Capacity Analysis of Subcarrier Pairing for OFDMA-Based DF Relay Link over Rayleigh Channel

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Abstract— In this paper we analyze the end-to-end capacity of subcarrier pairing strategies of OFDMA-based DF relay link over rayleigh fading channels. We compare fixed pairing strategy to optimal pairing strategy in the circumstance that the relay is fixed or mobile. The main contribution of this paper is deriving the closed-form expressions for average end-to-end SNR and capacity in these circumstances. The simulation result illustrates that the optimal pairing scheme can bring performance gain over fixed subcarrier pairing as expected.

Keywords- OFDMA; capacity; relay; subcarrier pairing

I. INTRODUCTION

The development of the wireless communications promotes the higher quality requirement of the users. Relayed transmission is becoming a promising technique that helps to extend coverage areas and enhances hotspot capacity in cellular networks[1]. Usually signal processing of the relays is divided into two types: One is called amplify-and-forward (AF) or non-regenerative relaying and the other is called decode-and-forward (DF) or regenerative relaying [2~4].

The orthogonal frequency division multiple access (OFDMA) relay network is defined as a broadband relay network where the node transmits data for different destination in parallel on different orthogonal subcarriers according to DF or AF protocol. In the network, the relay has to arrange which part of second-hop subcarriers are assigned for transmitting the data from the first-hop subcarriers to the destination. The subcarrier pairing strategy of the first-hop and the second-hop may greatly affect the end-to-end performance and capacity of the OFDMA-based relay link.

Subcarrier pairing selection in an OFDMA-based relay has been first mentioned in [5], where Hungarian algorithm was used for sub-optimal joint power allocation and subcarrier assignment. Subchannel assignment in a two-hop AF OFDM relay system was studied in [6] which concluded that a simple ranking of subcarrier will lead to the optimal assignment and presented full optimality proof for general AF relaying. In [7], the performance of the pairing strategy for a multi-user downlink OFDMA relaying system was studied by deriving closed-form expressions for average end-to-end signal-to-noise ratio (SNR) and capacity. The achievements of study above are all based on amplify-and-forward (AF) protocol. But for wireless relay-based communications the decode-and-forward (DF) protocol is more applicable. So in [8], the optimal pairing scheme in [6] was extended to 2-hop OFDMA-based DF relaying but the capacity analysis of the optimal pairing scheme was lacking.

In this paper we compare fixed pairing strategy to optimal pairing strategy in the circumstance that the relay is fixed or mobile over rayleigh fading channels and get the closed-form expressions for average end-to-end SNR and capacity. Then we simulate the performance of the two pairing strategy in these circumstances. The results illustrate the benefit of optimal pairing strategy compared to fixed subcarrier pairing strategy.

The paper is structured as follows. In Section II, we present the system model of the 2-hop OFDMA relay link. The expressions for average end-to-end SNR and capacity with two pairing strategy in different circumstances are derived in Section III, and the performance is simulated in Section IV. The section V concludes the paper.

II. SYSTEM MODEL

We consider a 2-hop OFDMA relay link that consists of a source node(SN), a relay node (RN) and a destination node(DN). In the multi-user access environment, the source node is the base station, the relay is fixed or mobile and the destination node may be an user mobile equipment.

The system is illustrated in Fig. 1. The relay operation allocates two orthogonal time channels of N subcarriers: one for S-R(source node to relay node) communication and one for R-D (relay node to destination node) communication.

![Figure 1 Schematic of a 2-hop OFDMA relay link](image)

We assume that the subcarriers number of source node and relay node allocated to each user is N , the total bandwidth allocated to each user is W, the wireless channel of each hop is frequency selective and the channel fading of each subcarrier is flat. The relay transmission scheme is as follow: In the first time slot, the source node sends the messages on the subcarriers to the relay node. In the second time slot, the data transmitted by the source node arrives at the relay ,then the...
relay node demodulates and decodes the received signal, recodes and remodulates the received messages on the subcarriers before send to the destination node.

