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Polarized optical feedback and noise effects on CO\textsubscript{2} laser polarization dynamics: spatial and temporal behavior

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We present the role of the discharge current and noise on the polarization dynamics in the CO\textsubscript{2} laser, showing a transition from stable to bistable states and the presence of coherent resonance. We also explore the role of an optical feedback to achieve polarization stabilization or alternation between first-order transverse modes.

Keywords: feedback; polarization dynamics; noise; coding

1. Introduction
Laser dynamics is commonly studied by considering the electric field as a scalar variable, since in most systems the polarization state is imposed by anisotropies of the cavity. However, in perfectly cylindrical laser cavities without any elements to select a preferred polarization, the study of the dynamics includes the necessity of considering the vector nature of the electric field. The polarization dynamics of an isotropic laser have been dealt with in several theoretical works, pointing out the important role played by the material variables in the selection of the polarization state [1]. In particular, the degeneracy of the angular momentum states of the laser transition sublevels has been considered as the coupling source between different polarization states. Dynamical models have been developed to explore the role of the anisotropy due to the laser medium [2–8]. In a quasi-isotropic laser, the medium as well as the optical cavity introduce anisotropy. When the induced anisotropy is of the same order of magnitude as that of the optical cavity we can observe a regime of competition between the two orthogonally polarized field components. The polarization dynamics during the switch-on transient of a quasi-isotropic CO\textsubscript{2} laser emitting on the fundamental mode and TEM\textsubscript{01} mode are reported [9,10]. Recently, using a quasi-isotropic CO\textsubscript{2} laser, it has been shown that it is possible to stabilize the emission of one of the two polarization modes by using an optical feedback and commute the polarization by a fast switch in the feedback arm [11]. In [12] experimental evidence of the in-phase and out-of-phase synchronization of the polarization in two coupled quasi-isotropic CO\textsubscript{2} lasers is reported.

In this work, we study the effects of noise-induced polarization bistability, including experimental evidence of coherence resonance [13–15]. We also show the effect of a polarized optical feedback to control the polarization state of a quasi-isotropic laser emitting on the first transverse mode.

2. Experimental results
The transient polarization dynamics of a quasi-isotropic CO\textsubscript{2} laser displays a competition between two distinct orthogonal polarization modes. The competition dynamics and the final steady state are related to residual cavity anisotropies. In a particular case, where the laser is close to the resonance condition, none of the polarization modes is favored by anisotropy and the system remains in a bistable condition, with spontaneous flips between the polarization states [9]. The experimental setup is shown in Figure 1. It consists of a quasi-isotropic CO\textsubscript{2} laser, with a Fabry–Pérot optical resonator defined by a totally reflective mirror and a partially reflective mirror mounted on a piezoelectric transducer (PZT) in order to control the laser detuning. The laser tube is terminated by an antireflection coated ZnSe window. Particular attention has been devoted to the insertion of two electrodes in the laser tube for preserving the cylindrical geometry of the discharge. The cavity length is 550 mm while the discharge length is 420 mm. The active medium is a mixture of 82% He, 13.5% N\textsubscript{2}, and
4.5% CO₂ gases and it is pumped by a dc discharge current. By means of the iris diaphragm the laser is operated on the fundamental mode (TEM₀₀ mode). The laser beam is directed toward the infrared camera (Pyrocam III) after passing through a linear polarizer fixed at 45° with the beam direction. The reflected beam from the polarizer is directed to a fast-response room temperature Mercury-Cadmium-Telluride (MCT) detector coupled with a digital oscilloscope.

In previous experiments [9,11], we have reported this bistable condition of the CO₂ laser polarization for different conditions, while here we present evidence that this phenomenon is related also to the laser pump condition which depends on the discharge current. The discharge current is slowly changed between an initial value (6 mA) where the laser is above threshold and no switches between polarization states are observed, to a high value (9 mA) where fast switching occurs. Figure 2 shows this transition between the stable condition and the bistable one where the switching spontaneously occurs. When the current is low, we have seen no change in the output mode structure (see Figure 3(a) for the spatial measurement and Figure 3(b) for the temporal measurement). These results show that the polarization bistability condition occurs when the laser is operated far from the threshold condition.

