



PERGAMON

Teaching and Teacher Education 17 (2001) 213–226

---

---

TEACHING  
AND TEACHER  
EDUCATION

---

---

www.elsevier.com/locate/tate

## Teachers' beliefs and practices related to mathematics instruction

Deborah J. Stipek<sup>a,\*</sup>, Karen B. Givvin<sup>b</sup>, Julie M. Salmon<sup>c</sup>, Valanne L. MacGyvers<sup>d</sup>

<sup>a</sup>Graduate School of Education & Information Studies, University of California at Los Angeles, Moore Hall, Box 951521, Los Angeles, CA 90095, USA

<sup>b</sup>LessonLab, Inc., 12436 Santa Monica Bl., Los Angeles, CA 90025, USA

<sup>c</sup>Arcadia Unified School District, 234 Campus Dr., Arcadia, CA 91007, USA

<sup>d</sup>Department of Psychology, University of Louisiana at Lafayette, Lafayette, LA 70504-3131, USA

Received 1 June 1999; received in revised form 13 December 1999; accepted 10 February 2000

---

### Abstract

Beliefs and practices related to mathematics were assessed for 21 fourth- through sixth-grade teachers. At the beginning and the end of the school year teachers' beliefs about (1) the nature of mathematics (i.e., procedures to solve problems versus a tool for thought), (2) mathematics learning (i.e., focusing on getting correct solutions versus understanding mathematical concepts), (3) who should control students' mathematical activity, (4) the nature of mathematical ability (i.e., fixed versus malleable), and (5) the value of extrinsic rewards for getting students to engage in mathematics activities were assessed. (6) Teachers self-confidence and enjoyment of mathematics and mathematics teaching were also assessed. Analyses were conducted to assess the coherence among these beliefs and associations between teachers' beliefs and their observed classroom practices and self-reported evaluation criteria. Findings showed substantial coherence among teachers' beliefs and consistent associations between their beliefs and their practices. Teachers' self-confidence as mathematics teachers was also significantly associated with their students' self-confidence as mathematical learners. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Teacher beliefs; Mathematics teaching; Elementary teachers; Classroom practices

---

### 1. The importance of teacher beliefs

A substantial body of research suggests that teachers' beliefs and values about teaching and learning affect their teaching practices (see reviews by Clark & Peterson, 1986; Fang, 1996; Kagan, 1992; Thompson, 1992). Influencing teachers' beliefs, therefore, may be essential to changing

teachers' classroom practices. This study examines teacher beliefs and practices that are directly related to inquiry-oriented mathematics instruction. The goal is to better understand the nature of teachers' beliefs about mathematics teaching and learning and the links between their beliefs and practices. Ultimately, we hope to inform the design of professional development approaches that will increase inquiry-based teaching of mathematics in classrooms.

The National Council of Teachers of Mathematics (NCTM, 1991) "Standards" stress that

---

\*Corresponding author. Tel.: +1-310-825-1801; fax: +1-310-206-4452.

E-mail address: stipek@gseis.ucla.edu (D.J. Stipek).

mathematics need to be taught as a dynamic tool for thought, not just as a set of operations to be learned. The NCTM Standards stipulate that students need opportunities to communicate math ideas and solve problems with others, that they should engage in mathematical activities with confidence and enthusiasm, and that teachers should use assessment strategies that focus on understanding rather than on right answers. Teachers are encouraged to value and reward students' effort and persistence, and to give children some discretion in how they approach mathematical problems and encourage them to use a variety of approaches to mathematics tasks.

This approach to mathematics instruction, referred to often as "inquiry-oriented" (also "constructivist" or "social-constructivist"), represents fundamental changes in teaching practices — a shift away from the exclusive use of more traditional textbook-based teaching, in which the teacher is in complete control and the students' only goal is to learn operations to get the right answer. This shift to including inquiry approaches is not made easily (see, for example, Prawat, 1992a).

Several researchers have suggested that professional development programs designed to help teachers implement inquiry-oriented mathematics instruction are minimally effective, in part because teachers filter what they learn through their existing beliefs. Cohen and Ball (1990), for example, observed in their study that teachers assimilated new practices to their more traditional beliefs about mathematics education. In their words, "New wine was poured, but only into old bottles" (p. 334; see also Schram & Wilcox, 1988; Skemp, 1978).

### *1.1. Teacher beliefs about mathematics and learning*

Most American teachers have a conception of mathematics<sup>1</sup> as a static body of knowledge,

---

<sup>1</sup>This conceptualization of mathematics is similar to what Skemp (1978) refers to as an instrumental concept of mathematics—a set of fixed plans for performing mathematical tasks involving step-by-step procedures. It is also similar to what Kuhs and Ball (1986) refer to as a content focus, in which students' mastery of mathematical rules and procedures are emphasized.

involving a set of rules and procedures that are applied to yield one right answer. "Knowing" mathematics means being skillful and efficient in performing procedures and manipulating symbols without necessarily understanding what they represent (Thompson, 1992). These beliefs, referred to heretofore as "traditional", about mathematics confer upon teachers the responsibility of transmitting those rules to students. Consistent with this conception of mathematics and mathematics learning, the teacher is in control. Research on prevalent practices finds that in a typical American lesson the teacher reviews or introduces a new procedure, provides students with step-by-step instructions, then assigns students problems on which to practice the procedure (Stigler & Hiebert, 1997; Thomson, 1984, 1985; Wood, Cobb, & Yackel, 1991).

Inquiry-oriented mathematics educators take a more dynamic view of mathematics, conceptualizing it as a discipline that is continually undergoing change and revision (Prawat, 1992b).<sup>2</sup> They embrace a conception of mathematics as a tool for problem solving and a set of cultural understandings that arise out of problem-solving activity (NCTM, 1991; Thompson, 1992). Accordingly, they recommend classroom practices that actively engage students in activities that will assist them to construct mathematical concepts, activities that require reasoning and creativity, gathering and applying information, discovering, and communicating ideas (Ball, 1993; Cobb, Wood, Yackel, & McNeal, 1993; Fennema, Carpenter, Franke, & Carey, 1993; Lampert, 1991; Thompson, 1992; Wood, Cobb, & Yackel, 1991). The role of the teacher is to support and guide this constructive process rather than to transmit discrete knowledge.

