Target–enclosing strategies for multi–agent using adaptive control strategy

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Abstract—In this paper, an adaptive control method for target-enclosing strategies is considered. The proposed enclosing control strategy achieves enclosing configurations based on virtual structures. We assume that there exist an uncertainty in the information exchange topology between an agent and all of its neighbors on the network. We regard that the uncertainty in the information exchange topology can be described by the perturbation of the elements of graph Laplacian matrix. Our proposed method can estimate the perturbation of the elements in the notion of adaptive control strategy. Numerical simulation results are given to illustrate the effectiveness of our proposed method.

I. INTRODUCTION

The purpose of this paper is to develop an adaptive control method for a target–enclosing strategy using information exchange among agents.

\[ x = x_d \]

Recently, there have been paid much attention to the formation and cooperative control for multi agent networked system [1], [4], [15]. Especially, to achieve a capturing formation around a target object by multiple agents using local area information, the coordination control strategies were given [2], [3], [8], [11], [12]. There have been proposed a lot of applications, but in short, the task of capturing target is summarized as the distributed cooperative control of multi–agent systems.

Capturing target behavior can be divided into two configurations, one is grasping and the other is enclosing. In [3], the grasping behavior is the object closure condition in decentralized form. On another front, the enclosing behavior is that multiple agents controlled to make an assigned formation which to enclose the moving target. In [5], the grasping control law which is based on the force–closure concept and the enclosing control law which is based on a gradient descent method for multiple agent were proposed. In this method, each agent must know the local information about target and neighbor agents placements. A distributed cooperative control method which is based on a cyclic pursuit strategy in a target–enclosing method for multi agent system was proposed [11]. But, in [5], [11], they assumed that the all agents have holonomic characteristics (e.g. the agent can change of course, instantaneously) and require the information about the target placement. Moreover, the information exchange topology between agents is only limited to the cycle graphs. These assumptions are easy to consider the problems, but, it is an obstacle for the practical applications.

For the target–enclosing control, the formation control with multiple agent systems is a key technology to achieve the objective. Some formation control strategy had already been proposed [6], [9], [13]. Their method is based on consensus algorithm citeseer04. In the consensus problem for multi agent system, an interaction rule that specifies the information exchange between an agent and all of its neighbors on the network is given. For the dynamics of multi agent system with information exchange topology, it is well known that its dynamics can be described by a graph Laplacian form [1], [7]. If the control input of each agent is given by the specified information exchange topology, then the consensus problem is achieved, e.g., target enclosing is achieved. To achieve the consensus problem, it is required that the information exchange topology does not have uncertainty.

But, for the practical applications, it should be concerned that the specified information exchange topology may be collapsed, because the breakdown of receiving or transmitting equipments of agent, the agent hide behind a something, and so forth. Such a situation can be represented as the switching topology of the graph. Using this representation, a compensation method had been proposed that the uncertainty of information exchange rule can be given as the discrete phenomena of switching topology of the graph. However, it is assumed that the graph topology must be satisfied the characteristics of graph Laplacian, such as ‘strongly connected’, and ‘balanced graph.’ [1], [4], [15], [10].

In this paper, we propose the target–enclosing strategy for multiple agents which have an uncertainty of information exchange topology. We show that the uncertainty of information exchange topology between agents can be regarded as the perturbation of the elements of the graph Laplacian. Moreover, we will show that such a perturbation of elements can be described by the unknown constant parameter vector. To compensate such an unknown parameter vector, we propose an adaptive control method. Using our proposed method, the agents can always reach a target–enclosing, even if the specified graph topology have been changed. A distinctive feature of this work is to address as follows: 1) does not prescribe of changing the graph topology; 2) adaptive control strategy which does not require the information of the state of another agents. We also consider that the nonholonomic agent systems and virtual structure concepts which was proposed in [13]. Numerical simulations are provided that demonstrate the effectiveness of our proposed method.
II. PROBLEM FORMULATION

