Optimal inventory control in a multi-period newsvendor problem with non-stationary demand

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ABSTRACT

The optimal control of inventory in supply chains plays a role in the competitiveness of a corporation. The inventory cost can account for half of company’s logistics cost. The classical inventory models, e.g., newsvendor and EOQ models, assume either a single or infinite planning periods. However, these models may not be applied to perishable products which usually have a certain shelf life. To optimize the total logistic cost for perishable products, this paper presents a multi-period newsvendor model, and the problem is formulated as a multi-stage stochastic programming model with integer recourse decisions. We extend the progressive hedging method to solve the model efficiently. A numerical example and its sensitivity analysis are demonstrated.

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1. Introduction

For any firm which needs to deliver goods to their customers, the control of inventory in supply chains plays a role in the competitiveness of a company. Inventory costs generally account for almost half of company’s total costs for logistics [1]. Although inventory may reduce product shortage, the excess inventory will cause leftovers and increase holding costs. Since inventory reduction can lead to a significant increase of return on investment [2], inventory control is of critical importance in the supply chain management.

In practice, many firms face inventory control problems with a perishable or short-life-cycle product [3]. In a single-echelon single-period inventory control problem, the newsvendor model is commonly used to determine the optimal inventory [4]. The model maximizes the expected profit through considering the trade-off between leftover and shortage costs under demand uncertainty. Several extensions of the newsvendor model (see [4,5]) are discussed in the literature.

In this paper, we extend a single-period newsvendor problem to a multi-period one. The multi-period newsvendor problem gives the optimal control of inventory for a perishable product over a finite number of time periods [6]. For example, a book store may order a monthly magazine multiple times within a month, but the leftover magazines are disposed at the end of the month. Thus, the book store replenishes its inventory multiple times over finite time periods to satisfy the uncertain demand. Tanaka et al. [7] pointed out that a company may financial loss through the short-life-cycle of product and the uncertainty of demand even for bestselling products. Prior works on the multi-period newsvendor problem commonly consider stationary demand [8–10]. However, the assumption of stationary demand may not be valid in practice, e.g., the situations of new products, uncertainty customer demand, economic conditions, or seasonal effects [11,12].

This research is motivated by the fashion goods which are perishable with long supply lead time and non-stationary demand [13]. Most suppliers are located in different countries so its supply lead time could be longer than a month [14]. A company may have only few batches of delivery from the suppliers during the whole selling season. Due to the supply chain complexity and uncertain demand, it is a challenge to find the optimal inventory control policy [15] as well as an accurate demand forecast [16]. In the research, we consider a two-echelon logistic system consisting of a distribution center and multiple retailers. To prevent stock out, the distribution center can replenish retailers over the finite time periods. Other retailers can also replenish a specific retailer which is called transshipment. We formulate the problem as a multi-stage stochastic programming model. Comparing to prior works related to the stochastic inventory model, the proposed research provides novel knowledge to optimize the trade-off between delivery, transshipment, shortage, and holding costs. In addition, the model supports decision making for allocating inventory of...
short-life-cycle products under the uncertainty. There are two main contributions. First, we proposed the mathematical model for multi-period newsvendor problem with transhipment and non-stationary demand. Secondly, we developed a multi-stage stochastic programming model to optimize the inventory control policy. The progressive hedging method is used to solve the problem.

The rest of the paper is organized as follows. Relevant works are reviewed in Section 2. Section 3 presents the proposed inventory control model. Section 4 describes the solution method. Numerical results are given in Section 5. Finally, we discuss our findings and future work.

2. Literature review

To the best of our knowledge, no model has been proposed combining the multi-period newsvendor problems with transhipment. Thus, this section reviews the literature of two issues separately: multi-period newsvendor and transhipment. For the newsvendor model, the interested reader might refer to [4,5] for the newsvendor model and extensions.

For multi-period newsvendor models, several works focus on the estimation of demand distribution. Bensoussan et al. [17] proposes an estimation method based on observed demands using dynamic programming and probability theory. Levi et al. [18] use the Monte Carlo simulations to estimate the demand distribution for both single newsvendor and multi-period newsvendor problems. Levina et al. [19] propose an online learning method.

