On the capacity of multiple input erasure relay channels

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***Abstract*— In this paper we consider a network that consists of two senders and two receivers. We further assume that each sender could act as a relay for other communications. The channel connecting these nodes is supposed to be an erasure channel where symbols are received correctly without any error, or lost. This setting could be used to analyze the capacity of a realistic Ad-hoc network. From the network layer point of view, packets arrive at destination or erased because of error detection or collision mitigation mechanisms at MAC layer. This model is realistic for many practical scenarios in the context of wireless and sensor networks.**

In the previous works, we have derived the capacity (degraded and non-degraded) for the single-sender relay channel, as well as the broadcast and point to point channels. In this paper, we extend the analysis to a multi-sender scenario and show that the cut-set bound can be reached through a practical coding scheme based on MDS codes. The approach is based on collaboration between the nodes to transfer optimally the multiple information sources to the receivers. It can be seen as a practical implementation of network coding idea. It is further shown that the rate attained here is higher than what can be achieved through the classical routing based schemes.

I. Introduction

Wireless networks consist of senders, receivers, and intermediate nodes that more or less collaborate to achieve a communications. An important problem in the context of wireless network consists of finding the best possible collaboration scheme between the nodes on the network such that the transferred information is maximized. Information theory aims toward finding the set of transfer rate that are ultimately achievable for any given scenario.

Most of the researches in the domain of ad-hoc networks or more generally, wireless networks, have focused on a simple type of collaboration. The nodes exchanges local information to define jointly routes with suitable properties (minimum cost, interference free, etc.). These routes have to be followed by packets going from any specific source to any destination. These approaches have led to several routing mechanisms such as OLSR (Optimized Link State Routing Protocol) [4] or AODV (Ad hoc On Demand Distance Vector) [15]. However, route definition is not the only collaboration method that one can imagine and some other methods might enable higher throughput.

Multi-user information theory explores ultimate capacity that could not be outpaced by any collaboration scheme. However, derivation of multi-user information theoretic achievable region have revealed to be much more difficult that the point-to- point case.

Recently network coding [1], [10], [13] has been proposed as a new paradigm to look at the issue of network capacity. The core notion of network coding is to allow and encourage mixing of data at intermediate network nodes. Network coding defines a new type of collaboration schemes which consists of mixing the received information through a coding scheme defined for each node and forwarding the encoded version. Classical forwarding scheme sends an exact copy of each received packet over another link is a very specific case of network coding where, coding reduces to copying a packet. Initially network coding has been developed in the context of communication graphs where no losses or errors occur for each transmission. Extending classical network coding to wireless network where losses can occur at each transmission is not straightforward. Moreover, most of network coding mechanisms are based on random coding scheme that might be difficult to use in a realistic situations. Some new efforts have been made in the past couple of year to provide practical network coding schemes. For example [2] proposed a distributed scheme for practical network coding which allows the intermediate nodes to merge received packets. However, the provided mechanism remains random as the network coding acts by randomly adding arriving packets and hoping with high probability. The received packets at the receivers create a sufficient number of linearly independent packets to be able to retrieve initial data. Another remarkable point is that the proposed coding scheme is only described for a single sender-multi receiver case.

This paper comes as an extension of the previous works of the authors on the area of wireless erasure channels. In [17] the capacity of a general stationary and ergodic broadcast erasure channel is derived that leads to a simple linear capacity bound. This bound can be achieved optimally through a simple time sharing mechanism called Priority Encoding Technique. In [8] and [7] the capacity of the single relay erasure channel is derived under degraded and non degraded hypothesis. A coding schemes based on a practical MDS code are provided that achieves the capacity under general hypothesis of stationarity and ergodicity, and without needs of any side information at the receiver. In [9] the results are extended to the case of cheap relay, where the relay cannot receive simultaneously from more than one source. This case happen in Ad-hoc Wifi networks. Some simulations are presented and the achieved throughput using the proposed coding scheme is compared with AODV results. It is notable that in [16] the capacity bound of the erasure relay channel is derived under perfect side information hypothesis at the decoder. The side information is provided in the form of the exact erasure pattern over **every link** in the network. The capacity bound is achieved through a random coding scheme, but it seems that the achievability is only valid under degraded hypothesis (even if not stated clearly in the paper).

