Option model for joint production and preventive maintenance system

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1. Introduction

The rapid changes in markets, the increasing uncertainty in customer demands, and the unexpected failures in unreliable machines have raised the cost of manufacturing in the current fast-paced industrial environment. To meet the uncertain future demands and to ensure the economic success of a production system, cost-effective and reliable production planning and maintenance scheduling is required.

The maintenance policies currently used in the industry are generally classified into reactive maintenance (RM, or corrective maintenance) and preventive maintenance (PM). RM is performed once a machine breaks down. The main task of such maintenance activities is to restore the machine to a desired condition. PM is like a "scheduled or planned machine breakdown" and it is defined as the maintenance work-orders performed on a machine while still in its operating state, which would bring the system to the "as good as new" condition (Sheu et al., 2001). It is observed that RM usually results in a higher downtime compared to PM due to unavailability of resources, causing logistic delays. Hence, the general cost of RM may be as high as three to four times as that of PM (Chitra, 2003). A large amount of existing literature has shown that implementing PM strategies in unreliable production facilities can effectively prolong machine life and reduce operating cost (Nakagawa and Yasui, 1991).

Generally, the risk of production and maintenance system can be managed both operationally and financially. Conventionally, PM policies implemented in manufacturing enterprise systems are operational-oriented. Operationally, an enterprise can do inventory-driven PM strategy, condition-based PM policy and time-triggered maintenances etc. There are numerous papers on operationally optimizing inspection and maintenance in...
production systems in the presence of machine failures. These optimization models are usually developed under consideration of production metrics such as throughput, reliability, availability and overall cost of a system. Srinivasan and Lee (1996) considered an (s, S) PM policy in which the PM work-orders are performed as soon as inventory reaches a certain threshold value, S. They formulated PM and safety stock strategies assuming that the PM tasks are scheduled every m units of time and derived m by minimizing expected cost per unit time. Sim and Endrenyi (1988) developed a minimal preventive-maintenance model where the optimal value of mean time for PM policy is determined by minimizing the unavailability of the machine due to machine breakdowns. Meller and Kim (1996) considered a control policy where the system undergoes repair once it breaks down. The production resumes immediately after repair, continuing until the inventory level reaches a threshold value. Dohi et al. (2001) extended the economic manufacturing quantity (EMQ) model by Cheung and Hausman (1997) to a new stochastic PM model to find an optimal policy, in which manufacturing quantity and safety stock are derived minimizing the cost per unit time. Kenné et al. (2007) formulated an analytical model for the joint determination of an optimal age-dependent buffer inventory and PM policy in a production environment that is subject to random machine breakdowns. Castro (2009), Zequeira and Bérenguer (2006) determined the optimal preventive maintenance schedules by considering two modes of failure (maintainable and non-maintainable) and the number of PM tasks are dependent on different failure rates of the system.

For operational maintenance planning, maintenance policy is usually time-triggered or periodical, which is either inefficient or excessive. It may happen that a PM task is scheduled right after the machine just resumes. High cost arises due to the unnecessary scheduled downtimes and expensive restoration. Another limitation of the maintenance policies presently implemented is the ignorance of the real dynamics of market demand, and lack of flexible mechanism to make a prompt response to a rapidly changing demand. Most of the researches focus on long-term planning policies that provide steady state plans, assuming that production systems have deterministic demand with constant rate (Lee and Rosenblatt, 1987; Groenevelt et al., 1992). Nevertheless, it is infeasible for the consumer industries with short product life cycle in today’s fast changing market.

In comparison to the rich and abundant literature available on operational planning and managing production and maintenance, seldom literature addresses the financial instrument to exploit intelligent strategies for an integrated production and maintenance system. Unlike conventional operational maintenance models, the novel idea of this paper is to incorporate financial engineering concept “option” into a manufacturing enterprise system to deal with the limitations abovementioned. Options will be rigorously applied to evaluate the joint production and maintenance system in order to effectively find the optimal maintenance policy.