---

<sup>2</sup>The reform view of the nature of mathematics is consistent with Hersh's (1986) view that mathematics is a creative, generative process. Knowing mathematics is making mathematics. It is also similar to Skemp's (1978) relational concept of mathematics, as a conceptual structure that enables individuals to construct plans for performing a task, and in which "the means becomes independent of particular ends to be reached" (p. 14), and to Kuhs and Ball's (1986) learner-focused concept of mathematics teaching, which emphasizes learners' personal construction of mathematical knowledge.

Thus, the NCTM Standards promote practices in which teachers give up some of their control over mathematical activity and allow students to initiate their own strategies to solve problems and grapple with contradictions.

Proponents of inquiry approaches are not proposing that teachers dispense altogether with developing students' fluency in mathematics computations, but rather that learning rules and applying them efficiently should not be the exclusive focus of mathematics education, as it is in many American classrooms. The new California Mathematics Framework (California State Department of Education, 1999), for example, points out the importance of a balance between conceptual understanding and computational proficiency. Thus, the two sets of beliefs are not necessarily contradictory, although they are usually pitted against each other in the literature.

Whether inquiry-oriented and traditional beliefs are "pitted against each other" in teachers' minds is unknown. Thus, the first question this study was designed to ask is whether beliefs related to the nature of mathematics (as rules and procedures to achieve one right answer or tools for thought and creative problem solving) and the teacher's role (teaching rules or guiding inquiry) cohere and are opposed to each other, as they are in much of the literature on mathematics education.

The study explored two other beliefs that might be expected to be associated with either traditional or inquiry-oriented beliefs. The first is related to the nature and stability of mathematics ability and the second concerns effective strategies for motivating children. Finally, the study assessed teachers' self-confidence and enjoyment of mathematics. These four additional teacher variables are explained below.

#### *1.1.1. Theories of ability*

Motivation theorists make a distinction between an entity theory of ability (ability is stable and limits the effectiveness of effort) and an incremental theory of ability (ability develops as a consequence of effort and learning (Dweck, 1986; see Stipek, 1998, for a review). Previous research suggests that an entity theory promotes a concern with perfor-

mance (i.e., getting right answers, looking smart) because students want to demonstrate their ability. As a consequence, an entity theory may undermine the focus on understanding that is advocated by mathematics education reformers (Dweck, 1986; Dweck, & Bempechat, 1983). Although there is no logical link between a more traditional view of mathematics teaching and learning and the view that mathematics ability is stable, we suspected that more traditional beliefs might be associated with an entity theory of ability because of their shared focus on correctness.

#### *1.1.2. Motivating students*

Although there is a vast literature on the effects of teaching practices on student motivation to learn, few studies have specifically examined teachers' beliefs about how to motivate students to engage in mathematics activities. One exception is a study by Nolan and Nicholls (1994), in which elementary and middle school teachers rated the value of various strategies to increase the motivation of disaffected students. The study did not assess associations between teachers' beliefs about motivating students and their beliefs about the nature of mathematics and mathematics learning. We hypothesized in the present study that traditional beliefs about mathematics teaching, which put the teacher in control of students' learning, would be associated with beliefs in the value of extrinsic, teacher-controlled motivational strategies, such as giving praise, rewards, or punishment. More inquiry-oriented beliefs were expected to be associated with the belief that motivation was based substantially on the qualities (e.g., challenge and interest level) of tasks.

#### *1.1.3. Teacher's self-confidence and enjoyment*

Inquiry-oriented mathematics teaching requires considerable knowledge of mathematics because teachers need to diagnose the concepts that underlie students' responses or problem-solving strategies and respond with appropriate scaffolding. Inquiry-oriented teaching may also require a high level of self-confidence (which is presumably correlated with their real understanding), given that teachers cannot rely on a textbook with step-by-step instructions. Accordingly, we predicted that

teachers' inquiry-oriented beliefs would be associated with greater self-confidence in their mathematics ability. Because self-confidence has been shown in many studies to be associated with greater enjoyment (Mac Iver, Stipek, & Daniels, 1991; see Stipek, 1998, for a review), we expected more traditional teachers' beliefs and practices to be associated with less enjoyment as well.

Mathematics education reformers have also stressed the importance of students being self-confident learners who enjoy mathematics activities (Fennema et al., 1993; McLeod, 1992; Middleton, 1995; Resnick, Bill, Lesgold, & Leer, 1991). It is likely that teachers who lack self-confidence and do not enjoy mathematics have difficulty fostering these beliefs and attitudes in their students, but this hypothesis has not heretofore been tested. The current study, therefore, assessed associations between teachers' and students' enjoyment and self-confidence related to mathematics.

In summary, this study was designed, first, to develop a measure of teacher beliefs about mathematics instruction that directly contrasts more traditional practices to practices suggested in the mathematics reform literature, and to assess associations among the different beliefs mentioned above. We expected coherence among the beliefs referred to above as traditional and coherence among the beliefs referred to as inquiry-oriented. We hypothesized that the more teachers embraced traditional beliefs about mathematics teaching and learning and the less they embraced inquiry-oriented beliefs, the more they would espouse: (1) an entity theory of mathematics ability, (2) the belief that the teacher needs to be in control of classroom instruction, and (3) the view that extrinsic, teacher-controlled rewards are effective strategies for motivating students to engage in mathematics activities, and the less they would express (4) self-confidence and (5) enjoyment related to teaching mathematics.