The proposed scheme in [7], [9] is a type of network coding approaches based on the collaboration of intermediate relays at the network layer to forward useful side information in the place of forwarding packets copies. Through this collaboration side information are diffused in space (similarly to CDMA were information are diffused in the whole frequency spectrum). The receiver receives information by combining relevant piece of information coming from all direction in space. The proposed method therefore does not needs any type of routing algorithm. In fact, this is the wireless channel which finds the best way to route the packets from source to the destination. The proposed coding scheme could be readily implemented on an actual WIFI based wireless network and it does not need any change to the physical layer.

The previous works have mainly dealt with the problem of sending a single information source to a set of receivers in the wireless network. The more general case of multi-sender multi-receiver situation has been rarely addressed, as it is much more difficult to handle. In this paper, we will analyze a simple communication scenario where two senders want to send different informations to two receivers (see Fig. 1). The problem dealt here is very similar to the cooperative diversity idea presented in [18], [19], [12]. Cooperative diversity is a new form of spatial diversity whereby diversity gains are achieved via the nodes cooperation. Up to our knowledge, this paper is the first work that tries to benefit from spatial diversity gain at the network level where the protocols stand. We consider the sub-case where cooperation is only done at the sender side and the receivers do not exchange their received information.

Similar to the previous works we consider the wireless channel as seen by the network layer as an erasure relay network. Each sender sends packets that might be received by the other nodes or be erased in the network because of transmission errors, collisions, or buffer overflows. We also suppose that the nodes are not able to benefit from any interference cancellation mechanisms. The interferences are suppressed by using different physical channels to reduce collisions (from the point of view of IP layer interference seems as collision). This approach might not suppress completely the collisions (because of far sender), however, MAC layer CSMA/CA mechanisms mitigate residual collisions.

In this paper, we first derive a cut-set type bound for the capacity of an erasure multi-sender erasure relay channel. We will therefore show that this bound is achievable through a

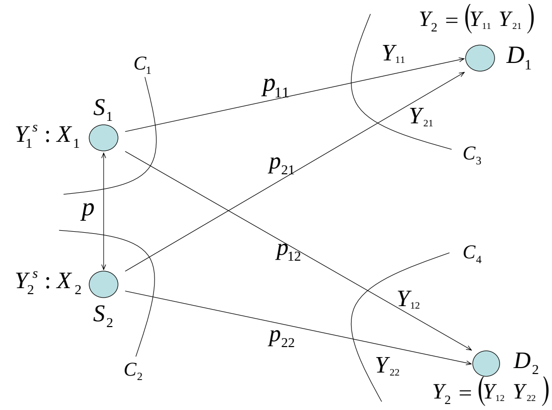

Fig. 1. Multi-sender relay Channel

practical coding scheme. Some comments and conclusion will be presented finally.

II. Theoretical Bound

The specific multi-sender relay network that will be studied in this paper is a network composed of two senders ($\mathcal{S}_1$ and $\mathcal{S}_2$) and two receivers ($\mathcal{D}_1$ and $\mathcal{D}_2$) as shown in Fig. 1. The sender $\mathcal{S}_i,\ i = 1, 2$ sends information to the two receivers $\mathcal{D}_j,\ j = 1, 2$. Simultaneously each sender might acts as a relay for the other sender. The Multi-sender relay channel can be described with 8 random variables $X_i,\ i = 1, 2$ representing the symbols sent by the sender, $Y_{ij},\ i, j = 1, 2$ representing symbols received from each sender by each receiver and Y_i^s representing the symbol received by sender i from the other sender. The conditional probability density function $p\left(y_{ij}, y_i^s\ i, j = 1, 2 | x_i\ i = 1, 2\right)$ defines the multi-sender relay channel. This last function gives the probability that when x_i is sent by $\mathcal{S}_i,\ i = 1, 2$, (y_{i1}, y_{i2}) are received at $\mathcal{D}_i$ and y_i^s is received at sender i. We further define $Y_j = (Y_{1j}, Y_{2j},\ j = 1, 2)$ as the total information received at $\mathcal{D}_j$. This description assumes that each receiver is linked to the senders through two separated channels. The separation of the two channels might be achieved by using different physical channel or by using time scheduling. We further assume that information send by a sender might be received by all receivers as well as the other sender.