The term “option”, originally a financial derivative, is defined as a “financial instrument whose payoffs and values are derived from or depend on something else” (Ross et al., 2002). An option is a contract giving the owner the “right but not the obligation” to buy or sell an asset at a fixed price before or on a future date. The term “real option”, coined by Myers (1977) and Copeland and Antikarov (2001), is defined as “the right but not the obligation to take an action at a predetermined cost called exercise price for a predetermined period of time”. In recent studies, many attempts have also been made to apply an option or real option to other areas such as manufacturing/production system. Bengtsson (2001) re-viewed manufacturing flexibility and real options from an industrial engineering and production management perspective. The value of option-based flexibility is considered at three levels: machine level, production system level, and manufacturing plant level.

In this paper, option is defined as the right of manufacturers to produce additional units at a lower price since PM can improve reliability and production efficiency. The manufacturing enterprise can balance the tradeoff between reduced risk from uncertainty that options afford and the increased price premium paid to invest PM resources. The proposed option-based PM policy can provide flexibility to adjust production output to satisfy the demand requirement. In this paper, the preliminary contribution is to introduce the new concept of “option” to formulate a joint production and maintenance system, and to extend the conventional periodical maintenance model to an option-based PM model with stochastic demand. The option described in this paper is an instrument derivative whose value is linked to demand uncertainty, which is easily modified to study options whose value is related to other uncertainties of manufacturing system. Using such options that increases the flexibility of the manufacturing operations and decreases the risk due to demand uncertainty, two main improvements can be addressed: (i) PM policy is changed from fixed periodical one to flexible one and (ii) conventionally deterministic demand is extended to uncertain demand.

The rest of this paper is organized as follows. In Section 2, an option-based PM optimization model is formulated and optimal decisions are derived. Section 3 discusses some important relationships among the cost parameters. Section 4 presents numerical studies. Finally, the conclusions and future work are given in Section 5.

2. Problem formulation and preliminary results

In modeling, we assume the production and maintenance decision can only be made at the beginning of production period. In addition, the deterioration of machines is assumed to be linearly increasing with the times of machine breakdowns.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>demand per period, supplied by the inventory</td>
</tr>
<tr>
<td>z</td>
<td>production rate (speed)</td>
</tr>
</tbody>
</table>
2.1. Option model with preventive maintenance

Consider a single-period, single-product production model, at the beginning of the production period, the manufacturer plans to produce after taking an order from a lower-stream retailer, and deliver finished products before the end of the period, \( \tau \), while shortages and overages are allowed.

Let \( D \) denote the demand, a random variable with cumulative distribution function \( F(\cdot) \), known at the beginning of the period. Each unit of the order will cost the manufacturer \( C_p \) to produce per unit time, which will be sold to the lower-stream retailer at a price of \( R \). At the end of period, whenever supply and demand are determined, any shortage incurs a penalty cost \( C_{p}\), and any surplus incurs a salvage value \( C_{s} \).

Consider the production operations with PM activities. At time \( t = 0 \), the planner can purchase an amount of PM options at a unit cost \( C_{p}\), which corresponds to the investment on PM resources such as skilled labor and equipment. Each of these PM options gives the system the right (but not the obligation) to produce one unit of product at a lower unit cost \( C_{pm}\).

The PM policy is described as follows: PM tasks are scheduled to be performed after every \( k \) RM tasks occur, from time 0 until time \( \tau \). The PM cycle is defined as the period between two consecutive PM activities. The production keeps at a specific rate \( z \). It is assumed that the stochastic demand \( D \) occurs continuously and only needs to be satisfied at the end of planning horizon \( \tau \). Since a complex system consists of many components, it is generally assumed that each RM task is treated as minimal repair to remove a failure at minimal effort. However, each PM task is a planned breakdown, which reduces uncertainty causing machine failures and improves system reliability.

