Transmission-Capacity Trade-Off for Coexisting Cellular and Mobile Ad Hoc Networks

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Abstract—In practice, radio spectrums allocated to cellular networks are usually underutilized. To increase the spectrum usage efficiency, we studied spectrum sharing between a cellular uplink and a mobile ad hoc networks. These networks use either all uplink frequency sub-channels or their disjoint sub-sets, called spectrum underlay and spectrum overlay, respectively. Given these methods, the capacity trade-off between the coexisting networks is analyzed based on the transmission capacity of a network with Poisson distributed transmitters. This metric is defined as the maximum density of transmitters subject to an outage constraint for a given signal-to-interference ratio (SIR). Using stochastic geometry, the transmission-capacity trade-off between the coexisting networks is derived, where both spectrum overlay and underlay as well as successive interference cancelation (SIC) are considered. In particular, for small target outage probability, the transmission capacities of the coexisting networks are proved to satisfy a linear equation. Its coefficients depend on the spectrum sharing method and whether SIC is applied. This linear equation shows that spectrum overlay is more efficient than spectrum underlay.

I. INTRODUCTION

Despite spectrum scarcity, most licensed radio spectrums are underutilized according to Federal Communications Commission [1]. In particular, in existing cellular networks based on frequency division duplex, equal bandwidths are allocated for uplink and downlink, even though the traffic load for downlink is much heavier than that for uplink [2]. For better utilization, uplink spectrum can be shared with other networks, which forms the theme of this paper.

Depending on whether holding a spectrum licence, a wireless network is called the primary (e.g. cellular networks) or secondary network (e.g. mobile ad hoc networks (MANETs)). Accessing a licensed band, the secondary transmitters must not cause significant interference to the primary receivers. One simple method of sharing a licensed band, called spectrum underlay, is to spread the signal energy radiated by each secondary transmitter over the whole band, suppressing the power spectrum density of the resultant interference to the primary receivers [3]. An alternative method for sharing licensed spectrum is called spectrum overlay, where secondary transmitters access frequency sub-channels unused by nearby primary receivers. Decentralized spectrum overlay can be realized using cognitive-radio algorithms [3], [4], which, however, are vulnerable to spectrum sensing errors and require complicated computation at secondary transmitters. For this reason, we consider spectrum overlay based on a simple random access protocol and relying on base stations for allocating sub-channels to coexisting networks.

In a unlicensed spectrum such as the industrial, scientific and medical (ISM) bands, all networks have equal priorities for spectrum access. Sharing unlicensed bands between competing networks has been studied using game theory [5], [6].

There exist few theoretical results on the capacity trade-off between networks sharing spectrum despite this being a basic question. Define the transmission capacity of a network as the maximum density of transmitters under an outage probability constraint for a target signal-to-interference-and-noise ratio (SINR) [7]. In [8], the transmission capacities of underlaid cellular and hot-spot networks are analyzed. Another capacity metric, called transport capacity, is defined in [9] as the end-to-end throughput per unit distance of a multi-hop network. In [10], the transport capacities of two coexisting ad hoc networks are shown to achieve the optimum scaling laws. In [8], [10], the network-capacity trade-off is not analyzed.

This paper targets a cellular uplink network and a mobile ad hoc network (MANET) sharing the uplink spectrum, where uplink users, base stations, and ad hoc transmitters all follow Poisson distributions. Our main contributions are summarized as follows. First, considering an interference-limited environment, bounds on the signal-to-interference-ratio (SIR) outage probabilities are derived for spectrum overlay and underlay with and without using successive interference cancelation (SIC) at receivers [11]. Second, for small target outage probability, the transmission-capacities of the coexisting networks are showed to satisfy a linear equation, whose coefficients depend on the spectrum sharing method and whether SIC is used. Define the capacity region as the set of feasible combinations of transmission capacities. Third, for small target outage probability, the capacity region for spectrum underlay is shown to be no larger than that for spectrum overlay. They can be equalized by setting transmission power as derived. Finally, we characterize the effects of different parameters on transmission capacities of the coexisting networks. For instance, the transmission capacity of the cellular network grows linearly with the base station density.

