Ergodic and Outage Capacities of Relaying Channels in Spectrum-Sharing Constrained Systems

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Abstract

This paper investigates the capacity of multiple relay channels in different fading and shadowing environments under spectrum-sharing constraints. We consider that a secondary user (SU) is allowed to share the spectrum band with a primary user (PU) provided that the SU’s transmit power remains below an interference power threshold set by the PU. Considering a scenario where the SU’s transmitter and receiver cannot communicate directly, a relay node, chosen among a set of $K$ terminals, helps transmitting data from the SU’s transmitter to the destination. The SU’s transmitter and chosen relay node adapt their corresponding transmission parameters so as to satisfy the interference-power constraint at the PU’s receiver. We derive closed-form expressions for the ergodic capacity of the SU’s channel in Rayleigh fading, Nakagami–m fading and Lognormal shadowing environments. We further obtain the outage capacity assuming the aforementioned environments and the above-mentioned spectrum-sharing limitations. Numerical results are provided to reinforce our theoretical derivations.

Index Terms

Cognitive radio, relay systems, spectrum sharing, ergodic capacity, outage capacity, opportunistic relaying, fading, shadowing.

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I. INTRODUCTION

As the demand for wireless services is exponentially growing, critical improvements in terms of robustness of communication and data transmission speeds are required. These changes make wireless devices greedy in spectrum resources. However, the frequency bands are not well exploited and, recently, measurements carried out by the federal communications commission (FCC) revealed that the allocated spectrum bands are not efficiently used in space and time. In particular, [1] highlights the importance of a better utilization of the spectrum white spaces.

Software defined radio (SDR) and cognitive radio (CR), terms coined by Mitola in 1991 and 1999, respectively, propose new conceiving approaches for wireless devices. SDR is a radio system whose main components, e.g., filters, mixers, amplifiers, modulators, etc . . . are built using softwares [2]. CR, on the other hand, is an architecture system based on SDR that can adapt itself to the environment in which it operates. In a spectrum-sharing context, CR can adapt its parameters to avoid inducing harmful interference to other devices. CR has been accepted as the technology that will help overcome the scarcity of the radio spectrum. Efforts have been converging towards standardization which led to the establishment of the IEEE 802.22 working group: its main task is to provide the specifications that will allow terminals to use the analog television (TV) bands without inducing harmful interferences to the primary users (PUs).

A new approach for assessing the performance of spectrum-sharing channels has been introduced in [3]. It relies on the assumption that a constraint on the received power at a third party’s receiver may be more relevant than the transmit power constraint. The capacity of point-to-point and multi-user additive white Gaussian noise (AWGN) channels under interference power constraint has been derived in [3]. This approach was further used in [4] to derive the capacity of a point-to-point channel under spectrum-sharing power limitations in various fading and shadowing environments while considering a constraint on the average interference-power at the PU receiver. These results were further extended in [6] by deriving the ergodic and outage capacities of Rayleigh fading channels considering joint constraints on the average and peak interference powers in spectrum-sharing systems.

In order to monitor the activity of the PU so as to improve the performance of a CR system, the SU uses sensing to either determine the presence or the absence of PU’s activity or to prevent harming the PU’s communications by controlling the transmission power. Power allocation for a system running under
joint peak and average-power interference constraints is studied in [7] wherein the authors utilize the benefits of soft-sensing to adapt the transmit power of the SU, and provide closed-form expressions for the constant rate transmission and the outage capacity. Moreover, the results of [8] reinforce the idea that the concept of soft-sensing arises as a key factor that enhances the data transmission by optimizing rate and power allocation at the SU’s transmitter.

As the SU terminals share a spectrum band with PU, they have to respect the criteria of interference temperature [9], by maintaining the transmit power below a certain limit not to create disturbance on the PU’s communication process. This power limitation, added to the random variations of the wireless communication medium and eventual obstacles, may cause the SU’s transmitter not to be able to communicate directly and reliably with its corresponding receiver. An intermediate solution, made of relay terminals, can solve this problem.

The three-terminal communication model in [10] and the equations’ setting developed in [11] were pioneering in investigating the relaying concept. Since then, this idea has widely been adopted to offer significant improvements in diversity and capacity (see [12], [13] and references therein). The concept of relaying channels has also been expanded to wireless terminals [14]. Moreover, the theoretical bases of wireless relaying networks for single user access and different relaying strategies such as decode-and-forward (DF) and amplify-and-forward (AF) have been presented in [15]. In the same context, several closed-form expressions of important statistics have been derived in [16] which studies the performance of relaying systems using partial relay selection technique. On the other hand, opportunistic relaying is attractive in the context of cognitive radio as it enhances the performance of wireless systems by optimizing the access to the limited available resources and reducing interference by selecting terminals that generate the least interference in space and time [17]. Besides, the capacity of relay-based systems in spectrum-sharing context under quality-of-service (QoS) requirements for several fading and shadowing environments is studied in [18].

In this paper, taking advantage of the benefits of the relaying concept, we study the ergodic capacity of SU’s relay channels under spectrum-sharing constraints for different fading and shadowing environments, and derive closed-form expressions for the ergodic and outage capacity metrics of the channel. More precisely, we consider that relay nodes function using the DF mode and that a single terminal among
them is selected to relay the information from the SU’s transmitter to the SU’s receiver. The SU adapts its transmit power such that the interference power inflicted on the PU’s receiver is below the predefined interference-power limit. The performance analysis is conducted for different fading, namely Rayleigh and Nakagami, and Lognormal shadowing models.

The remainder of the paper is organized as follows: The system model, the main hypotheses and notations are provided in Section II. In Section III, we derive closed-form expressions for the ergodic capacity of the system in Rayleigh and Nakagami–$m$ fading as well as Lognormal shadowing environments. Section IV presents the derivation of the outage capacity. Numerical results that assess our theoretical derivations are provided in Section V, before concluding the paper in Section VI.