The modeling objective is to determine the optimal number of PM work orders during one cycle \( \tau \) to maximize average profit per unit time. The objective function consists of five terms:

1. Revenue:
   \[
   R \cdot \min(D, Q + q)
   \]

2. Salvage value: produced goods, if exceeds demand, will have salvage value of
   \[
   C_{s}(Q + q - D)^{+}
   \]

3. Shortage cost: shortage is possible in case the demand exceeds the total production quantity due to the loss of production during PM period.
   \[
   C_{u} \cdot (D - (Q + q))^{+}
   \]

4. Production cost:
   \[
   C_{p} \cdot Q + C_{pm} \cdot \min(q, (D - Q)^{+})
   \]

5. PM cost:
   \[
   C_{s} \cdot q
   \]

As shown in Fig. 1, the option works as follows: the plant manager makes decision on the production quantity and the number of PM tasks for the production period at \( t = 0 \). The planning horizon is from the beginning of the project at \( t = 0 \) to the end at \( t = \tau \). After performing each PM task, the equipment is restored to “as good as new” condition. Let \( Q \) be the production quantity without PM effect and \( q \) be the additional production quantity when PM is involved during the planning horizon. Let \( C_{p}\) represent the unit cost per unit time for the normal production \( Q \), and \( C_{pm}\) represent the unit cost of purchasing options as PM investment. Then, we have \( Q \), units that are produced before the first PM task, \( Q_{i} \) is the production quantity without PM \( i \), and \( Q_{i} \) is the production quantity with PM \( i \). In each option, the right to produce \( q \) units of product at a cost of \( C_{pm}\) option exercise price. Under this arrangement, the enterprise will actually produce the quantity of \( Q + \min(q, (D - Q)^{+}) \) at unit cost

\[
\pi(Q, q) = E_{\tau} \left[ R \cdot \min(D, Q + q) + C_{s} \cdot (Q + q - D)^{+} - C_{p} \cdot Q - C_{s} \cdot q - C_{pm} \cdot \min(q, (D - Q)^{+}) - C_{u} \cdot (D - (Q + q))^{+} \right]
\] (1)

Note that
\[
(D - Q)^{+} = D - Q \quad \text{if} \quad D \geq Q
\]
\[
= 0 \quad \text{if} \quad D < Q
\] (2)

The above function can be rewritten as

\[
\pi(Q, q) = R \cdot E_{\tau} \left[ \min(D, Q + q) \right] + C_{s} \cdot E_{\tau} \left[ (Q + q - D)^{+} \right] - C_{p} \cdot E_{\tau} [Q] - C_{s} \cdot E_{\tau} [q] - C_{pm} \cdot E_{\tau} \left[ \min(q, (D - Q)^{+}) \right] - C_{u} \cdot E_{\tau} \left[ (D - Q - q)^{+} \right]
\] (3)
Using relations:

\[ E[\min(D, Q + q)] = Q + q - \int_0^{Q+q} F(x) \, dx \]  

(4)

\[ E[\min(q, (D - Q)^+)] = q - \int_0^{Q+q} F(x) \, dx \]  

(5)

The objective function (3) can be simplified to

\[
\pi(Q, q) = R(Q + q) + C_s \cdot \int_0^{Q+q} F(x) \, dx - C_p \cdot Q - C_q \cdot q + C_{pm} \left( \int_0^{Q+q} F(x) \, dx - q \right) + C_u \int_{Q+q}^{\infty} F(x) \, dx
\]

(6)

Let \( T = Q+q \).

\[
\pi(Q, T) = T \cdot (R - C_q - C_{pm}) + Q \cdot (C_q - C_p - C_{pm}) + R \cdot \int_0^T F(x) \, dx + C_{pm} \int_Q^T F(x) \, dx + C_s \cdot \int_0^T F(x) \, dx + C_u \cdot \int_T^{\infty} F(x) \, dx
\]

(7)

The manufacturer seeks to identify the \( Q \) and \( T \) that maximize \( \pi(Q, T) \), subject to the constraint that \( 0 \leq Q \leq T \), in order to decide the optimal number of PM work orders during the production period. It can be proved that the expected profit function is concave with respect to \( Q \) and \( T \), and thus has a unique maximum. Let \( Q^* \) be the optimal quantity of normal production, and \( T^* \) be the production quantity when PM options are implemented in production system. Taking partial derivatives of the objective function (7) and setting them equal to zero, we get

\[
F(T^*) = \Pr(D \leq T) = \frac{R - C_{pm} - C_q + C_u}{R - C_{pm} - C_s + C_u}
\]

