Mean square $H_\infty$ synchronization of coupled nonlinear delay stochastic partial differential systems

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Abstract: This paper considers the synchronization problem for the coupled nonlinear delay stochastic partial differential systems (SPDSs), both mean square asymptotical synchronization and mean square $H_\infty$ synchronization are studied. Making use of the Lyapunov-Krasoviskii functional method and Itô formula, sufficient conditions are derived which guarantee the mean square asymptotical synchronization of the coupled nonlinear delay SPDSs. When the external disturbances appear in the coupled nonlinear delay SPDSs, the mean square $H_\infty$ synchronization is dealt with and sufficient criterion is provided. Finally, numerical examples are given to illustrate the correctness of our results.

Key Words: Stochastic partial differential systems, delay, mean square asymptotical synchronization, $H_\infty$ synchronization, Lyapunove-Krasoviskii functional

1 Introduction

Synchronization is a collective behavior of complex systems, it has been attracted many attentions in the past two decades, being the wide existence in the real world and a great deal of applications in the engineering [1]. Stochastic disturbances are ubiquitous and they may degrade the system properties. The synchronization of stochastic systems has been a research focus, many exciting results have been reported, see [2, 3] and the references therein.

In a complex system, communications between the sub-systems are necessary. Usually, the bandwidth of communication channel is bounded, time-delay is inevitable when the data needed to be transmitted is large. Time-delay also can destroy the synchronization [4–6]. The synchronization of stochastic delay systems is well considered [7–11]. The topics of exponential mean square synchronization [7, 8], impulsive synchronization [9], almost surely synchronization [10, 11] are discussed, the Lyapunov functional method, invariant principle and stochastic techniques were used in these literatures.

As well known, in the chemical engineering, heat progressing, structure vibration and biological populations and so on, the system’s dynamics are usually described by partial differential equations. The study of synchronization for partial differential systems mainly focused on the systems of neural networks with reaction-diffusion terms [12–14]. The synchronization of stochastic neural networks with reaction-diffusion terms have attracted many attentions [15–21]. In this paper, we consider the following $N$-coupled nonlinear SPDSs with time delay

\[
\begin{align*}
dy^i(x, t) &= \{f(y^i(x, t), y^i(x, t - \tau)) \\
&+ B\nabla^2 y^i(x, t) + \sum_{j=1}^{N} g_{ij} y^j(x, t)\} dt \\
&+ h(y^i(x, t), y^i(x, t - \tau)) dB(t), \\
x \in U, t > 0, i = 1, 2, \cdots, N,
\end{align*}
\]  (1)

where $y^i(x, t) = (y^i_1, y^i_2, \cdots, y^i_n)^T$ is the system state of the $i$-th subsystem. $x = (x_1, x_2) \in U \subset \mathbb{R}^2$ and $t \geq 0$ are the

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spatial and time variable, respectively, \(|x_i| < l_i, i = 1, 2\).
The nonlinear functions \(f(\cdot)\) and \(h(\cdot)\) are smooth and the
time delay \(\tau\) is a positive constant. The coefficients \(B\) and \(D\)
are constant matrices. The constant \(c\) is the couple strength.

\(B(t)\) is a Brownian motion in a complete probability space
\((\Phi, F, P)\) with the filter \(F, G = (g_{ij})_{N \times N}\) is the couple
matrix of the network which satisfies the following condition:
if there exists a link between the node \(i\) and the node \(j,
\(i \neq j\), then \(g_{ij} = g_{ji} = 1\), otherwise \(g_{ij} = g_{ji} = 0\) and
the diagonal elements are defined as

\[ g_{ii} = -\sum_{j \neq i} g_{ij}, i = 1, 2, \cdots, N. \]

The Laplace operator \(\nabla^2\) is defined as

\[ \nabla^2 y_i(x, t) = \frac{\partial^2 y_i(x, t)}{\partial x_1^2} + \frac{\partial^2 y_i(x, t)}{\partial x_2^2}. \]

We pose the following boundary conditions and initial values
for system (1), for \(i \in \{1, 2, \cdots, N\},
\[ y_i(x, t) = 0, x \in \partial U, y_i(x, t) = \varphi_i(x, t), t \in [-\tau, 0]. \]