II. System Model

We consider an overlay framework where a SU is allowed to share and utilize the spectrum band with a PU under appropriate constraints. For the sake of simplicity, we refer to the SU’s transmitter (source) by $S$ and to the SU’s receiver (destination) by $D$. We assume that due to deep fading on the SU link, $S$ cannot directly communicate with $D$. A set of $K$ relay terminals are made available to help the SU for its data transmission. The relay nodes are labeled as $R_i, i \in \{1, 2, \ldots, K\}$. The channel coefficients between $S$ and $R_i$ and the ones between $R_i$ and $D$ are respectively denoted by $h_{SR_i}$ and $h_{R_iD}, i = 1, 2, \ldots, K$. We assume that the channel coefficients are independent of each others. The system works using time division multiple access (TDMA), so that the communication process occurs over two time slots (TS), and the $K$ relay nodes operate using DF technique. In order to efficiently use the available spectrum, a single relay is chosen, upstream, to relay the information from $S$ to $D$ using opportunistic (proactive) relay selection [17].

To help the smooth functioning of the system and respect the interference-power constraint at the PU receiver, the SU and the relay nodes must collect the required channel state information (CSI). CSI, in a cognitive context, can be either obtained by training-based [19] or blind [20] estimation approaches, or with the help of a permanent band manager [21]. The different CSI help $S$ to choose the best intermediate node $R_{b^*}$ to relay the data to $D$, taking into account $h_{SR_{b^*}}, h_{R_{b^*}D}, h_{SP}$ and $h_{R_{b^*}P}, i = 1, 2, \ldots, K$, where $h_{SP}$ and $h_{R_{b^*}P}$ are the channel gain coefficients between $S$ and $P$ and between $R_{b^*}$ and $P$, respectively.

As the SU is sharing the spectrum band with the PU, it has to keep its transmit power within a tolerable
limit not to induce harmful interference on the PU’s receiver. Let the PU’s receiver be denoted by $P$. During the whole duration of the communication process, the interference power received at $P$ has to remain below a predefined threshold, denoted $Q_{\text{peak}}$. The interference power constraints in the first and second TS can be formulated as:

$$P_S h_{SP} \leq Q_{\text{peak}} \quad \text{during TS}_1,$$  
(1a)

$$P_{R_b^*} h_{R_b^*P} \leq Q_{\text{peak}} \quad \text{during TS}_2,$$  
(1b)

where $P_S$ and $P_{R_b^*}$ are the transmit powers of $S$ and $R_b^*$, respectively, and where $h_{SP}$ and $h_{R_b^*P}$ are the channel power gain of links $S - P$ and $R_b^* - P$, respectively, which are independent of each others and independent of the SU’s link gains $h_{SR_b^*}$ and $h_{R_b^*D}$.

The best intermediate relaying node, chosen using opportunistic relay selection, results in the “best end-to-end” path, $S - R_b^* - D$. Thus, following the transmission of the data by $S$ during the first TS, the received signal at the chosen relay $R_b^*$ can be written as:

$$y_{R_b^*}[n] = \sqrt{h_{SR_b^*}[n]} x_S[n] + z_{SR_b^*}[n],$$  
(2)

where $n$ denotes the time index, $x_S$ is the signal sent by $S$ and $z_{SR_b^*}$ is the AWGN with zero-mean and power spectral density (PSD) $N_0$. During the second TS, the relay decodes the signal, re-encodes it, and then forwards it to node $D$. Hence, the signal received by $D$ is given by:

$$y_D = \sqrt{h_{R_b^*D}} x_{R_b^*} + z_{R_b^*D},$$  
(3)

where $x_{R_b^*}$ is the re-encoded signal sent by the relay node $R_b^*$, $z_{R_b^*D}$ is the AWGN with zero mean and PSD $N_0$ at $D$, and the time index is omitted as it is clear from the context.

In the two following sections, we study the long-term average achievable rate and the constant rate of the end-to-end SU channel for different radio propagation environments. For the sake of simplicity, we refer to these metrics as ergodic and outage capacities, respectively.

### III. Ergodic Capacity

In this section, we obtain the ergodic capacity of the end-to-end SU’s link under the interference power constraints shown in (1a) and (1b). Ergodic capacity is the maximum of long-term average achievable rate
with a small error probability while satisfying the constraint on the power [22]. Ergodic capacity is the relevant metric for the capacity of a system which is insensitive to transmission delays. As aforementioned, we consider that among the $K$ available relaying nodes, only the one that provides the “best end-to-end” path from $S$ to $D$ is chosen with the help of the CSI available at the SU. The “best end-to-end” relay index $b^*$ obeys the following equation:

$$b^* = \arg\max_{1 \leq i \leq K} \min \left\{ \frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_iD}}{h_{R_iP}} \right\}. \quad (4)$$

The ergodic capacity of the system under the constraints (1a) and (1b) is formulated, in [nats/s/Hz] (or in [bits/s/Hz] when dividing the ergodic capacity expression by $\log(2)$) by:

$$\frac{C_{er}}{B} = \frac{1}{2} \mathcal{E}_{h_{SR_i}, h_{R_iD}, h_{SP}, h_{R_iP}} \left[ \max_{1 \leq i \leq K} \min \left\{ \frac{C_{1,i}}{B}, \frac{C_{2,i}}{B} \right\} \right], \quad (5)$$

where the multiplicative constant $\frac{1}{2}$ comes from the fact that the communication occurs over two TSs, $B$ is the available bandwidth, $\mathcal{E}[.]$ is the expectation operator over the joint probability density function (PDF) of the channels’ coefficients $h_{SR_i}$, $h_{R_iD}$, $h_{SP}$ and $h_{R_iP}$, and we have

$$\frac{C_{1,i}}{B} = \log \left( 1 + \frac{h_{SR_i} Q_{\text{peak}}}{h_{SP} N_0 B} \right) \quad (6)$$

and

$$\frac{C_{2,i}}{B} = \log \left( 1 + \frac{h_{R_iD} Q_{\text{peak}}}{h_{R_iP} N_0 B} \right). \quad (7)$$

Thus, the ergodic capacity formula (5) can be reformulated according to

$$\frac{C_{er}}{B} = \frac{1}{2} \int_T \log(1 + \alpha x) f_T(x) dx, \quad (8)$$

where $\alpha = \frac{Q_{\text{peak}}}{N_0 B}$ and $T$ denotes the random variable that describes the “best end-to-end” path $S-R_{b^*}-D$ and is given by

$$T \triangleq \max_{1 \leq i \leq K} \min \left\{ \frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_iD}}{h_{R_iP}} \right\}. \quad (9)$$