The considered multi-sender relay channel consists of 2 separate erasure broadcast channel (as defined in [17]) $(X_i; Y_i^s, Y_{i1}, Y_{i2}),\ i = 1, 2$ and two erasure relay channels (as defined in [8]) $(X_1; Y_2^s : X_2; Y_{12}, Y_{22})$ and $(X_2; Y_1^s : X_1; Y_{11}, Y_{21})$. The loss probability between $\mathcal{S}_i$ and $\mathcal{D}_j$ is defined as p_{ij} and the loss probability between the two sender is supposed to be equal to p in the two direction (as shown in Fig. 1). This last assumption is for clarity sake, however the results might be extended straightforwardly to take in account possible asymmetry between the two senders.

Let's suppose that the total rate of information between $\mathcal{S}_i$ and $\mathcal{D}_j$ to be defined as R_{ij}. As said before we have two broadcast channels in the multi-sender scenario described in Fig. 1. The total rate R_{ij} could be splitted in two components : a private information rate R_{ij}^p which is the rate of information being sent from i only to j and a common information rate R_{ij}^c which is the rate that will be decodable jointly by the two receivers. In this paper we assume as in [11], [17] a degraded message-set to be sent over each broadcast channel, *i.e.* one receiver receives the private and common information and the other receives only the common information. Let's therefore suppose that the private information rate sent by $\mathcal{S}_i$ is equal to R_i^p and that the common information rate is R_i^c. In other terms, let us suppose that for example $\mathcal{D}_1$ have to receive private and common information sent by $\mathcal{S}_1$ and $\mathcal{D}_2$ have to receive only the common information. We have therefore $R_{11} = R_1^p + R_1^c$ and $R_{12} = R_1^c$. However, it is shown in [17] that one cannot do better than time-sharing for broadcast erasure channels. Meaning that there is a trade-off between the rate of private and common information. Larger private information rate means lower common rate and therefore lower reception rate for the receiver receiving only common information. This means that it is sound to suppose that all information are broadcasted as common information and private information to each receivers are sent through time sharing. Therefore, we will assume that there is only common information to be broadcasted by the senders to all receivers with rate R_i^*, i being the index of the sender.

Theorem 1 (Capacity region bound) *Under the hypothesis that X_1 and X_2 being independent the capacity region of multiple-Input relay channel in Fig. 1 is bounded by :*

$$\begin{cases} R_1^* \leq I(X_1; Y_2^s, Y_{11}, Y_{12}) \\ R_2^* \leq I(X_2; Y_1^s, Y_{21}, Y_{22}) \\ R_1^* + R_2^* \leq I(X_1; Y_{11}) + I(X_2; Y_{21}) \\ R_1^* + R_2^* \leq I(X_1; Y_{12}) + I(X_2; Y_{22}) \end{cases}$$

Proof. See the proof in appendix.

In this paper, we consider the special case of degraded channel in which the messages received at receivers are degraded version of messages received at each sender. In other term, we assume that all the information received at receiver have been received by the relay situated over the senders. This condition happens if the channel between the two senders is a stronger channel than the channel between senders and receivers ($p < \min p_{ij},\ i,j = 1,2$). Therefore $X^1 \rightarrow Y_S^2 \rightarrow (Y^{11}, Y^{12})$ and $X^2 \rightarrow Y_S^1 \rightarrow (Y^{21}, Y^{22})$ define Markov chains. Under such a situation the two first bound of the capacity bound in theorem 1 are changed to :

$$\begin{cases} R_1^* \leq I(X_1; Y_2^s) \\ R_2^* \leq I(X_2; Y_1^s) \end{cases}$$

where $I(X_1; Y_2^s, Y_{11}, Y_{12}) \leq I(X_1; Y_2^s)$ and $I(X_2; Y_1^s, Y_{21}, Y_{22}) \leq I(X_2; Y_1^s)$ follow from the properties of the markov chain.

The cut-set bound might be simplified thanks to the erasure nature of channels. The Shearer theorem is very helpful for the analysis of erasure channels.