This work builds on [12]. A new cellular network model based on Poisson tessellation is used in this paper, which captures geographic variation of cell sizes [13]. Moreover, SIC is considered in this paper but not in [12].
II. NETWORK MODEL

Network Architecture: The spectrum-sharing cellular and ad hoc networks are illustrated in Fig. 1. The transmitters in the MANET are modeled as a homogeneous Poisson point process (PPP) on the two-dimensional plane, denoted as $\Pi$ with the density $\lambda$. Each transmitter in the MANET is associated with a receiver located at a fixed distance $d$. The radiation power of transmitters is assumed fixed and denoted as $\rho$.

For the cellular network, the base stations and uplink users are modeled as two independent homogeneous PPPs denoted as $\Omega$ and $\Pi$, respectively. Their corresponding densities are $\lambda_b$ and $\lambda$. To enhance the long-term link reliability, each uplink user transmits to the nearest base station. Consequently, the cellular network forms a Poisson tessellation of the two-dimensional plane and each cell is known as a Voronoi cell [13]. Based on their distances from the serving base station, the users in each cell are separated into inner-cell and cell-edge users. Specifically, inner-cell users lie inside the largest disk contained in the corresponding Voronoi cell and other users in this cell are cell-edge users. Due to severe path-loss for direct links with the base stations, cell-edge users are assumed to rely on relay stations for uplink transmissions. For simplicity, relay-assisted transmission is assumed to keep the SIR outage probabilities of cell-edge users below those of inner-cell users.

Modulation: The uplink spectrum is divided into $M$ frequency-flat sub-channels by using orthogonal frequency division multiplexing (OFDM) [14]. In each network, a transmitter modulates signals using frequency-hopping spread spectrum, where signals hop randomly over all sub-channels assigned to the network [14].

Channel Model: Consider the link from a typical user $U_0$ to the serving base station $B_0$. A sub-channel accessed by $U_0$ consists of path loss and a fading factor $W$ such that the signal power received by $B_0$ is $\rho WD^{-\alpha}$, where $\rho$ is transmission power, $D = |U_0 - B_0|$, and $\alpha$ is the path-loss exponent. Similarly, the interference power from an interferer $X$ (user/ad hoc node) to $B_0$ is $P_X G_X R_X^{-\alpha}$, where $P_X \in \{\rho, \tilde{\rho}\}$, $R_X = |X - B_0|$ and $G_X$ is the fading factor.

For the ad hoc network, the received signal power for a typical receiver $T_0$ is $Wd^{-\alpha}$ where $W$ is the fading factor; the interference power from an interferer $X$ to $T_0$ is $P_X G_X R_X^{-\alpha}$, where $R_X = |X - T_0|$.

Spectrum Sharing Methods: For spectrum overlay, the $M$ sub-channels are divided into two disjoint subsets and assigned to two coexisting networks. Let $K$ and $\tilde{K}$ denote the numbers of sub-channels used by the cellular and ad hoc networks respectively, where $K + \tilde{K} = M$. Spectrum overlay requires initialization, where base stations broadcast to ad hoc nodes the indices of the available sub-channels and the allowable node density. Next, for spectrum underlay, both coexisting networks use all $M$ sub-channels. Compared with spectrum overlay, spectrum underlay has less initialization overhead as the cellular network need not inform the ad hoc network the indices of available sub-channels.

The transmission capacities of the coexisting networks can be increased by employing SIC at each base station and ad hoc receiver for reducing interference. The SIC model is simplified from that in [11]. Specifically, the interference power of each targeted interferer must exceed a threshold equal to the received signal power multiplied by a factor $\kappa > 1$. Increasing $\kappa$ decreases the average number of canceled interferers and vice versa. Perfect SIC is assumed.