Let function \(s(x, t)\) be the solution of an isolated node
which \(y_i(x, t)\)’s are expected to be synchronized. The function
\(s(x, t)\) satisfies the following equation

\[ ds(x, t) = (f(s(x, t), s(x, t - \tau)) + B\nabla^2(s(x, t)))dt + h(s(x, t), s(x, t - \tau))dB(t), x \in U, t > 0. \]

The boundary conditions and initial values are given as follows

\[ s(x, t) = 0, x \in \partial U, s(x, t) = \varphi_i(x, t), t \in [-\tau, 0]. \]

Taking \(e^i(x, t) = y_i(x, t) - s(x, t)\), a direct subtraction
yields the following error dynamic

\[ de^i = \{f(y^i, y^i_\tau) - f(s, s_\tau) + B\nabla^2 e^i + c \sum_{j=1}^{N} g_{ij} e^j\}dt + \{h(y^i, y^i_\tau) - h(s, s_\tau)\}dB(t), \]

\[ t > 0, i = 1, 2, \cdots, N. \]

For the sake of simplicity, here and in the sequel, the variables
\((x, t)\) are compressed, \(y^i\) and \(s_\tau\) denote \(y^i(x, t - \tau)\)
and \(s(x, t - \tau)\), respectively.

Denote

\[ e(x, t) = ((e^1)T, (e^2)T, \cdots, (e^N)T)T, \]

\[ F(y, y_\tau) = (f^T(y^1, y^1_\tau), \cdots, f^T(y^N, y^N_\tau))T, \]

\[ F(s, s_\tau) = (f^T(s, s_\tau), \cdots, f^T(s, s_\tau))T, \]

\[ H(y, y_\tau) = (h^T(y^1, y^1_\tau), \cdots, h^T(y^N, y^N_\tau))T, \]

and

\[ H(s, s_\tau) = (h^T(s, s_\tau), \cdots, h^T(s, s_\tau))T. \]

We obtain the following error dynamic

\[ de = (F(y, y_\tau) - F(s, s_\tau) + (I \otimes B)\nabla^2 e + c(G \otimes D)e)dt + (H(y, y_\tau) - H(s, s_\tau))dB(t). \]

Letting \(\dot{B} = I \otimes B\) and \(M = c(G \otimes D)\), the above error
dynamic can be rewritten as

\[ de = \{F(y, y_\tau) - F(s, s_\tau) + B\nabla^2 e + Me\}dt + (H(y, y_\tau) - H(s, s_\tau))dB(t), \]

\[ t > 0, i = 1, 2, \cdots, N. \]

Now we provide some definitions and lemmas which are
necessary for the further study.

**Definition 2.1.** The \(N\)-coupled nonlinear delay SPDSs (1)
achieve the mean square asymptotical synchronization if the error \(e^i(x, t)\) satisfies

\[ \lim_{t \to \infty} E[e^i(x, t)T e^i(x, t)] = 0 \]

for all \(x \in U\) and any \(i \in \{1, 2, \cdots, N\}. \) Here \(E(\cdot)\) denotes
the mathematical expectation.

**Lemma 2.2 (\[22]\))** Let \(U\) be a cube: \(|x| < l_i, i = 1, 2\) and
let \(z(x) = (z_1(x), z_2(x), \cdots, z_N(x))^T \in R^n\) be a function
belonging to \(C^2(U)\) which vanishes on the boundary \(\partial U\) of \(U\), then

\[ \int_U z^T(x)\nabla^2 z(x)dx \leq -(\frac{1}{l_1^2} + \frac{1}{l_2^2})\int_U z^T(x)z(x)dx. \]

**Lemma 2.3.** For any positive constant \(c > 0\) and vectors \(X, Y,\)
the following inequality holds

\[ X^TY + Y^TX \leq cX^TX + e^{-1}Y^TY. \]

**Assumption 2.4.** For any \(u_1, v_1, u_2, v_2 \in R^n, \) there exist
positive constants \(L_1, L_2, L_3\) and \(L_4\) such that the following inequalities hold

\[ \|f(u_1, v_1) - f(u_2, v_2)\|^2 \leq L_4\|u_1 - u_2\|^2 + L_2\|v_1 - v_2\|^2 \]

\[ \|h(u_1, v_1) - h(u_2, v_2)\|^2 \leq L_4\|u_1 - u_2\|^2 + L_4\|v_1 - v_2\|^2 \]

where \(\|u\|^2 = u^T u.\)