Theorem 2 (Shearer Theorem [3]) *Let X^n be a collection of n random variables and Z^n be a collection of n Boolean random variable, such that for each i, $1 \leq i \leq n$, $E\{Z_i\} = 1 - \tilde{C}$. If $X^n(Z^n)$ is a sub-collection containing the i^th random variable X_i if $Z_i = 1$. Then $E\{H(X^n(Z^n))\} \geq \tilde{C}H(X^n)$*

The theorem can be extended to conditional entropy as well. It can be shown thanks to this theorem that the mutual information over a stationary and ergodic point to point erasure channel with an erasure process Z have a very simple form given by [17]:

$$I(X^n; Y^n) = n\tilde{C}H(X) \tag{1}$$

Where $E\{Z_i\} = 1 - \tilde{C}$ is the average erasure probability on the channel. In other word the capacity of a stationary and ergodic channel is simply $\tilde{C}$. The simple form of this Equation (1) simplifies greatly the cut-set bound derived in theorem 2. We will describe here the more simple case of memoryless erasure probability. However, all result can be straightforwardly extended to stationary ergodic erasure channels. Using this theorem the capacity bound is simplified to :

Theorem 3 *The capacity region bound over a multiple-Input erasure relay channel is bounded as :*

$$\begin{cases} R_1^* \leq (1-p) \\ R_2^* \leq (1-p) \\ R_1^* + R_2^* \leq (1-p_{11}) + (1-p_{21}) \\ R_1^* + R_2^* \leq (1-p_{12}) + (1-p_{22}) \end{cases} \tag{2}$$

The first two bounds in this theorem, are constraints bounding the rate available for collaboration between the two senders. The two last bounds are bounding the amount of information coming in the receiver. In the next section, we will provide a coding scheme achieving the given bound.

III. Achievability and Coding Method

As explained before, because of error detection mechanisms at the link layer, the transmission channel of wireless networks as seen from higher layers can be modelled by an erasure channel acting over the large input alphabet formed by IP packets. Specific codes have been designed to deal with erasures in place of errors. It is worth mentioning that the class of Maximal Distance Separable (MDS) [21] code that leads to sphere packing codes for erasure channels. Let us suppose a systematic (n, k) linear code described by an encoding and decoding matrix. Let us suppose that a vector $S \in \mathcal{X}^k$ of information is encoded to a vector $Q \in \mathcal{X}^n$ by $S = Q\left[I_{k\times k}|A_{k\times(n-k)}\right]$. The vector S contains as its first k symbols the vector Q as well as $(n-k)$ redundant symbols. If all sub matrices of $\left[I_{k\times k}|A_{k\times(n-k)}\right]$ are invertible the resulting code will be MDS. Reed-Solomon codes are well known and widely applied members of this class of codes. A (n, k) MDS code with rate $R = \frac{k}{n}$ has the property that any combination of k encoded symbols out of the n encoded symbols enables to retrieve the initial k symbols. In other word MDS codes can correct at most $n-k$ erasures in a block of n symbols.

Before going further into the description of the coding scheme, we have to describe more carefully the action of an MDS code. Let us suppose that a (n, k) MDS codes is defined over a symbol set $\mathcal{X}$. For an erasure channel the output set of the channel will be $\mathcal{Y} = \mathcal{X} \cup \{\mathtt{e}\}$ where $\mathtt{e}$ is the erasure symbol. The MDS code divide the space $\mathcal{Y}^n$ with $(|\mathcal{X}|+1)^n$ points in $|\mathcal{X}|^n$ separated cossets containing each $\sum_{l=0}^{k}\binom{n}{n-l}$ points. The cossets need therefore an index with $n \log |\mathcal{X}|$ bits to be addressed. Each received packet in a block can be seen as a part of the index needed for finding the correct cossets and lead to a correct decoding.

The main idea of the coding schemes comes from the Slepian-Wolf coding[20]. The idea is the collaboration of the senders consists of exchanging enough information such that information in each sender become correlated. In this case, we land in the context of the Slepian-Wolf where are two separated correlated sources that have to be transferred to a common receiver. The idea of Slepian-Wolf coding is that the senders should send enough side information's about each variable such that the two correlated data are decoded at the receiver. Moreover we see that each packet encoded using an MDS code can be seen as a side information (an index) that might be reassembled to derive the initial packets. In [8], [7] we applied this idea in relay channel to derive practical deterministic (non-random) codes. Proposed coding scheme reaches the capacity of erasure relay channels in degraded and non-degraded case.