Other uncertainties in the newsvendor models include customer demand, supplier capacity and selling prices. Wang et al. [13] consider the uncertainties of customer demand and supplier capacity. Densing [8] models the uncertainty of selling prices in the multi-stage stochastic programming model for the hydropower plant. Nagarajan and Rajagopalan [20] consider the problem with two substitutable products with negatively correlated demand. They formulate the problem into the equilibrium with balancing between two competing firms [21].

For the transhipment models, several researches including [22–25] apply the optimization techniques to the transhipment problems. One technique used in the literature is the stochastic programming approach which can model the optimization problems with the uncertainty. Nasr et al. [26] formulate the transhipment problem as a simple distribution system with a distribution center and two retailers. They consider the stochastic supply interruption at the distribution center. Granot and Sosic [27] propose a transshipment model with multiple retailers. Each retailer aims to obtain its own optimal amount of transshipment. Archibald et al. [28] propose a heuristic method to decompose the multi-location transshipment problem into two location transshipment problems. Gong and Yucesan [29] suggest a transshipment model that considers multiple retailers and positive replenishment lead time with stationary demand. Slikker et al. [30] address the multiple-retailer news-vendor problems. They adopt the cooperative game theory and consider the transshipment between retailers. He et al. [31] propose a mixed-integer programming model for a multi-echelon supply chain. They consider deterministic customer demand, shortage costs, holding costs, handling costs, and transportation costs.

Table 1 summarizes the literature of the multi-period newsvendor and the stochastic transshipment models. Most works assume stationary demand, and do not take into account the other constraints such as replenishment, holding costs, and transshipment. When the customer demand is nonstationary, it significantly affects the decision of inventory control.

3. Optimal inventory allocation model in multi-period newsvendor problems

We formulate the multi-period newsvendor problem as a multi-stage stochastic programming problem. The uncertainty is discretized into a set of scenarios which denote the possible realizations of the random event [10] and its corresponding probability. Scenario-based models make stochastic constraints into regular (deterministic) constraints so the existing deterministic optimization methods can be applied.

The objective of the model is to allocate limited amount of inventory to minimize the total cost including delivery, transshipment, holding, and shortage costs. Due to considerable lead time of transshipments, delivery and transshipment decisions at each location should be made before the uncertainty is realized. Decisions are made at each period which makes it a multi-stage decision problem. Table 2 shows the data and decision processes in the multi-stage stochastic programming problem. Within a finite number of stages (t = 1, 1, . . . , m), denote a set of random variables

<table>
<thead>
<tr>
<th>Stage</th>
<th>Data process</th>
<th>Decision process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncertainty</td>
<td>Realized</td>
</tr>
<tr>
<td>0</td>
<td>(ξ0, ξ1, ξ2, . . . , ξn)</td>
<td>None</td>
</tr>
<tr>
<td>1</td>
<td>(ξ1, ξ2, . . . , ξm)</td>
<td>ξ0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>T</td>
<td>(ξm)</td>
<td>ξ0, ξ1, . . . , ξm−1</td>
</tr>
</tbody>
</table>

| Table 2 | Decision and data processes. |

<table>
<thead>
<tr>
<th>Demand</th>
<th>Time horizon</th>
<th>Multiple echelon</th>
<th>Multiple retailers</th>
<th>Transshipment</th>
<th>Multistage stochastic</th>
<th>Transshipment time/cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our proposed model</td>
<td>Non-stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Bensoussan et al. [17]</td>
<td>Non-stationary</td>
<td>Infinite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Densing [8]</td>
<td>Stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Halati and He [9]</td>
<td>Stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Levi et al. [18]</td>
<td>Non-stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Levina et al. [19]</td>
<td>Non-stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Nagarajan and Rajagopalan [20]</td>
<td>Deterministic</td>
<td>Both</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Nagarajan and Rajagopalan [21]</td>
<td>Stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Taaffe et al. [37]</td>
<td>Stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Wang et al. [13]</td>
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<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Prastacos [38]</td>
<td>Stationary</td>
<td>Infinite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Archibald et al. [28]</td>
<td>Stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Gong and Yucesan [29]</td>
<td>Stationary</td>
<td>Finite</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Granot and Sosic [27]</td>
<td>Stationary</td>
<td>Single</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Nasr et al. [26]</td>
<td>Constant</td>
<td>Single</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>
The multi-stage programming model is described as follows.