### *1.2. Beliefs and practices*

Second, the study was designed to address associations between teachers' beliefs and their classroom practices. Nearly all extant studies that have

examined associations between teachers' beliefs about mathematics and mathematics teaching and their classroom practices are based on qualitative case studies of one or a few teachers (see Thompson, 1992, for a review). These case studies suggest some congruence between beliefs and practices. In one of the few published quantitative studies, Peterson, Fennema, Carpenter, and Loef (1989) found very specific associations between beliefs and practices. For example, teachers who believed that children learn mathematics by constructing their own understanding in the process of solving problems (an inquiry-oriented view, which they refer to as "cognitively-based" beliefs) used more word problems in instruction and spent more time developing children's counting strategies before teaching number facts than teachers who believed that mathematics is learned by receiving knowledge about mathematical operations from the teacher in discrete units (a more traditional view).

Research on educational contexts suggests that students' concerns about performance are minimized when teachers stress effort, learning and understanding, and when they create a classroom climate in which risk-taking is encouraged and supported (i.e., wrong answers are treated as a natural part of learning that can be used to clarify misunderstandings) (see Stipek, 1996, 1998, for reviews). These practices, promoted by motivation researchers, are consistent with mathematics reformers' recommendation that teachers should emphasize process and encourage students to seek alternative solutions rather than to find a single correct solution (Stipek et al., 1998). The research on concepts of ability and concerns about performance suggests the value of focusing students' attention on learning by incorporating inadequate solutions into instruction, scaffolding responses to higher levels of understanding, and giving substantive feedback that can be used to guide future problem-solving efforts (see Carpenter & Fennema, 1991; Cobb, Wood, & Yackel, 1993; Lampert, 1991; Prawat, Remillard, Putnam, & Heaton, 1992b; Stipek et al., 1998). We expected, therefore, that the more teachers held what we refer to as traditional beliefs about mathematics instruction, the more they would emphasize performance and efficiency

(speed), convey to students that mistakes were something to be avoided, thus producing a high-risk environment, and be relatively controlling of students' mathematics activity. We expected that the more teachers supported inquiry-oriented, constructivist beliefs, the more they would emphasize effort and understanding rather than performance and speed, convey to students that mistakes are a natural part of learning (creating a low-risk environment), and allow some student autonomy. And we hypothesized that the high self-confidence and enjoyment that we predicted for teachers who held more inquiry-oriented beliefs would translate into the expression of greater enthusiasm during mathematics instruction in the classroom.

Finally, teachers' beliefs should have implications for their evaluation practices. We hypothesized that the more teachers held traditional beliefs, the more they would emphasize relative performance in evaluations given to parents. The more teachers held constructivist, inquiry-oriented beliefs, the more they were expected to emphasize effort, creativity, and independence in their student evaluations.

## 2. Method

### 2.1. Participants

#### 2.1.1. Teachers

Twenty-one fourth- through sixth-grade teachers from elementary schools throughout Los Angeles County (California) served as participants. They included all teachers who responded to a letter requesting volunteers to participate in a study of mathematics teaching, and who planned to teach fractions and measurement that year. Although teachers volunteered to participate in the study, they were paid for their participation, which may have been one incentive, and they varied substantially in their instructional approach. To our knowledge, the only variable that distinguishes them from other teachers was their willingness to participate in the study.

The levels of experience of the one male and 20 female teachers varied from one to more than 20 years. All of the teachers had teaching credentials

and taught in schools serving predominantly children from low-income families. A substantial number (over 25%) of the students in eight of the classrooms had limited English proficiency (predominantly Spanish speaking). Modal class size was high (about 32 children).

#### 2.1.2. Students

In each participating teacher's classroom, data were collected on all of the children present on the day of the student assessment. Only the 437 students who were present at both the beginning-of-the-year and the end-of-the-year administrations of the assessments were included in the analyses reported here (231 boys and 201 girls, and 5 students for whom we do not have information on gender). Subjects were predominantly Latino, but represented diverse ethnic backgrounds: there were 26 African Americans, 288 Latinos, 28 Asians, 66 Anglos, and 29 from other or mixed ethnic groups.

## 2.2. Measures

### 2.2.1. Teacher beliefs

At the beginning and end of the school year, teachers were asked to complete a 4-page survey of their "Beliefs About Mathematics and Teaching". The survey measured the strength of their agreement with 57 statements, including 14 fillers. Each item was measured on a 6-point Likert-type scale (1 = strongly disagree; 6 = strongly agree). The survey was designed to tap the beliefs below (see Table 1 for alphas and means). Items that were reversed to form scales have a "-" after them:

1. *Math as a set of operations versus a tool for thought*: 5 items, e.g., "Mathematics involves mostly facts and procedures that have to be learned"; "In mathematics you can be creative and discover things on your own" (-).
2. *Correct answers versus understanding as primary goal*: 7 items, e.g., "Students who aren't getting the right answers need to practice on more problems"; "It doesn't matter whether students get the right answer as long as they understand the math concepts inherent in a problem" (-).

Table 1  
Teacher beliefs: alphas, mean scores, and pre–post correlations

Belief dimensions	Alphas		Means (SD)		Pre–post correlation
	Pretest	Post	Pretest ( <i>N</i> = 21)	Post ( <i>N</i> = 20)	
Math as operations versus tool for thought	0.75	0.73	1.83 (0.69)	1.93 (0.95)	0.49 <sup>a</sup>
Focus on correctness versus understanding	0.68	0.72	2.21 (0.74)	2.09 (0.72)	0.81 <sup>b</sup>
Teacher control versus child initiation	0.79	0.80	2.24 (0.72)	1.99 (0.61)	0.81 <sup>b</sup>
Entity theory	0.84	0.71	2.46 (0.80)	2.32 (0.62)	0.50 <sup>a</sup>
Extrinsic motivation	0.67	0.63	2.16 (0.74)	2.01 (0.80)	0.75 <sup>b</sup>
Confidence	0.81	0.79	4.34 (0.91)	4.48 (0.88)	0.72 <sup>b</sup>
Enjoyment	0.83	0.44	4.76 (1.15)	4.87 (1.05)	0.47 <sup>a</sup>

<sup>a</sup>*p* < 0.05.