In this paper, we extend the approach to the multi-sender erasure relay channel . In this case, each node has two information parts to send, the originating part and relaying part. The sent codeword is a combination of these two parts. Each node must send its originating information while it must only send the partitioning index of the relaying information. For the decoding process, if the nodes receive sufficiently number of packets from the space they are able to decode information. We will present here a coding scheme based on MDS code for achieving the bound presented in the theoretical section.

A. Coding scheme description

We assume $L+1$ block of transmission. We also consider at each sender $\mathcal{S}_i$, L blocks of data each containing k_i information symbols (packets) $B^i = \{s_1^i, \ldots, s_{k_i}^i\}$. Let's suppose that at the end of block l we have been able to decode at $\mathcal{S}_1$, the l^{th} block of the message sent by $\mathcal{S}_2$ ($B_l^2 = (s_{l \cdot k_2}^2, s_{l \cdot k_2+1}^2, \ldots, s_{l \cdot k_2+k_2-1}^2)$). We will validate this hypothesis further. Moreover, let's assume that the $(l+1)^{\text{th}}$ block of message sent by $\mathcal{S}_1$ is $B_{l+1}^1 = (s_{(l+1)\cdot k_1}^1, s_{(l+1)\cdot k_1+1}^2, \ldots, s_{(l+1)\cdot k_1+k_1-1}^2)$.

Now let's define an MDS code with an encoding matrix $G^1_{(k_1+k_2)\times n}$. It generates the n encoded packets of the $(l+1)^{\text{th}}$ block from the k_1+k_2 given packets consisting of the k_1 packets of the $(l+1)^{\text{th}}$ block of message of $\mathcal{S}_1$ and k_2 packets of the $(l)^{\text{th}}$ block of message send by $\mathcal{S}_2$ and received in the previous block, *i.e.* $X_{l+1}^1 = [B_{l+1}^1 \ B_l^2] \times G^1$.

Sender $\mathcal{S}_2$ can easily decode the block $(l+1)$ broadcasted by $\mathcal{S}_1$, if it has received enough packets over the erasure channel connecting $\mathcal{S}_1$ to $\mathcal{S}_2$. As it have in memory the k_2 values in B_l^2, this happen if $n(1-p) > k_1$, *i.e.* asymptotically if the rate of the MDS code used at $\mathcal{S}_1$ is less $(1-p)$. The rate of the MDS code is equal to $R_1^* = \frac{k_1}{n}$ as out of the k_1+k_2 symbols used at the encoder input, k_2 of them are redundant (have been sent before over the channel). The decoded block B_{l+1}^1 is to be used in the next transmission block combined with block B_{l+2}^1 constructing X_{l+2}^2. As the initiation block we can use an all-zero block $B_0^j, \ j=1,2$ know to everybody in the network. Sender $\mathcal{S}_2$ encodes its proper information and mixes them with the information received from $\mathcal{S}_1$ using exactly the same mechanism but with a different encoding matrix G^2. With the same arguments we have $R_2^* = \frac{k_2}{n} \leq (1-p)$. We will choose the encoding matrix G^1 and G^2 such that $[G^1 \ G^2]$ defines an MDS code, *i.e.* each sub-matrix of $[G^1 \ G^2]$ is invertible.

Now let's see what happen at the receiver side when senders use such a coding schemes. At each receiver we will receive some packets coming from each sender. Asymptotically with large n, in each block of transmission containing, each receiver $\mathcal{D}_j$ will receive $n(1-p_{1j})+n(1-p_{2j})$ packets. In transmission block l, the packets received from the senders are a combination of B_l^1, B_l^2, B_{l-1}^1 and B_{l-1}^2. Clearly the decoding can not be done using only the information received in block l. The L-block Markov decoding technique can be applied. The idea is to do the decoding after reception of L blocks. Let suppose that the encoding matrix $[G^1 \ G^2]$ is rewritten as:

$$[G^1 \ G^2] = \begin{bmatrix} G_{11} & G_{21} \\ G_{12} & G_{22} \end{bmatrix}$$

where G_{ij} is a $k \times n$ matrix with the MDS property.