In term of this assumption, we can easily get

\[ (F(y, y_\tau) - F(s, s_\tau))^T (F(y, y_\tau) - F(s, s_\tau)) \leq L_4e^T e + L_2e^T_\tau e_\tau, \]

and

\[ (H(y, y_\tau) - H(s, s_\tau))^T (H(y, y_\tau) - H(s, s_\tau)) \leq L_4e^T e + L_4e^T_\tau e_\tau. \]

**Remark 2.5.** The existence and unique of the solution for
system (1) is a standing assumption in this paper.

### 3 Mean square asymptotically synchronization of
coupled nonlinear delay SPDSs

In this section, we consider the mean square asymptotic synchronization for coupled nonlinear delay SPDSs (1).

**Theorem 3.1.** If there exists a positive definite matrix \(P\)
such that the following inequalities hold

\[ P\dot{B} \geq 0 \]
Lemma 2.3, we get

\[ P^2 + L_1 I + Q - 2 \left( \frac{1}{l_1} + \frac{1}{l_2} \right) P \hat{B} + 2PM + \lambda_{\text{max}}(P)L_4 I < 0, \]

where \( Q = (L_2 + \lambda_{\text{max}}(P)L_4)P \), and \( \lambda_{\text{max}}(P) \) is the maximal eigenvalue of the matrix \( P \), then systems (1) achieve the mean square asymptotical synchronization.

**Proof.** Considering the following Lyapunov-Krasoviskii functional candidate

\[
V(\epsilon, t) = \int_U e(x, t)^T Pe(x, t) dx + \int_U \int_{t-\tau}^t e(x, s)^T Q e(x, s) ds dx,
\]

in term of the Itô formula, we have

\[
E(dV(\epsilon, t))) = E(\int_U 2e^T P F(y, t) - F(s, s_t) + B\nabla^2 e + Me) dx dt
+ E(\int_U 2e^T P (H(y, t) - H(s, s_t)) dB(t))
+ E(\int_U (H(y, t) - H(s, s_t))^T P (H(y, t) - H(s, s_t)) dx dt)
- E(\int_U (e^T Q e - e^T Q e_t) dx dt).
\]

Keeping Assumption 2.4 in mind and letting \( \epsilon = 1 \) in Lemma 2.3, we get

\[
2e^T P(F(y, t) - F(s, s_t))
= e^T P(F(y, t) - F(s, s_t)) + (F(y, t) - F(s, s_t))^T Pe
\leq e^T P^2 e + (F(y, t) - F(s, s_t))^T P (F(y, t) - F(s, s_t))
\leq e^T P^2 e + L_1 e^T e + L_2 e^T e_t
= e^T (P^2 + L_1 I)e + L_2 e^T e_t.
\]

A similar calculation yields

\[
(H(y, t) - H(s, s_t))^T P (H(y, t) - H(s, s_t))
\leq \lambda_{\text{max}}(P)(H(y, t) - H(s, s_t))^T (H(y, t) - H(s, s_t))
= \lambda_{\text{max}}(P) L_3 e^T e + L_4 e^T e_t.
\]

Since \( P \hat{B} \geq 0 \), there exists a matrix \( K \) satisfying \( P \hat{B} = K^T K \). Making use of Lemma 2.2, we get

\[
\int_U 2e^T P \hat{B} \nabla^2 e dx = \int_U 2e^T K^T K \nabla^2 e dx
= \int_U 2(K e)^T \nabla^2 (K e) dx
\leq -(\frac{1}{l_1} + \frac{1}{l_2}) \int_U (K e)^T (K e) dx
= -2(\frac{1}{l_1} + \frac{1}{l_2}) \int_U e^T \hat{B} e dx.
\]