We limit the total power of each hop to P, thus the transmitted power of each subcarrier is \( P/N \). We define \( X_{ss}(k) \), \( X_{sd}(k) \) as the transmitted signal on the kth subcarrier of the S-R link and the R-D link. The power of \( X_{ss}(k) \), \( X_{sd}(k) \) are normalized. Also we define \( Y_{ss}(k) \), \( Y_{sd}(k) \) as the received signal on the kth subcarrier of the relay node and destination node, the complex channel fading of the kth subcarrier is \( H_{ss}(k) \), \( H_{sd}(k) \) and the additive white gaussian noise is \( n_{ss}(k) \), \( n_{sd}(k) \). The variance of \( n_{ss}(k) \), \( n_{sd}(k) \) is \( \sigma_{ss}^2 = W \rho_{ss}/N \), \( \sigma_{sd}^2 = W \rho_{sd}/N \), \( \rho_{ss} \), \( \rho_{sd} \) is the power spectral density of S-R link and R-D link. Thus the expression of received signal on the kth subcarrier of relay node and destination node is given by

\[
Y_{ss}(k) = \frac{P}{N}H_{ss}(k)X_{ss}(k) + n_{ss}(k) \quad (1)
\]

\[
Y_{sd}(k) = \frac{P}{N}H_{sd}(k)X_{sd}(k) + n_{sd}(k) \quad (2)
\]

We define the receiver SNR on the kth subcarrier of relay node and destination node as \( \Gamma_{ss}(k) = \rho_{s} / (N \sigma_{ss}^2) \), \( \Gamma_{sd}(k) = \rho_{d} / (N \sigma_{sd}^2) \).

III. CAPACITY ANALYSIS OF THE SUBCARRIER PAIRING STRATEGY

In this section we will analyze the capacity of the subcarrier pairing strategy of the OFDMA-based decode-and-forward (DF) relay link.

We define the pairing strategy of the kth S-D link is \( <k,l> \) which means the data transmitted on the kth subcarrier of the source node and the lth subcarrier of the relay node. The end-to-end capacity of the kth S-D link is decided by the min capacity of the kth subcarrier of the S-R link and the lth subcarrier of the R-D link, that is

\[
C_{sd}(k) = \min \{ C_{ss}(k), C_{sd}(l) \} \quad (3)
\]

Thus the average S-D capacity of each user(\( \bar{C}_{sd} \)) is the sum of the average capacity of the kth S-D link(\( \bar{C}_{sd}(k) \)).

\[
\bar{C}_{sd} = \sum_{k=1}^{N} \bar{C}_{sd}(k) \quad (4)
\]

The relay of the link may be fixed or mobile, we will discuss the capacity expression in different circumstance separately.

A. fixed relay

Because the source node and the relay node are fixed, we assume that the channel of the S-R link remain static during the study time. The user node may mobile thus we consider channel of the R-D link time-varying rayleigh channel. In the multi-user access circumstance, we assume the channel gain of the subcarriers allocated to the users on R-D link are independent with each other, thus we know \( \Gamma_{sd}(k) \) is in exponential distribution and the probability density function (PDF) of it is as below,

\[
f_{\Gamma_{sd}(k)}(x) = \begin{cases} 1 \over \Gamma_{sd} - x & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (5)
\]

The cumulative distribution function (CDF) is also given by

\[
F_{\Gamma_{sd}(k)}(x) = \begin{cases} 1 - e^{-x/\Gamma_{sd}} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (6)
\]

\( \bar{\Gamma}_{sd} \) represents the average SNR on each subcarrier of the R-D link.

1) Fixed subcarriers pairing strategy

In this circumstance, the pairing strategy is \( <k,k> \) that is the data transmitted on the kth subcarrier of the source node is retransmitted on the kth subcarrier of the relay node. The expression of the end-to-end SNR of the kth relay link is \( z = \min \{ \Gamma_{ss}(k), \Gamma_{sd}(k) \} \).