Here we investigate the role of noise by adding to the discharge current a white noise with variable rms variance σ. Noise is added when the polarization state is stable at a low pump level (above the laser threshold current value, 4.5 mA). The switching condition between polarization states is also achieved in this way. The bistability appears when the noise signal (rms value) is around 5% of the steady-state value. In Figure 4 we show the polarization jumps, induced by noise with small (Figure 4(a)), and high levels, (Figure 4(b)), where it appears that the frequency of the jumps increases by increasing the noise signal level. This polarization bistability driven by noise shows evidence of coherence resonance [13–15], here reported for non-solid-state lasers. In order to obtain a quantitative measure of this phenomenon, we study the signal-to-noise ratio (SNR) of the power spectrum of a polarized intensity component for increasing values of the noise variance σ, measured in mV. This magnitude is defined as $\text{SNR}(\sigma) = 10 \log(P_s(\sigma)/P_n(\sigma))$, where $P_s(\sigma)$ is the maximum of the power spectrum of the experimental signal and $P_n(\sigma)$ is the corresponding value of the power spectrum for the extrapolated noise background. In order to clarify this measurement, in Figure 5 we plot an example of this experimental signal power spectrum $P_s(\sigma)$ corresponding to the signal plotted in Figure 4(b), as well as the corresponding extrapolated noise background power spectrum, showing the relevant points $[P_s(\sigma), P_n(\sigma)]$ used in the calculation of the $\text{SNR}(\sigma)$. As can be seen in Figure 6, a maximum of the $\text{SNR}(\sigma)$ is achieved for $\sigma = 600$ mV, corresponding to the situation presented in Figure 4(b).

When the iris diaphragm is slightly opened it is possible to have laser operation on the first transverse mode TEM₂₀ (annular mode), which is the superposition of the TEM₀₁ and TEM₁₀ modes. In such a case the dynamics is more complicated due to the interplay between laser multimode operation and polarization instabilities. Here we show the effect of a weak optical feedback when a mirror is inserted in the reflection arm (on the TEM₀₀ mode operation, a weak optical feedback leads to a polarization stabilization) [11].

During the oscillation of the feedback mirror, the laser output shape is alternating in an irregular way from the annular shape to two in-line circle modes, either horizontal or vertical. These two modes, with the...
same polarization direction closely resemble the two superposition modes giving rise the unperturbed annular mode (see Figure 7). The mode switching is evident from the time series in Figure 8, where the three signal levels refer to the three polarization modes. By adjusting the cavity detuning and the strength of optical feedback, we can get periodic switching between the horizontal and the vertical mode (without passing via the annular mode). The time evolution corresponding to this condition is shown in Figure 9.

3. Numerical results

The theoretical background of this work [9,12] is based on the theory of the isotropic laser. Taking into consideration the CO₂ laser, the condition is more complicated \( J = 19 - J = 20 \). In this model, the level splitting will be represented by different evolution equations for the electric, matter polarization, and population inversion fields in each polarization. The fields will be decomposed in a circularly polarized basis. The results are represented by two-level Maxwell–Bloch equations. By means of this model we reproduce most of the experimental observations reported in the previous section.

Numerical calculations of the steady state show a dependence on the detuning similar to that observed in the experiment [9]. The behavior of the total intensity shows a periodic modulation whose frequency depends only on the value of the detuning difference anisotropy.
between the second-order modes. The evolution of the total intensity reproduces the two consecutive transients corresponding to the onset of the fundamental and annular modes, respectively.

Our theoretical approach is based on the theory of the isotropic laser developed in [4] where the optical and quadrupole coherences between upper levels are considered.

Figure 6. Signal-to-noise ratio (SNR) of the maximum peak of the power spectrum of the polarized laser intensity as a function of the rms noise value applied to the pump.

Figure 7. Spatial structure of the alternating modes, TEM$_{01}$ and TEM$_{40}$ modes. (The color version of this figure is included in the online version of the journal.)

This theory was developed for the simplest case of a $J_1 - J_0$ transition, but it has been shown to predict also the behavior of lasers with a different level structure, such as the He–Ne and He–Xe laser. Despite the fact that the transition involved in our system is even more complicated ($J_{19} - J_{20}$) we have successfully applied this theory to explain all the polarization dynamics of a CO$_2$ laser when emitting on the TEM$_{00}$ mode [9]. In addition, we have deduced an effective value of the coherence relaxation rate for this laser. However, to our knowledge this theory has not been extended to include spatial degrees of freedom, i.e. to study how the presence of several transverse modes affects the polarization dynamics. In this part of the work, we present the theory in order to reproduce the experimental observations.
Therefore, the polarization dynamics due to an intrinsic anisotropy associated with the polarization in the active medium interacts with a spatial competition related to a geometrical origin. At a microscopic level, independent of the number of sublevels, the conservation of the angular momentum allows only two kinds of possible transitions (δm = ±1), which generate a split of the active population in two molecular ensembles in such a way that an anisotropy is induced in the active medium. In our model this splitting will be represented by different evolution equations for the electric, matter polarization, and population inversion fields in each polarization. The fields will be decomposed in a circularly polarized basis. The resulting two-level Maxwell–Bloch equations can be written as [1,7,9]

