Le Roux, K. (2014). Explaining mathematical meaning in “practical terms” and access to advanced mathematics. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education, 4, (pp. 65-72). Vancouver: PME.

# EXPLAINING MATHEMATICAL MEANING IN “PRACTICAL TERMS” AND ACCESS TO ADVANCED MATHEMATICS

Kate le Roux

University of Cape Town

*This paper uses a socio-political practice perspective of mathematics to investigate the action of first year university students as they work in small groups to explain the meaning of mathematical objects in “practical terms”. Written transcripts representing video recordings of the action were analysed using critical discourse analysis and focal analysis. I present a description of the micro-level action and locate this action relative to the mathematical discourses in the university space. This analysis raises questions about the relationship between problems requiring the use of the practical terms genre and access to advanced mathematics at university.*

## INTRODUCTION

The relationship between practical problems[1] and access to school mathematics has been in view in mathematics education research for the past two decades. The sociological work of Basil Bernstein and Paul Dowling has been used extensively in this research, with this and related research showing that solving practical problems involves recognising the practical/mathematical boundary (e.g. Straehler-Pohl, 2010) and the esoteric domain knowledge driving the recontextualization of the practical (e.g. Gellert & Jablonka, 2009). It has been argued that possession of these recognition rules is not equally distributed across social class (e.g. Cooper & Dunne, 2000), with Nyabanyaba (2002) arguing that student agency in choosing not to answer certain practical problems in high-stakes examinations is enabled by socio-economic status.

In this paper the relationship between practical problems and access to advanced mathematics[2] at university is in view. I focus on problems requiring students to explain the meaning of mathematical objects in “practical terms” (see Figure 1). The design of this problem draws on calculus reform texts of the 1990s. These texts argue that “formal definitions and procedures evolve from the investigation of practical problems” (Hughes-Hallet et al., 1994, p.vii) and that explanations in “practical terms” (p.vii) strengthen the meaning students attach to mathematical objects. The transition from intuitive to advanced calculus has long been a concern in mathematics education research. However, the role of practical problems in this transition has traditionally not been in view in the psychological research on advanced mathematics (e.g. Tall, 1991) and the untheorized evaluation studies of calculus reform curricula (e.g. Garner & Garner, 2001). A recent trend sees the uptake of sociological and systemic functional linguistic perspectives in research on undergraduate mathematics (e.g. Jablonka, Ashjari & Bergstrom, 2012), and this paper is part of this move.

> A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had, the disease $t$ days after the first case of flu was recorded.
>
> c) What does $P(4) = 1\,200$ mean in practical terms? (Your explanation should make sense to somebody who does not know any mathematics.)
>
> d) What does $\frac{P(7)-P(4)}{7-4} = 350$ mean in practical terms? Give the correct units.
>
> e) What does $P'(4) = 400$ mean in practical terms?
>
> *Course Answers:*
>
> *c) 4 days after the first recorded person got flu, 1200 people had the flu.*
>
> *d) From the 4$^{th}$ to the 7$^{th}$ day after the first recorded person got flu, the number of people on average who had the flu was increasing by 350 people per day.*
>
> *e) 4 days after the start of the epidemic, the number of people who had the flu was increasing by 400 people per day.*

Figure 1: The Flu Problem, questions (c) to (e), with course answers in italics.

This paper investigates the action of students on the questions in Figure 1. The students are enrolled in a first year university mathematics course (called *foundation mathematics*) at a South African university. The course aims to provide epistemological access to advanced mathematics for students considered educationally disadvantaged on the basis of their race, socio-economic status and language. A pass in this course provides formal access to advanced mathematics courses. I use a socio-political practice perspective of mathematics (a) to describe the student action, and (b) to ask whether this action reproduces or diverges from the university mathematical discourses. This analysis problematizes, at the level of the individual student, the relationship between using practical terms and advanced mathematics.