A. Rayleigh Fading

Here, we suppose that all channel coefficients are Rayleigh distributed with variance $\frac{1}{2}$, thus, the square of a channel coefficient is exponentially distributed. The ratio of two exponential random variables with the same mean parameter has a PDF defined by $f_Z(x) = \frac{1}{(1 + x)^2} U(x)$, where $U(x)$ is the step function
which equals 1 for positive values of $x$ and 0 otherwise [4]. Let $Y_i = \min \left\{ \frac{h_{SR, i}}{h_{SP}}, \frac{h_{RD, i}}{h_{RP}} \right\}$ be the random variable describing a bottleneck in term of end-to-end capacity of the $i^{th}$ link [23] and $X$ the random variable such that $X \overset{\Delta}{=} \max_{1 \leq i \leq K} \{Y_i\}$. One can show that $Y_i$ and $X$ have the following PDFs (see Appendix A):

$$f_{Y_i}(x) = \frac{2}{(1 + x)^3} U(x). \quad (10)$$

$$f_X(x) = \left[ \sum_{k=1}^{K} \binom{K}{k} \frac{2k(-1)^{k+1}}{(1 + x)^{2k+1}} \right] U(x). \quad (11)$$

Inserting (11) into (8) and computing integration by parts and partial rational fractions reduction, we obtain the following closed-form expression for the ergodic capacity:

$$C_{er} = \frac{1}{2} \sum_{k=1}^{K} \binom{K}{k} (-1)^{k+1} \left[ \frac{Q_{peak}^{2k}}{(Q_{peak} - N_0 B)^{2k}} \log \left( \frac{Q_{peak} - N_0 B}{Q_{peak}} \right) - \sum_{p=1}^{2k-1} \frac{Q_{peak}^p}{(2k-p)(Q_{peak} - N_0 B)^p} \right]. \quad (12)$$

B. Nakagami-$m$ Fading

In this case, the channel coefficients follow the Nakagami-$m$ distribution. As such, the square of the channel coefficients follow Gamma distribution. The ratio of two Gamma distributed random variables, with the same scale and shape parameters, has the following PDF [4]:

$$f_Z(x) = \frac{x^{m-1}}{B(m, m)(1 + x)^{2m}} U(x), \quad (13)$$

where $B(x, y) = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)}$ is the Beta function [24, Eq. 8.384.1] and $\Gamma(.)$ is the Euler Gamma function defined by $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt$ [24, Eq. 8.310.1]. We consider $2K$ independent random variables $Z_{1,1}, Z_{1,2}, \ldots, Z_{K,1}, Z_{K,2}$ following the distribution of (13). The PDF of $Z_i \overset{\Delta}{=} \min \{Z_{i,1}, Z_{i,2}\}$ is given by

$$f_{Z_i}(x) = \frac{2}{B(m, m)^2} \frac{2F_1 \left( \begin{array}{c} 2m, m; m+1; -\frac{1}{x} \end{array} \right)}{mx(1 + x)^{2m}}, \quad (14)$$

where $2F_1(., .; .; .)$ is the Gauss hypergeometric function [24, Eq. 9.10]. The proof of (14) is provided in Appendix B. Now, let $T$ be the random variable defined as

$$T \overset{\Delta}{=} \max_{1 \leq i \leq K} Z_i, \quad (15)$$

The cumulative distribution function (CDF) of $T$ can be expressed as

$$F_T(x) = \Prob{T \leq x} = \Prob{Z_1 \leq x, \ldots, Z_K \leq x}, \quad (16)$$
and as all the $Z_i$ are independent from each other and follow identical distribution, the CDF of $T$ simplifies to

$$F_T(x) = \prod_{i=1}^{K} F_{Z_i}(x) = \left[F_{Z_i}(x)\right]^K = \left[1 - \frac{2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)}{B(m, m)^2 x^{2m}}\right]^K.$$  \hspace{1cm} (17)

By taking the derivative of $F_T(x)$ with respect to $x$, we find the PDF, $f_T(x)$, according to

$$f_T(x) = 2K \left[1 - \frac{2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)}{B(m, m)^2 x^{2m}}\right]^{K-1} \frac{2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)}{m(m + 1)x^{2m+2}B(m, m)^2} \times \left[2mF_1\left(2m + 1, m + 1; m + 2; -\frac{1}{x}\right) - x(m + 1)2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)\right].$$  \hspace{1cm} (18)

Fig. 2 depicts different simulations that confirm the theoretical expression derived above for different values of the fading parameter $m$ and number of relays $K$.

The final expression of $f_T(x)$ will help us in finding the ergodic capacity, which is defined by (8).

Using integration by parts, we arrive to the following expression:

$$\frac{C_{er}}{B} = -\frac{1}{2} \int_0^{+\infty} \frac{\alpha}{1 + \alpha x} F_T(x) dx = \frac{\alpha}{2} \sum_{p=0}^{K} \binom{K}{p} (-1)^p \frac{2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)^{2p}}{m^{2p} B(m, m)^{2p}} \int_0^{+\infty} I_p \left(\frac{2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)^{2p}}{(1 + \alpha x)x^{mp}}\right) dx.$$  \hspace{1cm} (19)

The integral $I_p(\alpha, m)$ in (19) can be accurately calculated using the Gauss-Laguerre [25, Eq. 25.4.45] quadrature integration method as follows:

$$I_p(\alpha, m) \approx \sum_{i=1}^{n} w_i h_p(\alpha, m, x_i),$$  \hspace{1cm} (20)

where

$$h_p(\alpha, m, x) = e^x \frac{2 F_1\left(2m, m; m + 1; -\frac{1}{x}\right)^{2p}}{(1 + \alpha x)x^{mp}},$$  \hspace{1cm} (21)
$n$ is the number of interpolation points, $x_i$ is the $i$th zero of the Laguerre polynomial $L_n(x)$ [24, Eq. 8.97.2] and $w_i$ are the associated weights given by

$$w_i = \frac{(n!)^2 x_i}{(n+1)^2 [L_{n+1}(x_i)]^2}. \quad (22)$$

Finally, combining equations (19) and (20), we obtain the closed-form expression for the ergodic capacity [nats/s/Hz] of a relay channel in Nakagami$-m$ fading under spectrum-sharing constraints as

$$\frac{C_{er}}{B} \approx \frac{\alpha}{2} \sum_{p=0}^K \binom{K}{p} \frac{(-1)^p}{m^{2p} B(m,m)^{2p}} \sum_{i=1}^n w_i h_p(\alpha, m, x_i). \quad (23)$$

**C. Lognormal Shadowing**

In this case, channel gains follow Lognormal distribution with zero mean and variance $\sigma^2$. The ratio of two Lognormal random variables is also Lognormal distributed but with variance $2\sigma^2$ [4].