Transmission Capacity: As in [15], the networks are assumed to be interference limited and thus the reliability of received data packets is measured by the SIR. Let $\text{SIR}$ and $\tilde{\text{SIR}}$ represent the SIRs at the typical user $U_0$ and ad hoc receiver $T_0$, respectively. The correct decoding of received data packets requires the SIRs to exceed a threshold $\theta \geq 1$. In other words, the rate of information sent from a transmitter to a receiver is no less than $\log_2(1 + \theta)$ assuming Gaussian signaling. To support this information rate with high probability, the outage probability that SIR and $\tilde{\text{SIR}}$ are below $\theta$ must be no larger than a given threshold $0 < \epsilon \ll 1$, i.e.

$$P_{\text{out}}(\lambda) := \Pr(\text{SIR} < \theta) \leq \epsilon, \quad \tilde{P}_{\text{out}}(\tilde{\lambda}) := \Pr(\tilde{\text{SIR}} < \theta) \leq \epsilon$$

where $P_{\text{out}}$ and $\tilde{P}_{\text{out}}$ denote the SIR outage probabilities for the cellular and the ad hoc networks, respectively. The transmission capacities of the cellular and the ad hoc networks, denoted as $C$ and $\tilde{C}$ respectively, are defined as [7]

$$C(\epsilon) = (1 - \epsilon)\lambda_c, \quad \tilde{C}(\epsilon) = (1 - \epsilon)\tilde{\lambda}_c$$

where $\lambda_c$ and $\tilde{\lambda}_c$ satisfy $P_{\text{out}}(\lambda_c) = \epsilon$ and $\tilde{P}_{\text{out}}(\tilde{\lambda}_c) = \epsilon$.

III. OUTAGE PROBABILITIES

The outage probabilities for the coexisting networks are derived for spectrum overlay and underlay with and without SIC. Due to space limit, the proofs of the key results in this section are omitted but provided in the full-length paper [16].

A. Existing Analytical Approach

The analysis in subsequent sections adopts an existing approach for analyzing the SIR outage probability of a typical receiver in the presence of a Poisson field of interferers [7], [15], [17]. Due to the difficulty in direct analysis, existing
work focuses on deriving bounds on the outage probability as summarized below. The lower bound on the outage probability considers only the strong interferers that individually guarantees an outage event at the receiver; the upper bound results from applying Chebyshev’s inequality on bounding the distribution tail of the sum interference power from the remaining interferers.

For spectrum interferers, the SIRs can be written as

\[
\text{SIR} = \frac{\sum_{X \in \Pi_m \setminus \{U_0\}} G_X R_X^\alpha}{\sum_{X \in \Pi_m \setminus \{T_0\}} G_X R_X^\alpha}
\]

where the PPP \( \Pi_m \) with the density \( \frac{\lambda}{K} \) groups transmitters in \( \Pi \) that access the \( m \)th sub-channel; similarly the PPP \( \tilde{\Pi}_m \) is thinned from \( \Pi \) and has the density \( \frac{\lambda}{\tilde{K}} \) [18]. Using [15, Theorem 2], the bounds on \( P_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) in (1) are given in the following lemma.

**Lemma 1.** For spectrum overlay, the SIR outage probabilities \( P_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) are bounded as

\[
P_{\text{out}}^l \left( \frac{\lambda}{K} \right) \leq P_{\text{out}}(K, \lambda) \leq P_{\text{out}}^u \left( \frac{\lambda}{K} \right)
\]

\[
P_{\text{out}}^l \left( \frac{\tilde{\lambda}}{\tilde{K}} \right) \leq P_{\text{out}}(\tilde{K}, \tilde{\lambda}) \leq P_{\text{out}}^u \left( \frac{\tilde{\lambda}}{\tilde{K}} \right)
\]

where

\[
P_{\text{out}}(\lambda) = \mathbb{E} \left[ 1 - \exp \left( -\zeta \lambda W^{-\delta} D^2 \right) \right]
\]

\[
P_{\text{out}}^u(\lambda) = 1 - \mathbb{E} \left[ \xi(W, D, \lambda) \exp \left( -\zeta \lambda W^{-\delta} D^2 \right) \right]
\]

\[
\xi(W, D, \lambda) = \begin{cases} 
1 - \frac{\delta \zeta D^2 W^{-\delta} \lambda}{1 - \delta} & < 1 \\
0, & \text{otherwise}
\end{cases}
\]

and \( \zeta := \pi \theta \mathbb{E}[G^\delta] \) and \( \delta := \frac{2}{\alpha} \).