Substituting (7), (8) and (9) into (6), keeping \( Q = (L_2 + \lambda_{\text{max}}(P)L_4)P \) in mind, we have

\[
E(dV(\epsilon, t))) \leq E(\int_U (e^T (P^2 + L_1 I)e + L_2 e^T e_t)
- 2(\frac{1}{l_1} + \frac{1}{l_2}) e^T P \hat{B} e + 2PM e
t + \lambda_{\text{max}}(P)(e^T (P^2 + L_1 I)e + L_2 e^T e_t + e^T Q e - e^T Q e_t) dx dt)
\]

\[
= E(\int_U e^T (P^2 + L_1 I + Q - 2(\frac{1}{l_1} + \frac{1}{l_2}) P \hat{B} + 2PM + \lambda_{\text{max}}(P)(P^2 + L_3 I)e + e^T ((L_2 + \lambda_{\text{max}}(P)L_4) I - Q)e) dx dt)
\]

\[
= E(\int_U e^T (P^2 + L_1 I + Q - 2(\frac{1}{l_1} + \frac{1}{l_2}) P \hat{B} + 2PM + \lambda_{\text{max}}(P)(P^2 + L_3 I)e) dx dt).
\]

From condition (4), we know that \( E(dV(\epsilon, t))) < 0 \). In terms of the Lyapunov stability theory, we get that

\[
\lim_{t \to \infty} E[e^T (x, t) e(x, t)] = 0.
\]

This completes the proof. \( \square \)

### 4 Mean square \( H_\infty \) synchronization of coupled nonlinear delay SPDSs

In the real world, there exist external disturbances which can not be described by white noise. The \( H_\infty \) performance is an effective index for the disturbed system. Here, we consider the mean square \( H_\infty \) synchronization for the disturbed coupled nonlinear delay SPDSs. By virtue of the completing square method, the sufficient criterion is obtained for the mean square \( H_\infty \) synchronization.

The disturbed coupled nonlinear delay SPDSs are described as follows

\[
dy^i(x, t) = (f(y^i(x, t), y^i(x, t - \tau)) + B\nabla^2 (y^i(x, t))
+ e \sum_{j=1}^N g_{ij} y^j(x, t) + v_i(x, t)) dt
+ h(y^i(x, t), y^i(x, t - \tau)) dB(t)
\]

\[
x \in U, t > 0, i = 1, 2, \cdots, N,
\]

where \( v_i(x, t) \) is the external disturbance satisfying \( \int_0^T \int_U v_i^2(x, t) v_i(x, t) dt < \infty \), for a given positive constant \( t_f \).
Taking the notations defined as before, the coupled nonlinear delay SPDSs (10) can be written as
\[
d e(x, t) = (F(y, y_t) - F(s, s_t) + B\nabla^2 e + Me + v)dt
+ (H(y, y_t) - H(s, s_t))dB(t),
\]
x \in U, t > 0, i = 1, 2, \cdots, N,
where \(v(x, t) = \{v_1^T(x, t), v_2^T(x, t), \cdots, v_N^T(x, t)\}^T\).

**Definition 4.1.** The coupled time-delay SPDSs (10) are in the mean square \(H_{\infty}\) synchronization if the error \(e(x, t)\) and external disturbance \(v(x, t)\) satisfy the following inequality, for a given positive constant \(\gamma\), when \(e(x, s) = 0, s \in [-\tau, 0]\),
\[
\int_0^{t_1} \int_U E\|e(x, t)\|^2 dx dt \leq \gamma^2 \int_0^{t_1} \int_U E\|v(x, t)\|^2 dx dt.
\]

**Theorem 4.2.** If there exists a positive definite matrix \(P\) such that
\[
P B \geq 0
\]
and
\[
\frac{1}{\gamma^2} P^2 + P^2 + L_1 I + I + Q - 2\left(\frac{1}{l_1^2} + \frac{1}{l_2^2}\right)PB
+ 2PM + \lambda_{\max}(P)(P^2 + L_3 I) < 0
\]
where \(Q = (L_2 + \lambda_{\max}(P)L_4)P\), then the disturbed coupled nonlinear delay SPDSs (10) achieve the mean square \(H_{\infty}\) synchronization.