Let us consider now that $2K$ random variables $Z_{1,i}$ and $Z_{2,i}$, $i = 1, 2, \ldots, K$, follow the Lognormal distribution with zero mean and variance equal to $2\sigma^2$, and let $Y_i \triangleq \min\{Z_{1,i}, Z_{2,i}\}$. The PDF of $Y_i$ is given by

$$f_{Y_i}(x) = \frac{1}{2\sigma x \sqrt{\pi}} e^{-\frac{(\log x)^2}{4\sigma^2}} \text{erfc}\left(\frac{\log x}{2\sigma}\right) U(x), \quad (24)$$

where $\text{erfc}(.)$ is the complementary error function defined by $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$ [24, Eq. 8.250.4].

As done in the previous section, we can compute the CDF of the random variable $T \triangleq \max_{1\leq i\leq K} Y_i$ according to

$$F_T(x) = \text{Prob}\{Y_i \leq x\}^K = \left[1 - \frac{1}{4} \text{erfc}\left(\frac{\log x}{2\sigma}\right)^2\right]^K. \quad (25)$$

We can then determine the PDF by taking the derivative of $F_T(x)$ with respect to $x$, yielding

$$f_T(x) = K \frac{(\log x)^2}{2x\sigma \sqrt{\pi}} e^{-\frac{(\log x)^2}{4\sigma^2}} \text{erfc}\left(\frac{\log x}{2\sigma}\right) \left[1 - \frac{1}{4} \text{erfc}\left(\frac{\log x}{2\sigma}\right)^2\right]^{K-1}. \quad (26)$$

We illustrate the validity of our result in Fig. 3, which shows perfect match between the PDF’s theoretical formula and Monte-Carlo simulations.
Plugging the PDF of $T$ in the capacity formula allows us to find the ergodic capacity:

$$\frac{C_{\text{er}}}{B} = \frac{1}{2} \int_0^{+\infty} \log(1 + \alpha x) f_T(x) dx$$

$$= \frac{1}{2} \int_0^{+\infty} K \log(1 + \alpha x) \frac{(\log x)^2}{2x\sigma \sqrt{\pi}} \cdot \frac{\exp \left( \frac{-(\log x)^2}{4\sigma^2} \right)}{\text{erfc} \left( \frac{\log x}{2\sigma} \right)} \cdot \left[ 1 - \frac{1}{4} \text{erfc} \left( \frac{\log x}{2\sigma} \right)^2 \right]^{K-1} dx$$

$$= \frac{K}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} \log \left( 1 + \alpha e^{2u\sigma} \right) \exp \left( -u^2 \right) \text{erfc}(u) \left[ 1 - \frac{1}{4} \text{erfc}(u)^2 \right]^{K-1} du$$

$$= \frac{K}{2\sqrt{\pi}} \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{(-1)^k}{4^k} \int_{-\infty}^{+\infty} \log \left( 1 + \alpha e^{2u\sigma} \right) \exp \left( -u^2 \right) \text{erfc}(u)^{2k+1} du. \quad (27)$$

The integral $J_k(\alpha, \sigma)$ in (27) can be accurately evaluated using the Gauss-Hermite quadrature formula [25, Eq. 25.4.46]. Specifically, let

$$g_k(\alpha, \sigma, u) = \log \left( 1 + \alpha e^{2u\sigma} \right) \exp \left( -u^2 \right) \text{erfc}(u)^{2k+1}, \quad (28)$$

then,

$$J_k(\alpha, \sigma) \approx \sum_{i=1}^{n} \omega_i g_k(\alpha, \sigma, x_i), \quad (29)$$

where $x_i$ is the $i^{th}$ zero of the $n^{th}$ Hermite polynomial $H_n(x)$ and $\omega_i$ are the associated weights given by [25, Table 25.10]:

$$\omega_i = \frac{2^{n-1} n! \sqrt{\pi}}{n^2 [H_{n-1}(x_i)]^2}. \quad (30)$$

Finally, combining equations (27) and (29), we obtain the closed-form expression for the ergodic capacity of a relay channel subject to Lognormal shadowing and spectrum-sharing constraints, according to

$$\frac{C_{\text{er}}}{B} \approx \frac{K}{2\sqrt{\pi}} \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{(-1)^k}{4^k} \sum_{i=1}^{n} \omega_i g_k(\alpha, \sigma, x_i). \quad (31)$$

IV. OUTAGE CAPACITY

While ergodic capacity is adapted for systems that carry delay-insensitive applications, outage capacity is a more appropriate performance metric for delay-sensitive systems, e.g., real-time video and voice. Outage capacity is the constant rate that can be achieved under all fading states. The SU’s transmitter uses \textit{channel inversion with fixed-rate (cifr)} policy with the help of the provided CSI to send its data with a power that is inversely proportional to the channel gain. However, this technique remains inefficient
in severe fading environments, such as Rayleigh, where it achieves zero capacity. As an alternative, [26] proposed the truncated channel inversion with fixed rate (tifr) policy where the sender inverts the channel but with the transmission being suspended when the channel power gain does not fit within defined limits.

In this section, we determine the tifr capacity for the relaying system with constraints on the peak power received at the PU’s receiver. We consider that outage occurs when $h_{SR_i}$ becomes weak comparing to $h_{SP}$ during the first TS and when $h_{R_iD}$ is weaker than $h_{R_iP}$ to some threshold during the second TS.