A. Agent model

In this paper, the agent is a two–wheeled vehicle which is shown in Fig. 1 (framed in by line). We consider the networked multi–agent systems which consist of vehicles. We assume that the characteristic model of each vehicles are same. \( i_{th} \) agent is modeled by the nonlinear ordinary differential equations as follows:

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{\theta}_i
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i & 0 & v_i \\
\sin \theta_i & 0 & w_i \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_i \\
y_i \\
\theta_i
\end{bmatrix}
\] (1)

where \( r_i = [x_i, y_i]^T \in \mathbb{R}^2 \) is the position of \( i_{th} \) agent, \( \theta_i \in \mathbb{R} \) is the orientation, \( v_i, w_i \in \mathbb{R} \) is the forward velocity, and \( v_i, w_i \) are the input for \( i_{th} \) agent. In general, it is assumed that the friction force can be ignored, then there exist the velocity constraint which can be written as

\[
\dot{x}_i \sin \theta_i - \dot{y}_i \cos \theta_i = 0. 
\] (2)

This constraint means that these agents are nonholonomic systems.

B. Virtual Agent (: VA) and Virtual Target (: VT)

We consider the virtual agent (: VA) and virtual target (: VT) which is based on the virtual structure (: VS) concept [13].

The VA are placed as shown in Fig. 1 (framed in by dash line). If we give the positional relationship between agents and VA as Fig. 1, then the kinematic model of \( i_{th} \) VA is described as follows:

\[
\begin{bmatrix}
x_{\text{va}i} \\
y_{\text{va}i} \\
\theta_{\text{va}i}
\end{bmatrix} =
\begin{bmatrix}
x_i + x_{di} \cos \theta_i - y_{di} \sin \theta_i \\
y_i + x_{di} \sin \theta_i + y_{di} \cos \theta_i \\
\theta_i
\end{bmatrix} 
\] (3)

where \( r_{\text{va}i} = [x_{\text{va}i}, y_{\text{va}i}]^T \in \mathbb{R}^2 \) and \( \theta_{\text{va}i} \) are the position and heading angle of \( i_{th} \) VA, respectively. Moreover, \( r_d = [x_{di}, y_{di}]^T \in \mathbb{R}^2 \) is the distance between the real and the virtual agent.

Taking the time derivative of (3), then we have

\[
\begin{bmatrix}
x_{\text{va}i} \\
y_{\text{va}i} \\
\theta_{\text{va}i}
\end{bmatrix} =
\begin{bmatrix}
x_{i} + x_{di} \cos \theta_i - y_{di} \sin \theta_i \\
y_{i} + x_{di} \sin \theta_i + y_{di} \cos \theta_i \\
\theta_i
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_i & -x_{di} \sin \theta_i - y_{di} \cos \theta_i \\
\sin \theta_i & x_{di} \cos \theta_i - y_{di} \sin \theta_i \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
v_i \\
w_i
\end{bmatrix}
\] (4)

where \( v_i, w_i \) are the moving orientation of \( i_{th} \) target, \( v_{\text{obj}}, w_{\text{obj}} \) are the moving orientation of \( i_{th} \) virtual target, respectively.

C. Control Objective

In this paper, we define the enclosed–situation as follows: \( i_{th} \) VA encloses the target in a circular pattern which is shown as Fig. 3. We also define the enclosed position for \( i_{th} \) agent as \( R_i \in \mathbb{R}^2 \) that is distance between target and agent. Then, we only consider the equal convergence positions for all agents:

\[
\|R_1\| = \|R_2\| = \cdots = \|R_n\| = r = \text{const.} 
\] (5)

where \( \|\cdot\| \) is Euclidean norm and \( x_i \) is the enclosing radius. Besides, we define \( \phi_{\text{va}i} \) as the counterclock–wise angle of \( i_{th} \) virtual agent around the center of the virtual target.