\[
\begin{align*}
\text{Min} & \quad \sum_{i,t} c_i x_{i,t} + \sum_{i,s} p_s Q_i (a_{i,s}, b_{i,j,s}, x_{s,t}, l_{i,t}) \\
\text{s.t.} & \quad l_{i,t} = l_{i,t-1} - \sum_{j \in J} x_{i,j,t-1} - \sum_{s \in S} x_{i,s,t}, \\
& \quad \text{for all } t \in T, \forall i \in I, \\
& \quad l_{i,t} \leq l_{i,t}, \forall i \in I, \forall s \in S, \\
& \quad \sum_{t \in T} q_{i,t} \geq \alpha \sum_{t \in T} q_{i,t}, \forall i \in I, \forall s \in S, \\
& \quad q_{i,t} \geq 0, x_{s,t} \geq 0, \forall i \in I, \forall t \in T, \forall s \in S, \\
& \quad a_{i,s} \in \{0,1\}, b_{i,j,s} \in \{0,1\}, \forall i \in I, \forall j \in J, \forall t \in T.
\end{align*}
\]  

The objective function (3.1) is to minimize the total cost including delivery, transshipment, shortage, and holding costs at all retailers. We model the delivery and transshipment cost as a fixed cost since the products we considered in the model are relative small. The cost structure can be extended to other cost structure as well.

The first term of the objective function is the total cost in the first stage. The other term in the objective function is the expected cost in the later stages which are described as follows.

\[
Q_i (a_{i,s}, b_{i,j,s}, x_{s,t}, l_{i,t}) = \sum_{t \in T} \left( c_i x_{i,t} + r_i x_{s,t} + h_i l_{i,t} + \sum_{j \in J} m_{i,j} b_{i,j,s} \right)
\]

Constraint (3.2) is the inventory balance constraints for the distribution center. Constraints (3.3) and (3.4) are the inventory balance constraints for the retailers. Constraints (3.5) and (3.6) formulate the transportation cost and transportation quantity. Constraint (3.7) shows the inventory capacity constraints at retailers. Constraint (3.8) ensures the required service level. Constraint
(3.9) enforces the constraints between the demand, the shortage, and the served demand. Constraints (3.10) and (3.11) are non-negativity constraints.

At the beginning of period, it is reasonable to assume there are no prior order, transshipment, and inventory at retailers. Hence, the inventory at retailer and transshipment can be set at zeros, i.e., \( l_{0,i} = 0 \) and \( x_{0,j,i} = 0 \), \( \forall i,j \). Under this initial setting,\n
\[
\begin{align*}
l_{i,j,i} & = 0, \quad \forall t \in T, \quad t < l, \quad \forall s \in S \tag{3.14} \\
x_{t,i,j,i} & = 0, \quad \forall t \in T, \quad t < l, \quad \forall i,j \in I 
\end{align*}
\]

under uncertainty.

4. Progressive hedging method

To solve the proposed multi-stage programming model, we extend the progressive hedging method to solve the model efficiently. The progressive hedging method is developed by Rockafellar and Wets [32] which solves stochastic models through scenario decomposition and is commonly used to solve multistage stochastic programs [33]. Comparing to Benders decomposition, the progressive hedging method is not limited to problems with convexity and can be applied to the stochastic mixed integer program without the special scheme of convexification which transfers or approximates the nonconvex function to convex one. The idea of the progressive hedging is to find the scenario independent solution which is obtained by aggregating the solutions of scenario subproblems. With nonanticipativity constraints, the progressive hedging method steers the scenario solutions to converge into the scenario invariant solution. The detail proof of optimality and convergence of the algorithm is in [32].

The progressive hedging method is illustrated in Fig. 1. We decompose the problem into a number of scenario sub-problems which are denoted as \( S1, S2, \ldots, Sm \). The method moves the nonanticipativity constraints to the objective function as a quadratic penalty term with Lagrange multipliers [34]. All scenario sub-problems are solved independently until the nonanticipativity constraints are satisfied within a pre-specified tolerance.