Let $S$ invert the channel during the first TS while observing the constraint on the peak interference power at the PU’s receiver $P$. The same scenario happens during the second TS: the chosen relay node $R_i$ uses the CSI to invert the channel while maintaining its transmit power within the associated spectrum-sharing constraint. Suppose that the cutoff threshold for $S-R_i$ and $R_i-D$ is $\frac{\gamma_{1,i}}{N_0B}$ and $\frac{\gamma_{2,i}}{N_0B}$, respectively, $\forall i \in \{1, 2, \ldots, K\}$ where $\gamma_{1,i}$ and $\gamma_{2,i}$ are threshold constants. The power allocation policy is then given by [6]:

$$P_S = \begin{cases} \frac{\alpha_{1,i} h_{SP}}{h_{SR_i}} & \frac{h_{SP}}{h_{SR_i}} \leq \frac{\gamma_{1,i}}{N_0B} \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

and

$$P_{R_i} = \begin{cases} \frac{\alpha_{2,i} h_{R_iP}}{h_{R_iD}} & \frac{h_{R_iP}}{h_{R_iD}} \leq \frac{\gamma_{2,i}}{N_0B} \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

where $\alpha_{1,i}$ and $\alpha_{2,i}$ are constants associated to links $S-R_i$ and $R_i-D$, respectively. Following the same approach in [6], but without the constraint on the average interference power at the PU’s receiver, we obtain the tifr capacity, given by

$$C_{\text{tifr}} = \frac{1}{B} \max_{1 \leq i \leq K} \log \left( 1 + Q_{\text{peak}} \max_{1 \leq i \leq K} \min \left\{ \frac{1}{\gamma_{1,i}}, \frac{1}{\gamma_{2,i}} \right\} \right) \left( 1 - P_{\text{out}} \right). \tag{34}$$

where $P_{\text{out}}$ is the probability that the “best end-to-end” path goes into outage.

A. Rayleigh Fading

We now derive closed-form expression for the capacity under the tifr policy in Rayleigh fading environment. We start by computing the outage probability $P_{\text{out}}$. The system goes to outage when either one of the selected links $h_{SR_i}$ or $h_{R_iD}$ is in outage, or when both of them experience outage. In other words, the event $\{\text{system in outage}\}$ is complementary to the event $\{\text{no outage on both links of the “best end-to-end}$
path”]. Let \( W_i \) denote the different end-to-end paths \( \forall i = 1, 2, \ldots, K \), i.e. \( W_i \) refers to \( S - R_i - D \), and let \( W_{b^*} \) be the “best end-to-end path” where \( b^* \) is given by equation (4). Let \( X_i, i = 1, 2, \ldots, K \), and \( X_{b^*} \) be the random variables that describe the \( S - R_i - D \) and the “best end-to-end” links, respectively. Therefore, we can write

\[
X_i = \min \left\{ \frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_i D}}{h_{R_i P}} \right\}
\]

and

\[
X_{b^*} = \max_{1 \leq i \leq K} X_i.
\]

The system experiences outage whenever \( X_{b^*} \) falls below a certain threshold \( \gamma_{th} \). Thus, we have:

\[
P_{\text{out}} = \text{Prob} \left\{ X_{b^*} \leq \gamma_{th} \right\}
= \text{Prob} \left\{ \max_{1 \leq i \leq K} X_i \leq \gamma_{th} \right\}.
\]

As we consider that all the paths from \( S \) to \( D \) are independent, equation (37) can be written as:

\[
P_{\text{out}} = \prod_{i=1}^{K} \text{Prob} \left\{ X_i \leq \gamma_{th} \right\}.
\]

For a certain path \( S - R_i - D \), the probability that this path is in outage can be written as:

\[
\text{Prob} \left\{ X_i \leq \gamma_{th} \right\} = \text{Prob} \left\{ \min \left\{ \frac{h_{SR_i}}{h_{SP}}, \frac{h_{R_i D}}{h_{R_i P}} \right\} \leq \gamma_{th} \right\}
= 1 - \text{Prob} \left\{ \frac{h_{SR_i}}{h_{SP}} \geq \frac{N_0 B}{\gamma_{1,i}} \right\} \text{Prob} \left\{ \frac{h_{R_i D}}{h_{R_i P}} \geq \frac{N_0 B}{\gamma_{2,i}} \right\}.
\]

Therefore, the outage probability can be expressed as:

\[
P_{\text{out}} = \prod_{i=1}^{K} \left[ 1 - \int_{\frac{N_0 B}{\gamma_{1,i}}}^{+\infty} \frac{dt}{(1+t)^2} \int_{\frac{N_0 B}{\gamma_{2,i}}}^{+\infty} \frac{dt}{(1+t)^2} \right]
= \prod_{i=1}^{K} \left[ 1 - \frac{1}{1 + \frac{N_0 B}{\gamma_{1,i}}} \frac{1}{1 + \frac{N_0 B}{\gamma_{2,i}}} \right].
\]

Accordingly, as we considered the same variations on all channel links, we can suppose that we have the same cutoff threshold over all links:

\[
\gamma_0 = \gamma_{1,i} = \gamma_{2,i} \quad \forall i = 1, 2, \ldots, K.
\]
Therefore, we obtain the outage probability as
\[ P_{\text{out}} = \frac{(N_0 B)^K (2\gamma_0 + N_0 B)^K}{(\gamma_0 + N_0 B)^{2K}}. \]  \hspace{1cm} (42)

Fig. 4 shows the variations of the outage probability for opportunistic relaying versus the threshold \( \gamma_0 \) for different values of \( K \). We clearly see the contribution of the relaying nodes. The more we have, the better the communication is and the less the outage will be.

The previous assumptions simplify the tifr capacity of the system as follows:
\[ \frac{C_{\text{tifr}}}{B} = \frac{1}{2} \max_{\gamma_0} \log \left(1 + \frac{Q_{\text{peak}}}{\gamma_0} \right) \left(1 - \frac{(N_0 B)^K (2\gamma_0 + N_0 B)^K}{(\gamma_0 + N_0 B)^{2K}}\right). \]  \hspace{1cm} (43)

The optimum value \( \gamma_0^* \) that maximizes the objective function in (43) cannot be found in closed-form for any \( K \geq 2 \), but in case \( K = 1 \), a closed-form expression can be derived and we obtain
\[ \gamma_0^* = \frac{2Q_{\text{peak}}N_0 B W_0 \left(\frac{Q_{\text{peak}} - N_0 B}{2N_0 B \sqrt{e}}\right)}{Q_{\text{peak}} - N_0 B - 2N_0 BW_0 \left(\frac{Q_{\text{peak}} - N_0 B}{2N_0 B \sqrt{e}}\right)}, \]  \hspace{1cm} (44)

where \( W_0(.) \) is the Lambert function [27]. The derivation of \( \gamma_0^* \) is detailed in Appendix C.