<sup>b</sup>*p* < 0.01.

3. *Teacher control versus some child autonomy in classroom lessons*: 6 items, e.g., “It’s important for students to complete assignments exactly as the teacher planned”; “Students should construct many of their own math problems” (–).
4. *Entity versus incremental view of intellectual ability*: 11 items, e.g., “Mathematical ability is something that remains relatively fixed throughout a person’s life”; “All of my students would be good at math if they worked hard at it” (–).
5. *Extrinsic versus intrinsic motivation*: 5 items, e.g., “Giving rewards is a good strategy for getting students to complete math assignments”; “Students will work hard on interesting and challenging math tasks, whether or not their work is graded” (–).
6. *Confidence in teaching math*: 6 items, e.g., “I am confident that I understand the math material I teach”; “When I teach math I often find it difficult to interpret students’ wrong answers” (–).
7. *Enjoyment of math*: 3 items, e.g., “Math is my favorite subject to teach”; “I don’t enjoy doing math” (–).

### 2.2.2. Teacher practices

2.2.2.1. *Videotapes*. Teachers’ classroom practices were coded from a series of videotapes of at least two instructional periods for each teacher. To

eliminate the effects of the particular mathematical concept being taught, videotaping in all classrooms was done during instruction on adding and comparing fractions.

A coding system was developed to characterize teachers’ practices. Ratings for each dimension were done on a scale, from 1 (“not at all like this teacher”) to 5 (“very much like this teacher”), based on a detailed description of practices related to the dimension. Reliable codes were developed for the following seven dimensions:

1. The degree to which the teacher emphasized *performance* outcomes (e.g., getting correct answers, high grades), such as by praising or criticizing performance, referring to grades, or recommending that students avoid problems that they might not be able to do. A low score was given if the teacher rarely made comments about performance outcomes.

2. The degree to which teachers emphasized *speed* in completing tasks (e.g., “hurry up, you should be able to finish this quickly”, “you should be done by now”). A low score was given if the teacher made no mention of speed or did not encourage students to complete tasks quickly.

3. The type of environment the teacher fostered: high scores were given when the social context was psychologically *high risk* and threatening (e.g., teacher ignored wrong answers and asked another child, threatened tests, made comments conveying

low expectations (e.g., I didn't expect you to get that right), tolerated put-downs by students). A low score was given when there was no risk of being embarrassed or put down for mistakes or lack of understanding (teacher conveyed that mistakes are OK, scaffolded a student having difficulty, refused to tolerate students putting each other down).

4. The degree to which the teacher encouraged and gave opportunities for students to work *autonomously*, such as by pointing out resources in the room, encouraging students to engage in self-evaluation, giving students choices in how to solve problems, and refraining from offering unnecessary help. A low score was given when the teacher controlled all aspects of math activities (e.g., the nature of tasks, when they were done, the strategies used to complete problems).

5. The degree to which the teacher *emphasized student effort* and conveyed the message that effort will eventually pay off (e.g., by praising effort or encouraging students to keep trying, giving them instrumental help that facilitates their progress, giving plenty of time). A low score was given when teachers made no reference to effort and conveyed that effort may not pay off for all students (e.g., "just do what you can").

6. The degree to which the teacher emphasized and encouraged students to *focus on understanding* and mastery (e.g., by encouraging students to try alternative strategies, asking them to explain their strategies, asking them to apply concepts in new contexts, focusing comments on problem-solving strategies, or using inadequate solutions in instruction). Teachers were given a low score if they did not emphasize learning and mastering basic concepts or conveyed to students that they were not necessarily expected to understand the concepts behind the operation.

7. The level of *teachers' enthusiasm* and interest in mathematics: a high score was given when teachers appeared to enjoy mathematics and showed interest and personal engagement with the task. Teachers received a low score if they showed negative attitudes toward mathematics in general or for the task at hand (e.g., "I know math isn't fun, but I had to learn it and so do you"; "I never liked doing fractions").

Raters observed videotapes as many times as they believed necessary to make reliable ratings. Each lesson was given two sets of ratings, one that reflected all of the time the teacher was involved in instruction with the whole class, and one for the periods in which children worked on activities or problems in small groups or individually. These two parts of the instructional period were coded separately because it seemed possible that teachers might behave differently when they were directing whole-class lessons from when they were supervising student work. In most lessons these two segments were clearly distinguishable. A common format was for the teacher to present a lesson to the whole class at the beginning of the time set aside for mathematics and for students subsequently to work collaboratively or independently on problems or activities. In some classrooms the teacher returned to a whole-class format at the end of the mathematics period. A few of the teachers moved back and forth from a more teacher-dominated, whole-class format to a format that involved students working individually or in small groups.

Twelve of the lessons were rated independently by two raters. Table 2 shows the level of agreement between two raters (percentage of ratings that were exact matches or were within one point of a match), and the correlations between the whole-class and student-activity segments. As can be seen from the correlations, teachers' instructional styles were fairly consistent across these two settings. An average of the two ratings for each dimension was therefore used in analyses of teachers' classroom practices.

*2.2.2.2. Student evaluation.* A second source of information about teachers' practices came from a questionnaire teachers completed on their criteria for formal evaluation. They were asked, specifically, to what degree they used (1) effort, (2) relative scores, (3) creativity, and (4) independence, "... in determining students' term grades" (or, if they did not give grades, which did they stress in written reports to parents). Responses were on a 1- to 5-point Likert scale (1 = not at all; 5 = mostly).