**B. Outage Probabilities: Spectrum Underlay**

For spectrum underlay, the interferers for each receiver include transmitters in both coexisting networks. The SIRs can be modified from (3) accordingly. The bounds on \( P_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) are given in the following proposition. Its proof uses the superposition property of the PPPs grouping interferers from different networks.

**Proposition 1.** For spectrum underlay, the outage probabilities \( P_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) are bounded as

\[
P_{\text{out}}^l \left( \frac{\lambda + \eta^{-\delta} \tilde{\lambda}}{M} \right) \leq P_{\text{out}}(\lambda, \tilde{\lambda}) \leq P_{\text{out}}^u \left( \frac{\lambda + \eta^{-\delta} \tilde{\lambda}}{M} \right)
\]

\[
P_{\text{out}}^l \left( \frac{\eta \lambda + \tilde{\lambda}}{M} \right) \leq \tilde{P}_{\text{out}}(\lambda, \tilde{\lambda}) \leq \tilde{P}_{\text{out}}^u \left( \frac{\eta \lambda + \tilde{\lambda}}{M} \right)
\]

where \( \eta := \rho / \tilde{\rho} \), and \( P_{\text{out}}^l \) and \( P_{\text{out}}^u \) are defined in Lemma 1.

Proposition 1 shows that the outage probability for each network depends on the transmitter densities of both networks. This coupling is due to spectrum underlay and the resultant mutual interference between the coexisting networks. As shown in Section IV, such coupling may reduce the transmission capacities for spectrum underlay with respect to those for spectrum overlay. Moreover, Proposition 1 also shows that the outage probabilities for spectrum underlay depend on the transmission power ratio \( \eta \). The effect of \( \eta \) is also characterized in Section IV.

Finally, the probability density function (PDF) of the distance \( D \) between an inner-cell user and the serving base station is given in the following lemma, required for computing the bounds on \( P_{\text{out}} \) for both spectrum overlay and underlay.

**Lemma 2.** The PDF of \( D \) for an inner cell user is given as

\[
f_D(t) = -8\pi \lambda_d Ei(-4\pi \lambda_d t^2)
\]

where the exponential integral \( Ei(x) = \int_{-\infty}^{x} t^{-1} e^t dt \).

It can be observed from (4) that the key parameter of the PDF of \( D \) is the density of base station \( \lambda_b \). Intuitively, increasing the density of base stations reduces the cell sizes and thus \( D \) and vice versa.

**C. Outage Probabilities: Spectrum Sharing with SIC**

Recall from Section II that an interferer \( X \) is canceled at the typical base station or ad hoc receiver if the interference power exceeds \( \kappa \) times the received signal power. Using this model, the outage probabilities for coexisting networks using SIC are bounded in the following proposition.

**Proposition 2.** For spectrum sharing with SIC, the bounds on outage probabilities \( P_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) can be modified those in Lemma 1 and Proposition 1 by replacing \( P_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) with \( \hat{P}_{\text{out}} \) and \( \tilde{P}_{\text{out}} \) as follows

\[
\hat{P}_{\text{out}}^l(\lambda) = P_{\text{out}}^l(\lambda \chi)
\]

\[
\hat{P}_{\text{out}}^u(\lambda) = 1 - \mathbb{E} \left[ \xi(W, D, \lambda)e^{-\chi \lambda W^{-\delta} D^2} \right]
\]

where \( \chi := (1 - \theta^{-\delta} \kappa^{-\delta}) \).