\[
E_R = \kappa (P_R - E_R) + \delta E_E + (\alpha + i \beta) E_L + i\alpha (\Delta - 4\rho^2) E_R,
\]

\[
\dot{E}_L = \kappa (P_L - E_L) + \delta E_E + (\alpha + i \beta) E_R + i\alpha (\Delta - 4\rho^2) E_L,
\]

\[
\dot{P}_R = -\gamma _E [P_R - D_R E_R - E_L C],
\]

\[
\dot{P}_L = -\gamma _E [P_L - D_L E_L - E_R C^*],
\]

\[
\dot{C} = \gamma_c C - \frac{\gamma_0}{4} (E_L^* P_R + E_R P_L^*),
\]

\[
\dot{D}_R = -\gamma _E \left[ D_R - r + \frac{1}{2} (E_R P_L^* + E_L^* P_R) + \frac{1}{4} (E_L P_L^* + E_R^* P_R) \right],
\]

\[
\dot{D}_L = -\gamma _E \left[ D_L - r + \frac{1}{2} (E_L P_R^* + E_R^* P_L) + \frac{1}{4} (E_L^* P_R + E_R P_L^*) \right],
\]

(1)

where \( E_R (r, t), E_L (r, t) \) are the slowly varying electric fields. \( \dot{P}_R (r, t), \dot{P}_L (r, t) \) stand for the matter polarization fields, \( D_R (r, t) \) and \( D_L (r, t) \) are the respective population inversions and \( C (r, t) \) is the coherence field. \( r(t) \) is the rescaled pump. The transverse coordinates are rescaled as \((\nu, \eta) = (x, y)/w_0\), \( w_0 = \lambda (L(r_2 - L)/\pi)^{1/2} \) being the minimum beam waist. The factor \( \rho^2 = \nu^2 + \eta^2 \) is the transverse distance to the mirror center. Therefore, \( \Delta = \sigma_x^2 + \sigma_y^2 \) is the transverse Laplacian and \( \alpha = \lambda c / (4\pi w_0^2) \) is the diffraction coefficient. The parameter \( \delta \) represents the detuning between the cavity and the atomic transition frequencies. We denote by \( \omega_0 \) the central frequency of the only active atomic transition, which is the \( P(20) \) line. We denote as \( \omega \) the field frequency without taking into account the transverse contribution to the eigenfrequency. Thus, \( \delta = (\omega_0 - \omega)/\gamma_{\text{per}} \) is the rescaled cavity detuning of the modes from the central atomic transition frequency, apart from the transverse frequency shift, which will be considered later. The parameters \( \alpha = (\kappa_y - \kappa_H)/2 \) and \( \beta = (\delta_H - \delta_V)/2 \) stand for the linear anisotropies in the losses and detuning with respect to the cavity \( H-V \) axes, respectively. Here \( \kappa_y, \kappa_H \) are the losses in the \( H \) and \( V \) axes, respectively, and \( \delta_y, \delta_H \) are the corresponding detuning.

In our homogeneously broadening CO\(_2\) laser, the polarization decay is \( \gamma_{\text{per}} = 4.4 \times 10^{8} \) s\(^{-1}\) and the inversion decay rate is \( \gamma_I = 1.95 \times 10^{7} \) s\(^{-1}\) [16]. The \( \gamma_c \) parameter represents the coherence decay rate, whose value should be chosen between \( \gamma_{\text{per}} \) and \( \gamma_I \) [1]. This parameter cannot be directly measured, but an extensive comparison between simulation and experiment allowed us to deduce an effective value \( \gamma_c \approx \gamma_I \) in all cases, which is also consistent with the observation that just linearly polarized states are found in the experiment [9]. From the experimental results we know that just the TEM\(_{00}\), TEM\(_{01}\), and TEM\(_{10}\) take part in the dynamics, and therefore the general expressions of the variables as linear combinations of these three modes are

\[
E_{R(L)} (v, \eta, \tau) = \sum \limits_{j=0}^{2} A_j(v, \eta) e_{j, R(L)}(\tau),
\]

\[
P_{R(L)} (v, \eta, \tau) = \sum \limits_{j=0}^{2} A_j(v, \eta) p_{j, R(L)}(\tau),
\]

\[
D_{R(L)} (v, \eta, \tau) = \sum \limits_{k=0}^{5} B_k(v, \eta) d_{k, R(L)}(\tau),
\]

\[
C(v, \eta, \tau) = \sum \limits_{k=0}^{5} B_k(v, \eta) c_k(\tau),
\]