#### *2.2.3. Student reports of competence and enjoyment*

Students were asked, at the beginning and the end of the school year, questions about their own

Table 2

Reliabilities, mean scores, and correlations between whole-class and student work settings ( $N = 21$ )

Dimension	% agree <sup>a</sup>	Means (SD)		$r$ : whole class and student work <sup>b</sup>
		Whole class	Student work	
Emphasis on performance	83/70	3.54 (1.15)	3.48 (1.04)	0.76 <sup>c</sup>
Emphasis on speed	92/90	1.85 (0.84)	1.85 (0.78)	0.62 <sup>c</sup>
Low-risk environment	83/80	3.00 (1.06)	3.12 (1.03)	0.88 <sup>d</sup>
Encourage autonomy	92/80	2.57 (1.28)	2.92 (0.97)	0.82 <sup>d</sup>
Emphasis on effort	92/90	2.86 (1.10)	3.40 (0.79)	0.69 <sup>d</sup>
Emphasis on understanding	92/100	3.01 (1.03)	3.08 (0.97)	0.82 <sup>d</sup>
Teacher enthusiasm	92/100	3.13 (0.66)	3.13 (0.70)	0.82 <sup>d</sup>

<sup>a</sup>Percentage of agreement (within one point) for two raters, for class/student work.<sup>b</sup>Correlation coefficients between whole-class and student independent or small-group work ratings.<sup>c</sup> $p < 0.01$ .<sup>d</sup> $p < 0.001$ .

competence in mathematics and about how much they enjoyed doing mathematics. All responses were on 1–6 Likert scales. For self-perceptions of mathematics ability, students were asked three questions (e.g., “How good are you at math?”). They responded to three additional questions for enjoyment (e.g., “How much do you like doing math?”).

### 3. Results

#### 3.1. Stability of teacher beliefs

In part to test the reliability of the scales, the first analyses assessed the degree to which the teacher beliefs that we measured were stable from the beginning to the end of the school year. The correlation coefficients, shown in Table 1, suggest a fair degree of stability for all dimensions assessed, especially for the degree to which they focused on correctness versus understanding, how much control over math activities they believed teachers should exercise, their belief in the value of extrinsic motivation, and their own confidence in mathematics.

#### 3.2. Coherence in teacher beliefs

Analyses were then conducted to examine associations among teachers' beliefs. A principal compo-

Table 3

Results of factor analysis of teacher's beliefs

Teacher beliefs	Factor 1	Factor 2
Math as operations	<b>0.82</b>	0.21
Focus on correctness	<b>0.72</b>	0.46
Teacher control	<b>0.84</b>	0.37
Entity theory	<b>0.81</b>	–0.05
Extrinsic motivation	<b>0.89</b>	0.06
Confidence	–0.51	<b>0.76</b>
Enjoyment	–0.63	<b>0.68</b>

nents factor analysis resulted in two interpretable factors, shown in Table 3, which accounted for 57% of the variance (eigenvalue, 4.01), and 21% (eigenvalue, 1.44), of the variance, respectively. The factor analysis reported was based on the average of pre- and post-test teacher belief scores. Analyses conducted separately for pre- and post-test scores resulted in the same two factors with the same items loading on each factor.

The factor structure shows two groupings of variables. Loading on the first factor comprised dimensions for which high scores were presumed to be associated with traditional, transmission theories of teaching (that mathematics is a set of operations used to get correct solutions to problems rather than a tool for thought; that correct answers were the primary goal for students; that teachers



Table 4  
Correlations between teacher's beliefs and their practices ( $N = 19$ )

Classroom practices	Teacher beliefs				
	Math as operation	Focus on correction	Teacher control	Entity theory	Extrinsic motivation
Performance emphasis	<b>0.75<sup>a</sup></b>	<b>0.41<sup>b</sup></b>	<b>0.70<sup>c</sup></b>	<b>0.53<sup>d</sup></b>	<b>0.59<sup>c</sup></b>
Speed emphasis	<b>0.50<sup>d</sup></b>	0.10	0.28	<b>0.14</b>	<b>0.26</b>
Low-risk environment	– <b>0.59<sup>c</sup></b>	– 0.25	– <b>0.48<sup>d</sup></b>	– <b>0.29</b>	– <b>0.53<sup>d</sup></b>
Autonomy	– <b>0.64<sup>c</sup></b>	– 0.29	– <b>0.59<sup>c</sup></b>	– <b>0.43<sup>b</sup></b>	– <b>0.50<sup>d</sup></b>
Effort emphasis	– <b>0.51<sup>d</sup></b>	– 0.15	– 0.36	– <b>0.33</b>	– <b>0.25</b>
Understanding emphasis	– <b>0.66<sup>c</sup></b>	– 0.26	– <b>0.60<sup>c</sup></b>	– <b>0.43<sup>b</sup></b>	– <b>0.58<sup>c</sup></b>
Teacher enthusiasm	– <b>0.55<sup>d</sup></b>	– 0.32	– <b>0.53<sup>d</sup></b>	– <b>0.40<sup>b</sup></b>	– <b>0.54<sup>d</sup></b>

<sup>a</sup> $p < 0.001$ .

<sup>b</sup> $p < 0.10$ .

<sup>c</sup> $p < 0.01$ .

<sup>d</sup> $p < 0.05$ .

need to be in complete control; that grades and rewards are useful to motivate students' effort). Loading on this factor also was the belief that math ability is stable and limits the effect of effort, rather than malleable and based on effort and practice. Confidence in teaching mathematics and enjoyment of math loaded on a second factor. These two factors were negatively correlated to each other,  $r = -0.51$ ,  $p < 0.05$ . Thus, teachers scoring high on the more traditional beliefs were less self-confident about teaching mathematics and enjoyed it less.

An Alpha statistic, computed for the scales that loaded on each of the two factors, provided further evidence of the coherence of the various beliefs measured. The alpha for the five dimensions that loaded on the first factor — for which high scores reflected traditional beliefs and low scores reflected inquiry-oriented beliefs — was 0.90. The alpha was 0.84 for the two variables (confidence and enjoyment) that loaded on the second factor.

It is important to note that this coherence was found on a small sample of teachers. The replication (pre- and post-test) lends some credibility to the findings, but it will be important in future research to replicate the findings also on a larger sample of teachers.