Then, the control objective for virtual structure based enclosed situation as follows:

(1) \( \lim_{t \to \infty} \| r_{\text{va}i}(t) - r_{\text{obj}}(t) \| = \xi = \text{const.} \)

(2) \( \lim_{t \to \infty} \| \phi_{i+1}(t) - \phi_i(t) \| = \frac{2\pi}{n} \)

(3) \( \lim_{t \to \infty} \| \theta_{\text{va}i}(t) - \theta_{\text{obj}}(i) \| = 0, i, j = 1, 2, \cdots, n (i \neq j) \)
If we consider \( i = N \), then \( N + 1 = 1 \). We also make assumptions for the information exchange rules and the motion of agents and target as follows:

**Assumption 1**

Information exchange topology between agents is connected graph or a balanced graph.

**Assumption 2**

The target moves at the forward speed \( v_{obj} \neq 0 \) and all agents can acquire the position of target \( r_{obj} \) and its derivative \( \dot{r}_{obj} \) from the target.

### III. TARGET ENCLOSING STRATEGIES BASED ON VIRTUAL STRUCTURE USING ADAPTIVE CONTROL METHOD

#### A. Previous enclosing control strategy

In this subsection, we show the target–enclosing control strategy which achieves the aforementioned control objectives (1) – (3) for any initial relative positions between all agents and the target [14]. The control strategy for \( i_{th} \) agent is given as follows:

\[
u_i = B_i^{-1} \left[ -k_i (r_{va} - r_{obj} - R_i) - k_a \sum_{j \in N_i} (r_{va} - R_i) - (r_{va} - R_j) + \dot{r}_{obj} \right]
\]

where \( k_i, k_a \in \mathbb{R} \) are positive constants, design parameters, \( N_i \) is the set of neighboring agents of \( i_{th} \) agent, \( B_i^{-1} \) is the inverse matrix of \( B_i \).

**Theorem 1** [14] Consider the system of \( n \) virtual agents (4) and the virtual target (5). If we apply the enclosing control strategy (7) to the system with assumptions 1, 2, then the control strategy (7) asymptotically achieves the control objectives (1) – (3).

**Proof**

We give the another variables as follows:

\[
\begin{align*}
\dot{r}_{va}' &= r_{va} - R - \dot{r}_{obj} \\
\dot{r}_{va} &= \dot{r}_{va} - 1 \otimes \dot{r}_{obj} - R = \dot{r}_{va} - 1 \otimes \dot{r}_{va}\n\end{align*}
\]

where \( r_{va} = [r_{va1}^T \cdots r_{vaN}^T]^T \) and \( R = [R_1^T \cdots R_N^T]^T \). From (8), enclosing control strategy \( u \) for \( N \) agents can be described as

\[
u = \sum_{i=1}^{N} B_i^{-1} \left\{-k_{i} r_{va} - k_{a} L_{\Sigma} r_{va} + 1 \otimes \dot{r}_{obj}\right\}
\]

where \( L \) is graph Laplacian matrix of information exchange topology of the agents and \( L_{\Sigma} = L \otimes I_2 \). Then the virtual agents (4) can be described by \( N \) virtual agents form as

\[
\dot{r}_{va} = \sum_{i=1}^{N} B_i u
\]

where \( \sum_{i} a_i \) has diagonal block elements \( a_i \). Substitute (9) into (10), then we have

\[
\dot{r}_{va}' = -k_{a} L_{\Sigma} r_{va}' = -\rho r_{va}'
\]

where \( k = \sum_{i} k_{i} a_i \) and \( \rho \equiv \sum_{i} k_{i} a_i + k_{a} L_{\Sigma} \). Then we give the positive definite function \( V \) as

\[
V(r_{va}') =\frac{1}{2} (r_{va}')^T r_{va}' > 0
\]

Taking time derivative of (12) along the solution of (11), then we have

\[
\dot{V}(r_{va}) = (r_{va})^T r_{va}' = -\rho (r_{va}')^2 < 0
\]