After receiving L blocks $\mathcal{D}_j$ we will receive a sub-sequence (because of erased symbols) $Y_0^L(Z_{ij}^{2Ln})$, where Z_{ij}^{2Ln} is the loss process observed over erasure channel going from $\mathcal{S}_i$ to $\mathcal{D}_j$ during the $2Ln$ transmissions of L blocks. Y_0^L is vector of length $2nL$ symbols obtained as in Eq. 3. The obtained L-block code of rate $R^* = \frac{(k_1+k_2)L}{2nL}$ (as the first block $[B_1^0, B_2^0]$ is known) is still an MDS code code as every sub-matrix of the encoding matrix will be invertible. Asymptotically, we will receive at each receiver $nL(1-p_{1j}) + nL(1-p_{2j})$ packets. The MDS code can be decoded if $nL(1-p_{1j}) + nL(1-p_{2j}) > (k_1+k_2)(L+1)$, in other term if $\frac{L+1}{L}(R_1^*+R_2^*) < (1-p_{1j}) + (1-p_{2j})$. We can see therefore that the proposed coding scheme achieves the capacity bound under degraded hypothesis when $L \to \infty$.

IV. Practical comments

The proposed coding scheme is original from several perspectives. It provides a practical and simple way of doing Slepian-Wolf coding in the context of erasure channels. In place of sending the information independently from $\mathcal{S}_1$ and $\mathcal{S}_2$, we send an index obtained by mixing information coming from the two senders. By doing this we reduce the amount

$$Y_0^L = [B_1^0\ B_2^0\ B_1^1\ B_2^1 \dots B_1^L\ B_2^L] \begin{bmatrix} G_{11} & G_{21} & 0 & 0 & \dots & 0 & 0 \\ G_{12} & G_{22} & G_{11} & G_{21} & \dots & 0 & 0 \\ 0 & 0 & G_{12} & G_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & G_{11} & G_{21} \\ 0 & 0 & 0 & 0 & \dots & G_{12} & G_{22} \end{bmatrix} \quad (3)$$

of information to be sent by each sender and we reach a collaboration gain. Moreover, in this setting all symbols received over the multicast channel are useful to decode the final information.

Up to now we have considered the case where all the sent information are common information. Now if one want to send private information he might use a simple time sharing scheme. Let's suppose that we want to send with a rate R_{ij} from $\mathcal{S}_i$ to $\mathcal{D}_j$. The cut-set bound become equal to :

$$\begin{cases} R_{11} + R_{12} \leq (1-p) \\ R_{21} + R_{22} \leq (1-p) \\ R_{11} + R_{21} \leq (1-p_{11}) + (1-p_{21}) \\ R_{12} + R_{22} \leq (1-p_{12}) + (1-p_{22})\} \end{cases}$$

Under this situation it would be possible to design a time-sharing mechanism based on the proposed coding scheme, sending information from $\mathcal{S}_i$ to $\mathcal{D}_j$ a proportion of time equal to $\frac{R_{ij}}{R_{1j}+R_{2j}}$. This time sharing mechanism achieves the cut-set bound, meaning that the previous coding scheme can be applied even in the case that private information is to be send to the received.

It is noteworthy that the decoding process of MDS code has a $\mathcal{O}(n)\log n$ complexity. This can be reduced to a linear complexity through use of Tornado codes that are almost-MDS [14]. The main drawback of the method is large decoding delay at receiver that comes from the fact that we have to wait until reception of L block before decoding. However, there is a trade-off between rate efficiency and delay. Through the choice of L, one can trade-off between the decoding delay and rate; larger L Leads to larger rates but larger delays and *vice-versa*.

Comparing the obtained capacity region with other approaches for sending information in a multi-sender relay network is helpful. We will compare the achieved capacity with two scenarios. In the first scenario, we are not using the collaboration between the two senders and we do in parallel a broadcast from source $\mathcal{S}_j$. In the second scenario, we might use a time-sharing of four simple single sender relay channels; two relay channels $(\mathcal{S}_1, \mathcal{S}_2, \mathcal{D}_j),\ j = 1,2$ and two other $(\mathcal{S}_2, \mathcal{S}_1, \mathcal{D}_j),\ j = 1,2$.