### 3.3. Teacher beliefs and practices

#### 3.3.1. Classroom instruction

Correlations were computed between the five teacher belief scales that concerned mathematics

teaching and learning and the seven ratings of classroom practice. Each teacher belief score represented the average of the pre- and post-test assessments. The classroom practice scores were the average of the whole-class and student-work periods.

Table 4 shows that several teacher beliefs were consistently associated with observed practices. Three beliefs — that mathematics is a set of operations and procedures to be learned, that the teacher should be in complete control, and that extrinsic reinforcements are effective strategies for motivating students to engage in mathematics activities — were positively associated with an emphasis on performance rather than learning in the classroom, and negatively associated with an emphasis on understanding, student autonomy, a low-risk environment, and teacher enthusiasm. The conceptualization of mathematics as operations and procedures was also positively associated with teachers' emphasis on speed and negatively associated with their emphasis on effort. An emphasis on performance rather than understanding was also associated with the belief that mathematics ability is stable and not very amenable to change.

#### 3.3.2. Evaluation criteria

The next set of analyses examined associations between teachers' beliefs (pre- and post-test combined) and their criteria for formal evaluations. The analyses summarized in Table 5 show that the more

Table 5  
Correlations between teacher beliefs and their evaluation practices ( $N = 20$ )

	Criteria for evaluation			
	Effort	Relative performance	Creativity	Independence
Math as operations	– 0.27	0.05	– 0.36	0.35
Focus on correctness	– <b>0.53<sup>a</sup></b>	– 0.14	– <b>0.49<sup>a</sup></b>	<b>0.56<sup>b</sup></b>
Teacher control	– <b>0.48<sup>a</sup></b>	– 0.09	– <b>0.58<sup>b</sup></b>	<b>0.59<sup>b</sup></b>
Entity theory	– 0.11	– 0.06	– 0.03	<b>0.53<sup>a</sup></b>
Extrinsic motivation	– 0.29	– 0.10	– <b>0.41<sup>c</sup></b>	<b>0.60<sup>b</sup></b>

<sup>a</sup> $p < 0.05$ .

<sup>b</sup> $p < 0.01$ .

<sup>c</sup> $p < 0.10$ .

teachers believed they should control instruction and the more they claimed to focus on correctness rather than understanding, the less likely they were to consider effort and creativity, and the more likely they were to consider independence in students' term grades or reports to parents. Consideration of student independence was also relatively high for teachers who endorsed the belief that grades and rewards are effective motivation strategies.

### 3.3.3. Student self-confidence and enjoyment

Two partial correlation coefficients were computed, the first between teachers' confidence as mathematics teachers and the classroom average of students' self-confidence as mathematics learners at the end of the year, with students' self-confidence at the beginning of the year covaried ( $n = 18$ ,  $r = 0.54$ ,  $p < 0.05$ ). The second partial correlation computed was between teachers' and students' enjoyment related to mathematics activities at the end of the year, with students' enjoyment at the beginning of the year covaried ( $r = 0.05$ ). The classroom averages of students' relevant pretest scores were partialled out in both correlations.

## 4. Discussion

### 4.1. Coherence and stability of teacher beliefs

The factor analysis and alphas revealed a coherent set of beliefs about the nature and learning of

mathematics, the role of the teacher, effective motivation strategies, and the nature of mathematical ability. Five dimensions of beliefs were strongly associated with each other: (1) mathematics is a set of operations to be learned; (2) students' goal is to get correct solutions; (3) the teacher needs to exercise complete control over mathematics activities; (4) mathematics ability is fixed and stable; and (5) extrinsic rewards and grades are effective strategies for motivating students to engage in mathematics. Or, looking at the opposite end of the dimensions, there was coherence in the belief that mathematics is a tool for thought, that students' goal is to understand, that students should have some autonomy, that mathematics ability is amenable to change, and that, in the absence of rewards, students will want to engage in mathematics if the tasks are interesting and challenging. In addition to being coherent, these beliefs were also fairly consistent over the course of a school year.

No previous study has examined relationships among this particular set of beliefs. However, one previous study observed, as we did, an association between need for control and an entity theory of ability (Midgley, Feldlaufer, & Eccles, 1988). In that study, teachers who considered math ability to be a fixed trait perceived themselves to be less efficacious and had stronger needs to control student behavior than did teachers who believed that math ability can readily change. Perhaps teachers who believe that students' ability is relatively fixed do not want to allow students much autonomy

because they assume that students who are low in mathematics ability will not be able to use it productively.

As predicted, teachers who embraced more traditional beliefs about mathematics and learning had lower self-confidence and enjoyed mathematics less than teachers who held more inquiry-oriented views. We suspect that less confident teachers are drawn to a set of beliefs and practices that require relatively less teacher judgment and decision-making. Teachers who focus on procedures and correct answers can teach in a very prescribed way, following the procedures described in the textbook and even using the answer sheets in the teacher's manual to correct student work. Teachers who focus on students' own or socially constructed understanding of mathematics need to analyze the meaning of students' errors and strategies and to provide instructional input that is directly linked to those analyses. If our speculation is valid, building teachers' self-confidence in math, which requires building their mathematical understanding, could be an important, if not necessary, ingredient in moving them toward more inquiry-oriented beliefs and practices.

#### 4.2. *Beliefs and practices*

The associations between teachers' beliefs and their classroom practices were all in the predicted directions. More traditional beliefs were associated with more traditional practices. Thus, for example, the higher teachers scored on the traditional beliefs, the more they emphasized performance (e.g., getting correct answers, getting good grades) and speed in their classrooms, rather than learning and understanding. Teachers who held the more traditional beliefs also gave students relatively less autonomy and maintained a social context in which mistakes were something to be avoided.