As we can see, \( \dot{V} \) is negative definite, therefore \( V \) is Lyapunov function and we have

\[
r_{va} - r_{obj} \rightarrow R_i \text{ as } t \rightarrow \infty
\]

If the enclosed position is given as follows

\[
R_i = |\xi| e^{i \theta} = |\xi| e^{i \frac{2 \pi (i-1)}{N}}
\]

then we can achieve the control objective (1) and (2). Moreover, from (4) and (7), we have

\[
\dot{\theta}_va = -v_{obj} \sin(\theta_{va} - \theta_{obj})
\]

Therefore, it implies that

\[
\theta_{va} \rightarrow \theta_{obj} \text{ as } t \rightarrow \infty.
\]

Consequently, we can accomplish the control objective (3).

**B. Problems of the previous method**

From Theorem 1, the agents can enclose the target using control strategy (7). As we can see (11), the graph Laplacian plays important role in the accomplishment of the target–enclosing. Therefore, the information exchange topology must be kept. But, in the practical situation, it can not be always possible to exchange the information between an agent and all of other agents with specified network protocols. To solve this problem, there has been analyzed the two cases: 1) directed and undirected networks with communication time–delays [1]; 2) directed networks with switching topology [1], [4], [10], [15]. The uncertainty of information exchange between agents is a frequent phenomenon and it is equivalent to a change of graph topology. In the
previous researches, it is assumed that the property of the graph Laplacian is maintained even if the graph topology had been changed, i.e., it does not change to be strongly connected. However, it should be considered the situation that the communication topology between agents does not change, but the communication is cut off. For example, receiving or transmitting equipments of agent cut out, the agent hide behind a something, and so forth. When we consider the communication networks with directed graph, it is equivalent to the situation that only the communication of a limited direction is possible. To solve this problem, we propose an adaptive control method for the consensus problem.

C. Information exchange uncertainty

In a practical applications, if the transmitting equipment of the agent $j$ cut off, then the signal from agent $i$ can not receive the state information of agent $j$. As for the graph Laplacian description, such a situation can be represented that the related element of the adjacency matrix would be $a_{ij} = 0$. This perturbation can be regarded as a changing the element of the adjacency matrix $A$ of graph Laplacian $L$.

If the information exchange topology does not change, then the dynamics of agent can be described as follows [1]:

$$\dot{x} = -Lx$$  \hspace{1cm} (18)

But, if the information exchange topology is perturbed, then the dynamics of agent can be written by:

$$\dot{x} = -\tilde{L}x = -(L - A_p)x$$  \hspace{1cm} (19)

where $L$ is graph Laplacian with uncertainty in the adjacency matrix. $A_p$ represents the perturbation of adjacency matrix which has same matrix structure with adjacency matrix $A$. From the definition of the graph description of the multi network agents, the element of $A_p$ is 0 or −1. If the value of the element of $A_p$ became −1, then the agent which depended on the element number is cut off. It means that the corresponding agent can not send or receive the information to the other agent.

Next, we show that the dynamics of consensus problem is always stable, even if there exists the perturbation of the adjacency matrix $A$.

Theorem 2 [16] All eigenvalues of graph Laplacian $-\tilde{L}$ with perturbed adjacency matrix $A$ which shown as (19) are located in a closed–left half plane with the zero eigenvalue.

From this theorem, the state of each agent which its dynamics is given by (19) converges to equilibrium point. Therefore, the dynamics of the consensus problem (19) always stable, even if the adjacency matrix has uncertainty.

Proposition 1 All state of the agent which dynamics is given by (19) always converge to some constant value, even if the adjacency matrix $A$ has been perturbed.

Based on the representation of information exchange uncertainty such as (19), the virtual target system (11) can be described as follows:

$$\dot{r}_v = -kr_v - k_uL\dot{r}_v - k_uL\dot{r}_v (-1 \otimes r_{obj})$$

$$= -kr_v - k_u(L - A_p) \otimes I_2 (1 \otimes r_{obj})$$  \hspace{1cm} (20)

Therefore, the structure of the virtual target system (11) change into (20), we have to design another control input for $i_{th}$ agent.