A. Scenario 1: parallel broadcast

In this case, the achievable rate is governed by the capacity region of the erasure broadcast derived in [17]. We still assume that we are in the context where only common information is send. As explained before time-sharing might extend the approach to the situation of private and common information. The bound is :

$$\begin{cases} R_1^* \leq \min\{(1-p_{11}), (1-p_{12})\} \\ R_2^* \leq \min\{(1-p_{21}), (1-p_{22})\} \end{cases}$$

B. scenario 2: time-sharing of single-sender relay

Let suppose that the use a time-sharing between the two relay channels $(\mathcal{S}_1, \mathcal{S}_2, \mathcal{D}_j),\ j = 1,2$ and $(\mathcal{S}_2, \mathcal{S}_1, \mathcal{D}_j),\ j = 1,2$. Let us α being the proportion of time one uses the relay channel. Now the achievable rate is governed by the capacity region of the degraded erasure relay channel [8]. Using this capacity bound we have :

$$\begin{cases} R_1^* \leq \alpha(1-p) \\ R_2^* \leq (1-\alpha)(1-p) \\ R_1^* \leq \alpha \min\{(1-p_{11}) + (1-p_{21}), (1-p_{12}) + (1-p_{22}))\} \\ R_2^* \leq (1-\alpha) \min\{(1-p_{11}) + (1-p_{21}), (1-p_{12}) + (1-p_{22})\} \end{cases}$$

Comparing the bounds obtained in the previous two scenarios, with the bounds obtained in theorem 3, could be instructive. Clearly the bounds, given in theorem 3 outperforms the bounds given in the first scenario, if the transmission is not bounded by the channel between the two senders, *i.e.* if p is small. The comparison between scenario 2 and bounds obtained in theorem 3 is also straightforward. Let supposes that. Adding the two last bounds obtained in scenario 2, we have :

$$R_1^* + R_2^* \leq \min\{(1-p_{11}) + (1-p_{21}), (1-p_{12}) + (1-p_{22})\}$$

This means that for time-sharing of single-input erasure relay channel the rate available for collaboration between the two senders is lower than proposed coding scheme.

V. CONCLUSION

We have presented here a capacity region for the degraded multi-sender erasure relay channel. We first derive a version of the cut-set bound for the proposed scenario, and we proposed a coding scheme based on MDS code achieving the capacity of this channel. The proposed scheme is original from several perspectives. It provide a practical and simple way of doing Slepian-Wolf coding in the context of erasure channels. In place of sending the information independently from sender $\mathcal{S}_1$ and $\mathcal{S}_2$, we send an index obtained by mixing information coming from the two senders.

Moreover, we showed that the capacity region of the proposed coding scheme is larger than the achievability region of parallel broadcast and time-sharing of single-sender relay hypothesis. The capacity region of a non-degraded multi-sender erasure relay channel would be the objective of the forthcoming paper.

APPENDIX

Let's W_1 and W_2 the messages sent by $\mathcal{S}_1$ and $\mathcal{S}_2$. Let's further assume that they are independent and chosen randomly (uniformly) over the sets of integers $\mathcal{W}_1 = \{1, 2, .., 2^{nR_1^*}\}$ and $\mathcal{W}_2 = \{1, 2, .., 2^{nR_2^*}\}$. The rate R_1^* can then bound as :

$$
\begin{aligned}
nR_1^* &= H(W_1) \\
&\overset{(a)}{=} H(W_1|W_2) \\
&= I(W_1; Y_2^{sn}, Y_{11}^n, Y_{12}^n|W_2) + H(W_1|Y_2^{sn}, Y_{11}^n \\
&\quad , Y_{12}^n, W_2) \\
&\overset{(b)}{\leq} I(W_1; Y_2^{sn}, Y_{11}^n, Y_{12}^n|W_2) + n\epsilon_n \\
&\overset{(c)}{=} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}|Y_2^{s\,i-1}, Y_{11}^{i-1}, Y_{12}^{i-1}, W_2) \\
&\quad -H(Y_{2i}^s, Y_{11i}, Y_{12i}|Y_2^{s\,i-1}, Y_{11}^{i-1}, Y_{12}^{i-1}, W_1, W_2) \\
&\quad +n\epsilon_n \\
&\overset{(d)}{\leq} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}) - H(Y_{2i}^s, Y_{11i}, Y_{12i}| \\
&\quad Y_2^{s\,i-1}, Y_{11}^{i-1}, Y_{12}^{i-1}, W_1, W_2, X_{1i}, X_{2i}) + n\epsilon_n \\
&\overset{(e)}{\leq} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}) - H(Y_{2i}^s, Y_{11i}, Y_{12i}| \\
&\quad X_{1i}, X_{2i}) + n\epsilon_n \\
&\overset{(f)}{=} \textstyle\sum_{i=1}^n H(Y_{2i}^s, Y_{11i}, Y_{12i}) - H(Y_{2i}^s, Y_{11i}, Y_{12i}| \\
&\quad X_{1i}) + n\epsilon_n \\
&= \textstyle\sum_{i=1}^n I(X_{1i}; Y_{si}^2, Y_{11i}, Y_{12i}) + n\epsilon_n \\
&= \textstyle\sum_{i=1}^n I(X_{1q}; Y_{sq}^2, Y_{11q}, Y_{12q}|Q = i) + n\epsilon_n \\
&= nI(X_{1Q}; Y_{sQ}^2, Y_{11Q}, Y_{12Q}|Q) + n\epsilon_n \\
&\overset{(g)}{=} nI(X_1; Y_2^s, Y_{11}, Y_{12}|Q) + n\epsilon_n \\
&\overset{(h)}{\leq} nI(X_1; Y_2^s, Y_{11}, Y_{12}) + n\epsilon_n
\end{aligned}
$$