(9)

Remark:

\[ Q^* < T^* \] implies that \( \left( \frac{R - C_{pm} - C_q + C_u}{R - C_{pm} - C_s + C_u} \right) > \left( \frac{C_q - C_p + C_{pm}}{C_{pm}} \right) \)

and

\[ C_q < \frac{(R - C_{pm} + C_u) \cdot C_{pm}}{(C_{pm} - C_p)(R - C_{pm} + C_u - C_s)(R - C_s + C_u)} \]

This shows that if the option cost \( C_q \) is too high, the enterprise will not purchase any options, which means no PM is needed. Throughout this paper, it is assumed that the cost parameters always satisfy this constraint.

There are several natural feasible conditions in the option-based modeling:

(i) \( C_q + C_{pm} \geq C_p \)
(ii) \( C_q < C_{pm} < C_p < C_u \)
(iii) \( R > C_{pm} + C_s - C_u \)

(10)

If condition (i) is violated, \( C_q + C_{pm} < C_p \), it would be advantageous for the planner to only purchase options. In other words, it would cost less to produce after doing PM (buy options) than to do normal production (without doing any PM), which would make the normal production without PM useless. Condition (ii) states that penalty cost \( C_q \) is greater than normal production cost \( C_p \), and the normal production cost \( C_p \) is always greater than the option exercise price \( C_{pm} \), which is always greater than salvage value \( C_s \). Condition (iii) must hold to ensure profit for the production project.

**Proposition 1.** The objective function in (6) is jointly concave in \((Q, q)\). Consequently, the optimal decision of \((Q, q)\) is obtained as:

\[
\text{if } \left( \frac{R - C_{pm} - C_q + C_u}{R - C_{pm} - C_s + C_u} \right) > \left( \frac{C_q - C_p + C_{pm}}{C_{pm}} \right) 
\]
holds, then optimal $Q$ and $q$ are as follow:

$$Q^* = F^{-1}\left(\frac{C_q - C_p + C_{pm}}{C_{pm}}\right)$$

$$q^* = T^* - Q^* = F^{-1}\left(\frac{R - C_{pm} - C_q + C_u}{R - C_{pm} - C_s + C_u}\right) - F^{-1}\left(\frac{C_q - C_p + C_{pm}}{C_{pm}}\right)$$

(11)\hspace{1cm}(12)

**Proof.** The joint concavity in $(Q, q)$ follows directly from the $\pi(Q, q)$ expression in (3), taking into account that $|x|^* \geq f(x)$ is an increasing and convex function. Concavity of objective functions is proved as follow:

$$\pi(Q, T) = T \cdot (R - C_q - C_{pm}) + Q \cdot (C_q - C_p - C_{pm}) + R \cdot \int_0^T f(x) dx + C_{pm} \int_0^T f(x) dx + C_s \cdot \int_0^T f(x) dx + C_u \cdot \int_T^\infty f(x) dx$$

Then

$$\frac{\partial \pi(Q, T)}{\partial Q} = (C_q - C_p - C_{pm}) - C_{pm} \cdot f(Q)$$

and $\frac{\partial^2 \pi(Q, T)}{\partial Q^2} = -C_{pm} \cdot f(Q) < 0$ imply that $\pi(Q, T)$ is concave w.r.t $Q$.

Similarly, since

$$\frac{\partial^2 \pi(Q, T)}{\partial T^2} = -(R - C_{pm} + C_u - C_s) \cdot f(T) < 0$$

(13)\hspace{1cm}(14)

and the concavity of $\pi(Q, T)$ w.r.t $T$ is also convinced when condition (iii) in (10) holds. \qed

2.2. Decision on the optimal number of PM tasks

The optimal number of PM tasks can be derived based on the previous results. Let $n$ denote the number of PM tasks performed during the period $[0, \tau]$. PM tasks are undertaken at times $t_1, t_2, \ldots, t_n, n \geq 1$. Relation $t_n = \tau$ is enforced. There is always a PM task done at the end of period (instant $\tau$) in order to ensure that the process is restored to “as good as new” condition at the beginning of each new production cycle.