We assume the PDF of the end-to-end SNR of the kth S-D link is \( f_{z}(z) \). It’s easy to know that when \( z < 0 \) or \( z > \Gamma_{ss}(k) \) , \( f_{z}(z) = 0 \), when \( 0 \leq z \leq \Gamma_{ss}(k) \),

\[
f_{z}(z) = \frac{1}{\Gamma_{ss}} e^{-z/\Gamma_{ss}} + e^{-z/\Gamma_{sd}} \delta(z - \Gamma_{ss}(k)) \quad (7)
\]

The expression of the capacity of the kth relay link is:

\[
C_{sd}(k) = \frac{W}{2N} \log_{2}(1 + z) \quad (8)
\]

Base on (8) we know that when \( \Gamma_{sd}(k) \leq \Gamma_{ss}(k) \), the capacity of the S-D link is determined by the SNR of the R-D link and is variable, when \( \Gamma_{sd}(k) > \Gamma_{ss}(k) \), the capacity of the link is determined by the SNR of the S-R link and is constant.

We assume the probability density function of the capacity of the kth S-D link is \( \psi_{d}(x) \), thus the range of x of the kth S-D link is \( [0, (W/2N) \log_{2}(1 + \Gamma_{ss}(k))] \).

By (5),(6),(7),(8) the \( \psi_{d}(x) \) is given by

\[
\psi_{d}(x) = f_{\Gamma_{sd}(k)}(x) \frac{1}{\Gamma_{ss}} e^{-x/\Gamma_{ss}} + (1 - f_{\Gamma_{sd}(k)}(\Gamma_{ss}(k))) \delta(x - \frac{W}{2N} \log_{2}(1 + \Gamma_{ss}(k)))
\]

\[
= \frac{1}{\Gamma_{ss}} e^{-x/\Gamma_{ss}} + e^{-x/\Gamma_{sd}} \delta(x - \frac{W}{2N} \log_{2}(1 + \Gamma_{ss}(k)))
\]

In (9), the \( \delta(*) \) presents the unit impulse function. Then the average capacity of the kth S-D link is given:
\( C_{sD}(k) = \frac{W}{2N} \log_2(1 + \gamma_{sD}(k)) \)

\[
C_{sD}(k) = \frac{1}{T_{RD}} \int_0^{x} x \psi_1(x)dx
\]
\[
= \frac{1}{T_{RD}} \int_0^{x} xe^{\frac{W}{2N} e^\frac{1}{T_{RD}} \log_2(1 + \gamma_{sD}(k))} dx + \frac{W}{2N} e^\frac{1}{T_{RD}} \log_2(1 + \gamma_{sD}(k))
\]
\[
(10)
\]

Then from (4),(10) we can get the average S-D capacity of each user in this circumstance.

2) Optimal subcarriers pairing strategy

In [8], the optimal pairing strategy of the subcarriers of OFDMA-based DF relay is proved. We state that the optimal pairing strategy of the subcarriers of source node is also given. From above it is obvious that the SNR \( \gamma_{sD}(k) \) of the kth largest sample is as below:

\[
\text{CDF of } \gamma_{sD}(k) = \int x \psi_1(x)dx
\]

\[
(13)
\]

From (7),(12),(13), the PDF of the capacity of the kth S-D link is given.

for \( x < 0 \) or \( x > \frac{W}{2N} \log_2(1 + \gamma_{sD}(k)) \),

\[
\psi_1(x) = 0
\]

for \( 0 \leq x \leq \frac{W}{2N} \log_2(1 + \gamma_{sD}(k)) \),

\[
\psi_1(x) = f_{\gamma_{sD}(k)}(x) \left( 2^{\frac{x}{2N}} - 1 \right) + \left( 1 - F_{\gamma_{sD}(k)}(\Gamma_{sD}(k)) \right) \delta(x - \frac{W}{2N} \log_2(1 + \gamma_{sD}(k)))
\]

Also we can get the average capacity of the kth relay-link.

\[
C_{sD}(k) = \frac{W}{2N} \int_0^{x} \psi_1(x)dx
\]

\[
= \int_0^{x} x f_{\gamma_{sD}(k)}(x) \left( 2^{\frac{x}{2N}} - 1 \right) dx + \frac{W}{2N} \log_2(1 + \gamma_{sD}(k)) (1 - F_{\gamma_{sD}(k)}(\Gamma_{sD}(k)))
\]

(15)

B. Mobile relay

Because the relay node is mobile, thus we consider channel of the S-R link and the R-D link are both time-varying rayleigh channel, so the the SNR \( \Gamma_{sD}(k) \) and \( \Gamma_{sD}(k) \) are both in exponential distribution and independent with each other. But the average SNR may be different, we assume the average SNR of the S-R link is \( \overline{\Gamma}_{sD} \) and the average SNR of the R-D link is \( \overline{\Gamma}_{RD} \).