B. Nakagami-m Fading

We now consider the Nakagami-\( m \) fading case. Thus, the square of the channel coefficients follow Gamma distribution and their ratio follow the distribution of equation (13). Following the same procedure as in the previous section and considering the same cutoff \( \gamma_0 \) on the different hops, we compute the outage probability as follows:
\[ P_{\text{out}} = \prod_{i=1}^{K} \left[1 - \text{Prob} \left\{ \frac{h_{SR_i}}{h_{SP}} \geq \frac{N_0 B}{\gamma_0} \right\} \text{Prob} \left\{ \frac{h_{R_iD}}{h_{R_iP}} \geq \frac{N_0 B}{\gamma_0} \right\}\right] \]
\[ = \prod_{i=1}^{K} \left[1 - \left(\int_{\frac{N_0 B}{\gamma_0}}^{\infty} \frac{t^{m-1}}{B(m, m)(1 + t)^{2m}} \, dt\right)^2\right] \]
\[ = \prod_{i=1}^{K} \left[1 - \frac{1}{B(m, m)^2 m^2(N_0 B)^{2m}} \gamma_0^{2m} 2F_1 \left(2m, m; m + 1; -\frac{\gamma_0^2}{N_0 B}\right)\right] \]
\[ = \left(1 - \frac{1}{B(m, m)^2 m^2(N_0 B)^{2m}} \gamma_0^{2m} 2F_1 \left(2m, m; m + 1; -\frac{\gamma_0^2}{N_0 B}\right)\right)^K. \]  \hspace{1cm} (45)

Using the identity of the sum of a geometric series
\[ \sum_{k=0}^{K-1} x^k = \frac{1 - x^K}{1 - x}, \]  \hspace{1cm} (46)
the non-outage probability can be written as

\[ 1 - P_{\text{out}} = \sum_{k=0}^{K-1} \frac{1}{\mathcal{B}(m, m)^{2k+2} m^{2k+2} (N_0 B)^{2m(k+1)}} \gamma_0^{2m(k+1)} 2F_1 \left( 2m, m + 1; -\frac{\gamma_0}{N_0 B} \right)^{2k+2}. \]  

(47)

As such, the tifr capacity of the system under Nakagami-m fading becomes

\[ C_{\text{tifr}} = max_{\gamma_0} \sum_{k=0}^{K-1} \frac{1}{\mathcal{B}(m, m)^{2k+2} m^{2k+2} (N_0 B)^{2m(k+1)}} \log \left( 1 + \frac{Q_{\text{peak}}}{\gamma_0} \right) 2F_1 \left( 2m, m + 1; -\frac{\gamma_0}{N_0 B} \right)^{2k+2}. \]  

(48)

C. Lognormal Shadowing

In Lognormal shadowing environments, the random variable describing the ratio of two channel coefficients follows the Lognormal distribution with zero mean and variance \(2\sigma^2\). In this case, the outage probability of the system under spectrum-sharing constraints is given by

\[
P_{\text{out}} = \prod_{i=1}^{K} \left[ 1 - \text{Prob} \left\{ \frac{h_{SR_i}}{h_{SP}} \geq \frac{N_0 B}{\gamma_0} \right\} \text{Prob} \left\{ \frac{h_{R_i}}{h_{R,p}} \geq \frac{N_0 B}{\gamma_0} \right\} \right]
\]

\[ = \prod_{i=1}^{K} \left[ 1 - \left( \int_{\frac{N_0 B}{\gamma_0}}^{\infty} \frac{1}{2\sigma \pi \sqrt{x}} \exp \left( -\frac{\log(x)^2}{4\sigma^2} \right) dx \right)^2 \right]
\]

\[ = \prod_{i=1}^{K} \left[ 1 - \left( \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^2 \right]
\]

\[ = \left( 1 - \left( \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^2 \right)^K,
\]

(49)

where \(\text{erf}(.)\) is the error function defined by \(\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt \) [24, Eq. 8.250.1].

Using the identity (46), the probability of non-outage \((1 - P_{\text{out}})\) can be simplified as:

\[ 1 - P_{\text{out}} = 1 - \left( 1 - \left( \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^2 \right)^K
\]

\[ = \frac{1}{4} \left( 1 - \left( \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^2 \right)^{K-1} \sum_{k=0}^{K-1} \left[ 1 - \left( \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^2 \right]^{K-1-k}
\]

\[ = \sum_{k=0}^{K-1} \sum_{p=0}^{k} \left( \frac{k}{p} \right) \left( -\frac{1}{4} \right)^{p} \left( 1 + \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^{2p+2}
\]

\[ = \sum_{k=0}^{K-1} \sum_{p=0}^{k} \sum_{\ell=0}^{2p+2} \left( \frac{k}{p} \right)^{2p+2} \left( \frac{(-1)^p}{4^{p+1}} \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0 B} \right) \right) \right)^{\ell},
\]

(50)

where \(\left( \frac{q}{p} \right)\) is the binomial coefficient defined by \(\left( \frac{q}{p} \right) = \frac{q!}{p!(q-p)!}\).
Inserting the result of equation (50) in the \( \text{tifr} \) capacity expression, we obtain

\[
\frac{C_{\text{tifr}}}{B} = \max_{\gamma_0} \sum_{k=0}^{K-1} \sum_{p=0}^{k} \sum_{\ell=0}^{2p+2} \binom{k}{p} \left( \frac{2p+2}{\ell} \right) \left( \frac{\ell}{4p+1} \right) \log \left( 1 + \frac{Q_{\text{peak}}}{\gamma_0} \right) \text{erf} \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0B} \right) \right) \left( \frac{1}{2\sigma} \log \left( \frac{\gamma_0}{N_0B} \right) \right)^{\ell}.
\] (51)

Note that for all the environments, the optimum threshold \( \gamma_0 \) cannot be found in a closed-form and have to be evaluated numerically.

V. NUMERICAL RESULTS

In this section, we plot the different results obtained in the preceding sections and go further by studying the effect of the availability of \( K \) relay nodes and other key parameters, such as the fading parameter \( m \) and the shadowing variance \( \sigma \), on the performance of the system under study.

As for the ergodic capacity, Fig. 5 depicts the evolution of the capacity, as a function of the peak-power limit \( Q_{\text{peak}} \), in a relay-based system with spectrum-sharing constraints for the different considered environments and different values of the number of available relay nodes \( K \). We plotted the numerical results (solid lines) along with the Monte-Carlo simulations based on the approach in [28]. Simulation results are represented by a serie of un-connected symbols (star, dot and cross in Fig. 5(a); star and cross in Fig. 5(b)). For the results of Nakagami–\( m \) fading and Lognormal shadowing where we used the gaussian quadrature approximations, we set the value of the number of interpolation points to \( n = 20 \), which gives an accuracy of order \( 10^{-5} \). In general, we note that the capacity increases with \( K \). This shows that the more relay nodes are available, the better performance can be achieved at the SU’s side.