(2)

where \( e_{j, R(L)}(\tau), p_{j, R(L)}(\tau) \) with \( j = 0, 1, 2, d_{k, R(L)}(\tau) \) and \( c_k(\tau) \) with \( k = 0, 1, \ldots, 5 \) are the temporal evolution profiles of the variables. The mode spatial functions \( A_j(v, \eta) \) are the standard Gauss–Hermite modes TEM\(_{00}\), TEM\(_{01}\), and TEM\(_{10}\), respectively:

\[
A_0(v, \eta) = \left( \frac{2}{\pi} \right)^{1/2} \exp(-\rho^2),
\]

\[
A_1(v, \eta) = 2\eta A_0(v, \eta),
\]

\[
A_2(v, \eta) = 2\nu A_0(v, \eta).
\]

(3)

The full details of this model have been reported in [10]. By means of this model we reproduce most of the experimental observations reported in the previous section. In order to reduce the large number of parameters involved in the simulations, we consider the cavity losses as perfectly isotropic, taking \( a = 0 \) in all cases. Numerical calculations of the steady state show a dependence on the detuning similar to that observed in the experiment. The behavior of the total intensity shows a periodic modulation whose frequency depends only on the value of \( \omega_0 \), the detuning difference anisotropy between the second-order modes. Therefore, we can associate this stable modulation with the degeneration frequency break of the
annular modes. Unfortunately, we have no experimental control over this parameter, in contrast with the case studied in [17]. According to Tamm, a progressive decrease of \( d \), approaching the resonance condition, yields a chaotic fluctuation of the intensity, which has never been observed in the experiment. This gives us an effective measure of the value of the symmetry breaking in our experiment. In order to obtain the same frequency, around 2 MHz, observed in our experiment, we set this parameter to \( d = 0.007 \), a very small percentage compatible with spontaneous symmetry breaking. As was observed in the experiment, the modulation percentage is smaller for small detuning values (\( \delta = 0.1 \), Figure 10(c)) than for large detuning values (\( \delta = 0.7 \), Figure 10(d)) which indicates that the symmetry breaking slightly increases when \( \delta \) increases.

Once the steady state has been characterized and contrasted with the experiment, the reproduction of the transient dynamics is explored. In this case, the evolution of the total intensity reproduces the two consecutive transients corresponding to the onset of the fundamental and annular modes, respectively. When the intensity of each polarized component is calculated, we observe out-of-phase oscillations at \( \approx 100 \text{ kHz} \).

Additionally, our system is quasi-isotropic, and therefore the possible linear anisotropies are reduced to the unavoidable residual mechanical vibration, minimal misalignments or even nonuniform heating. All of these possible effects behave as an effective noise that is at the base of the observed bistability. This behavior cannot be faithfully reproduced with a static linear loss parameter, which is the usual form in the literature [1]. Then, in our model the linear anisotropy parameter is included as a noise. Finally, the model reads as [1,9,11]

\[
\dot{E}_R = [\kappa(D_R - 1) + i\delta]E_R + (\kappa C - \omega)E_L,
\]

\[
\dot{E}_L = [\kappa(D_L - 1) + i\delta]E_L + (\kappa C - \omega)E_R,
\]

\[
\dot{C} = -\gamma_l \left[ C \left( 1 + \frac{1}{4}(|E_L|^2 + |E_R|^2) \right) - \frac{1}{4}E_RE_L^*(D_R + D_L) \right],
\]

\[
\dot{D}_R = -\gamma_l \left[ D_R(1 + |E_R|^2) - r + \frac{1}{2} \left( D_L|E_L|^2 + \frac{3}{2}(E_RE_L^*C + E_LE_R^*C) \right) \right].
\]