1) Fixed subcarriers pairing strategy

In this circumstance, the data transmitted on the kth subcarrier of the source node is also retransmitted on the kth subcarrier of the relay node.

On the basis of (8) we know that the end-to-end capacity of the S-D link is determined by the min SNR of the S-R link and the R-D link. We assume \( z \) is the end-to-end SNR of the S-D link. Then the probability density function of the capacity of the kth link is \( \psi_z(x) \) where \( x = \frac{W}{2N} \log_2(1 + z) \). The PDF of \( z \) is given.

When \( z < 0 \), \( f(z) = 0 \).

When \( z \geq 0 \), we know that the PDF of \( z \) as follows.

\[
f(z) = f_{\Gamma_{sD}(k)}(z) (1 - F_{\gamma_{sD}(k)}(z)) + f_{\Gamma_{sD}(k)}(z) (1 - F_{\gamma_{sD}(k)}(z))
\]

\[
= \left( \frac{1}{\overline{\Gamma}_{sD}} + \frac{1}{\overline{\Gamma}_{RD}} \right) \delta(z - \frac{W}{2N} \log_2(1 + z))
\]

(16)

The CDF of \( z \) is also given.
The subcarrier got the $k$th ($k = 1, \ldots, N$) largest SNR is independent with each other, the probability that a link that gets the $k$th largest SNR follows:

$$F(z) = \begin{cases} 1 - e^{-\frac{1}{T_{SR}}} & z \geq 0 \\ 0 & z < 0 \end{cases}$$ (17)

The $\psi_r(x)$ is given as follows:

$$\psi_r(x) = \begin{cases} 0 & x < 0 \\ \left(1 + \frac{1}{T_{SR}}\right) e^{-\frac{1}{T_{SR}}} e^{\frac{x^2}{2}} & x \geq 0 \end{cases}$$ (18)

$$C_{SR}(k) = \int_{0}^{\infty} x \psi_r(x) dx$$
$$= \left(1 + \frac{1}{T_{SR}}\right) \int_{0}^{\infty} x e^{-\frac{1}{T_{SR}}} e^{\frac{x^2}{2}} dx$$ (19)

2) Optimal subcarrier pairing

Because the $N$ subcarriers of the source node are all equal and independent with each other, the probability that each subcarrier got the $k$th ($k = 1, \ldots, N$) largest SNR is $\frac{1}{N}$. Then from (12), we get the PDF of the SNR on each subcarrier of the S-R link that gets the $k$th largest SNR as follows:

$$f_{SR-kth}(x) = \begin{cases} \frac{1}{N} \sum_{j=0}^{N} \delta(j,k) e^{-\frac{x}{T_{SR}}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$ (20)

The CDF of the SNR on each subcarrier that gets the $k$th largest SNR is also given:

$$F_{SR-kth}(x) = \begin{cases} \frac{1}{N} \sum_{j=0}^{N} \delta(j,k) \left(1 - e^{-\frac{x}{T_{SR}}}\right) & x \geq 0 \\ 0 & x < 0 \end{cases}$$ (21)

From (12), (13), (16), and (20), the PDF of the SNR on each subcarrier of the S-D link is given by

$$f(z) = \sum_{k=1}^{N} f_{SR-kth}(z)(1 - F_{SR-kth}(z)) + f_{RD-kth}(z)(1 - F_{RD-kth}(z))$$ (22)

The PDF of the end-to-end capacity on each subcarrier of the S-D link $\psi(x)$ is given as follows.

$$\psi(x) = \sum_{k=1}^{N} f_{SR-kth}(2^{\frac{x}{2T_{SR}}} - 1)(1 - F_{SR-kth}(2^{\frac{x}{2T_{SR}}} - 1))$$
$$+ \sum_{k=1}^{N} f_{RD-kth}(2^{\frac{x}{2T_{RD}}} - 1)(1 - F_{RD-kth}(2^{\frac{x}{2T_{RD}}} - 1))$$ (23)

From (15) and (23), we can also get the average capacity of the $k$th relay-link $C_{SR}(k)$ and the average S-D capacity of each user $C_{SD}$.