where (a) follows from the independence of W_1 and W_2, (b) follows from Fano's inequality, (c) from the chain rule and definition of mutual information, (d) from the fact that removing conditioning increase the first term and conditioning reduces the second term, (e) from the fact that $Y_i = (Y_{si}^2, Y_{11i}, Y_{12i})$ depends only on the current symbol X_{1i} and X_{1i} [5] by the memoryless property of the channel, (f) from the fact that Y_i is the received vector message if X_{1i} is sent over the channel. Also based on the definition of relay channel [6] X_{2i} only depends on the past received symbols and W_2. Therefore at the transmission time i, the received vector Y_i only depends on X_{1i} and conditionally is independent from X_{2i} (also note that X_{2i} send over a channel physically separated from X_{1i}), (g) by defining $X_1 \triangleq X_{1Q}$, $Y_2^s \triangleq Y_{2Q}^s$, $Y_{11} \triangleq Y_{11Q}$ and $Y_{12} \triangleq Y_{12Q}$ as the new random variable where $Q \rightarrow X_1 \rightarrow (Y_s^2, Y_{11}, Y_{12})$ for $|\mathcal{Q}| \leq \min\{|\mathcal{X}_1|, |\mathcal{Y}_s^2|, |\mathcal{Y}_{11}|, |\mathcal{Y}_{12}|\}$, and (h) from the Markov chain properties.

With the same argument we can show that $R_2^* \leq I(X_2; Y_1^s, Y_{21}, Y_{22})$ which leads us to the two first terms of the capacity bound of theorem1.

At the receiver side we use the cut-set bound defined in [5] and we have :

$$
\begin{cases} C_3 : & R_1^* + R_2^* \leq I(X_1, X_2; Y_{11}, Y_{21}) \\ C_4 : & R_1^* + R_2^* \leq I(X_1, X_2; Y_{12}, Y_{22}) \end{cases} \tag{4}
$$

As said before, the nodes transmit over the physically separated channel. From the point of view of $\mathcal{D}_1$ (resp. $\mathcal{D}_2$) the channel can be modeled by two point to point channel $\mathcal{S}_1 - \mathcal{D}_1$ and $\mathcal{S}_2 - \mathcal{D}_1$ (resp. $\mathcal{S}_1 - \mathcal{D}_2$ and $\mathcal{S}_2 - \mathcal{D}_2$). Under this hypothesis the maximum of $R_1^* + R_2^*$ achieve if $\mathcal{S}_1$ and $\mathcal{S}_2$ send independent codeword over these two independent channel. This lead to the maximum of $I(X_1, X_2; Y_{11}, Y_{21})$ being equal to $I(X_1; Y_{11}) + I(X_2; Y_{21})$ and maximum of $I(X_1, X_2; Y_{12}, Y_{22})$ being equal to $I(X_1; Y_{12}) + I(X_2; Y_{22})$. In other term, the collaboration between the sender and the relay reduces to ensuring that the variable sent by $\mathcal{S}_1$, X_1, and $\mathcal{S}_2$, X_2, are independent from each other but are still complementary to enable a maximal rate at receiver. This leads to the two last terms of the capacity bound of theorem 1□.

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