**Proof.** Take
\[
V(e(\cdot, t)) = \int_U e^T P e dx + \int_U \int_{t-\tau}^{t} e(x, s)^T Q e(x, s) ds dx.
\]
The same calculation as done in the previous section yields
\[
E(dV(e(\cdot, t))) \leq E\left[\int_U e^T(P^2 + L_1 I + Q + 2e^T P v)
- 2\left(\frac{1}{l_1^2} + \frac{1}{l_2^2}\right)PB + 2PM + \lambda_{\max}(P)(P^2 + L_3 I)e\right] dx dt.
\]
Using the above inequality, we find
\[
E\left(\int_0^{t_1} \int_U (e^T e - \gamma^2 v^T v) dx dt\right)
= E(V(0, e(\cdot, 0)) - E(V(t_1, e(\cdot, t_1))) + E(\int_0^{t_1} dV dt)
+ E\left(\int_0^{t_1} \int_U (e^T e - \gamma^2 v^T v) dx dt\right)
\leq E\left(\int_0^{t_1} \int_U \left\{ e^T(P^2 + L_1 I + Q - 2\left(\frac{1}{l_1^2} + \frac{1}{l_2^2}\right)PB
+ 2PM + \lambda_{\max}(P)(P^2 + L_3 I)e\right\} dx dt\right)
= E\left(\int_0^{t_1} \int_U \left\{ -2(\frac{1}{l_1^2} + \frac{1}{l_2^2})PB + \lambda_{\max}(P)(P^2 + L_3 I)e\right\} dx dt\right)
\leq E\left(\int_0^{t_1} \int_U \left\{ -2(\frac{1}{l_1^2} + \frac{1}{l_2^2})PB + \lambda_{\max}(P)(P^2 + L_3 I)e\right\} dx dt\right).
\]
In light of condition (13), we get the mean square \(H_{\infty}\) synchronization performance.

**5 Examples.**
In this section, we provide two examples to illustrate the correctness of our results.

**Example 5.1.** For the case of asymptotical synchronization, the following coupled nonlinear SPDSs are considered
\[
d y_i = [10^{-3} \arctan(y_i(x, t - 0.5))] + 0.003 \nabla^2 y_i
+ 2 \sum_{j=1}^{N} g_{ij} y_j dt + 10^{-3} y_i(x, t - 0.5)dB(t)
\]
x = (x_1, x_2) \in U = [0, 1] \times [0, 1], i = 1, 2, 3.

The synchronization function \(s(x, t)\) satisfies
\[
ds = (10^{-3} \arctan(s(x, t - 0.5))) + 0.003 \nabla^2 s dt
+ 10^{-3} s(x, t - 0.5)dB(t).
\]
The boundary conditions are given as follows, for \(i = 1, 2, 3,\)
\[
y_i(0, x_2, t) = y_i(1, x_2, t) = y_i(x_1, 0, t) = y_i(x_1, 1, t) = 0,
s(0, x_2, t) = s(1, x_2, t) = s(x_1, 0, t) = s(x_1, 1, t) = 0.
\]
The initial conditions are given as follows, for \(t \in [-0.5, 0],\)
\[
y_i(x, y, t) = \sin(2\pi x) * \sin(2\pi y) * \cos(2),
y_i(x, y, t) = \sin(2\pi x) * \sin(2\pi y) * \sin(2),
y_i(x, y, t) = \sin(2\pi x) * \sin(3\pi y) * \cos(2),
s(x, y, t) = \sin(2\pi x) * \sin(4\pi y) * \sin(2)\).
\]
The adjacency matrix is defined as
\[
G = \begin{pmatrix}
-2 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2
\end{pmatrix}
\]
It is easy to see that \(\tau = 0.5, B = 0.003\) and \(L_1 = L_3 = 0, L_2 = L_4 = 10^{-6}, l_1 = l_2 = 1.\) If we take \(P = 10^{-3} I,\) then \(Q = (10^{-9} + 10^{-12}) I,\) it is not hard to verify the inequalities (4). Therefore, systems (14) achieve the mean square asymptotical synchronization. The errors are shown in Figure 1, Figure 2 and Figure 3 at time \(t = 0, t = 1, t = 5,\) respectively. Through these figures, we can read out the asymptotical synchronization. Moreover, when some spatial variable is fixed, e.g., \(x_2 = 0.2,\) we also show the errors along time \(t\) in Figure 4. It illustrates the asymptotical synchronization again.
Fig. 1: The errors of coupled SPDSs (14) at \( t = 0 \).