Figure 10. Numeric steady configuration. Near resonance for \( \delta = 0.1 \): (a) average spatial profile polarized along the \( H \) eigendirection, (b) laser total intensity time dependence for (a). Far from resonance for \( \delta = 0.7 \): (b) spatial profile polarized along the \( H \) eigendirection; (d) laser total intensity time dependence for (c) static.
\[ \dot{D}_L = -\gamma_I \left[ D_L(1 + |E_L|^2) - r + \frac{1}{2} \left( D_R|E_R|^2 + \frac{3}{2} (E_R E_R^* C + E_L E_R^*) \right) \right], \]  

where \( E_R(t), E_L(t) \) are the slowly varying electric fields, \( D_R(t) \) and \( D_L(t) \) are the respective population inversions. Subscripts \( R \) and \( L \) stand for right and left, respectively, indicating that a circularly polarized basis is adopted for the fields. The losses are \( \kappa = -c \ln(R_1 R_2)/4L = 6.74 \times 10^5 \text{s}^{-1} \). In our low pressure CO\(_2\) laser, the polarization decay is \( \gamma_{\text{polar}} = 4.4 \times 10^5 \text{s}^{-1} \), the inversion decay rate is \( \gamma_{\text{II}} = 1.95 \times 10^5 \text{s}^{-1} \), and the coherence decay rate is fitted to \( \gamma_c = \gamma_{\text{II}} \) (14). Therefore, the experimental system corresponds to a class B laser, and then the polarization matter variables have been adiabatically eliminated. \( C(t) \) represents the optical coherence between the upper sublevels in the case of the transition from a state \( J=1 \) to \( J=0 \). In a matrix density description, \( C \) corresponds to the off-diagonal matrix elements coupling different angular momentum states of the upper level (5).

The parameter \( \delta \) represents the detuning between the cavity and the atomic transition frequencies. In our numerical study, we fit \( \delta = 0 \) since in the experiment the laser remains close to the resonant condition. Parameter \( \alpha(t) \) represents the linear anisotropies in the losses with respect to the laser eigendirections in the following \( XY \) axes, see Figure 1.

Numerical simulations have been carried out in order to analyze the effects of the noise in the pump parameter in the polarization switching induced by the cavity losses. We have observed that for a fixed value of \( \beta \), depending on the mean pump value \( r_0 \) (or, what is equivalent, on the laser power), the system changes from a stable polarization to a bistable condition of polarization switching. Thus, depending on the available energy, the linear anisotropies in the cavity losses can induce polarization jumps or not. In Figure 6 we give numerical results of the temporal evolution of the \( Y \) polarized intensity component together with the temporal evolution of the parameter \( \alpha \) (magnified for better observation), for increasing values of the mean pump parameter \( r_0 \), while \( \beta \) is constant during the simulation with a value of about 1% of the global pump. It can be noticed that \( \alpha \) is about 0.1% of the total losses \( \kappa \), which fits well with an effective noise. We see that very close to the lasing threshold the polarization rarely switches (Figure 11(a)), but for increasing values of \( r_0 \), the jumps occur more often (Figure 11(b)) and finally, for sufficiently high values of \( r_0 \) (Figure 11(c)), the linear anisotropies in the cavity losses guide completely the transitions between the two polarizations. This can be compared with its experimental counterpart, Figure 2, where the experimental route from low to high energy is plotted. This model also reproduces the experimental response of the polarization flips when increasing white noise is added to the pump, shown in Figure 4. The equivalent numerical results can be seen in Figure 12, where the \( Y \)
intensity is plotted for two values of the maximum noise amplitude $\beta$ (1% and 3%) with $r_0$ fixed. Unfortunately, in spite of the fact that it reproduces most of the observed features, it is clear that our model is not able to reproduce the coherent resonance observed in the experiment, since the model must fit the unknown cavity anisotropies through binomial noise in the $\alpha$ parameter. Therefore, for a certain value of $\alpha$, the noise can induce polarization flips but they are not coherent unless the perturbation in the losses is periodic.

4. Conclusions

We have shown the noise induced transition from monostable to bistable polarization of an isotropic laser. For certain values of the noise, the polarization flips are maximally coherent. This is one of the few experimental evidences of coherent resonance in non-feedback bistable optical systems.

By means of a polarized optical feedback it is possible to exploit interesting spatio-temporal polarization dynamics including periodic switching between two linearly polarized transverse modes. This kind of periodic alternation is crucial for polarization coding transmission links using CO$_2$ lasers.

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References