## THEORETICAL FRAMEWORK

The socio-political practice perspective of mathematics used in this paper is based on Fairclough's (2001, 2003, 2006) critical linguistics, work that draws on Bernstein's sociology and the systemic functional linguistics of Halliday. For the description of *mathematical discourse* I use Morgan (1998), Moschkovich (2004) and Sfard (2008). In this perspective, the student action as text is located in the socio-political practice of *foundation mathematics*. Language (or *mathematical discourse*) is used in the practice to represent mathematics, with the *discourse type* of foundation mathematics involving particular ways of talking about and looking at mathematical objects, of operating on objects, and of making and evaluating arguments. Language is used to interact communicatively (the *genre*), making discursive links between texts and practices, and between students. Lastly, language identifies the students as particular types of people (the *style*).The term *practical* in this paper recognises the "relationship of recontextualization" (Fairclough, 2006, p.34) between foundation practice and the virus in epidemiology, with the discourse types, genres and styles in the latter practice "filtered" (Fairclough, 2003, p.139) by the recontextualising mathematical practice.

In this perspective, using practical terms to describe mathematical objects is a *genre*. Using the genre involves, for example, looking at the spread of the flu mathematically as an increasing function and looking operationally at the subtraction $P(7)–P(4)$ as the change in the number of people who have or have had the flu (Le Roux & Adler, 2012). The absence of mathematical words like "rate" is a key evaluation criterion in the genre. Practical terms are sourced via links to the problem text and the course lecture text where the genre was demonstrated prior to answering the Flu Problem. Students interact socio-politically as they talk in small groups. At the university at which this study was conducted this genre is restricted to foundation mathematics, its use thus identifying the students in the style of foundation students.

The student action as text that is the focus of this paper is, according to Fairclough (2001), shaped by / a repetition of the mathematical discourse of the foundation practice. In fact, the students are enabled to act, provided this is within the constraints of the discourse. Yet the text also creates meaning and how it represents mathematics, interacts communicatively, and identifies students may diverge from the discourse type, genre and style respectively of the foundation practice. This is a result of the text cutting across practices and the agency of students. Fairclough (2001) points to likely asymmetries in the extent to which students resist the constraints of the discourse.

Thus from this perspective, answering the research questions in this paper involves, firstly, describing the mathematical, discursive and socio-political action of the students. Second, I consider whether this meaning reproduces or diverges from the discourse types, genres and styles of the university mathematical discourses.

## METHODOLOGY

The texts used in this paper are from a wider study of the use of practical problems and a learner-centred pedagogy in the foundation course (Le Roux, 2011).[3] Video recordings of students working in small groups were transcribed to represent the non-verbal action (shown in bracketed *italics*) and the verbal action. For the latter, pauses are represented using three points …, emphasis is marked by underlining, rising intonation shown by the up arrow ↑, and square brackets [ ] enclose overlapping talk.

For the analysis, the theoretical view of text as both repetition and creation was operationalized using Fairclough's three-stage method of critical discourse analysis. The specific action on mathematical objects was brought into view using an adapted version of focal analysis (Sfard, 2000). First, the descriptive stage involved working line by line through the transcript to identify what students are looking at (the *attended focus*) and saying or writing (the *pronounced focus*). The latter was analysed in detail to identify textual features such as the choice of words, the tense and the mood. The interpretation stage involved working across longer pieces of text to identifying what meaning these features give to the mathematical discourse. For example, specialist terms may represent the text as mathematical, and rising intonation at the end of a statement may identify the student's claim as tentative. Finally, I consider the mathematical discourse of the students in relation to the discourse-types, genres and styles of the foundation and advanced mathematics practices.

In this paper I present the texts of two groups of five students each, with pseudonyms Bongani, Lungiswa, Mpumelelo, Siyabulela and Vuyani used in Group A and Hanah, Jane, Jeff, Lulama and Shae for Group B. The similarities and differences between the groups as they work with the function and both the average and instantaneous rate of change of this function enable a rich description of the student action. I note here that these groups are mixed with respect to gender, home language, socio-economic status, race and schooling, a point I return to at the end of this paper.