To decompose the whole problem into separable scenario sub-problems, the first stage variables are also included in the scenario-based sub-problems. The objective function is revised as follows.

\[
\sum_{s \in S} \left( \sum_{i \in I} \left( \sum_{j \in J} c_{a_{i,j,i}} + \sum_{j \in J} m_{j} b_{j,i,j} \right) + Q_{l}(a_{i,j}, b_{j,i,j}, x_{s,i,j}, l_{i,j}) \right) \tag{4.1}
\]

To link all scenario-based sub-problems, the following nonanticipativity constraints are added into the model:

\[
a_{i,j,i} = a_{i,j}, \quad \forall i, \forall s \tag{4.2}
\]

\[
b_{j,i,j,i} = b_{j,i,j}, \quad \forall i,j, \forall s \tag{4.3}
\]

\[
x_{s,j,i} = x_{0,j,i}, \quad \forall i, \forall s \tag{4.4}
\]

\[
x_{s,i,j} = x_{i,j}, \quad \forall i,j, \forall s \tag{4.5}
\]

The values \( a_{i,j}, b_{j,i,j}, x_{0,j,i} \) and \( x_{i,j} \) are the estimates of the scenario invariant solutions, and are commonly set as the expected values of the optimal solutions of the scenario subproblems. Since the problem is not fully separated by scenarios due to the nonanticipativity constraints, the progressive hedging method moves the nonanticipativity constraints into the objective function as penalty terms with Lagrange multipliers. Therefore, each scenario subproblem has the following objective function.

\[
\begin{align*}
\text{Min} \quad & \sum_{i \in I} \left( c_{a_{i,j,i}} + \sum_{j \in J} m_{j} b_{j,i,j} \right) + Q_{l}(a_{i,j}, b_{j,i,j}, x_{s,i,j}, l_{i,j}) \\
& + \sum_{i \in I} \left[ \rho_{x_{i,j}^2} \left( a_{i,j,i} - a_{i,j} \right) + \sum_{j \in J} \rho_{x_{i,j}^2} \left( b_{j,i,j} - b_{j,i,j} \right) \right] \\
& + \frac{\rho}{2} \sum_{j \in J} \left[ \rho_{x_{i,j}^2} \left( x_{0,j,i} - x_{i,j} \right) + \sum_{i \in I} \rho_{x_{i,j}^2} \left( x_{s,i,j} - x_{s,i,j} \right) \right] \\
& + \frac{\rho}{2} \left[ \rho_{x_{i,j}^2} \left( x_{0,j,i} - x_{i,j} \right) + \sum_{i \in I} \rho_{x_{i,j}^2} \left( x_{s,i,j} - x_{s,i,j} \right) \right] \tag{4.6}
\end{align*}
\]

where \( \rho \) is a penalty parameter and \( \lambda \) is a Lagrange multiplier.

The proposed progressive hedging algorithm is as follows.

**Algorithm 1** (Progressive Hedging Method)

1. Set \( \rho_{x_{i,j}^2}, \rho_{x_{i,j}^2}^2, \rho_{x_{i,j}^2} \), and \( \gamma_{e,j} > 1, k := 0, tol = 1.0e - 5 \) (termination threshold).

2. For all \( s \in S \), solve scenario sub-problems without nonanticipativity constraints.

3. Let the solution \( a_{i,j}, b_{j,i,j}, x_{0,j,i} \) and \( x_{i,j} \) for all \( i, j \).

4. \( \lambda_{x_{i,j}} = \lambda_{x_{i,j}} + \gamma_{e,j} \lambda_{x_{i,j}} + \gamma_{e,j} \lambda_{x_{i,j}} \) for all \( i, j \).

5. Update \( \lambda_{x_{i,j}} = \lambda_{x_{i,j}} - \gamma_{e,j} \lambda_{x_{i,j}} - \gamma_{e,j} \lambda_{x_{i,j}} \) for all \( i, j \).

6. If \( \theta_{e} \) is less than \( tol \) and \( \theta_{x} \) is less than \( tol \), then terminate. Otherwise, go to step 5.