Note that \( \hat{P}_{\text{out}}^l \) and \( \hat{P}_{\text{out}}^u \) differ from \( P_{\text{out}}^l \) and \( P_{\text{out}}^u \) only by the factor \( \chi \). The factor \( \chi < 1 \) represents the SIC advantage of reducing outage probabilities with respect to the case of no SIC (\( \chi = 1 \)). Moreover, decreasing the SIC factor \( \kappa \) reduces \( \chi \) and thus outage probabilities. Nevertheless, \( \kappa \) being too small may invalidate the assumption of perfect SIC. Specifically, small \( \kappa \) implies small SIR for the process of decoding interference prior to its cancelation and potentially results in significant residual interference after SIC.

**IV. Network Capacity Trade-Off: Asymptotic Analysis**

Using the results in the preceding section, the trade-off between the transmission capacities \( C \) and \( \tilde{C} \) defined in (2) is characterized in the following theorem for small target outage probability \( \epsilon \to 0 \). Its proof is given in [16].
\textbf{Theorem 1.} For $\epsilon \to 0$, transmission capacities of the coexisting networks satisfy
\begin{equation}
\tilde{\mu}C + \mu C = \frac{M}{\varphi} \epsilon + O(\epsilon^2)
\end{equation}
where the weights $\mu$ and $\tilde{\mu}$ are given as\footnote{The subscripts $o$ and $u$ identify spectrum overlay and underlay, respectively.}

(Overlap) $\tilde{\mu}_o = \zeta \mathbb{E}[W^{-\delta}] \varphi^2$, $\mu_o = \frac{\zeta \mathbb{E}[W^{-\delta}]}{8\pi \lambda_b}$

(Underlay) $\tilde{\mu}_u = \tilde{\mu}_0 \vee (\eta^{-\delta} \mu_o)$, $\mu_u = (\eta^{-\delta} \tilde{\mu}_0) \vee \mu_o$.

If SIC is used, $1 - \theta^{-\delta} \kappa^{-\delta} \leq \varphi \leq \frac{2}{\varphi - \delta} - \theta^{-\delta} \kappa^{-\delta}$; otherwise $\varphi = 1$.

Theorem 1 shows that the trade-off between $C$ and $\tilde{C}$ follows a linear equation. In particular, the slope at which $\tilde{C}$ increases with decreasing $C$ is $-\mu/\tilde{\mu}$, which depends on different network parameters according to (8). The results in Theorem 1 are interpreted using several corollaries as follows.

To facilitate discussion, define an 	extit{outage limited} network as one whose transmission capacity is achieved with the outage constraint being active. For instance, the cellular network is outage limited if $C = (1 - \epsilon) \lambda_c$ with $P_{\text{out}}(\lambda_c) = \epsilon$. For spectrum overlay, both the coexisting networks are outage limited. Nevertheless, for spectrum underlay, it is likely that only one of the two networks is outage limited as explained shortly. As implied by the proof of Theorem 1 in [16], for spectrum underlay, both networks are outage limited only if $\mu_u = \tilde{\mu}_u$, where $\mu_u$ and $\tilde{\mu}_u$ are in (8). Otherwise, $\mu_u > \tilde{\mu}_u$ correspond to only the cellular network being outage limited; $\mu_u < \tilde{\mu}_u$ indicates that only the MANET is outage limited.

Spectrum overlay is shown to be more efficient than spectrum underlay as follows. Define the capacity region of the coexisting networks as the region enclosed by the capacity trade-off curve in (7) and the positive axes of the $C$-$\tilde{C}$ coordinates. This region contains all feasible combinations of transmission capacities of coexisting networks. Thus, the size of the capacity region measures the spectrum sharing efficiency. The capacity regions for spectrum overlay and underlay are compared in the following corollary.

\textbf{Corollary 1.} For $\epsilon \to 0$, the capacity region for spectrum underlay is no larger than that for spectrum overlay. They are identical if and only if the transmission-power ratio is
\begin{equation}
\eta = \left( \frac{\mu_o}{\tilde{\mu}_o} \right)^{\frac{1}{\delta}}
\end{equation}
where $\mu_o$ and $\tilde{\mu}_o$ are given in (8).

Corollary 1 shows that spectrum overlay is potentially more efficient than spectrum underlay due to network coupling for the latter. Specifically, the possibility that a network is not outage limited compromises the efficiency of spectrum underlay. This can be avoided by setting $\eta$ as in (9), ensuring that both spectrum-underlaid networks are outage limited.