Also as predicted, teachers' support for an entity theory of mathematics ability was significantly associated with an emphasis on performance in the classroom (Dweck, 1986; Dweck & Bempechat, 1983), but an entity theory was not associated with other practices assessed. The belief that mathematics ability is something you "have or don't have and

you can't do much to change it" may, however, have implications for instructional practices that were not assessed in this study. Previous studies suggest that an entity theory may minimize a teacher's effort and persistence with students whom they have identified as low in ability (Dweck, 1986; Dweck & Bempechat, 1983). Prawat (1992b) proposes that a focus on individual differences in ability also undermines teachers' attention to subject-matter learning. Instead of attending to children's understanding related to a particular mathematics problem in a particular context, teachers who hold an entity theory of ability may focus primarily on students' overall skill level. Accordingly, they may be more likely to group by ability and adjust assignments and teaching between groups but not within groups. Their attention would be more on how much students knew in general, relative to other students, rather than on students' interpretations and understandings of particular math concepts.

It is not obvious why teachers who held the more traditional beliefs claimed to enjoy mathematics less and exhibited relatively less enthusiasm in their classrooms. Perhaps teachers who did not enjoy and showed less enthusiasm for mathematics themselves assumed that extrinsic motivation and teacher control were important for students because they were not very interested in mathematics and would not do it if it was not required. Enthusiasm in the classroom may also be affected by teachers' practices. For example, the more varied and personalized interactions with students involved in an inquiry approach may have contributed to teachers' enthusiasm and interest.

Three of the teacher belief scales were consistently associated with criteria teachers reported to use in their formal evaluations of students. The more teachers claimed to believe that teacher control and correctness were important, the less they emphasized effort and creativity and the more they emphasized student independence in their evaluations. Teachers who believed in the value of extrinsic motivation strategies were also less likely to emphasize creativity and more likely to emphasize independence in their grading. An entity belief was also associated with a greater emphasis on independence. These more traditional beliefs were not,

however, positively associated with using relative performance as a criterion for evaluation.

Including creativity as a criterion for evaluation is more consistent with an inquiry-oriented, constructivist view of mathematics learning — in which students are encouraged to explore mathematics problems and attempt multiple strategies — than with a traditional view, which focuses on learning procedures to get correct answers. Because we initially conceptualized independence in terms of student initiative and control, we had expected independence to be associated with inquiry-oriented beliefs as well. But we did not articulate our conceptualization in our measure. Our interpretation, in the light of the findings, is that the participating teachers conceptualized independence as the opposite of dependence on the teacher (e.g., asking questions, asking for help). Inquiry-oriented mathematics teachers should value this kind of independence less than traditional teachers because reform advocates emphasize the value of supportive (e.g., using scaffolding) interactions between the learner and a more knowledgeable person — either a peer or an adult. Independence would not be emphasized or highly valued if social interaction was considered to be critical to learning.

#### *4.3. Teachers' and students' self-confidence and enjoyment*

Teachers' enjoyment of mathematics did not predict students' enjoyment at the end of the school year, perhaps in part because teacher enjoyment was relatively less stable. But teachers' self-confidence as mathematics teachers was significantly correlated with students' perceptions of their own competence as mathematics learners, even when students' beginning-of-the-year perceived competence scores were partialled out. Teachers may have influenced students' beliefs directly by simply modeling self-confidence themselves. Or, the effect could be indirect. Perhaps teachers who judged their competencies as mathematics teachers to be high were, in fact, better teachers who produced greater learning, and consequently more self-confidence in students. Although open to interpretation, our finding of a link between teacher and student

self-confidence suggests the value of further exploration of how teachers' self-perceptions might influence students' self-perceptions. The finding provides further support for the importance of teachers having good mathematical understanding. Increased understanding of mathematics should enhance the quality of their teaching and give them self-confidence, that may, directly or indirectly, contribute to their students' self-confidence as math learners.

#### *4.4. Implications for teacher training*

Our findings indicate that teachers had a fairly coherent set of beliefs which predicted their instructional practices. Inquiry-oriented mathematics teaching, as proposed in the NCTM (1991) Standards, appears to conflict with more traditional beliefs about the nature of mathematics and of mathematics teaching and learning. Assuming that beliefs influence practices, as these data suggest, the beliefs of many teachers may need to be changed to achieve broader implementation of inquiry-oriented approaches. But how is this to be achieved?

Previous research suggests that teachers' beliefs tend not to change much from the time they enter until they leave pre-service training programs and that their beliefs are generally not influenced by reading and being asked to apply findings of educational research. As mentioned above, beliefs probably persist in part because they serve as filters through which new information is processed (Cohen & Ball, 1990; Duffy & Roehler, 1986; Hollingsworth, 1989; Kagan, 1992; Thompson, 1984).

Reflection on classroom experiences, however, has been shown to be effective in changing teachers' beliefs. Cohen and Ball (1990), for example, describe a very traditional teacher who was taught to implement an inquiry-oriented fractions lesson. The teacher did so, precisely as he was instructed. Reflecting on his students' reactions, the teacher indicated that he was amazed to find that his fifth graders could think and reason in such advanced ways. The authors speculate that if this teacher continued his efforts to implement inquiry-oriented lessons, while engaging in guided reflection, he might ultimately change his beliefs about

mathematics teaching and learning. Case studies conducted by Wood, Cobb, and Yackel (1991), and Prawat (1992a) provide additional examples of teachers' beliefs and practices changing as they resolved conflicts that arose between previously established beliefs and their observations of students' responses to their attempts to change their practices (see also, Cobb, Wood, & Yackel, 1990). Thus, we agree with many current researchers that for meaningful and lasting change to occur, teachers need to engage in practical inquiry (Franke, Carpenter, Fennema, Ansell, & Behrend, in press) — to move back and forth among a variety of settings to learn about new instructional strategies, to try them out in their own classrooms, and to reflect on what they observed in a collaborative setting (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Franke et al., in press; Kagan, 1992; Peterson, Fennema, Carpenter, & Loef, 1989; Wood, Cobb, & Yackel, 1991). Whatever approach is used, it is clear that beliefs and practices are linked, and emphasis in teacher professional development on either one without considering the other is likely to fail.

## Acknowledgements

The authors are grateful to the many teachers and students who participated in this project.