## DESCRIPTION OF THE STUDENT ACTION

### Describing the function value $P(4) = 1\,200$ in practical terms

The students write practical descriptions of $P(4) = 1\,200$ relatively quickly. They evaluate one another's verbal pronouncements about the two variables, for example, "After 4 days" for the independent variable. Shae looks at the problem text to identify that "*t* is in days", but the students do not repeat the description of this variable in that text, where *t* represents the number of days "after the first case of flu was recorded". The use of the preposition "after" for the time when four days have passed reproduces the course lecture text.

To describe the dependent variable, some students repeat the problem text ("1 200 people will have it or have had it", Jane). It is not possible to tell whether they are simply copying the problem text or actually looking mathematically at $P(t)$ as an increasing function, a gaze needed in some parts of the Flu Problem. Other students reword the problem text to people who "have been infected" (Hanah) or "are infected" (Lungiswa). The students' lack of attention to variations in tense may suggest that they are acting in the style of students solving word problems; Gerofsky (2004) argues that, since these problems are only a pretence of the real world, inconsistencies in tense are not considered a problem by students familiar with the word problem genre.

While co-constructing answers in practical terms, Siyabulela and Lungiswa in Group A express surprise at the large number of people infected; Lungiswa laughs as she says "That's too much". Referring to the nameless "somebody" in question (c), Shae in Group B asks Jeff, "If he doesn't know any mathematics, don't we have to teach him numbers and stuff?" They link this action to the "yellow books" called "Maths for Dummies". Thus these students give significance to mathematical action, with Siyabulela and Lungiswa only pretending that the flu virus exists (Gerofsky, 2004) and Shae and Jeff representing the practical terms genre as targeting "dummies".

## Describing the average rate of change in practical terms

Characteristic of the action in Group A, in lines 375 to 381 Siyabulela and Lungiswa co-construct the first verbal pronouncements, seeking feedback (the rising intonation ↑) and giving feedback ("ja"). The words "from four to seven days" suggest that they are looking operationally at the denominator of the fraction $\frac{P(7)-P(4)}{7-4}$, with the subtraction representing the change in time. Their choice of prepositions "from… to…" reproduces their lecture text, although they do try out "between". Siyabulela's use of the unit "people" rather than "people per day", and his evaluation of his peers' pronouncements as described later in this section, suggests that his operational view does not extend to the numerator as the change in the number of people infected and the division as the change in the number of people infected relative to time.

375 Lungiswa: What does mean in practical terms↑° ((*Reads text*))... From four to seven days

376 Siyabulela: Oh... ai that one is bet... that one is between seven uh

377 Lungiswa: Four to seven days

378 Siyabulela: Oh ja… four to seven days… the number of people infected

379 Lungiswa: Uh huh↑ Between↑… ja

380 Siyabulela: Ja [from four to seven days]… 350 people were infected↑

381 Lungiswa: [from four to seven days]

Vuyani enters the discussion with a tentative suggestion (using the imperative mood and the negation "aren't"); "Aren't we supposed to include the word... average?" He is looking structurally at the fraction as an object and linking to the lecture text where "the… word average" is used for such objects. This prompts other students to look structurally at the fraction ("this one" and "<u>this</u>") as an average rate of change; "It's the average this one", and "<u>This</u> is a rate of change". However, Lungiswa reminds her peers about using "practical terms", and the students proceed to verbally insert "the word... average" into Siyabulela's first attempt in line 380, as in "the average in the number of people were… " (unidentified), and "… between that period of 4 and 7... there were like … how many people infected? Ja like 350 people ... on average average" (Bongani). However, trying to produce what sounds right in the lecture text ends in frustration, suggested by Bongani's loud, "Aargh".