7. \( \theta_{e} = \sqrt{\sum_{i \in I} \left( a_{i,j} - b_{j,i,j} \right)^2} \)

8. \( \theta_{x} = \sqrt{\sum_{i \in I} \left( x_{0,j,i} - x_{i,j} \right)^2} \)

In Step 3, we calculate the initial estimates from the scenario sub-problem solutions by moving the nonanticipativity constraints into the objective function. Initial parameters are also set according to the initial estimates in Step 4. With these initial estimates and parameters, Step 5 solves the quadratic programming problem with penalty and Lagrange multipliers terms. The estimates and parameters are updated by the solution from the quadratic scenario sub-problems in Step 6. In Step 7, the parameter \( \gamma_{e,j} \) is used to increase the penalty parameter in each iteration. The termination conditions check if the decision values are scenario invariant through the pre-specified tolerance denoted by \( tol \).
and $P_{k}^c = P_{k}^s$.

The results are stable.

In the experiments, we solve the problem with 14 retailers, 10 scenarios, and 7 stages. The nonanticipativity constraints are used to restrict the stage decision variables. By the augmented Lagrange multipliers, we penalize the stage decision variables if it violates the nonanticipativity constraints using the quadratic terms in the objective function of each scenario and are all converged into scenario invariant solutions with the given tolerance.

### 5. Numerical results

In this section, we present a numerical example of the multi-period newsvendor model. To illustrate the benefits of the proposed model, we compare it with economic order quantity (EOQ) and newsvendor models. We also conduct the sensitivity analysis of delivery, holding, shortage, and transshipment costs.

The model is implemented in C# programming language with CPLEX 12.5 and solved by the enhanced progressive hedging method. The penalty parameter, $\rho$, is 0.001 and it increases 200% every five iterations. The tolerance of termination is set as 0.01. These parameters are used for the convergence of the solution in the modified progressive hedging algorithm.

#### 5.1. Data generation

The numerical data is generated based on a fashion goods company which needs to control the inventory in a distribution network. Poisson distributions are used to represent the demand of perishable products. We employed a non-stationary Poisson process over the finite time horizon. Thus, the parameter $\lambda$ of the Poisson distribution changes over time. The $\lambda$ is specified using the company's current deterministic demand data, which is between 3 and 30. Similar data generation processes can be found at He et al. [31], Petruzzi and Monahan [35], and Zhao and Zheng [36]. We use the historical delivery, holding, and transshipment costs and the current supply chain configuration. In the experiments, we test the number of scenarios until all computational results are stable.

#### 5.2. Solution time

The experiments are conducted on a computer with Intel(R) Core(TM) i7-2600 CPU 3.4 GHz and 8 G bytes of memory. The solution times for various sizes of problems are summarized in Table 3. In the experiments, we solve the problem with 14 retailers, 10 scenarios, and 7 stages. The nonanticipativity constraints are used to restrict the stage decision variables. By the augmented Lagrange multipliers, we penalize the stage decision variables if it violates the nonanticipativity constraints using the quadratic terms in the objective function of each scenario and are all converged into scenario invariant solutions with the given tolerance.

#### 5.3. Comparison with other methods

The results of the multistage stochastic programming (SP) are compared with economic order quantity (EOQ) and newsvendor model. The result is shown in Table 4. The single-period newsvendor model has the least total delivery cost since it only delivers once. On the other hand, its holding cost is higher than other methods. The delivery and holding costs of the proposed method, SP, are closed to the EOQ model, but its shortage cost is much less than the shortage cost in the EOQ model.

#### 5.4. Sensitivity analysis

We analyze the parameters including the initial inventory, holding, delivery, transshipment, and shortage costs as shown in Fig. 2. As the unit holding cost increases, both the delivery cost and transshipment cost increase. The total shortage cost is very stable.

**Table 3**

Solution time by problem size.