## References

- Ball, D. (1993). With an eye toward the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal*, 93, 373–397.
- Borko, H., Mayfield, V., Marion, S., Flexer, R., & Cumbo, K. (1997). Teachers' developing ideas and practices about mathematics performance assessment: Successes, stumbling blocks, and implications for professional development. *Teaching and Teacher Education*, 13, 259–278.
- California State Department of Education (1999). *Mathematics framework for California public schools: Kindergarten through grade twelve*. Sacramento, CA: Author.
- Carpenter, T., & Fennema, E. (1991). Research and cognitively guided instruction. In E. Fennema, T. Carpenter, & S. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 1–16). Albany, NY: SUNY.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. Wittrock (Ed.), *Handbook of research in teaching* (3rd ed.) (pp. 255–296). New York: MacMillan.
- Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. Davis, C. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics, Journal for Research in Mathematics Education Monograph* (pp. 125–146). Reston, VA: National Council of Teachers of Mathematics.
- Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. Forman, N. Minick, & C. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91–119). New York: Oxford University Press.
- Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1993). Mathematics as procedural instructions and mathematics as meaningful activity: The reality of teaching for understanding. In R. Davis, & C. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 119–134). Boston: Allyn & Bacon.
- Cohen, D. K., & Ball, D. L. (1990). Relations between policy and practice: A commentary. *Educational Evaluation and Policy Analysis*, 12(3), 331–338.
- Duffy, G., & Roehler, L. (1986). Constraints on teacher change. *Journal of Teacher Education*, 36, 55–58.
- Dweck, C. (1986). Motivational processes affecting learning. *American Psychologist*, 41, 1040–1048.
- Dweck, C., & Bempechat, J. (1983). Children's theories of intelligence: Consequences for learning. In S. Paris, G. Olson, & H. Stevenson (Eds.), *Learning and motivation in the classroom* (pp. 239–255). Hillsdale, NJ: Erlbaum.
- Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, 38(1), 47–65.
- Fennema, E., Carpenter, T., Franke, M., & Carey, D. (1993). Learning to use children's mathematical thinking: A case study. In R. Davis, & C. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 93–117). Boston, MA: Allyn & Bacon.
- Franke, M., Fennema, E., Carpenter, T., Ansell, E., & Behrend, J. (1998). Understanding teachers' self-sustaining, change in the context of professional development. *Teaching and Teacher Education*, 14, 67–80.
- Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymocxko (Ed.), *New directions in the philosophy of mathematics* (pp. 9–28). Boston: Birkhauser.
- Hollingsworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal*, 26, 160–190.
- Kagan, D. M. (1992). Implications of research on teacher belief. *Educational Psychologist*, 27, 65–90.
- Kuhs, T., & Ball, D. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills, and dispositions*. East Lansing: Michigan State University, Center on Teacher Education.
- Lampert, M. (1991). Connecting mathematical teaching and learning. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 121–152). New York: SUNY.
- Mac Iver, D., Stipek, D., & Daniels, D. (1991). Explaining within semester changes in student efforts in junior and senior high school courses. *Journal of Educational Psychology*, 83, 201–211.

- McLeod, D. B. (1992). Research on affect in mathematics: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: MacMillan.
- Middleton, J. A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal constructs approach. *Journal for Research in Mathematics Education*, 26, 254–279.
- Midgley, C., Feldlaufer, H., & Eccles, J. (1988). The transition to junior high school: Beliefs of pre-and post-transition teachers. *Journal of Youth and Adolescence*, 17, 543–562.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- Nolan, S. B., & Nicholls, J. G. (1994). A place to begin (again) in research on student motivation: Teachers' beliefs. *Teaching and Teacher Education*, 10, 57–69.
- Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. F. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction*, 6(1), 1–40.
- Prawat, R. (1992). Are changes in views about mathematics teaching sufficient? The case of a fifth-grade teacher. *The Elementary School Journal*, 93(2), 195–211.
- Prawat, R. S. (1992a). Teachers' beliefs about teaching and learning: A constructivist perspective. *American Journal of Education*, 100(3), 354–395.
- Prawat, R. S., Remillard, J., Putnam, R. T., & Heaton, R. M. (1992b). Teaching mathematics for understanding: Case studies of four fifth-grade teachers. *The Elementary School Journal*, 93, 145–152.
- Resnick, L., Bill, V., Lesgold, S., & Leer, M. (1991). Thinking in arithmetic class. In B. Means, C. Chelemer, & M. S. Knapp (Eds.), *Teaching advanced skills to at-risk students: Views from research and practice* (pp. 27–67). San Francisco: Jossey-Bass.
- Schram, P., & Wilcox, S. (1988). Changing preservice teachers' conceptions of mathematics learning. In: M. Behr, C. Lacampagne, & M. Wheeler (Eds.), *PME-NA: Proceedings of the tenth annual meeting*, November (pp. 349–355). DeKalb, IL: Northern University.
- Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, 26(3), 9–15.
- Stigler, J., & Hiebert, J. (1997). Understanding and improving classroom mathematics, instruction: An overview of the TIMSS video study. *Phi Delta Kappan*, September 12–21.
- Stipek, D. (1996). Motivation and instruction. In D. Berliner, & R. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 85–113). New York: Macmillan.
- Stipek, D. (1998). *Motivation to learn: From theory to Practice* (3rd ed.). Needham Heights, MA: Allyn and Bacon.
- Stipek, D., Salmon, J., Givvin, K., Kazemi, E., Saxe, G., & MacGyvers, V. (1998). The value (and convergence) of practices promoted by motivation researchers and mathematics education reformers. *Journal for Research in Mathematics Education*, 29, 465–488.
- Thompson, A. (1984). The relationship of teachers' conceptions of mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15, 105–127.
- Thompson, A. (1985). Experience, problem solving, and learning mathematics: Considerations in developing mathematics curricula. In E. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 189–236). Hillsdale, NJ: Erlbaum.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.
- Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal*, 28(3), 587–616.