As usual, Siyabulela acts as an authority in the foundation practice by, firstly, critiquing Mpumelelo's use of "a mathematical term" in "rate of infection". Yet his laughter mocks the practical terms genre. Secondly, he dismisses as "the derivative" Bongani's use of "increasing" in "the number of people were increasing that were infected by 350". Since Bongani may be looking operationally at the subtraction $P(7)–P(4)$ as required in the genre, Siyabulela diverges from his usual role as the student who enables the action in Group A. Not knowing how to proceed, the students discuss the prepositions and whether their choices are "bad English" (Siyabulela).

The Group B students initially view the fraction $\frac{P(7)-P(4)}{7-4}$ as an object linked to the words “average” and “rate of change”. They also insert the word “average” into talk about the function (“the average... people who will be infected is 350... from”, Jane). Two actions enable a shift from testing what sounds right relative to the course. Firstly, Hanah’s attention to the units (“the average amount... per day”) suggests she is looking operationally at the division. Secondly, Lulama links the words “average” and “rate of change”; “is this an average or an average rate of change?” Attention to the voices of Lulama and Hanah is rare in this group, yet progress is made in the practical terms genre, for example, “From 4 to 7 days the average number of people infected per day are 350 people”. Shae is the exception, using the “rate of change” in his answer.

**Describing the instantaneous rate of change in practical terms**

In Group A Siyabulela and Bongani link the symbols $P'(4)$ to their earlier talk about “the derivative”. Bongani avoids mathematical words by naming it “my thing thing”. As usual, Lungiswa and Siyabulela begin to co-construct verbal answers in practical terms. Lungiswa’s initial description of the independent variable (“After each four days”) reproduces the preposition in the lecture text, with Siyabulela refining this to “After four days”. Siyabulela then adds the meaning of the dependent variable, using Bongani’s word “increasing”; “the number of people… who were infected were increasing by… 400 <u>per</u> day”. The other students join in, and considerable effort is made to produce practical wording that sounds right (irrespective of the tense).

In contrast, the Group B students complete question (e) at different times and use mathematical words. As usual, Jeff and Shae begin and, after stating words like “rate of change” and “instantaneous” settle on “infection rate” in their writing. Answering question (e) later, Jane and Hanah identify Jeff as the authority in the practice, relying on him to confirm or rework their words. In one such version Jeff uses practical terms only (“<u>at</u> day 4, 400 people <u>per day</u> are being infected), although he gives significance to mathematical terms in his writing. For the time variable the students switch between the prepositions “after” of the lecture text and their own “at” for “instantaneous”. Lulama does not talk with his peers and writes “average rate of change”.

## DISCUSSION

Using practical terms involves students verbally stating the meaning of the variables and reworking and adding to one another’s talk, action that is initiated by the students who identify and are identified as authorities in the foundation practice. Words from the problem and lecture texts are combined to sound right. Progress is enabled or constrained by the group interaction, as students invest in or resist the style of students who collaborate in groups and as the authorities in the practice control who has a voice.

Viewing this action relative to the discourse-types, genres and styles of the university mathematical discourses raises concerns about the relationship between the practical terms genre and advanced mathematics. Firstly, it is possible that, by repeating words in the problem and lecture texts and ignoring inconsistent tenses, students use the practical terms genre without recognising the esoteric domain knowledge of calculus as intended (Gellert & Jablonka, 2009). Secondly, the students' jokes about the genre suggest that they recognise the practical/mathematical boundary (Straehler-Pohl, 2010). Yet they still invest considerable time using the genre, sometimes unsuccessfully. It may be that not repeating the genre involves, in Lulama's words, being "wrong" in the foundation practice, a practice that provides formal access to advanced mathematics. Paradoxically, using this genre requires producing meaning in the public domain which does not provide access to the vertical mathematics practice (Dowling, 1998) and, in question (d), looking operationally at an object rather than structurally as valued in the vertical practice.

Finally, some students (e.g. Shae and Jeff) appear to resist using the practical terms genre more easily than others (e.g. Lulama and Lungiswa). It may be that, like the students in Nyabanyaba's (2002) study, Shae and Jeff choose not to reproduce foundation questions targeting "dummies" in the knowledge that they can still pass the course. I note at this point that Shae and Jeff would not be classified as educationally disadvantaged in the South African context and only joined the foundation class after performing poorly in the first assessments in the mainstream mathematics course. While the conceptualization of this study does not allow claims in this regard, there is scope for an investigation of whether the foundation practice acts in reproductive ways with respect to race, socio-economic status and language.