Let us denote $X_i = t_{i+1} - t_i - s - k \cdot E[T_i]$ to be the real production time between two PM tasks subtracting RM time and PM time. $T_i$ is the time of performing one reactive maintenance task. $X_i$ indicates the real production time and follows an identical and independent Erlang distribution (which is a summation of exponentially distributed production time between two consecutive PM tasks). The PM policy is described as follows: PM frequency is determined by the number of RM activities. There are $k$ RM activities between two PM tasks, and thus $k \cdot n$ up–down cycles during the time period $[0, \tau]$. Mean value of the Erlang distributed time interval between two consecutive PM tasks is $k$ times of MTBF. Assuming the number of breakdowns is $k$, we have

$$E[X_i] = (k + 1)/\lambda_2 = \eta$$

(15)

$$T^* = Q^* + q^* = \alpha \cdot \sum_{i=1}^n E[X_i] = \alpha \cdot n \cdot E[X_1] = \alpha \cdot n \cdot (k + 1)/\lambda_2$$

(16)

where $\tau = n \cdot s + k \cdot n \cdot E[T_i] + n \cdot E[X_i]$. Note: $\theta$ denote value of $E[X_i]$. Hence, the value of $k$ is given as

$$k = \frac{\tau - n \cdot s - \frac{n}{\lambda_2}}{n \cdot \theta + \frac{n}{\lambda_2}}$$

(17)

which gives

$$n^* = \frac{\tau}{s - \theta} - \frac{(Q^* + q^*)(\lambda_2 \theta + 1)}{\alpha(s - \theta)}$$

Therefore, we find the optimal number of PM activities $n^*$ that maximizes the expected profit per unit time during $[0, \tau]$.

3. Discussions

In this section, three more propositions are addressed based on the option model obtained. The Proposition 2 shows how the optimal decisions depend on the cost parameters. The Proposition 3 is the comparison between the basic model and option model with PM, which gives an encouraging conclusion. The last one presents the advantage of option-based model over the conventional periodic PM policy in terms of productivity and profit.

**Proposition 2.** (a) $Q$ increases as $C_{pm}$ or $C_q$ increases $q$ decreases as $C_{pm}$ or $C_q$ increases and $T = Q + q$ also decreases as $C_{pm}$ or $C_q$ increases. (b) $Q \leq Q_0 < Q + q$, where $Q_0$ is obtained from the production system without PM, while $Q$ and $q$ are obtained from option model. (c) The expected profit per unit time decreases in $(C_{pm}, C_q)$.

**Proof.** The argument of $F^{-1}$ in Eq. (11) can be rewritten as $1 - C_q - C_s/C_{pm}$. Hence, $Q$ increases as $C_{pm}$ or $C_q$ increases. Similarly, the argument of $F^{-1}$ from the first term in Eq. (12) can be written as: $1 - C_q - C_s/(R - C_{pm} + C_u - C_s)$ which decreases as $C_{pm}$ or $C_q$ increase. Thus (b) and (c) are proved. \qed

**Proposition 3.** In the PM option model, the manufacturer’s expected profit per unit time is larger than that of the case without PM option.

**Proof.** When $q = 0$ and $Q = Q_0$, it is exactly the basic case that there is no PM options. Let $\pi_0(Q_0)$ denotes the objective value of the basic case without PM. We have

$$\pi_0(Q_0) = (R + C_u - C_p)Q_0 - (R + C_u - C_s) \times E[Q_0 - D]^+ - C_u \cdot E[D]$$

(18)

To show the statement, it is sufficient to show $\pi(Q_0, q) > \pi_0(Q_0)$. For a certain $q$, the left hand side is derived by the objective value of the option-based model.
Comparing (6) and (18), we get
\[
\pi(Q_0, q) - \pi_0(Q_0) = (R - C_{pm} - C_q + C_u)q
- (R - C_{pm} + C_u - C_s)
\times \int_q^{Q_0 + q} F(x) \, dx
\]
(19)