Fig. 2: The errors of coupled SPDSs (14) at \( t = 1 \).

Fig. 3: The errors of coupled SPDSs (14) at \( t = 5 \).

Fig. 4: The errors of coupled SPDSs (14) at \( x_2 = 0.2 \).

**Example 5.2.** For the case of \( H_\infty \) synchronization, the following coupled nonlinear SPDSs are considered

\[
\begin{align*}
\frac{dy^1}{dt} &= (0.1 \arctan(y^1(x, t - 0.5)) + \nabla^2 y^1 + 2 \sum_{j=1}^{N} g_{ij} y^j) \\
&\quad + e^{-t} \sin(xy) \cos(t) dt + 0.1 y^1(x, t - 0.5) dB(t) \\
\frac{dy^2}{dt} &= (0.1 \arctan(y^2(x, t - 0.5)) + \nabla^2 y^2 + 2 \sum_{j=1}^{N} g_{ij} y^j) \\
&\quad + e^{-t} \sin(2xy) \cos(t) dt + 0.1 y^2(x, t - 0.5) dB(t) \\
\frac{dy^3}{dt} &= (0.1 \arctan(y^3(x, t - 0.5)) + \nabla^2 y^3 + 2 \sum_{j=1}^{N} g_{ij} y^j) \\
&\quad + e^{-t} \sin(3xy) \cos(t) dt + 0.1 y^3(x, t - 0.5) dB(t)
\end{align*}
\]

\( x = (x_1, x_2) \in U = [0, 1] \times [0, 1], i = 1, 2, 3. \) (16)

The synchronization function \( s(x, t) \) satisfies

\[
\begin{align*}
ds &= (0.1 \arctan(s(x, t - 0.5)) + \nabla^2 s) dt \\
&\quad + 0.1 s(x, t - 0.5) dB(t)
\end{align*}
\]

The boundary conditions are given as follows, \( i = 1, 2, 3, \)

\[
y^i(0, x_2, t) = y^i(1, x_2, t) = y^i(x_1, 0, t) = y^i(x_1, 1, t) = 0, \\
s(0, x_2, t) = s(1, x_2, t) = s(x_1, 0, t) = s(x_1, 1, t) = 0.
\]

The initial conditions are given as follows, \( t \in [-0.5, 0], \)

\[
y^1(x, y, t) = y^2(x, y, t) = y^3(x, y, t) = s(x, y, t) \\
= \sin(2\pi x) \sin(2\pi y) \cos(2).
\]

The adjacency matrix is defined as (15). From equation (16) we can get that the external disturbance are defined as follows:

\[
\begin{align*}
v_1(x, t) &= e^{-t} \sin(xy) \cos(t), \\
v_2(x, t) &= e^{-t} \sin(2xy) \cos(t), \\
v_3(x, t) &= e^{-t} \sin(3xy) \cos(t).
\end{align*}
\]

It is easy to see that \( \tau = 0.5, B = 1 \) and \( L_4 = L_3 = 0, L_2 = L_4 = 0.1, l_1 = l_2 = 1. \) If we take matrix \( P = I \)
then $Q = 0.2I$, it is not difficult to verify the inequalities (13). Taking $\gamma = 1$ and $t_f = 10$, we find
\[
E \int_0^{t_f} \int_U e^2(x, t)v(x, t)dxdt = 0.0144^2 < 1^2. \quad (17)
\]
This shows the $H_\infty$ synchronization performance. The conservativeness may be caused by the choices of the external disturbances in this example and the choice of the Lyapunov-Krasoviskii functional in the theorem.

6 Conclusions

In this paper, we consider the synchronization problem for the coupled nonlinear delay stochastic partial differential systems. Both mean square asymptotical synchronization and mean square $H_\infty$ synchronization are studied. Making use of the Lyapunov-Krasoviskii functional method and Ito formula, we obtain the sufficient conditions which guarantee the mean square asymptotical synchronization of the coupled nonlinear delay SPDSs. When the coupled nonlinear delay SPDSs are disturbed by the external disturbances, we provide the sufficient criterion for the mean square $H_\infty$ synchronization of coupled delay SPDSs. Finally, we provide numerical examples to illustrate the correctness of our results.

References