## Notes

1. My use of this term is consistent with its use in calculus reform texts (e.g. Hughes-Hallet et al., 1994). The literature variously uses *realistic problems*, *real-world problems*, and *word problems.*

2. I use this term for the formal abstract mathematical practice at university (called *advanced mathematical thinking* in much of the literature, e.g. Tall, 1991). At South African universities, first year mathematics is preparation for advanced mathematics in the second year of undergraduate study.

3. I thank Jill Adler for her supervision of the wider study. I also acknowledge the research funding provided by the National Research Foundation (Grant No. TTK2006040500009). The opinions, findings and conclusions are those of the author and the NRF does not accept any liability in regard thereto.

## References

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham, England: Open University Press.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts*. London: Falmer Press.

Fairclough, N. (2001). *Language and power* (2nd ed.). Harlow, England: Longman.

Fairclough, N. (2003). *Analysing discourse: Textual analysis in social research*. London: Routledge.

Fairclough, N. (2006). *Language and globalization*. London: Routledge.

Garner, B.E., & Garner, L.E. (2001). Retention of concepts and skills in traditional and reformed applied calculus. *Mathematics Education Research Journal*, *13*(3), 165-184.

Gellert, U., & Jablonka, E. (2009). "I am not talking about reality": Word problems and the intricacies of producing legitimate text. In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modelling verbal descriptions of situations* (pp.39-53). Rotterdam: Sense.

Gerofsky, S. (2004). *A man left Albuquerque heading east: Word problems as genre in mathematics education*. New York: Peter Lang.

Hughes-Hallet, D., Gleason, A.M., Flath, D.E., Gordon, S.P., Lomen, D.O., Lovelock, D., et al. (1994). *Calculus*. New York: John Wiley & Sons.

Jablonka, E., Ashjari, H., & Bergsten, C. (2012). Recognising knowledge criteria in undergraduate mathematics education. In C. Bergsten, E. Jablonka, & M. Ramon (Eds.), *Proceedings of the Eighth Swedish Mathematics Education Research Seminar* (pp.101-110). Linköping: Swedish Society for Research in Mathematics Education.

Le Roux, C.J. (2011). *A critical examination of the use of practical problems and a learner-centred pedagogy in a foundational undergraduate mathematics course*. Unpublished doctoral dissertation, University of the Witwatersrand, South Africa.

Le Roux, K. & Adler, J. (2012). Talking and looking structurally and operationally as ways of acting in a socio-political mathematical practice. In Tso, Tai-yih (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education*, 3 (pp. 51-58). Taipei: PME.

Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. London: Falmer Press.

Moschkovich, J. (2004). Appropriating mathematical practices: A case study of learning to use and explore functions through interaction with a tutor. *Educational Studies in Mathematics*, *55*, 49-80.

Nyabanyaba, T. (2002). *Examining examination: The ordinary level (O-level) mathematics examination in Lesotho and the impact of recent trends on Basotho students' epistemological access*. Unpublished doctoral dissertation, University of the Witwatersrand, South Africa.

Sfard, A. (2000). Steering (Dis)course between metaphors and rigor: Using focal analysis to investigate an emergence of mathematical objects. *Journal for Research in Mathematics Education*, *31*(3), 296-327.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge: Cambridge University Press.

Straehler-Pohl, H. (2010). Recognizing what the talk is about: Discussing realistic problems as a means of stratification of performance. In U.Gellert, E. Jablonka, & C.Morgan (Eds.), *Proceedings of the Sixth International Mathematics Education and Society Conference* (pp.447-456). Berlin: Freie Universität Berlin.

Tall, D. (Ed.). (1991a). *Advanced mathematical thinking*. Dordrecht: Kluwer.