D. Adaptive enclosing control strategy

In this section, we will propose an adaptive enclosing control strategy which can achieve the target enclosing even if the information exchange topology is collapsed.

A new control input for $i_{th}$ agent is given as

$$v_i = [v_{k_v} v_y]^T$$  \hspace{1cm} (21)

Substitute (21) into (7), then we have new control input for $i_{th}$ agent as follows:

$$u_i = B_i^{-1} \left[ -k_v (r_v - r_{obj} - R_v) 
- k_e \sum_{j \in N_i} \{ (r_v - R_v) - r_{v_j} - R_j \} 
+ r_{obj} + 1 \otimes v \right]$$  \hspace{1cm} (22)

Furthermore, we substitute (22) into (10), then we have

$$\dot{r} = -kr_v - k_uL\dot{r}_v + k_uA_pe\dot{\beta}_e + k_uA_p^T\dot{\beta}_d + 1 \otimes v$$  \hspace{1cm} (23)

where $\beta_e = [\beta_1, \ldots, \beta_N]^T$ and $\beta_d = [\beta_d, \ldots, \beta_dN]^T$ are unknown parameter vectors, $A_p^T \otimes \text{diag} \{ r_{obj}, \ldots, r_{obj} \}$ and $A_p^T \otimes \text{diag} \{ r_{obj}, \ldots, r_{obj} \}$.

Using adaptive control method, we propose an adaptive enclosing control strategy. The proposed new control input $v$ which can compensate the unknown parameter vector $\beta_e$ and $\beta_d$ and estimation strategies are given as

$$\begin{align*}
v & = -k(A_p^T \dot{\beta}_e + A_p^T \dot{\beta}_d) \\
\dot{\beta}_e & = k\Gamma_e A_p^T \dot{r}_v \\
\dot{\beta}_d & = k\Gamma_d A_p^T \dot{r}_v
\end{align*}$$  \hspace{1cm} (24)

where $\Gamma_e$ and $\Gamma_d$ are positive definite matrices which have appropriate dimensions. Here, we have the following Theorem.

Theorem 3 Consider the system of $N$ virtual agents (4) and the virtual target (5). We apply the adaptive enclosing control strategies (22) and (24) to the system. The control strategy satisfies $k > 0$ and assumption 2, then control objectives (1) − (3) are achieved, even if the information exchange topology is collapsed.

Proof : We give the positive definite function as follows:

$$V = \frac{1}{2} r_v^T \dot{r}_v + \frac{1}{2} \dot{\beta}_e^T \Gamma_e^{-1} \dot{\beta}_e + \frac{1}{2} \dot{\beta}_d^T \Gamma_d^{-1} \dot{\beta}_d$$  \hspace{1cm} (25)
TABLE I
INITIAL VALUES OF VEHICLES

<table>
<thead>
<tr>
<th>vehicle: i</th>
<th>initial values: (x_i, y_i, θ_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,0,π/4)</td>
</tr>
<tr>
<td>2</td>
<td>(1,1,π/4)</td>
</tr>
<tr>
<td>3</td>
<td>(1,0,π/3)</td>
</tr>
<tr>
<td>4</td>
<td>(0,1,π/5)</td>
</tr>
</tbody>
</table>

where $\dot{\beta}_e = \dot{\beta}_e - \beta_e$, $\dot{\beta}_d = \dot{\beta}_d - \beta_d$. Taking time derivative of (25), then we have

$$V = r_{va}^T r_{va} + \dot{\beta}_e^T \Gamma_e^{-1} \dot{\beta}_e + \dot{\beta}_d^T \Gamma_d^{-1} \dot{\beta}_d$$

$$= r_{va}^T (-kL_{2} r_{va} + kA_p^T c \beta_e + kA_p^T \beta_d + 1 \otimes v)$$

