group members is rarely present (or even desired) is the topic of current work. This paper focused on single decision maker problems, but its applicability to product design processes can be expanded with a sound consistency (or inconsistency) model for group decisions.

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REFERENCES