### IV. NUMERICAL SIMULATION

In this section, we will compare the performance of fixed pairing strategy and optimal strategy for a 2-hop OFDMA-based relay link over Rayleigh channel by numerical simulation. The simulation parameters are selected as Table I.

**TABLE I LIST OF SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum bandwith</td>
<td>10MHz</td>
</tr>
<tr>
<td>number of subcarriers</td>
<td>64</td>
</tr>
<tr>
<td>length of cyclic prefix</td>
<td>1/4</td>
</tr>
<tr>
<td>carrier frequency</td>
<td>2.5GHz</td>
</tr>
<tr>
<td>channel</td>
<td>six path Rayleigh</td>
</tr>
<tr>
<td>SNR of S-R link</td>
<td>10dB ~ 30dB</td>
</tr>
<tr>
<td>SNR of R-D link</td>
<td>5dB ~ 40dB</td>
</tr>
<tr>
<td>signal processing type</td>
<td>DF</td>
</tr>
</tbody>
</table>

We assume that all nodes of the 2-hop downlink OFDMA system are perfectly synchronized in time and frequency and the data processing of the nodes works properly. Then we can focus on the end-to-end SNR and capacity analysis of the S-D link. We assume SD-SNR, SR-SNR, and RD-SNR represents the average value of the SNR of the S-D link, S-R link, and R-D link separately.

First, we assume that the relay is fixed and the user is mobile. We know that the S-R link is static relatively and the R-D link is time varying in this circumstance. We set the mean SNR on all the subcarriers of S-R link to 30dB and sort the subcarriers according to their SNR in descending order. The figure 2 and figure 3 below illustrate the end-to-end SNR and the capacity on each subcarrier under different R-D link SNR conditions and pairing strategy when given N SNR samples on the subcarriers of S-R link.

From figure 2 and figure 3 we can see that the performance of optimal pairing strategy is better than fixed pairing strategy when SR-SNR is high. When SR-SNR is equal or lower to a certain extent, the performance of optimal pairing strategy may be deteriorated. Because the subcarrier having the $k$th largest SR-SNR is always paired with the subcarrier having the $k$th largest RD-SNR, the RD-SNR on the paired subcarrier of the S-R link having low SNR is also low which reduces the possibility of SNR improvement on the whole S-D link.

Second, we study the performance of subcarrier pairing strategy assuming that the relay and the user are both mobile. In this circumstance, the channels of S-R link and R-D link are both time-varying Rayleigh channels. We set SR-SNR to 10dB, 20dB, 30dB and R-D SNR 5dB ~ 40dB, from figure 4, figure 5 we can see that the overall performance of the optimal pairing strategy is always better than fixed pairing strategy. When the SR-SNR is equal to RD-SNR, we can get the largest capacity gain (>1bps/Hz).

### V. CONCLUSION

In this paper, we analyze the end-to-end SNR and capacity of subcarrier pairing strategies over Rayleigh fading channels. We compared fixed pairing strategy to optimal pairing strategy in the circumstance that the relay is fixed or mobile.
The main contribution of this paper is deriving the closed-form expressions for average end-to-end SNR and capacity in these circumstances.

At last we simulate the performance of the pairing strategies in different circumstances. The simulation results show that if the relay is fixed, the performance of optimal pairing strategy is better than fixed pairing strategy when the SNR of the S-R link is high and when the SNR of the S-R link is low to a certain extent, the performance of optimal pairing strategy may be deteriorated. The results also show that if the relay is mobile, the overall performance of the optimal pairing strategy is always better than fixed pairing strategy.

References