Another conclusion from Fig. 5(a) is that the value of the Nakagami fading parameter \( m \) affects the rapidity of the increase in capacity. In fact, when \( m \) increases, the capacity gain resulting from having more relay nodes available is reduced as the capacity is larger for small values of the parameter \( m \). This can be explained by the fact that \( m \) describes the fading severity of the environment (the smaller \( m \) is, the greater the severity is). Thus, the communication process will be affected. In fact, in very severe channel conditions, the SU will be able to transmit in a more relaxed manner as fading will make the power received at the PU’s receiver fall down. In contrast, in less severe fading environment, the SU’s transmitter will use the CSI to control more strictly its transmit power as it will affect in a more significant way the PU’s receiver.
In Fig. 5(b), we can see that the standard deviation $\sigma$ of the Lognormal shadowing affects the $tifr$ capacity especially for very constrained (low) values of $Q_{\text{peak}}$: when $\sigma = 6$ dB, capacity reaches the same value as for $\sigma = 4$ dB but with a smaller number of relaying nodes.

Investigating the effect of the number of relay nodes available to help the SU, let $\rho(K)$ be the ratio of the capacity of the relay channel when $K$ relay nodes are available over the capacity of the same channel when using a single relay:

$$\rho(K) = \frac{\text{Capacity with } K \text{ relays}}{\text{Capacity with 1 relay}}.$$  \hspace{1cm} (52)

This parameter will help us understand how the availability of more relay devices can be helpful, or in contrast useless, to the system. In particular, Fig. 6 includes the plots for $\rho(K)$ versus the peak-power limit $Q_{\text{peak}}$ for Rayleigh fading, Nakagami–$m$ fading and Lognormal shadowing environments, under spectrum-sharing system constraints. We notice that increasing the number of relays allows us to have a certain gain in capacity but that this gain diminishes as $Q_{\text{peak}}$ increases. The value of the ratio $\rho(K)$ tends to converge toward 1 as $Q_{\text{peak}}$ increases, but $\rho(K)$ never reaches the limit value of 1, which means that for relaxed systems, little capacity improvements will not be noticeable.

Dealing with the $tifr$ capacity, Fig. 4 shows that the outage probability $P_{\text{out}}$ decreases when the number of available relaying nodes $K$ increases, thus reinforcing the advantages of the use of relay terminals to improve the communication process in a wireless system.

Fig. 7 depicts the evolution of the $tifr$ capacity in Rayleigh fading, Nakagami fading and Lognormal shadowing scenarios when the number of available relay nodes $K$ increases. The same conclusions drawn when studying the ergodic capacity of the system with spectrum-sharing constraints apply here, as we see that the capacity increases when $K$ increases.

As we did for the ergodic capacity, we can evaluate the impact of the number of available relay nodes by using the ratio in (52). Fig. 8 illustrates the variations of $\rho(K)$ versus the peak-power limit $Q_{\text{peak}}$. Generally, we notice the contribution of the relaying nodes as the ratio $\rho(K)$ increases with the value of $K$. From Fig. 8(a), we notice that for higher value of the fading parameter $m$, the value of $\rho(K)$ is slightly lower than for smaller values of $m$. We can provide here the same explanation we gave some paragraphs above to explain the evolution of the ergodic capacity in Nakagami-$m$ fading channels. In fact, as the severity of the environment increases, the transmit signal will fade away drastically before reaching the
PU’s receiver, allowing the SU’s transmitter to transmit with higher power levels. Besides, we see in Fig. 8(b) that the ratio $\rho(K)$ remains constant before falling toward 1, and that this constant value affects a wider range of the peak-power constraint $Q_{\text{peak}}$ for higher values of $K$. Thus, a system with an important number of relays operating in a Lognormal shadowing environment under spectrum-sharing limitations will provide a significant capacity gain.

VI. CONCLUSIONS

In this paper, we studied relay channels under spectrum-sharing constraints for different fading and shadowing environments. We considered opportunistic relaying scheme and assumed that a secondary user is authorized to share the spectrum with a primary user as long as the former respects the constraint on the received-power at the latter. We derived the ergodic and outage capacities in closed form for different fading and shadowing environments. Numerical results were also provided and show how capacity evolves when the choice of the relaying node is wider. For instance, ergodic capacity for different fading and shadowing environments increases as the number of available relaying terminals gets higher. On the other hand, we showed that the outage probability diminishes as the number of relaying nodes increases taking advantage of the opportunistic relaying strategy. As a result, the outage capacity increases with the number of relaying terminals. Besides, the severity of fading arises as a factor that affects the capacity. We showed that, in Nakagami-$m$ fading environments, both the ergodic and the outage capacity increases faster for smaller values of $m$ as the severity of the fading allows the secondary user to transmit with higher power. Finally, it is worthwhile mentioning that the analytical expressions obtained in this article can also serve as a starting point to find other results that can lead to the study of the performance of similar systems, being therefore a stepping-stone in the design of transmissions approaches to satisfy quality-of-service requirements in relay-based spectrum-sharing systems.
Consider two independent and identically distributed random variable $Z_{1,i}$ and $Z_{2,i}$. The PDF of $Y_i = \min \{Z_{1,i}, Z_{2,i}\}$ is derived by taking the derivative of the CDF $F_{Y_i}(x)$ which is given by:

$$F_{Y_i}(x) = \text{Prob} \{Z_i \leq x\}$$

$$= \text{Prob} \{\min \{Z_{1,i}, Z_{2,i}\} \leq x\}$$

$$= 1 - \text{Prob} \{\min \{Z_{1,i}, Z_{2,i}\} \geq x\}$$

$$= 1 - \text{Prob} \{Z_{1,i} \geq x\} \text{Prob} \{Z_{2,i} \geq x\}$$

$$= 1 - (\text{Prob} \{Z_{1,i} \geq x\})^2. \quad (53)$$

with

$$\text{Prob} \{Z_{1,i} \geq x\} = \int_x^{+\infty} \frac{dt}{(1 + t)^2}$$

$$= \frac{1}{1 + x}. \quad (54)$$

Inserting (54) in (53), and taking the derivative with respect to $x$ leads to Eq. (10).