The next corollary specifies the effects of several parameters on transmission capacities of the coexisting networks.

\textbf{Corollary 2.} For $\epsilon \to 0$, transmission capacities vary with network parameters as follows.

1. (Spectrum overlay) $C$ increases linearly with the base station density $\lambda_b$, $\tilde{C}$ increases inversely with the ad hoc transmitter-receiver distance $d$.

2. (Spectrum underlay) If the cellular network is outage limited, both $C$ and $\tilde{C}$ increase linearly with the base station density $\lambda_b$. Otherwise, $C$ and $\tilde{C}$ increase inversely with the ad hoc transmitter-receiver distance $d$.

3. For both spectrum sharing methods, $C$ and $\tilde{C}$ increase linearly with $\epsilon$ and the number of sub-channels $M$, and inversely with $\varphi$ related to SIC.

Finally, we analyze the transmission-capacity gains due to spatial diversity gains contributed by multi-antennas [14]. To obtain concrete results, the fading factors $W$ and $\tilde{W}$ are assumed to follow the chi-squared distributions with the degrees of freedom $L$ and $\tilde{L}$, respectively, which are the diversity gains. These fading distributions can result from using spatial diversity techniques such as beamforming over multi-antenna i.i.d. Rayleigh fading channels [14]. Thus
\begin{equation}
\mathbb{E}[W^{-\delta}] = \frac{\Gamma(L - \delta)}{\Gamma(L)}, \quad \mathbb{E}[\tilde{W}^{-\delta}] = \frac{\Gamma(\tilde{L} - \delta)}{\Gamma(\tilde{L})}.
\end{equation}

The following corollary is obtained by combining Theorem 1, (10) and Kershaw’s Inequalities [19].

\textbf{Corollary 3 (Spatial Diversity Gain).} Consider the diversity gains per link of $L$ and $\tilde{L}$ for the coexisting cellular and ad hoc networks, respectively.

1. (Spectrum overlay) The spatial diversity gains multiply $C$ by a factor between $(L - 1)^8$ and $L^8$, and $\tilde{C}$ by a factor between $(\tilde{L} - 1)^8$ and $L^8$.

2. (Spectrum underlay) The spatial diversity gains multiply both $C$ and $\tilde{C}$ by a factor between $(L - 1)^8$ and $L^8$ if the cellular network is outage limited, or otherwise between $(\tilde{L} - 1)^8$ and $L^8$.

\textbf{V. SIMULATION AND NUMERICAL RESULTS}

In this section, the asymptotic transmission capacity trade-off curves derived in the preceding section are compared with simulation results. The simulation procedure follows that in [20]. For simulations, the distance between the typical ad hoc transmitter and receiver is $d = 5$ m, the required SIR $\theta = 3$ or 4.8 dB, the path-loss exponent $\alpha = 4$, the base station density $\lambda_b = 10^{-3}$, the SIC factor $\kappa = 2$ dB, and the transmission-power ratio $\eta = 5$ dB.

Fig. 2 compares the asymptotic transmission-capacity trade-off curves in Theorem 1 and those generated by simulations for the target outage probability $\epsilon = 10^{-2}$. In Fig. 2(b) for the case of SIC, the bounds on the asymptotic trade-off curves correspond to those on $\varphi$ as given in Theorem 1. By comparing Fig. 2(a) and Fig. 2(b), the capacity regions for spectrum overlay are larger than those for spectrum underlay.
The transmission-capacity trade-off for the coexisting cellular and ad hoc networks is analyzed for different spectrum sharing methods. Bounds on outage probabilities are derived for spectrum overlay and underlay with and without SIC. For small target outage probability, the transmission capacities of the coexisting networks are shown to satisfy a linear equation. This result provides a theoretical basis for adapting the size of the ad hoc network to the traffic dynamics of cellular uplink. The trade-off equation also provides guidelines for controlling network parameters to increase network capacities.

VI. CONCLUSION

References