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Number of Retailers</th>
<th>Number of Scenarios</th>
<th>Number of Stages</th>
<th>Number of variables</th>
<th>Number of constraints</th>
<th>Number of iterations</th>
<th>Number of subproblems solved</th>
<th>Solution time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>5</td>
<td></td>
<td>Continuous</td>
<td>110</td>
<td>60</td>
<td>120</td>
<td>208</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
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<td>230</td>
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<td>440</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>6</td>
<td></td>
<td>Continuous</td>
<td>468</td>
<td>336</td>
<td>672</td>
<td>901</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>6</td>
<td></td>
<td>Continuous</td>
<td>708</td>
<td>540</td>
<td>1080</td>
<td>1371</td>
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<tr>
<td>11</td>
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<tr>
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<td>7</td>
<td></td>
<td>Continuous</td>
<td>1554</td>
<td>1274</td>
<td>2548</td>
<td>3030</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td>7</td>
<td></td>
<td>Continuous</td>
<td>1771</td>
<td>1470</td>
<td>2540</td>
<td>3458</td>
</tr>
</tbody>
</table>

**Table 4**

Performance comparison between the proposed algorithm, EOQ and newsvendor model.

<table>
<thead>
<tr>
<th>Problem size</th>
<th>Delivery cost</th>
<th>Holding cost</th>
<th>Shortage cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re (retailers), Sc (Scenarios), St (Stages), SP (Multi-Stage Stochastic Programming), EOQ (Economic Order Quantity), and NV (Newsvendor).</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>13.5</td>
<td>13.5</td>
<td>8.5</td>
<td>48.62</td>
</tr>
<tr>
<td>5</td>
<td>19.0</td>
<td>22.5</td>
<td>16.5</td>
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<td>33.0</td>
<td>16.5</td>
<td>317.72</td>
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<tr>
<td>9</td>
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<td>45.0</td>
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<tr>
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<td>70.5</td>
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</tr>
<tr>
<td>13</td>
<td>70.5</td>
<td>75.0</td>
<td>28.5</td>
<td>1046.20</td>
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<tr>
<td>14</td>
<td>81.5</td>
<td>77.0</td>
<td>34.0</td>
<td>1168.77</td>
</tr>
</tbody>
</table>

The cost is relatively high, a retailer may order a smaller amount. As a result, the numbers of delivery and transshipment increase due to the higher unit holding cost.

Fig. 4 shows the corresponding delivery, shortage, holding, transshipment and total costs when we increase the unit transshipment cost. When the unit transshipment cost increases, the total holding cost will also increase. The other costs are relatively stable.

Fig. 5 shows the total delivery, holding, shortage, transshipment costs when we have more initial inventory at the distribution center. We use the total expected demand as our base point. As shown in the figure, all costs decrease with respect to the increasing initial inventory. When we have 4% more initial inventory, the total shortage cost decreases to zero. The total shortage cost can be reduced if we have more initial inventory.

Fig. 6 shows all costs when increasing the unit delivery cost. We denote “[i, j]” as a uniform distribution between i and j. As shown in the figure, when the delivery cost increases, the total holding and transshipment costs also increase. The shortage costs are the same in all different cases.

Fig. 7 shows all costs when increasing the unit delivery cost. The number of deliveries decreases until the delivery cost is uniform(1, 4). Then it will become stable and is equal to the minimum number of deliveries, i.e., one-time delivery per retailer.

6. Conclusions

This paper proposed a multi-period newsvendor model with non-stationary demand. We formulated it as a multi-stage stochastic programming model and extended the progressive hedging algorithm to optimize the model efficiency. The model can be applied to other similar supply chains with non-stationary demand such as newly launched products. The experimental results show that the proposed multi-stage stochastic programming model performs better than the EOQ and single-period newsvendor models. The total cost can be reduced if we consider transshipments. Several sensitivity analyses are conducted on the holding, delivery, transshipment, and shortage cost.
In the research, we have addressed perishable or short-life-cycle product supply chains. The experimental results have shown that the transshipment can be a good strategy to minimize the total cost as well as the number of stock-outs. Although transshipments incur additional costs, they also improve the number of stock-out cost. Our proposed method provides an efficient approach to optimize the trade-off between delivery, transshipment, and stock-out costs.

One potential direction for future research is to improve the computational time. Under the progressive hedging framework, we can implement the algorithm through parallel computing. Another direction is the extension of the current algorithm to other supply chains such as smart phone, food, and energy supply chain. It is also important to consider more than one distribution center in a perishable product supply chain.

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References