Let now consider $X = \max_{i=1,2,...,K} Y_i$. As $Y_i$, for $i = 1, 2, \ldots, K$ are independent and identically distributed, the CDF of $X$ is given by:

$$\text{Prob} \{X \leq x\} = \left(\text{Prob} \{Y_i \leq x\}\right)^K$$

$$= \left(1 - \frac{1}{(1 + x)^2}\right)^K$$

$$= \sum_{k=0}^{K} \binom{K}{k} \frac{(-1)^k}{(1 + x)^{2k}}, \quad (55)$$

where the last expression comes from the binomial expansion. Taking the derivative with respect to $x$ of (55) leads to the result of Eq. (11).

**APPENDIX B**

This section details the derivation of equation (14). We consider two identical and independent random variables $Z_{1,i}$ and $Z_{2,i}$ that follow the PDF of equation (13). The CDF of $Z_i = \min \{Z_{1,i}, Z_{2,i}\}$ is derived
the same way as in Appendix A and we obtain:

\[
\text{Prob}\{Z_i \leq x\} = 1 - (\text{Prob}\{Z_{1,i} \geq x\})^2.
\]  

(56)

Taking the derivative of \(\text{Prob}\{Z_i \leq x\}\) with respect to \(x\) leads to the PDF \(f_{Z_i}(x)\) which can be written as

\[
f_{Z_i}(x) = 2f_{Z_{1,i}}(x)\text{Prob}\{Z_{1,i} \geq x\},
\]  

(57)

where

\[
\text{Prob}\{Z_{1,i} \geq x\} = \int_x^{+\infty} \frac{t^{m-1}}{B(m, m)(1 + t)^{2m}} dt,
\]  

(58)

which can be solved using [24, Eq. 3.194.2] so that we have

\[
\int_x^{+\infty} \frac{t^{m-1}}{B(m, m)(1 + t)^{2m}} dt = \frac{2F_1 \left(2m, m; m+1; -\frac{1}{x}\right)}{mx},
\]  

(59)

where \(B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}\) is the Beta function [24, Eq. 8.384.1] and \(\Gamma(.)\) is the Euler Gamma function defined by \(\Gamma(x) = \int_0^{+\infty} t^{x-1}e^{-t}dt\) [24, Eq. 8.310.1].

Finally, combining (57) and (59) leads to the desired result.

**APPENDIX C**

This section demonstrates the derivation of (44). We consider the function \(g\) defined by

\[
g(K, x) = \log \left(1 + \frac{\delta}{x}\right) \left(\frac{x}{x + \beta}\right)^{2K},
\]

where we used \(x = \gamma_0\), \(\delta = Q_{\text{peak}}\) and \(\beta = N_0B\) for the sake of simplicity. Finding the maximum of function \(g(.,.)\) implies taking the derivative with respect to \(x\) and then solving

\[
\frac{\partial g(K, x)}{\partial x} = 0,
\]

which after some algebraic manipulations leads to the following expression:

\[
2K\beta(x + \delta) \log\left(1 + \frac{\delta}{x}\right) = \delta(x + \beta).
\]  

(60)
Now, let \( y = 1 + \frac{\delta}{x} \). Equation (60) can then be reexpressed as follows:

\[
2K\beta \left( \frac{1}{y-1} + 1 \right) \log y = \delta \left( \beta + \frac{\delta}{y-1} \right)
\]

\[
\Leftrightarrow 2K\beta y \log y = \beta(y-1) + \delta
\]

\[
\Leftrightarrow \log y = \frac{1}{2K} - \frac{1}{2K} \left( 1 + \frac{\delta}{\beta} \right) \frac{1}{y}
\]

\[
\Leftrightarrow y = \exp \left( \frac{1}{2K} \right) \exp \left( \frac{\delta - \beta}{2K\beta} \right)
\]

\[
\Leftrightarrow \frac{\delta - \beta}{2K\beta} \exp \left( -\frac{1}{2K} \right) = \frac{\delta - \beta}{2K\beta} \exp \left( \frac{\delta - \beta}{2K\beta} \right),
\]

which is the exact definition of the Lambert function \([27]\). Then, we can write

\[
\frac{\delta - \beta}{2K\beta} \exp \left( -\frac{1}{2K} \right) = W_0 \left( \exp \left( -\frac{1}{2K} \right) \right)
\]

\[
\Leftrightarrow y = \frac{\delta - \beta}{2K\beta W_0 \left( \exp \left( -\frac{1}{2K} \right) \right)}.
\]

Finally using \( x = \frac{\delta}{y-1} \), we achieve the result of (44).

REFERENCES


Figure 1. System Model: the SU sender (S) communicates with the SU destination (D) through a single relay chosen by opportunistic relaying among the $K$ relaying terminals. Transmit power of $S$ and $R_k$ have to meet peak power constraint at the PU receiver $P$.

Figure 2. Analytical results and Monte-Carlo simulations for the PDF $f_T(x)$ in Rayleigh and Nakagami fading and with multiple relays.
Theoretical formula
Simulation

Figure 3. PDF $f_{T}(x)$ in Lognormal case: comparison between analysis and Monte-Carlo simulation (200000 samples).

Figure 4. Outage probability of opportunistic relaying versus threshold $\gamma_0$ for different values of $K$. 

$K = 4, \sigma_{dB} = 4 \text{ dB}$

$K = 8, \sigma_{dB} = 6 \text{ dB}$
(a) Rayleigh ($m = 1$) and Nakagami-$m$ fading

(b) Lognormal shadowing

Figure 5. Ergodic capacity of relay channels under spectrum-sharing constraints (marks indicated simulations).

(a) Rayleigh ($m = 1$) and Nakagami-$m$ fading

(b) Lognormal shadowing

Figure 6. Gain in ergodic capacity by use of multiple relay nodes.
(a) Rayleigh ($m = 1$) and Nakagami-$m$ fading

(b) Lognormal shadowing

Figure 7. Evolution of $\text{tifr}$ capacity for a multirelay system with spectrum-sharing limits.

(a) Rayleigh ($m = 1$) and Nakagami-$m$ fading

(b) Lognormal shadowing

Figure 8. Gain in $\text{tifr}$ capacity for a multirelay system with spectrum-sharing limits.