$$+ \dot{\beta}_e^T \Gamma_e^{-1} \dot{\beta}_e + \dot{\beta}_d^T \Gamma_d^{-1} \dot{\beta}_d$$

$$= -k\lambda_2 \| r_{va} \|^2 + kA_p^T A_p^T \beta_e + kA_p^T \beta_d + r_{va}^T (1 \otimes v)$$

$$+ \dot{\beta}_e^T \Gamma_e^{-1} \dot{\beta}_e + \dot{\beta}_d^T \Gamma_d^{-1} \dot{\beta}_d$$

$$\leq -k\lambda_2 \| r_{va} \|^2 + r_{va}^T (kA_p^T \beta_e + kA_p^T \beta_d + 1 \otimes v)$$

$$+ \dot{\beta}_e^T \Gamma_e^{-1} \dot{\beta}_e + \dot{\beta}_d^T \Gamma_d^{-1} \dot{\beta}_d$$

$$= -k\lambda_2 \| r_{va} \|^2 - kr_{va}^T A_p^T \beta_e - kr_{va}^T A_p^T \beta_d + \dot{\beta}_e^T \Gamma_e^{-1} \dot{\beta}_e$$

$$+ \dot{\beta}_d^T \Gamma_d^{-1} \dot{\beta}_d$$

$$= -k\lambda_2 \| r_{va} \|^2 - \dot{\beta}_e^T (kA_p^T r_{va} - \Gamma_e^{-1} \dot{\beta}_e)$$

$$- \dot{\beta}_d^T (kA_p^T r_{va} - \Gamma_d^{-1} \dot{\beta}_d)$$

$$\leq -k\lambda_2 \| r_{va} \|^2 \leq 0$$

Therefore, we can show $r_{va} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and the boundedness of $\dot{\beta}_e$ and $\dot{\beta}_d$. Finally, we can show $\lim_{t \to \infty} r_{va} = 0$ and we can conclude that the target-enclosing is achieved.

IV. NUMERICAL SIMULATIONS

To show the effectiveness of our proposed method, we apply the proposed adaptive control method to the target enclosing control problem. We consider a group of 4 agents and assume that all of the agents have same kinematic model (1). Fig. 4 shows the network structure of the agents. We set the parameters as follows: $N = 4$, $r_{di} = [0.2, 0]^T$, $R_i = 0.5[\cos \pi/2, \sin \pi/2]^T$, $m_i = 0.03\text{[m/s]}$, $k_i = 0.3$, $k = 1.0$, $\Gamma_{\beta e} = 1.0$. The initial positions for each agents are given as Tab 1. As for case 1, we give the simulation results using proposed control strategy (22) and (24) without communication uncertainty. The simulation results are depicted as Fig. 5 (each number means agent number and ‘T’ means target). All of agents are converges to appropriate position which enclose the target. Next, as for case 2, we consider the situation such as: after 30[sec] of start, the receiving and transmitting equipment of agent 1 is breakdown until the end. The simulation result for such a situation is shown as Fig. 6. As we can see from this figure, we can also achieve the target enclosing, even if the communication uncertainty exists. Fig. 7 shows the agent positions for x and y axis, and orientations. As we can see these figures, all agents moved under kinematic constraints. Moreover, from Fig. 8, 9 the estimation strategies also work well.

V. CONCLUSIONS

In this paper, we considered the target enclosing problem for multi agent system using virtual structure method. We assumed that the information exchange of each agents has uncertainty, then adaptive compensation strategy had been proposed. Using the graph Laplacian description for the dynamics of multi agent system, it was shown that the uncertainty of information exchange was able to represent as the perturbation of the adjacency matrix. Moreover, we
showed that the dynamics of multi agent was always stable, even if the element of the adjacency matrix had changed. Based on this property, the perturbation of adjacency matrix can be compensated by the proposed adaptive control strategy. Numerical simulation results were given to show the effectiveness of our proposed method.

REFERENCES