When \( q = 0 \), the left hand side is equal to zero. Taking derivative of the above function at \( q = 0 \), we have
\[
\frac{R - C_{pm} - C_q + C_u}{R - C_{pm} + C_u - C_s} > \left( \frac{C_u - C_p + C_{pm}}{C_{pm}} \right)
\]

From the proof of Proposition 1, we know that the above inequality is exactly the same inequality in Proposition 1 when it holds as a strict inequality. \( \square \)

**Proposition 4.** Comparing the option model with the conventional periodic PM model by performing the same number of PM tasks and RM tasks, the option model outperforms periodic PM model with higher productivity and expected profit per unit time within the same production period.

**Proof.** From Eq. (17),
\[
n^* = \frac{\tau}{s - \theta} - \frac{(Q^* + q^*)\lambda_2\theta + 1}{n(a(s - \theta)}
\]
we obtain the optimal number of PM tasks given the optimal production quantity \( Q^*+q^* \). Let the production time between two PM tasks be \( \eta_1, \eta_2, \ldots, \eta_{n^*} \), respectively. It is known that
\[
\theta = \mathbb{E}[X] = \eta_1 + \eta_2 + \cdots + \eta_{n^*}
\]

From the relation \( \eta_1 + \eta_2 + \cdots + \eta_{n^*} \geq n \cdot a \sqrt{\frac{\eta_1 \cdot \eta_2 \cdot \cdots \cdot \eta_{n^*}}{n^*}} \), the equality is obtained if and only if \( \eta_1 = \eta_2 = \cdots = \eta_{n^*} \), which is exactly the periodic PM policy. This implies that the production time in option PM model is larger than that in the periodic PM model, and so is the production quantity (because the production rate is constant). From Proposition 2 part (c), the profit per unit time increases as \( T \) increases. Therefore, we can conclude that PM option model yields better profit per unit time than the periodic PM policy. \( \square \)

### 4. Numerical case studies

In this section, the optimal decisions are numerically calculated. The demand is assumed to follow a normal distribution w.r.t. \( D = \mu + \sigma z \), where \( z \) is the standard normal variate, and \( \Phi \) denotes the c.d.f. associated with \( z \).

Consider an example with the following data:
\[
C_1 = 0, \quad R = 100, \quad C_p = 40, \quad C_{pm} = 35, \quad C_q = 20, \quad C_u = 45
\]

To investigate how demand volatility affects the optimal value for the option model, we calculate the production quantity under both the optimal options model and basic case without PM options. By substituting these cost parameters into Eqs. (11) and (12), the optimal values \( Q^* \) and \( q^* \) are derived. The expected profit per unit time of the basic model \( \pi_0 \) is computed as follows:
\[
\pi_0(Q_0) = \mathbb{E}(R \cdot \min(D, Q_0) + C_s(Q_0 - D))^+ - C_p \cdot Q_0 - C_u \cdot (D - Q_0)^+
\]
(20)

Since the objective function in (20) is concave in \( Q_0 \), the solution yields the optimal value \( Q_0^* \), where \( Q_0^* = F^{-1}(R + C_u - C_p)/(R + C_u - C_s) \).

Given the same mean value of demand but with increasing variation, the decisions of the manufacturer \( (Q^*, Q^*+q^*) \) in the option model and the corresponding optimal production decision \( Q_0^* \) of the basic model are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Manufacturer’s decisions</th>
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<th></th>
<th>Manufacturer’s decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>Option</td>
<td>Basic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>(Std)Dev.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu = 100.00 )</td>
<td>( \sigma = 0 )</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( \mu = 100.00 )</td>
<td>( \sigma = 30 )</td>
<td>95</td>
<td>127</td>
<td>118</td>
</tr>
<tr>
<td>( \mu = 100.00 )</td>
<td>( \sigma = 50 )</td>
<td>91</td>
<td>145</td>
<td>130</td>
</tr>
</tbody>
</table>

Table 1 indicates that with the same mean of demand, the optimal option quantity \( q^* \) increases as the variation (std.) of demand increases. This implies that more options should be purchased and exercised with the increment of demand uncertainty. Consequently, more flexibility is involved in reducing the risk from unmet and excessive demand.

In Table 2, the example is given with the same value of \( R, C_p, C_q \) and \( C_u \) as in Table 1 to show the change of optimal normal production quantity and additional quantity with increasing \( C_{pm} \) and \( C_q \) according to Proposition 2. The cost parameter \( C_{pm} \) and \( C_q \) chosen that vary in the first and fifth column, the variation both the optimal quantity to produce \( Q^* \) and \( q^* \) indicates the sensitive trade-off relationship between “option cost” and “payoff” from the options.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Manufacturer’s decisions</th>
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<th>Manufacturer’s decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{pm} )</td>
<td>( Q^* )</td>
<td>( Q^<em>+q^</em> )</td>
<td>( C_q )</td>
<td>( Q^* )</td>
</tr>
<tr>
<td>35</td>
<td>68</td>
<td>68</td>
<td>136</td>
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<td>36</td>
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<td>40</td>
<td>83</td>
<td>46</td>
<td>129</td>
<td>30</td>
</tr>
</tbody>
</table>

Based on the results in Table 2, it is found that the optimal decisions \( Q^* \) increases and \( q^* \) decreases as the option exercise price \( C_{pm} \) increases, while the total quantity \( Q^*+q^* \) also decreases. It is intuitively reasonable to buy fewer options when the option price or option exercise price goes higher.

Repeating the above example, for \( C_q = 0, \mu = 100 \) with \( \sigma = 20, 30, 40 \) and 50, the results are summarized
in Table 3. It is observed that the performance of the option model improves while the performance of the basic model degrades with the increment of demand variation. Furthermore, the profit improvement of option model over basic model \( \Delta \) dramatically increases as \( \sigma \) linearly increases. Hence, Proposition 3 indicates the advantage of option function in reducing the risk of uncertain demand.

### 5. Conclusions and future work

The primary objective of this study is to determine the optimal number of preventive maintenance tasks and production quantity for a manufacturing enterprise system to improve the cost-effectiveness and system performance using financial instrument “option”. The option-based mathematical model for the joint production and maintenance system provides useful maintenance decisions in the environment of uncertain demand. The resulting PM policy is found to eliminate the risk of stopping a machine from operating status that would cause production losses as the traditional periodic maintenance policy does, and reduce the risk of shortage or overage of demand when the conventional assumption of constant demand is generalized to stochastic demand.

In addition, comparisons among the basic model without option, traditional periodic PM model and the option-based PM model have shown the encouraging results that the option model is a more flexible model under uncertain demand and leads to greater profit than basic model and periodic PM policy.

Further extensions, such as considering safety stock, cost of product defects, breakdowns in bottleneck resources, and restoration cost, would be included in the option-based maintenance model to provide avenues for future research.

### References


### Table 3
Comparisons of optimal profits (\( C_s = 0 \)).

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<thead>
<tr>
<th>( D )</th>
<th>Manufacturer’s decisions</th>
<th>Manufacturer’s profit</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu = 100, \sigma = 20 )</td>
<td>( Q^* )</td>
<td>( Q^<em>q^</em> )</td>
<td>( \pi )</td>
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<tr>
<td>( \mu = 100, \sigma = 30 )</td>
<td>79</td>
<td>124</td>
<td>7100</td>
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<tr>
<td>( \mu = 100, \sigma = 40 )</td>
<td>68</td>
<td>136</td>
<td>7750</td>
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<tr>
<td>( \mu = 100, \sigma = 50 )</td>
<td>57</td>
<td>147</td>
<td>8308</td>
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<tr>
<td>( \mu = 100, \sigma = 60 )</td>
<td>47</td>
<td>160</td>
<td>8975</td>
</tr>
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